The Two Revolutions: Land Elites and Education during the Industrial Revolution

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Abstract: The understanding of the unique event that was the Industrial Revolution has been subject to a passionate debate. The process of transition from a Malthusian equilibrium to today's Modern Economic Growth has been subject to several theories. This paper pretends to add more insights on the process of industrialization and the demographic transition that followed this period. By applying the perspective of interest groups to land elites and by analyzing elites' incentives to allow education or not, it is showed that land elites and their political power to decide on supporting education are important for events during Industrial Revolution. Besides, contributions are made on the discussion around the existence and role of the Agricultural Revolution and land fertility to the process of industrialization. It is showed that Agricultural Revolution, as a positive force on elites' incentives, played a significant role on fastening the process of industrialization by allowing an early emergence of education. A model and its numerical simulation are presented to show these results.

Keywords: Industrial and Agricultural Revolution; Demographic Transition; Education; Interest Groups.

Jel Classification: N53, O13, O14, O43, O50

1. Introduction

The Great Divergence, which started two centuries ago, has been one of the main research challenges economists have been facing in growth and development fields of study. The understanding of this unique event has been subject to a passionate debate. Many hypotheses have been put forward to explain the process of transition from a Malthusian equilibrium to a Post Malthusian phase until today's Modern Economic Growth. Unified Growth Theory has attempted to understand and put forward explanations on the behavior of the economies in this particular time period. Comparative economic development has considered factors such as geographical, institutional, ethnic, religious, human capital formation and colonization as main explanatory elements for the different timing of the transition from Malthusian to Modern Economic Growth. Meanwhile, the processes of declining fertility, educational and human capital formation, and the agricultural transformation were intimately related with the onset of the Industrial Revolution as Unified Growth Theory has consistently showed.

In this paper, by applying tools from Unified Growth Theory, it is advanced the suggestion that the way land elites (landlords) observe their gains and losses from the process of education and therefore if they agree and contribute to that process, changes the ability and willingness of individuals to educate their children and, hence, the timing of the provision of education among population. This force preventing the rise of education will, on one hand, delay the process of demographic transition and, on the other hand, delay the real take off of the industrial sector that cannot fully achieve its full potential without a major level of education. Indeed, the lack of human capital promoting institutions (public schooling, child labor regulations and other time reducing cost institutions) reduces the rate and the timing of the transition from an agricultural to a fully industrial economy. Therefore, it contributes to the emergence of the great divergence in income per capita across countries even during the process of industrialization. Along with this theory, it is also proposed in this paper that the improvement in the agricultural processes, which are believed to have happened in the century previous to the Industrial Revolution, may have contributed to accelerate the process of industrialization as well as contribute to a more favorable decision of land elites to allow education to emerge since the risk of losing rents and their share in the economy to the industrial sector is lower because of the prevailing higher productivity of land. By the same token the role of initial land fertility is examined as a byproduct of the present model, the main conclusion is that land fertility may have a positive effect on education as well, although it goes against the conclusions of other existing studies.

The Industrial Revolution as a process of transformation of an agricultural economy to an industrial one had a different timing on different countries. This differential timing on the take-off and demographic transition led to the so called "Great Divergence" in income per capita as well as on population growth across regions. Although in the end of the first millennium Asia was the world leader in both wealth and knowledge, at the 1800's, during the Industrial Revolution, Europe had already surpassed those societies (Pomeranz, 2000; Galor, 2011). Empirical analysis on this period and afterwards shows that besides England, where Industrial Revolution first took place, most of countries in continental Europe followed the trend and had their own process of industrialization. France, Belgium, Prussia and the Netherlands are some examples of western countries that witnessed this revolution just after England. Along with these countries, the Western offshoots, such as the US, Canada and Australia also saw their economies develop sooner surpassing the European countries. As for all other countries in the world, most of their economies remained stagnant for almost the last two centuries (Landes, 1998; Maddison, 2003).

The divergence that began in the Industrial Revolution delimited the end of the Malthusian era in a path towards the Modern Growth regime. The Malthusian epoch was characterized by a continuous struggle of population for survival. Income per capita during all the period between 1 AC to 1500 AC was kept almost constant (Maddison, 2003). This feature was mainly due to the interconnections between technology and population which were running in this Malthusian epoch. In fact, only breakthroughs of technology could lead to temporary income per capita gains. Improvements of technology initially had a positive effect on productivity and, hence, income but ultimately these gains would vanish. Since fertility increased, causing population to grow, then income per capita would decrease again to its Malthusian equilibrium levels. In a nutshell, any technology gains were in this period channeled to population growth while income per capita maintained almost constant, resulting in a stagnant economic environment.

Besides this, during this period other forces were said to be influencing the economy mainly in England. What was later called Agricultural Revolution, is supposed to have led to the improvement of agricultural productivity and, hence, standards of living of population.

The importance of Agricultural Revolution in the creation of the modern world is, for some, greater than the Industrial Revolution itself. Indeed, between 1700 to 1850, it made possible output per acre and output per worker increase to levels far from those verified during the medieval ages (Clark, 1993). There is quite controversy on the real dimension of the Agricultural Revolution and if it really happened in England. It is argued that way before the Industrial Revolution english farmers were already quite productive (Mokyr, 2009).

Looking at the estimative of rents in different studies for England, we observe that the usual results present an upsurge of rents in the beginning of the 17th century and then there is a slow motion on the growth of rents until the beginning of the 19th century (Allen, 1988; Clark, 2002). Nevertheless, there were some significant changes in the english process of farming that amounted for a rise in the output and productivity of land. Two levels are considered to allow for the increase of output: intensity on the usage of land and efficiency on its usage (Brown, 1991; Clark, 1993; Mokyr, 2009). We observe an increase on cultivated land during the 18th century. On the efficiency gains we observe during this period a change in the crops used on plantations, more productive ones and with higher value. A new rotation system, more intensive and less restricted on the selection of land "in rest", was adopted by farmers. The agricultural knowledge also spread across the country with the adoption of new tools and new methods of farming. The enclosure process that started already in the beginning of the 18th century had a decisive impact on the economy in the late 18th century early 19th century.

On the expansion of cultivated area, we observe during this period and increase of arable land and pasture land due to the process of enclosure on one side and due to the decrease of "wasted land" – land that was neither used for grazing nor for farming. Arable land increased from 11 million to 14.6 million acres and pasture increased from 10 to 16 million acres in the period between 1700 and 1850 (Brown, 1991; Mokyr, 2009). At the same time the cultivated area increased also its quality improved - new methods of farming were implemented. New crops were introduced in the new land taken from waste or fallow. Only the best seeds were selected to use on land. Only the animals with the best reproductive characteristics were chosen to breed and some species were brought from abroad to improve livestock cattle. This had a great impact by increasing the productivity of land and also the supply of food. New rotation systems were also introduced adding to the boost of production. The improvement of soil fertility was also accomplished by the increase of usage of manure and other natural fertilizers, such as marl and lime. In addition, water meadows were very common as well as the drainage of fens (Brown, 1991; Allen, 2009).

Finally, the enclosure process, which took place mostly in England, has been a very controversial issue since there is an ongoing discussion on its real effects previous to the Industrial Revolution. The process of enclosure meant the enclosure of open fields. These open fields were divided between the several cultivators of the village and each cultivator had its individual right to cultivate his share of land until the harvest. The rest of land, pasture, waste and woods were all common around the year. The enclosure started in the mid 1500s and until 1700 half of the cultivated had already been enclosed. Still in the late 1700s there was a boom on enclosure and the last lands were enclosed (Brown, 1991). Although it was thought that the open field system could be the cause of inefficiencies on agricultural production, the weight given to the open field system as a retarding force was too excessive. Allen (2009) discusses the advances on innovative methods in open field land where actually there was some modernization by farmers, leaving the idea that enclosed fields did not boosted innovation so much more than open fields. Indeed, most of the fields suffered improvements during the period of 1700 to 1850 and were capable of productivity growth and technological progress (McCloskey, 1972). Agreements between all farmers in open fields were much harder to achieve but they were still possible and, in fact, they occurred quite often during this period. Of course, the enclosure process was a mean to achieve higher production in the sense that it allowed for a more structured organization of those acres, easier agreements on new production techniques, and an increase on the size of the average agricultural holding. These new private fields were run by professional managers, following market principles (Mokyr, 2009).

Independently of having or not influenced England and other countries during the whole eighteenth century, a new social and economic phase had began in late eighteenth century – the Post- Malthusian phase. It differs from the last by breaking the stagnating equilibrium and by preparing the ground for the transition to the Modern Growth regime. From Maddison (2003), we observe that in this period (early 1800's) both population and income per capita start to increase simultaneously. There are no checks to income per capita as before - gains in income are not entirely allocated to fertility. So, income per capita was not diluted in time. As for fertility, fertility rates continue to increase even more until the middle 1800's (Dyson and Murphy, 1985; Lee, 2003). Along with these demographic trends, another important feature is the continuous and progressive process of industrialization. This process starts early in England and spreads throughout western countries during the middle 1800's (Bairoch, 1982). As these forces start to be pervasive in the western countries, the Malthusian trap becomes less and less powerful and with the rise of demand for human capital this trap is overcome definitely (Galor, 2011).

Despite education was already regarded as an asset in the eighteenth century it had a minor role during the first phase of industrialization. Only after the middle 1800's, when demand for education was reaching a fever pitch did education starts to rise and become essential for the definite take-off of the industrial sector. Although in the first phase of industrialization demand for skilled workers was tiny, because the requirements to work on industry were still very simple - illiteracy was still very common among workers, as industrialization moved on, working in industry became more and more demanding and a higher level of education was required. Despite educational reforms were taking place during the eighteenth and nineteenth centuries, the most important ones, which led to a real increase in the educational level of people, only emerged in the late nineteenth century. This was pernicious for the economy by the time since despite the high demand for education and capital formation, each country had his own pace on placing educational reforms (Cubberley, 1920; Galor, 2011). Laws regarding schools and education started to emerge in several countries in Europe in the eighteenth century. For instance, in England, the process of education started only in the 1850's when several reforms were effective in promoting education among children. After dominating the industrial field since the beginning of the Industrial Revolution, England began to fell behind to continental countries and so after the 1850's several acts were approved to try to retake the leadership in manufacturing technology (Flora *et al*, 1983; Green, 1990). In Prussia, on the contrary, the first laws regarding the establishment and organizing schools were issued in the early 1700's. Many measures were taken to oblige the attendance to schools for the children. New codes were issued to coordinate the provision of education among the existent schools, mainly managed by the church. Nevertheless, these new codes met with resistance everywhere since there was no acceptance by population in general, and landlords in particular, to cope with the financial burden. This conducted to a slow advance of the measures that would be expected to be taken. Only after, in the middle 19^{th} century and with state intervention did the reorganization of the educational system from elementary school to university became effective (Cubberley, 1920).

The same happened in France and Italy, where the influence of the French revolution, and the new tendencies on education changed the way of education was envisaged. Starting with the substitution of church schools to state schools, small steps were given during Napoleon's period. But after his fall and the fall of the following monarchy, education regressed by the imposition of restrictions to state schools and the enhancement of church schools and private schools that promoted a more favorable teaching for the dictatorship by the time. Only later education regained its previous position on society's priorities (Cubberley, 1920; Green, 1990).

Besides education, another key trend was emerging in this period: the decline of fertility rates. This decline characterized the demographic transition in most countries along the last two centuries. In Western countries this reduction of population growth started in the late nineteenth century while in Latin America and Asia this phenomenon began much later in the middle twentieth century. This transition has continued through the last century and has contributed to fertility reach the limit of the replacement level (Lee, 2003).

More interestingly is how several studies show that education and fertility are interconnected. The decline of fertility was dominated by investment on education so that there was a negative correlation between both factors (Flora *et al*, 1983). This negative correlation is associated in several studies with the trade-off between child quantity and quality. Becker et al (2010) and Becker et al (2012) found evidence of this trade-off in the nineteenth century Prussia while Murphy (2010) finds evidence for France in the late nineteenth century. If this is true, any explanation of the transition during the Industrial Revolution must account for this phenomenon.

Eventually, as these processes of education and demographic transition took place, the onset of a Modern Growth era arrived. Following this decline on fertility and the rise of education and, hence, human capital formation, income per capita increases consistently over the years until nowadays. And, as it is observed today in all developed countries these trends persist and are even more accentuated in countries where Industrial Revolution happened in the nineteenth century while other countries started this industrialization process in the middle twentieth century, such as Latin American and Asian countries, while in Africa the process barely started.

From the previous paragraphs three features excel: first, the Agricultural Revolution had an impact on the seventeenth century economies, namely England; education only effectively emerged in the late nineteenth century which also depended on the willingness of the state and government for promote and support; finally, the demographic transition which has behind the quantity- quality trade-off. The question that subsists is how to reconcile these facts with all the elements present in the Industrial Revolution period and how they are interconnected. Why did the education lag remain for so long? Did the Agricultural Revolution contributed to the onset of the industrial revolution and education of population, and, if so, to what extent? Can the forces behind these developments be uncovered? Had they fastened or delayed education to emerge? How can we show elites decision options under this period?

Some authors have shown that small interest groups promote a blockage of new technologies and better institutions in order to keep their own power and their rent extraction. As Mancur Olson teach us: "…small groups in a society will usually have more lobbying and cartelistic power per capita…" (pp. 41, Olson, 1982). Indeed, small groups organize in order to pursue their own interest disregarding society as a whole, blocking and delaying any process of development and the shift of institutional or technological environment when it does not suit their interests (Olson, 1982; Acemoglu and Robinson, 2000; Lizzeri and Persico, 2004; Acemoglu and Robinson, 2008)

The period of Industrial Revolution was no exception for the rise of these groups. Land elites were a small group in pre-industrial societies. They were too powerful and their initial incentives were of halting the process of education, and, hence, the complete take-off of the Industrial Revolution (Galor *et al*, 2009). As referred before, this group was the one the state recurred to finance education. This power and unwillingness to support education was the main reason for the conflict between the emerging capitalist class and the old landowners. In fact, the transition from an agricultural to an industrial economy has changed the pervasive agrarian economy conflict to the industrial conflict. While the former had the landlords and the masses as main protagonists, in the latter the competition for power was taken between these land elites and the emerging industrialist elites. The fight for more education in these centuries was one of the main points of divergence between these two groups. While the industrialists wanted more educated masses to boost their production, the landlord elites would perceive the loss of land workers to cities and so opposed determinately to educate them. The power of these elites on this period of time was enough strong to prevent the dissemination of education. The financial influence of landlord elites, the big and richest group by the time, in countries, implied that most of the decisions kings and princes could do depended on the own advantages elites could have (Ekelund and Tollison, 1997; Lizzeri and Persico, 2004). In fact, the dependence of kings on the money of landlords for warfare and other expenses made it easy for landlords to impose the king the concession of monopolies, private businesses, patents, and other advantageous businesses to elites where we could include a less disseminated and public education. For instance, it is known that on this epoch, deriving from the mercantilist era, the power of the state was indeed flooded with private interests and interest groups that managed efforts to conduce policies in the way it suited them most (Ekelund and Tollison, 1997).

Therefore, what is proposed in this paper is to show that since elites have the power to support financially education, they would only do that if, and only if, they find it profitable to do. And it will only be profitable if the revenues they will earn from their rents compensate the initial financial loss[1]. While other approaches suggest that education would mostly harm land elites by minimizing their rents (Galor *et al*, 2009), we suggest that it indeed may harm initially so that they set taxes to zero in a first moment, but as time goes by, they will find it profitable to tax and provide education. Provided that gains on rents, due to the slower growth of land productivity and the increasing marginal spillover gains from the industrial sector, become higher than the cost of taxes, elites will have an economic incentive to allow and abide to give funds to education. Therefore, one of the main novelties of this paper is to explore the predictions and the validity of an alternative but

[1] It will be assumed that elites provide financial support by setting a tax on their wealth.

complementary explanation for the rise of education where land elites agree to the provision of education. While in most of literature it is always agreed that they are against these measures but in the case of small landowners (Galor *et al*, 2009), here, rather than being always against there is a threshold where they by their own intention force the education of population. The fact that elites can be better off with taxation is an interesting conclusion and clears the perspective of an always negativist perspective of elites. It also provides an alternative explanation for the education time lag since it attributes to the decisive political power of elites to determine when education is provided, although it will keep the quality level on the hands of population, allowing for the quantity-quality trade-off to still be a characteristic of this epoch.

In addition, and following the patterns and the evolution during mainly the transition from the Malthusian regime to the Modern Growth regime, it is argued in this paper that the process of Agricultural Revolution and natural fertility of land have an impact on the way landlords agree with the process of education of population. In fact, it is showed that continuous technology advancements make regions, and even countries, be sooner able to allow education to emerge. So, a contribution can be added to the debate on the positive and negative impacts of the Agricultural Revolution on the Industrial Revolution, as well as discuss when did it actually occurred. Indeed, we may ask how the rise of productivity of land may have induced a negative and a positive effect for the effectiveness of the Industrial Revolution. The first effect, concerns the higher marginal gains for farm workers compared to urban workers, hence, contributing to a lower pace of migration from the countryside to urban areas. The second one relates with the willingness of elites to provide education. Since their rents do not decrease or remain at a significant low level and instead increase to higher levels, due to a more productive labor force, the loss driven from the higher financial support to turn population more educated and productive and the loss of workers to the industrial sector becomes less significant. The risk of being overtaken by industry vanishes faster since the benefits of externalities from industry become bigger than the prejudices of allowing industry to complete its take-off at an early phase. Although elites do not immediately allow for education, the necessary incentives arrive earlier since agricultural technology in the Industrial Revolution is already more developed[2]. It will be also explored the hypothesis of land endowments and how they affect elites' decisions. Again by the same token of Agricultural Revolution, a more favorable land endowment causes productivity to be higher in the agricultural sector and, therefore, elites find less risky to allow education to emerge at an earlier stage. This conclusion runs against some of the existing literature (e.g. Engerman and Sokoloff, 2000; Litina, 2012) but it is assumed that this result is more a byproduct of the model than a main result and it represents a positive force among all the possible existing forces land endowment imposes in this period.

To sum up, the proposed argument in this paper aims to add to existent literature a complementary perspective on the role of elites by joining elites' decisions to Unified Growth Theory in an explicit way. And, hence, it advances a rather different explanation for the delay of education and the differences among countries on its emerging timing but never forgetting all the features and trends inherent to this time period, for instance the Malthusian trap, and the demographic, economic and technological transitions during the Post-Malthusian phase and afterwards during the Modern Growth regime. Finally it contributes to the discussion about the Agricultural Revolution on its positive and negative impacts to elites' decisions and, consequently, on education. Land endowments effects are also discussed under this frame.

To undertake this study, this paper is organized as follows. In section 2, the set-up of the model is defined as well as the main assumptions. Section 3 provides the analysis of the main predictions of the model from the period before to the period after the Industrial Revolution and a discussion is drawn on the main results. Finally, some concluding remarks are made in the closing section.

2. Model Setup

Consider an overlapping-generations economy in which activity extends over infinite discrete time. Before the industrial sector emerges, every period the economy produces a single homogeneous good, using land and labor as inputs. After the emergence of the industrial sector, the economy produces two goods

[2] Besides these effects, we have also the positive effects highlighted in the literature on the provision of more and cheaper goods as well as labor to urban population which would sustain and expand it (Overton, 1996; Allen, 2009).

(agricultural and manufactured), using as inputs land and labor, and efficiency units of labor for the agricultural and manufactured good, respectively. The supply of land is exogenous and fixed over time. The number of efficiency units of labor is determined by households' decisions in the preceding period regarding the number and level of human capital of their children.

The model comprises two types of individuals: workers and elites. Workers reproduce themselves asexually so that each individual has a single parent. The number of offspring depends on the decisions of workers. Elites, alternatively, have one child each. They own land, consume, leave a bequest to their child and use their power to set taxes. They do not work but their income comes from land rents and the bequest left by their parents. Therefore, in every period t, a generation of a continuum of L_t economically identical workers enters the labor force. Individuals of generation t live for two periods.

2.1. Production

To produce a good in this economy, each worker supplies inelastically one unit of labor in every time period. The aggregate supply of workers evolves over time at the endogenously determined rate of population growth. Land is exogenous and fixed over time. In early phase, Malthusian period, the agricultural sector is the only operating, whereas the industrial sector is not yet economically viable. Since technology in the industrial sectors grows over time, at some point the productivity threshold will be reached and then both sectors will operate in the economy – emergence of the industrial sector.

2.1.1. Production in both sectors

The output produced in the agricultural occurs according to a constant-returns-to-scale technology. In period t, Y_t^A is determined by land, X_t; labor employed in the agricultural sector, L_t^A; and by agricultural technology A_t^A, determined endogenously. There is also an additional factor that measures initial land fertility $\Upsilon > 0$.

$$Y_t^A = (\Upsilon A_t^A X_t)^\alpha (L_t^A)^{1-\alpha} \quad \text{for} \quad 0 < \alpha < 1 , \tag{1}$$

$L_t^A = (1 - \lambda_t)L_t$ where $(1 - \lambda_t)$ is the share of workers in the agricultural sector. $(1 - \lambda_t) \epsilon (0,1)$ but it is one $(\lambda_t = 0)$ until the emergence of the industrial sector.

The output of the industrial sector will have two structures. When no workers are employed in the sector (before the emergence) and when workers are employed in the sector. This change of structure is applied in order to allow for the characterization of the periods before and after the emergence of the industrial sector. It is a method to show the transition process and to avoid any nuisance with the performance and movements of workers to the industrial sector. Therefore, before the emergence of the industrial sector we have a linear constant-returns-to-scale production function depending on technology A_t^I at period t, and on efficient labor H_t at period t:

$$Y_t^I = A_t^I H_t \,, \tag{2}$$

With $H_t = \lambda_t h_t L_t$, where h_t is the level of human capital and again, λ_t is the share of workers in the industrial sector.

After the emergence of the industrial sector we consider that there are decreasing returns to scale in the production function so that the gains of more efficiency units will decrease over time. The same happens to technology gains. This will imply that now both sectors will always be open - λ_t always higher than zero (marginal productivities tend to infinity when number of workers tends to zero). The elements will keep the same: technology and efficient labor:

$$Y_t^I = (A_t^I)^{1-\theta}(H_t)^{\theta} \quad \text{for} \quad 0 < \theta < 1 \,, \tag{3}$$

Finally, the total labor force is given by the sum of the number of workers in both sectors:

$$L_t = L_t^A + L_t^I \,, \tag{4}$$

Where $L_t^I = \lambda_t L_t$ and $L_t > 0$ in every period t.

2.1.2. Factor prices, Labor Market and the technology threshold

The economy has two types of agents: workers and elites. Workers receive their wages according to their productivity in the sector they are working on. As for elites receive rents from land, since they own their property rights. So, return to land is not zero. Property rights are not transmissible to other elite members or workers. They are inherited by the child of each member of the elite.

Rents are determined as the marginal gains for each unit of land held by an elite member. We define $\bar{x}^i > 0$ as the share of land held by an i elite member, and we assume that all members have the same share of land. Therefore, rent received by the i elite member is such that:

$$\rho_t = \alpha(\Upsilon A_t^A)^\alpha (X_t)^{\alpha-1}(L_t^A)^{1-\alpha} , \tag{5}$$

We are going to assume a fixed value for land $X_t = 1$. From above we know that rents are positively related with technology and land fertility and negatively related with the number of workers allocated to the agricultural sector: $\rho_A(\Upsilon, A_t^A, L_t^A) > 0$, $\rho_\Upsilon(\Upsilon, A_t^A, L_t^A) > 0$ and $\rho_{L^A}(\Upsilon, A_t^A, L_t^A) > 0$ for any $\Upsilon, A_t^A, L_t^A > 0$.

Depending on the period the economy is (before or after the emergence of industrial sector) wages can be only earned in the agricultural sector or in either the agricultural or the industrial sector. The market for labor is perfectly competitive and, hence, wages are given by the marginal productivity of labor in each sector. Given (1), the marginal product and, hence, the inverse demand of labor in the agricultural sector is given by:

$$w_t^A = (1-\alpha)(\Upsilon A_t^A)^\alpha (X_t)^\alpha ((1-\lambda_t)L_t)^{-\alpha} , \tag{6}$$

Where w_t^A is the wages of agriculture workers.

As for wages in the industrial sector, from (2) the potential earnings before the industrial sector rises is given by:

$$w_t^I = A_t^I h_t , \tag{7}$$

Where w_t^I is the potential wage in the industrial sector, for the total human capital each worker has. In the second phase, marginal productivity is determined using (3):

$$w_t^I = \theta(A_t^I)^{1-\theta}(H_t)^{\theta-1}h_t = \theta(A_t^I)^{1-\theta}(\lambda_t L_t)^{\theta-1}(h_t)^\theta , \tag{8}$$

From (6) and (7) we observe that productivity of the industrial sector is finite and initially low (if we consider initial low technology values for industrial technology) whereas productivity in the agricultural sector tends to infinity for low initial levels of employment. So, the agricultural sector will be open in every period, whereas the industrial sector will emerge only when labor productivity in this sector exceeds the marginal productivity of labor in the agricultural sector, considering that the entire labor force is employed in the agricultural sector. When the emergence takes place, the new production structure will also be applied in the industrial sector. Then, (6) and (8) will have to equalize to guarantee the perfect labor mobility assumption, and, hence, determine the share of workers in each sector. To establish conditions in industrial technology necessary for the emergence of the industrial sector we will set Lemma 1.

Lemma 1: If wages are determined by (6) and (7), there exists a threshold value for industrial technology $\hat{A}_t^I$ for which the industrial sector becomes economically viable if and only if:

$$\hat{A}_t^I > \frac{(1-\alpha)(\Upsilon A_t^A X_t)^\alpha}{L_t^\alpha h_t}$$

See proof in the appendix.

From here, we conclude that the moment the threshold is exceeded the industrial sector emerges. When this happens λ_t is no longer zero. As follows from Lemma 1, if $A_t^I < \hat{A}_t^I$ then the agricultural sector is the only open sector and so wages are set equal to the marginal product of the agricultural sector $w_t = w_t^A$. Otherwise ($A_t^I \geq \hat{A}_t^I$), by the perfect mobility of workers, marginal products equalize $w_t = w_t^A = w_t^I$ and so wages are set to be equal to the marginal product of the industrial sector, (8). The equilibrium share of labor between the two sectors at period t is given by[3]:

$$\lambda_t = \begin{cases} 0 & if \quad A_t^I < \hat{A}_t^I \\ \dfrac{\theta^{\frac{1}{\alpha}} A_t^I (h_t)^{\frac{\theta}{\alpha}}}{\Upsilon A_t^A X_t (1-\alpha)^{\frac{1}{\alpha}} + \theta^{\frac{1}{\alpha}} A_t^I (h_t)^{\frac{\theta}{\alpha}}} & if \quad A_t^I \geq \hat{A}_t^I \end{cases}, \tag{9}$$

And,

$$w_t = \begin{cases} (1-\alpha)(\Upsilon A_t^A)^\alpha (X_t)^\alpha ((1-\lambda_t)L_t)^{-\alpha} & if \quad A_t^I < \hat{A}_t^I \\ \theta (A_t^I)^{1-\theta} (\lambda_t L_t)^{\theta-1} (h_t)^\theta & if \quad A_t^I \geq \hat{A}_t^I \end{cases}, \tag{10}$$

[3] Note that, for the easy tractability of the equilibrium we assume that $\theta = 1 - \alpha$.

2.2. Workers

As for workers, in the first period (childhood) of their lives they are raised by their parents and may be educated and, hence, acquire human capital. In the second period of their lives (adulthood), individuals supply their efficiency units of labor and allocate the resulting wage income. The preferences of members of generation t (those born in period $t-1$) are defined over consumption above a subsistence level $\tilde{c} > 0$ in period t as well as over the potential aggregate income of their children *i. e.* the number of their children, their acquired level of human capital and their correspondent wages (observed in period $t+1$). They are represented by the utility function:

$$u_t = c_t^{\gamma}(h_{t+1}n_t)^{1-\gamma} \quad \text{for } 0 < \gamma < 1\,, \tag{11}$$

Where c_t is consumption, h_{t+1} is the level of human capital of each child and n_t is the number of children of individual t. Following Galor and Weil (2000), the individuals function is strictly monotonically increasing and strictly quasi-concave, satisfying the conventional boundary conditions that ensure that, for sufficiently high income, there exists an interior solution for the utility maximization problem. For a sufficiently low level of income the subsistence consumption constraint is binding. Let $z_t = w_t h_t$ be the level of potential income, which is divided between expenditure on child-rearing (quantity as well as quality). We will define $\tilde{z}$ as the level of potential income below which subsistence consumption is binding.

Let $\tau^r > 0$ be the time endowment cost faced by a member of generation t for raising a child, regardless of quality, and let and $g(\tau^e, T_t) > 0$ be the time endowment cost necessary for each unit of education for each child. Consider that the function $g(.)$ depends positively of $\tau^e > 0$ and negatively on $T_t \geq 0$. This function depicts the intervention of elites on the process of education. T_t is the amount of revenues raised by elites among themselves to reduce the cost of education in order to incentive parents (workers) to educate their children. $T_t = f(t_t, b_t)$ which depends positively of both variables, tax rate (t_t) and elites received bequest (b_t) - see below section 3.2.

Regarding the level of human capital in the second period of life, it is determined by the units of education received during childhood. The level of human capital is an increasing, concave, function of education. The more education the higher the level of human capital but the gains associated to each additional unit have diminishing returns.

$$h_{t+1} = h(e_{t+1})\,, \tag{12}$$

Where $h(0) = 1$, $\lim_{e\to\infty} h^{'}(e_{t+1}) = 0$, $\lim_{e\to 0} h^{'}(e_{t+1}) = \Upsilon < \infty$. In the absence of education, individuals possess basic skills - one efficiency unit of human capital.

We can now sketch the budget constraint faced by parents in the second period:

$$c_t + w_t h_t n_t(\tau^r + g(\tau^e, \iota_t)e_{t+1}) \leq w_t h_t\,, \tag{13}$$

2.2.1. Optimization

Members of generation t maximize their utility subject to the budget constraint. They choose the number of their children and the level of education of each child and their own consumption. Substituting (13) into (11), the optimization problem for a member of generation t reduces to:

$$(n_t, e_{t+1}) = argmax\{w_t h_t(1 - n_t(\tau^r + g(\tau^e, T_t)e_{t+1}))\}^{\gamma}\{(h_{t+1}n_t)\}^{1-\gamma}, \tag{14}$$

Subject to

$$w_t h_t\big(1 - n_t(\tau^r + g(\tau^e, T_t)e_{t+1})\big) \geq \tilde{c}$$
$$n_t, e_{t+1} \geq 0$$

It follows from the optimization process:

$$n_t = \begin{cases} \dfrac{1-\frac{\tilde{c}}{w_t}}{(\tau^r + g(\tau^e,\iota_t)e_{t+1})} & if \;\; z_t < \tilde{z} \\ \dfrac{1-\gamma}{(\tau^r + g(\tau^e,\iota_t)e_{t+1})} & if \;\; z_t \geq \tilde{z} \end{cases}\,, \tag{15}$$

For a binding consumption constraint $z_t < \tilde{z}$, the optimal number of children for a member of generation t is an increasing function of individual t's income. This mimics one of the fundamental features of a Malthusian epoch. The individual consumes the subsistence level $\tilde{c}$, and uses the rest of the time endowment for child-rearing. The higher the wage he earns, the lower the time he needs to spend in the labor force so that the time spent rearing his children increases.

Independently of the division between time devoted to consumption and child rearing, the units of education for each child only depend on the relative weight of raising costs and educating costs. While the raising costs are constant, the educating costs depend on the willingness of elites to devote resources to foster education. The higher the resources devoted to education by elites the higher the units of education given to children. Using (14) and (15), the optimization with respect to e_{t+1} shows this as the implicit function $E(.)$ only depends of e_{t+1} and T_t:

$$E(e_{t+1}, T_t) = h'_{t+1}(\tau^r + g(\tau^e, T_t)e_{t+1}) - h_{t+1}g(\tau^e, T_t)\,, \qquad (16)$$

Where $E_e(e_{t+1}, T_t) < 0$. $E_T(e_{t+1}, T_t) = g'(\tau^e, T_t)[h'_{t+1}e_{t+1} - h_{t+1}] > 0$ for a specific set of equations. To guarantee that for a positive level of T_t the chosen level of education is higher than zero, it is assumed that:

$$E(0,0) = h'_{t+1}(0)\tau^r - h_{t+1}(0)g(\tau^e) = 0\,, \qquad (A\ 1)$$

Lemma 2: If (A 1) is satisfied then, for the specific set of equations referred above, the level of education of generation $\boldsymbol{t}$ is a non decreasing function of $\boldsymbol{T_t}$.

$$e_{t+1}\begin{cases} = 0 & if \quad T_t \le 0 \\ > 0 & if \quad T_t > 0 \end{cases} \quad \text{and} \quad e'_{t+1}(T_t) > 0 \quad \text{for} \quad T_t > 0$$

See proof in the appendix.

From the above information and (15) we can draw some conclusions on the behavior of education and the number of offspring.

Proposition 1: From Lemma 2, (15), (16) and (A 1):

(A) The number of offspring and level of education are affected by the level of T_t. An increase of $\mathrm{T_t}$ results in a decline in the number of offspring and in an increase in their level of education: $\frac{\partial n_t}{\partial T_t} < 0$ and $\frac{\partial e_{t+1}}{\partial T_t} > 0$

(B) The number of offspring is affected by changes in the potential income of parents if the subsistence consumption constraint is binding while the level of education is not affected. Otherwise, none of the two variables are affected:

$$\begin{cases} \dfrac{\partial n_t}{\partial z_t} > 0 \text{ and } \dfrac{\partial e_{t+1}}{\partial z_t} = 0 & if \quad z_t < \tilde{z} \\ \dfrac{\partial n_t}{\partial z_t} = \dfrac{\partial e_{t+1}}{\partial z_t} = 0 & if \quad z_t \ge \tilde{z} \end{cases}$$

2.3. Elites

As referred before, elites have one child each. There are no decisions on the quantity or quality of children by elites. In the first period (childhood) of their lives they are raised by their parents. They receive their bequest and decide how to spend it. Namely, they can use part of their bequest to support education or use it to consume and leave a bequest in adulthood to their children. Hence, in the second period of their lives (adulthood), elites divide the value of rent from land and the bequest left from period one into consumption and the bequest to their children. The preferences of members of generation t (those born in period $t-1$) are defined over consumption as well as over the bequest left to their children. They are represented by the utility function:

$$u_t = c_{t+1}^{\mu}(b_{t+1})^{1-\mu} \quad \text{for } 0 < \mu < 1\,, \tag{17}$$

Where c_{t+1} is consumption in period two, b_{t+1} is the bequest for the child. The elites function is again strictly monotonically increasing and strictly quasi-concave, satisfying the conventional boundary conditions. For the sake of simplicity we will assume that $\mu = \gamma$.

As we referred, the available income to use as consumption and bequest is defined by the period two rent and the remains of the bequest after taking the amount to support education. This amount (T_t) is taken in the first period, when the bequest is received. Bequest keeps the same value through periods – interest rate equals zero. $T_t = f(t_t, b_t)$ where $f_t(t_t, b_t) > 0$ and $f_b(t_t, b_t) > 0$. We will assume $f_{tt}(t_t, b_t) < 0$ and $f_{bb}(t_t, b_t) < 0$ since a concave reaction of the amount spent in decreasing the cost of education appears to be reasonable. A boundary to the gains of more spending seems to be is plausible to avoid infinite gains. The rent depends on the amount of land each elite member has. $\bar{x}^i$ establishes how land is divided among elites. The higher the value the less dispersed is land.

We can now sketch the budget constraint faced by elites in the first period:

$$c_{t+1} + b_{t+1} \leq \bar{x}^i \rho_{t+1} + (1 - t_t) b_t\,, \tag{18}$$

2.3.1. Optimization

Members of generation t maximize their utility subject to the budget constraint. They choose the tax level, their consumption in next period and the next period bequest for their children. Substituting (13) into (11), the optimization problem for a member of generation t reduces to:

$$(t_t, b_{t+1}) = argmax\{\bar{x}^i \rho_{t+1} + (1 - t_t) b_t - b_{t+1}\}^{\gamma} \{(h_{t+1} n_t)\}^{1-\gamma}, \tag{19}$$

Subject to

$b_{t+1} \geq 0$

and $t_t \in [0,1]$

It follows from the optimization process:

$$b_{t+1} = (1-\gamma)\left(\bar{x}^i \rho_{t+1} + (1-t_t)b_t\right) \ , \tag{20}$$

Elites spend $(1-\gamma)$ of their income in giving a bequest to their children.

Using (5) and (19), the optimization with respect to t_t shows that the implicit function $G(.)$ depends on t_t, Υ, A_t^A, A_t^I and L_t:

$$G(A_t^A, \Upsilon, A_t^I, L_t, t_t) = \bar{x}^i \frac{d\rho_{t+1}}{dt_t} - b_t = 0 \ , \tag{21}$$

The implicit function will have two different characterizations depending if we are before or after the structural break. There are effects with different signals that influence the decision of elites and are reflected in $G(.)$. The primary effect is the "bequest effect". The higher the bequest, the higher is the amount transferred to support education. This effect is always negative. The "rent effect" is the other effect that can be divided between two main effects: "technology effect" and "workers effect". The technology effect establishes a positive effect since more amount spent in supporting education will increase the externality of the industrial technology on the agriculture technology. The worker effect, on the contrary, establishes an ambiguous effect. With more education: agriculture share can increase due to the externalities that increase productivity in this sector, while the industrial sector can benefit from more human capital that also increases marginal productivities in this sector. Therefore, depending on the strength of forces, the share of workers in industry and agriculture may increase. Now, since elites' decision depends on an implicit function and $t_t \in [0,1]$, we can draw some conclusions on the decision, depending on $G(.)$.

Lemma 3: The decision to set taxes higher than zero depends on the value of the implicit function. Since $G(.)$ can take different values in [0,1], then:

If for the entire interval [0,1]:

$$\begin{cases} G(.) < 0, & \Rightarrow \quad t_t = 0 \\ G(.) > 0, & \Rightarrow \quad t_t = 1 \end{cases}$$

If in the interval [0,1]:

$$G(.) = 0, \qquad \Rightarrow \qquad t_t \in [0,1]$$

See proof in the appendix.

From the previous lemma it is possible to understand which the possible decisions of elites are and how they affect the economy. When G(.) does not equalize to zero then either an increase of t_t increases the utility at a maximum of $t_t = 1$ or it decreases the utility so that the best choice is $t_t = 0$. Otherwise, $t_t \in [0,1]$ so that the best choice is an interior solution[4].

From (5), (6), (7) and (21) we can determine the implicit function before the structural break:

$$G(.) = \frac{\alpha \bar{x}^i \Upsilon (L_t)^{\varepsilon} (A_t^A)^{\delta}}{(1+e_{t+1})^{\frac{\beta(1-\alpha)}{\alpha}}} \left[\frac{(1-\alpha)^{\frac{1}{\alpha}}}{((1+h_t L_t^A)^{\zeta} A_t^I)^{\frac{1}{\alpha}}} \right]^{1-\alpha} \frac{de_{t+1}}{dt_t} \left((A_t^I)^b - \frac{\beta(1-\alpha)}{\alpha} \frac{(1+e_{t+1}(A_t^I)^b)}{(1+e_{t+1})} \right) - b_t \ , \quad (22)$$

Where only e_{t+1} depends on t_t. Before the break, the decision of elites on education depends on the technology effect being higher than the workers effect and the bequest effect.

Lemma 4: Before the structural break, for $G(.) \geq 0$ the condition $(A_t^I)^b - \frac{\beta(1-\alpha)}{\alpha} \frac{\left(1+e_{t+1}(A_t^I)^b\right)}{(1+e_{t+1})}$ must be positive.

Proof: follows directly from (22).

After the structural break, when industrial revolution takes off, the decision rule differs due to the new industrial structure. The decision continues to contemplate the same three effects and one more. In contrast with the previous decision rule, the "population effect" now does not vanish, and therefore the rent effect has now three effects within it. The population effect relates to education since the more education the less time endowment workers have to raise children so that the quantity – quality trade-off becomes instrumental. The implicit function after the structural break is derived from (5), (6), (8) and (21) – see appendix for the full equation.

Lemma 5: After the structural break, for G(.) ≥ 0 it must be true that:

$$\left(1 - (1-\alpha) \frac{\Upsilon(L_t)^{\varepsilon} \left(A_t^A\right)^{\delta} X_{t+1} (1-\alpha)^{\frac{1}{\alpha}} + \theta^{\frac{1}{\alpha}} \left(1+h_t L_t^A\right)^{\zeta} A_t^I (1+e_{t+1})^{\beta \frac{\theta}{\alpha}} \left(\beta \frac{\theta}{\alpha} \frac{1}{(1+e_{t+1})\left(A_t^I\right)^b} \right)}{\Upsilon(L_t)^{\varepsilon} \left(A_t^A\right)^{\delta} X_{t+1} (1-\alpha)^{\frac{1}{\alpha}} + \theta^{\frac{1}{\alpha}} \left(1+h_t L_t^A\right)^{\zeta} A_t^I (1+e_{t+1})^{\beta \frac{\theta}{\alpha}} \left(\frac{1}{\left(1+e_{t+1}\left(A_t^I\right)^b\right)} \right)} \right) \text{ is positive.}$$

[4] We show by simulation that $G(.)$ is a decreasing function with respect to t_t.

Proof: follows directly from implicit function condition presented in the appendix.

Regarding these decision rules, elites will define when do they support and allow education on the economy. Decisions will depend on the macroeconomic environment and main variables, such as land fertility and agricultural technology. These implications will be analyzed in section 4.2.

2.4. Dynamical Paths

The economy is governed by three main macroeconomic variables: agricultural productivity A_t^A, industrial productivity A_t^I and the evolution of working population L_t.

2.4.1. Population Dynamics

From (15), the size of the labor force in period $t+1$, L_{t+1}, is determined by:

$$L_{t+1} = L_t n_t = \begin{cases} \dfrac{1 - \dfrac{\tilde{c}}{w_t}}{(\tau^r + g(\tau^e, \iota_t) e_{t+1})} L_t & if \;\; z_t < \tilde{z} \\ \dfrac{1-\gamma}{(\tau^r + g(\tau^e, \iota_t) e_{t+1})} L_t & if \;\; z_t \geq \tilde{z} \end{cases}, \tag{23}$$

Where the initial historical size of the adult population, $L_0 > 0$, is given.

2.4.2. Technology Dynamics

The level of each technology is affected by its level in the previous period.

Agricultural technology at time $t+1$ is affected by two elements: the externality of the "learning by doing effect" and general knowledge effect of population in technology; and the external effect from the gains of educating the youngsters in the period jointly with the existent level of industrial technology. This latter effect allows for interconnections between technology and education and existent working population and level of agricultural technology. The law of motion of agricultural technology is such that:

$$A_{t+1}^A = (1 + e_{t+1}(A_t^I)^b)(L_t)^\varepsilon (A_t^A)^\delta, \tag{24}$$

Where $(L_t)^\varepsilon (A_t^A)^\delta$ captures the "learning by doing effect" and general externalities of growing population in agricultural technology. The factor

$e_{t+1}(A_t^I)^b(L_t)^\varepsilon(A_t^A)^\delta$ is the effect external effect of industrial technology and education.

We assume that $\varepsilon > 0$ and $\delta > 0$ and $\varepsilon + \delta < 1$, which implies that population has decreasing effect in knowledge creation, and also it implies a "fishing out" effect, namely the negative effect of past discoveries on making discoveries today. In addition, $b > 0$ so that when people are educated, externalities of industrial technologies are spilled to technology in agriculture.

Evolution in industrial technology is given by the past period level of technology and the improvement of knowledge driven by working population size measured by its level of human capital. The more human capital and the more the number of workers in the economy, the more the gains to industrial technology driven by learning by doing and externalities associated with human capital.

$$A_{t+1}^I = (1 + h_t L_t^\Delta)^\zeta A_t^I \,, \tag{25}$$

Where $\zeta \in (0,1)$ as well as $\Delta \in (0,1)$. Equation (25) shows that industrial technology advances according to the expansion of the existent level of technology due to increasing population and human capital level but in a diminishing returns fashion.

The initial historical levels of both technologies, $A_0^A, A_0^I > 0$, are given.

3. Dynamics of the Development Process

Now, it will be examined how the structure of the economy and agents' decisions shape the evolution of the process of development of the economy. It will show how the economy can evolve from a pre-industrial equilibrium to a state of sustained economic growth and how land fertility, agricultural technology and elites' decisions affect the economic equilibrium during the different states. This section shows that countries with lower land fertility have elites more open to support education. By the same token, more positive shocks in agriculture technology due to a presumable agricultural revolution also cause the delay of the education process. The demographic transition timing is then a consequence of elites' decisions on education.

3.1. Before the Industrial Revolution

In this section it will be studied the transition from an agricultural to an industrial economy that is it will be described the evolution of the system within the Malthusian epoch, as well as the endogenous transition to industrialization. It will be shown that the process of transition is very straightforward but will depend on the initial level of land fertility, agricultural technology as well as on the elites' decision on supporting education. These causes will hence affect also the rise of education in the economy delaying the process of deep industrialization.

During the malthusian phase, the economy is governed by the dynamical system given by equations (23), (24) and (25) which yield the sequence of state variables $\{A_t^A, A_t^I, L_t\}_{t=0}^{\infty}$. Initial values (A_0^A, A_0^I, L_0) are given.

Following Ashraf and Galor (2011), the pre-industrial equilibrium can be analyzed by the behavior of the two variables $\{A_t^A, L_t\}$ and the distance to the Industrialization frontier. The industrial technology variable does not affect the pre-industrial equilibrium since until the emergence of the industrial sector it is just a latent variable that does not interfere directly with any macroeconomic variable. It may only interfere on the elites' decisions but as it can be shown in the simulations it does not affect it before the take-off of the industrial sector. Also, it must be stressed that under this pre-industrial period the economy is under the Malthusian regime, i.e., the economy evolves under the assumption that the subsistence consumption constraint is binding and so fertility depends on income of workers. So, as it will be shown, under a steady state equilibrium the economy will be trapped on the Malthusian regime and on a binding consumption constraint.

3.1.1. The Industrialization Frontier

The Conditional Industrialization Frontier (CIF) gives the frontier between the agricultural economy and the industrial economy. It is a geometric locus, in $\{A_t^A, L_t\}$ space, for a given A_t^I where workers are indifferent between working or not in the industrial sector. Once the economy's trajectory crosses the frontier, the industrial sector becomes operative. The CIF is then given by:

$$CIF\big|A_t^I \equiv \left\{(A_t^A, L_t) : L_t = \hat{L}(A_t^A, A_t^I)\right\}, \tag{26}$$

We can establish the following lemma:

Lemma 6: If $\{A_t^A, L_t\}$ belongs to the CIF then, for a given A_t^I,

$$L_t = \frac{(1-\alpha)^{\frac{1}{\alpha}} \Upsilon A_t^A X_t}{(h_t A_t^I)^{\frac{1}{\alpha}}}$$

Where $\frac{\partial \hat{L}(A_t^A, A_t^I)}{\partial A_t^A} > 0$ and $\frac{\partial \hat{L}(A_t^A, A_t^I)}{\partial A_t^I} < 0$.

Proof: Follows directly from Lemma 1, (6), (7) and (26)

The CIF is upward sloping where in the region strictly below the frontier agriculture in the only sector open, whereas in the region above both sectors are open. The more A_t^I, the more close we are from the trigger and from surpassing the CMF.

For the case of the agricultural technology locus, we set it for all the pairs $\{A_t^A, L_t\}$ such that A_t^A it is in steady state.

$$AA \equiv \{(A_t^A, L_t) \colon A_{t+1}^A - A_t^A = 0\}\,, \tag{27}$$

Lemma 7: If $\{A_t^A, L_t\}$ belongs to AA then,

$$L_t = (A_t^A)^{\frac{1-\delta}{\varepsilon}} \equiv L^{AA}(A_t^A)$$

Where $\frac{\partial L^{AA}(A_t^A)}{\partial A_t^A} > 0$ and $\frac{\partial^2 L^{AA}(A_t^A)}{\partial (A_t^A)^2} > 0$.

Proof: Follows directly from (24) and (6) using the steady state equilibrium condition and (27).

The AA locus is a convex, upward sloping curve. If we are above L^{AA} then the number of workers is big enough to ensure the expansion of the technology frontier overcoming the erosion effects of imperfect intergenerational transmission of knowledge. If it is below the L^{AA} then workers are not enough to overcome the latter effect, shrinking the technology level.

The population locus (LL) is the set of all pairs $\{A_t^A, L_t\}$ such that L_t is in steady state, regarding that the CIF is not surpassed and taking into account the subsistence consumption constraint. Since the Malthusian epoch is characterized by a direct interconnection between fertility and income, then it should be considered the population locus (population equilibrium) when the subsistence consumption constraint is binding.

$$LL \equiv \left\{(A_t^A, L_t): L_{t+1} - L_t = 0 \middle| L_t < \hat{L}; z_t < \tilde{z}\right\}, \tag{28}$$

Lemma 8: If $\{A_t^A, L_t\}$ belongs to LL then,

$$L_t = \left[\frac{(1-\tau^r)(1-\alpha)}{\tilde{c}}\right]^{\frac{1}{\alpha}} \Upsilon A_t^A X_t \equiv L^{LL}(A_t^A)$$

Where $\frac{\partial L^{LL}(A_t^A)}{\partial A_t^A} > 0$ and $\frac{\partial^2 L^{LL}(A_t^A)}{\partial (A_t^A)^2} = 0$.

Proof: Follows from (23) using the steady state equilibrium condition and (28).

Hence, the LL locus is an upward sloping linear function. L_t grows over time below the LL locus ($L_{t+1} > L_t$) when for a lower population size wages increase and, hence, allows for fertility above replacement. Otherwise, wages become lower and so, due to the consumption constraint, it reduces resources available for fertility and then $L_{t+1} < L_t$. In addition we can draw the relationship between the LL locus in Lemma 8 and the CIF in Lemma 6.

Lemma 9: For $A_t^I > 0$ and for all A_t^A such that $(A_t^A, \hat{L}(A_t^A, A_t^I)) \in CIF | A_t^I$ and $\left(A_t^A, L^{LL}(A_t^A)\right) \in LL$

$$\hat{L}(A_t^A, A_t^I) \gtreqless L^{LL}(A_t^A) \qquad \text{if and only if} \qquad A_t^I \lesseqgtr \frac{\tilde{c}}{(1-\tau^r)h_t}$$

Proof: Follows from comparing $\hat{L}(A_t^A, A_t^I)$ and $L^{LL}(A_t^A)$ in Lemma 6 and Lemma 8, respectively.

So, for $A_t^I < \frac{\tilde{c}}{(1-\tau^r)h_t}$ the CIF is above the LL locus. Nevertheless, the more A_t^I increases the more the trigger of the industrial sector is close and so when $A_t^I = \frac{(1-\alpha)\tilde{c}}{(1-\tau^r)h_t}$ the CIF equalizes the LL locus. After this point, $A_t^I > \frac{(1-\alpha)\tilde{c}}{(1-\tau^r)h_t}$ the CIF is below the LL locus – the industrial sector emerges.

3.1.2. Equilibrium and Global Dynamics

If we consider the economy in the pre-industrial Malthusian equilibrium than we have to assure that the condition $A_t^I < \frac{(1-\alpha)\tilde{c}}{(1-\tau^r)h_t}$ in Lemma 9 verifies as well as the subsistence consumption constraint is binding $z_t < \tilde{z}$. Following these conditions, the malthusian steady state is characterized by a globally stable steady state equilibrium $\{A_{ss}^A, L_{ss}\}$. Using Lemma 7 and Lemma 8 the pre-industrial steady-state values of productivity in the agricultural sector, A_{ss}, and the size of working population, L_{ss}, are given by:

$$A_{ss}^{A} = \left[\frac{(1-\tau^r)(1-\alpha)}{\tilde{c}}\right]^{\frac{\varepsilon}{\alpha(1-\delta-\varepsilon)}} \Upsilon^{\frac{\varepsilon}{(1-\delta-\varepsilon)}} X^{\frac{\varepsilon}{(1-\delta-\varepsilon)}}, \tag{29}$$

$$L_{ss} = \left[\frac{(1-\tau^r)(1-\alpha)}{\tilde{c}}\right]^{\frac{1-\delta}{\alpha(1-\delta-\varepsilon)}} \Upsilon^{\frac{1-\delta}{(1-\delta-\varepsilon)}} X^{\frac{1-\delta}{(1-\delta-\varepsilon)}}, \tag{30}$$

By ruling out the unstable equilibrium at the origin ($L_0 > 0$ and $A_0^A > 0$) we keep the globally stable equilibrium $\{A_{ss}^A, L_{ss}\}$. At initial stages of development, agriculture is the pervasive sector since the latent industrial sector has a very low level of productivity and so it is not sufficiently attractive so that the economy operates exclusively in the agricultural sector. Therefore the CIF locus is located above the LL locus. And the above mentioned dynamics of L_t and A_t^A are valid.

To guarantee that the discrete dynamical system is globally stable and that the convergence to the steady state takes place monotonically over time,

Lemma 10: If $A_t^I < \frac{(1-\alpha)\tilde{c}}{(1-\tau^r)h_t}$ then the equilibrium in the dynamical system:

(1) is globally stable if the Jacobian matrix $J(A_{ss}^A, L_{ss})$ has real eigenvalues with modulus less than 1;

(2) and the convergence to the steady state is monotonically stable.

See proof in the appendix.

The economy is initially in an early stage of development meaning that the economy evolves in the pre-industrial regime and both macroeconomic variables (A_t^A, L_t) gravitate at the steady state values. In order to guarantee this pre-industrial equilibrium remains until the latent industrial sector emerges, we must assure that the subsistence consumption constraint remains binding during this regime so that:

$$z_t | A_{ss}^A, L_{ss} = w_{ss} h_t < \tilde{z}, \tag{A 2}$$

For initial $h_t = 1$. With only the agricultural sector is operative, the all workers are employed in the sector, and therefore from (1) it follows that the steady-state level of income per worker is given by:

$$y_{ss} = (\Upsilon A_{ss}^A X)^{\alpha} (L_{ss})^{-\alpha}, \tag{31}$$

Using (29) and (30), the steady-state level of income per worker agrees with the dynamics under the Malthusian epoch - in the long-run, the level of income is independent of the level of technology and it is constant $\hat{y}_{ss} = 0$. It is crucial to note that income per capita in the model is also affected by the level of natural land endowment, which gives rise to different levels of cooperation thereby implying different long-run levels of income per capital across countries with variations in land endowment.

3.2. The Industrial Revolution

As the economy evolves under the Malthusian epoch within the pre-industrial steady state, it operates exclusively in the agricultural sector, although the latent and endogenous process of industrialization continues in the background. And the latter implies the take-off to a state of sustained economic growth will happen in a near future. This section will examine the transition from the Malthusian regime, through the Post-Malthusian Regime, to the demographic transition and Modern Growth.

3.2.1. Dynamics

This section focuses on the description of the two different potential regimes after the emergence of the industrial sector. It was assumed that under the pre-industrial period the subsistence consumption constraint was binding. Therefore, the economy is still under the Malthusian regime and it is trapped in this regime since there is a stable steady-state equilibrium that dismisses any chance of moving out of that trap - see (A 2). However, after the emergence of the industrial sector, in the first regime the subsistence consumption constraint is still binding, as in the Malthusian phase. Despite of this, since we are no longer under the steady state equilibrium of the previous regime, wages increase and so the subsistence constraint will vanish in time. From before, the Post-Malthusian phase starts and population grows faster although income still has an effect on fertility. As for the second regime, the subsistence constraint is no longer binding. This means that there will be no direct effect of income on fertility. Population grows at a constant level that will only be affected by choices of workers on education due to elites' decision of supporting education – see (23).

In the first regime the economy will be governed by a four-dimensional non-linear first-order autonomous system:

$$\begin{cases} A_{t+1}^{A} = (1+e_{t+1}(A_t^I)^b)(L_t)^{\varepsilon}(A_t^A)^{\delta} \\ A_{t+1}^{I} = (1+h_t L_t^A)^{\zeta} A_t^I \\ e_{t+1} = e(T_t(A_t^A, A_t^I, L_t)) \\ L_{t+1} = \dfrac{1-\dfrac{\tilde{c}}{w_t^I}}{(\tau^r + g(\tau^e, \iota_t)e_{t+1})} L_t \end{cases} \quad \text{for } z_t < \tilde{z} \tag{32}$$

In the second regime, since the subsistence consumption constraint is no longer binding, the regime is governed by the same four-dimensional system although now population growth does not depend on income of workers:

$$\begin{cases} A_{t+1}^{A} = (1+e_{t+1}(A_t^I)^b)(L_t)^{\varepsilon}(A_t^A)^{\delta} \\ A_{t+1}^{I} = (1+h_t L_t^A)^{\zeta} A_t^I \\ e_{t+1} = e(T_t(A_t^A, A_t^I, L_t)) \\ L_{t+1} = \dfrac{1-\gamma}{(\tau^r + g(\tau^e, \iota_t)e_{t+1})} L_t \end{cases} \quad \text{for } z_t \geq \tilde{z} \tag{33}$$

In these regimes an analytical analysis is not straightforward since there are four interconnected differential equations. Nevertheless, some inferences can be drawn on the passage from one to another regime. Namely, the transition between the two regimes is given by the distance to the Malthusian frontier. As explained previously - (32) and (33), the economy departs from the first regime when potential income z_t exceeds that level. So, $\{A_t^A, A_t^I, L_t, e_{t+1}\}$ belongs to MF if

$$MF \equiv \left\{(A_t^A A_t^I, L_t, e_{t+1}) \colon \theta(A_t^I)^{1-\theta}(\lambda_t(A_t^A, A_t^I, L_t)L_t)^{\theta-1}(h_t)^{\theta} = \tfrac{\tilde{c}}{1-\gamma}\right\}, \tag{34}$$

Lemma 11: The economy surpasses the Malthusian regimes if:

$$w_t^I = \theta(A_t^I)^{1-\theta}(\lambda_t(A_t^A, A_t^I, L_t)L_t)^{\theta-1}(h_t)^{\theta} \geq \frac{\tilde{c}}{1-\gamma}$$

Proof: Follows from (8), definition of z_t and $\tilde{z}$, and (34).

3.3. From Malthusian Epoch to Modern Growth

The economy evolves from the Malthusian epoch to Modern Economic Growth passing through the Post Malthusian phase and the demographic transition. This path of evolution derives from section 4.1 and the two regimes explained above.

Consider an economy trapped in the pre-industrial equilibrium. Population is quite small and agricultural technology is stagnant. On what the main macroeconomic variables are concerned we have a globally stable steady state equilibrium on agricultural technology and population (Lemma 10). Provision of education is initially not supported by elites and therefore workers do not provide it. There is only the latent industrial sector whose productivity is growing slowly over time. Income per capita is constant as well. This is the typical Malthusian stagnation regime. Under Lemma 9 and Lemma 10 we can characterize this globally stable conditional steady state equilibrium and characterize the moment when the take off takes place. As for education, from Lemma 4 and equation (22) we know that only if these conditions are satisfied do elites have incentives to support and allow education. They are only verified if $G(.)|A_{ss}, L_{ss}, A_t^I \geq 0$

As productivity grows in the latent sector and later on the emergence of the industrial sector occurs, the economy changes to the Post-Malthusian regime. Now workers will split between the two sectors, and with the structural break we know there will always be workers in both sectors. As the equilibrium conditions break, population starts to grow over time as well as agricultural technology. From (24) and (25) we know that growing population will have a scale effect on both technologies. And there will be an interconnection between variables since, now, more population and technology lead to higher wages. As income increases, and the economy still is in a Post - Malthusian phase, it will affect positively fertility. More income means higher fertility and, hence, there is a boost in population. All the three state macroeconomic variables grow over time. Therefore, the more income available the less restrictive is the budget constraint so that consumption increases over time. In reaction to increasing disposable income the subsistence consumption constraint vanishes. As this occurs the economy moves to the second regime. Here, population grows at a steady rate given in (33) - income does not affect fertility. Fertility is, then, only dependent on the quality-quantity trade-off. Elites have a word to say on this, since again they are who decides if there is education or not (see Lemma 2 and Proposition 1). From section 3.3 we know that only if $G(.) \geq 0$ and, hence, if Lemma 5 applies, elites support education. Since industrial technology is growing and the share of workers is mostly in the industrial sector, the marginal gains from the technology effect will at some point exceed the workers effect and the population effect (if negative). As this condition applies, the overcoming of the bequest effect, which is negatively affected by growing industrial technology, will soon follow. The moment this occurs, elites will have an incentive to promote education since they will gain more from technology improvements than they will lose from transfers of a share of their bequest. Besides the gains on technology from education, which will enhance industrial and agricultural technology, the outcome of this decision is a demographic transition. As it was explained above fertility, now, only depends on the quality-quantity trade-off derived from workers' decisions. Therefore, the more education is given to children the lower is the number of offspring, causing the decrease of population growth rates.

The rise of the industrial sector and the posterior rise of education have a virtuous effect on the economy. The industrial sector allows workers to earn more and to have more resources available to allocate to children quantity and quality. The moment education is allowed by elites the amount allocated to just quantity now splits also to quality decreasing population growth rate but increasing technology and, hence, increasing productivity levels of workers in both sectors, from (24) and (25), which amounts to more earnings that will induce more available income for consumption.

From simulations of the model, shown in the next subsection, as the economy evolves, the main macroeconomic variables take a constant behavior: population continues to grow at a small rate, productivity in both sectors increase over time with industrial productivity growing more than agricultural productivity. As for the share of workers, we observe a shift of most of population to the industrial sector. As for education, it increases over time but it remains almost stable after the initial boost.

Now it must be understood the interaction between many of the features referred previously in the introduction, the Agricultural Revolution, land fertility the Industrial Revolution, elites' behavior and education. Firstly, it will be showed how the model behaves by itself and how elites behave in their willingness to allow the provision of education to children and consequently cause the demographic transition. Then, the relationship between land endowments and the onset of industry and education must be examined to verify how they affect the industrial take off as well as the elites' decisions on education. Finally, it will be discussed what the possible role of Agricultural Revolution was and how it can account for the onset and continuous process of industrialization and education.

From here we can draw the main hypotheses advanced in this paper:

H1 - The emergence of education and, hence, the demographic transition depends on the decision of elites: elites delay the emergence of education even after the onset of the Industrial Revolution;

H2 – Agricultural Revolution has a positive effect on the emergence of education;

H3 – Land Fertility affects positively the emergence of education.

3.3.1. Model Calibration: Education and Demographic Transition

This section will begin with the simulation of the model and all its properties. Galor (2011), Lagerlöf (2006) and Voigtländer and Voth (2006) provide a quantitative analysis of the Unified Growth models which have similarities with this one, so their calibration of parameters will be followed closely when possible.

Firstly, some specific functional forms for human capital and the cost of education function will be specified for the calibration in order to conform to Lemma 2. From (12):

$$h_{t+1} = (1 + e_{t+1})^{\beta} , \tag{35}$$

With $0 < \beta < 1$. It is an increasing, strictly concave function, of the investment on education, e_{t+1}.

As for the time endowment cost necessary for each unit of education and for each child, $g(\tau^e, T_t)$, is given by:

$$g(\tau^e, T_t) = (\tau^e + T(t_t, b_t))^{\phi} , \tag{36}$$

Where $-1 < \phi < 0$ and $T(t_t, b_t) = \frac{t_t b_t}{1+t_t b_t}$ is an increasing, concave function, in t_t and b_t. And so, $g(.)$ is decreasing in t_t and b_t.

If the parameters are chosen as in Table 1:

Table 1 – Parameter calibration

Parameter	Values
Land share (α) /Labor share ($1-\alpha$)	0.4 / 0.6
Human Capital share (θ)	0.6
Land Fertility (Υ)	1
Land (X)	1
Division of land among elites	1
Weight on children in utility function ($\gamma = \mu$)	0.645
Fixed time cost of raising children (τ^r)	0.34
Time cost of educating children[5] ($g(\tau^e)$)	0.119
Subsistence consumption ($\tilde{c}$)	1
Human capital (β)	0.35
Time endowment cost concavity (ϕ)	-0.9
Weight of population on agricultural "learning by doing effect" (ε)	0.05
Weight of agricultural technology on agricultural "learning by doing effect" (δ)	0.07
Externality of industrial technology (b)	0.80
Weight of population on industrial "learning by doing effect"(Δ)	0.05
Diminishing returns effect on industrial dynamical path (ζ)	0.06

Besides this, the initial conditions of the model are given by the equilibrium values for the pre-industrial period of A_0^A and L_0 as well as for fertility, education, industrial productivity, share of workers and bequest. See Table 2 below.

Table 2 – Initial Conditions

Parameter	Values
Population (L_0)	0.0861
Agricultural productivity (A_0^A)	0.8726
Industrial productivity (A_0^I)	0.6
Fertility (n_0)	1
Education (e_0)	0
Share of workers (λ_0)	0
Bequest (b_0)	1

Using these parameterization and initial values, the patterns of the benchmark economy resemble closely the expected patterns referred above as well as the patterns observed in modern history. As depicted in Figure 1 and Figure 2, initially

[5] It is considered that taxes are initially zero. Since for taxes equal to zero there is no education, from (35) and (36) and Lemma 2, then $g(\tau^e)$must be equal to that calibrated value.

the economy is in a pre-industrial Malthusian equilibrium with population and agricultural productivity and income per capita constant over time, while industrial productivity keeps increasing. From both figures, it is possible to observe the take off of the industrial sector after about 40 periods.

In figure 1, the pre-industrial Malthusian regime vanishes after the beginning of the industrial phase so that population growth rates are higher than before until the demographic transition. Consistent with the empirical evidence, this occurs after about 100 periods when education is allowed by elites and it starts increasing over time. Before, education levels were always zero. It is observed the transition from the pre industrial Malthusian regime where fertility depends on income of workers and has a positive correlation with it and where education is not provided to a Modern Growth regime where fertility no longer depends on income, education emerges after elites allow for it and the demographic transition begins where from then on fertility rates become much lower and education grows over time. In the transition process, we have the Post Malthusian regime where fertility is higher than in the previous pre-industrial phase but it still depends on income, and education is still not provided. These features are showed in figure 1.

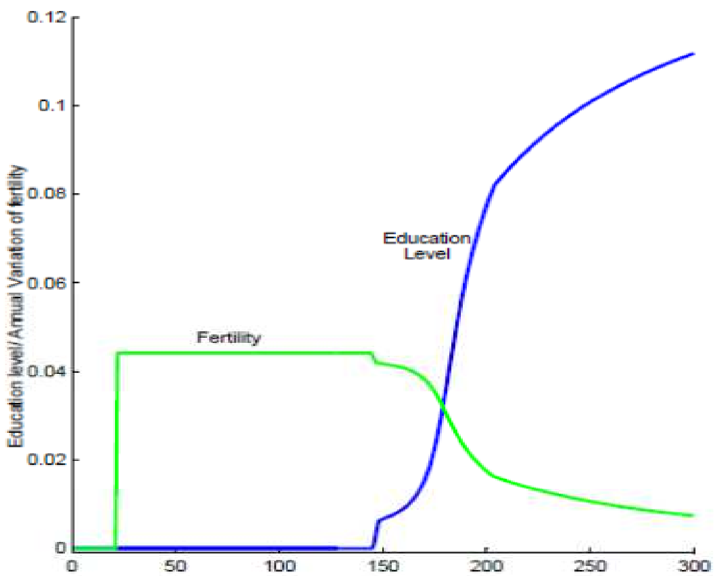

Figure 1: Quantitative analysis of calibrated model – education and fertility

In Figure 2, after the pre-industrial Malthusian equilibrium, agricultural and industrial productivities increase continuously as well as population. In this process both sectors productivities have a boost due to the effect of education on them. Namely, agricultural productivity suffers a peak after education starts to be provided with high rates of growth rates in this period while industrial productivity suffers a small increase but keeps its ascending path almost constant. As time goes by, the industrial sector and income per capita continue to grow at increasing rates, although agricultural productivity grows at a slower pace than the industrial one. Figure 2 presents these features:

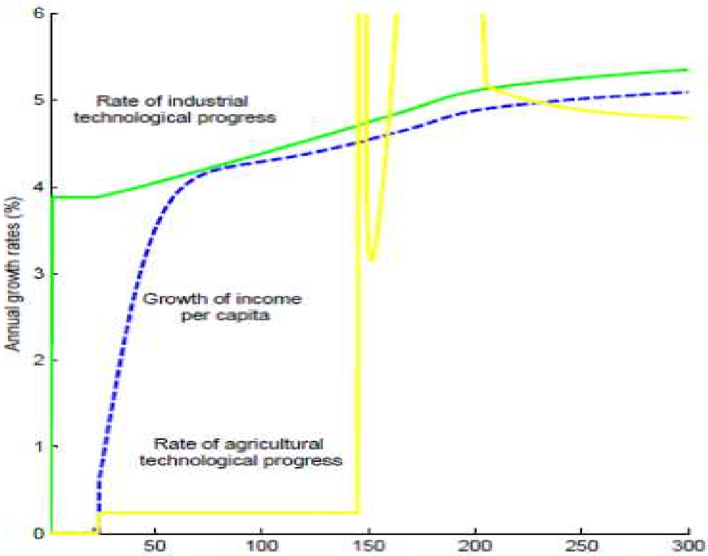

Figure 2: Quantitative analysis of calibrated model – productivity rates and income per capita

From the above explanations, the model and its simulations, as well as the above mentioned lemmas, it is possible to derive the following proposition.

Proposition 2: Elites aversion to education does not persist in time. The decision to support education from elites themselves occurs at some point in time. Elites may delay education but they do not prevent education to arise indefinitely.

Proof: It follows from the numerical simulation of the model and can be derived from Lemma 4, Lemma 5, (22), elites' decision after structural break, (35) and (36).

What must be held in mind from this proposition is that, in contrast with many papers and many theories, it is possible that elites had an incentive to allow for education. Nevertheless, and agreeing with those same theories elites also had the power to prevent education to emerge sooner due to their own choices. This means that although they decide to support education they only decide later in time which is consistent with the delayed process of education verified in history: the Industrial Revolution took place in the late 1700's and education only spread in the middle/late 1800's. This delay, although many other causes are behind them, can be traced on the power and willingness of elites to support it. If, as it is argued in the in the initial sections, elites were a small group of interest which had decision power on society and was the only group with enough economic resources to provide the means to educate people, they had, then, the power to inhibit or not education. From the above analysis, they had incentives to block education right after industrial emergence, but had also incentives afterwards to support it, when it was economically beneficial to them. Complementary to the existent literature, it can be showed that elites are not always against education and industrial enhancement.

3.3.2. Agricultural Revolution

Following the understanding of the factors influencing the outcomes of the model, the effect of a possible agricultural revolution will be examined. The agricultural revolution subject is highly discussed and has not had a final outcome. The hypothesis advanced in this paper suggests that increasing technology in agriculture eases the willingness of elites to allow for education.

Higher levels of agricultural productivity mean higher initial rents available for elites during the process of industrialization. Although there is the bequest effect, which increases, the technology effect will be higher than the bequest effect sooner because gains from externalities of industry will be boosted by having in the recipient technology an already higher level. In other words, the higher level of agricultural technology will provide the ground to the industrial externalities have an even more enhancing effect on technology of agriculture.

From the discussion on agricultural revolution, there is the debate on if agricultural revolution really happened, and if true, when it happened. From the model, we can advance the intuition that having shocks on agriculture productivity would cause a faster positive decision of elites on education, and a negative effect on the time of industrialization. Analytically and numerically it is possible to advance the following proposition:

Proposition 3: Agricultural Revolution has a positive impact on elites decision to educate population. The higher productivity of agriculture during the process of industrialization the more elites are prone to support education.

Proof: It follows from the numerical simulation of the model and can be derived from Lemma 1, Lemma 4, Lemma 5, (22), elites' decision after structural break, (35) and (36).

From the simulated model it is showed how shocks in agriculture affect the decision of elites in supporting education as well as there is a small fastening on the take off of the industrial sector with higher agricultural technology shocks. It is considered a positive random shock to agriculture, using a random uniform distribution to simulate an exogenous increase in agricultural technology. It is then clear that higher shocks have a positive impact on the early onset of education.

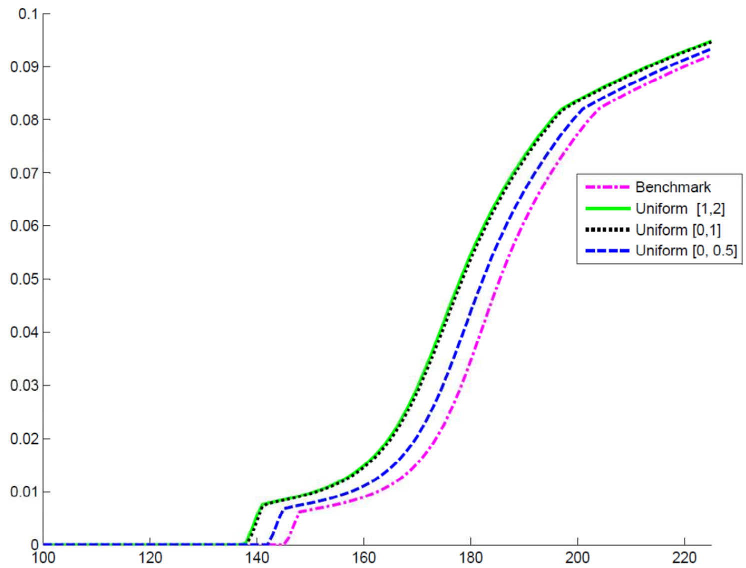

Figure 3: Period of take-off of education for different scenarios of shocks in technology of agriculture

From here, some insights can be added to the debate. Considering the economy is in a Malthusian equilibrium before the take-off, if the shock on agriculture productivity takes place in a time far from the take-off, the gains vanish over time (equilibrium is globally stable) and so there are no effective impact on the outcomes in the economy. But, if there are constant shocks in the economy so that the level of population and agriculture technology increase consistently above the equilibrium levels at the time of the take off, it implies there is an effect on rents as well as on bequests and, hence, the willingness of elites to provide education. It is a virtuous cycle in the economy that will then imply a faster boost in the economy due to more education and therefore higher industrial and agricultural technology growth rates. This means that countries that suffered from an Agricultural Revolution, which was mainly England and some Continental countries, but in a smaller scale, benefited from an earlier take off of education and an earlier economic boom. The other countries lagged behind which may have contributed to the divergence process in industrialization verified in this time period.

From the debate going on, it is possible to argue that there must have been agricultural technology shocks in the 1700's so that the take-off of the industrial sector and education took place earlier in countries such as England and the Netherlands. Shocks in the late 1600's or in the middle/later 1700's may not have had the necessary impact because of happening too early in time. The argument followed here points to a consistent level of ongoing shocks in the 1700's period must have been essential to have had a stronger industrial revolution and an early escape to Modern Economic Growth.

3.3.3. Land Fertility

The land fertility hypothesis is a byproduct of the model that runs at some stage against the existing literature. Land fertility is said to have a negative effect on the ability of countries to take off and develop. The hypothesis advanced in this paper suggests that high levels of fertility of land imply a fast process of provision of education by elites. The main reason is that more land fertility decreases the risk of elites in providing education the same way as the Agricultural Revolution did - higher levels of land fertility mean higher rents available for elites during the process of industrialization. And so, the rise of education will allow for a stronger externality of industrial technology on the agricultural one. It has again to do with the balance of the bequest effect and the technology effect. The more land fertility, the more the rents are and the higher the bequest but technology effect will sooner be more significant in their decision than the bequest effect. As it is observed from several simulations, the period of onset of education is anticipated, see Figure 4:

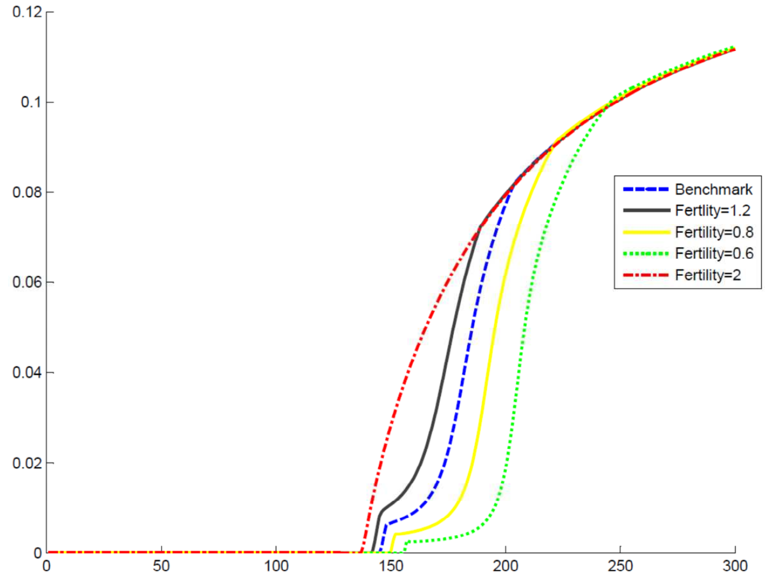

Figure 4: Period of emergence of education for different scenarios on land fertility

From Figure 4 we can observe how indeed land fertility has a positive effect on elites' decisions. Also there is an almost insignificant effect on the take off of the industrial sector since with fertility of land equal to two the onset is given one period earlier while with the other values the effect is null. Nevertheless, it must be stressed that this result is not a main conclusion and, from many other research studies, it is more a representation of a positive force involved in the process of growth than a definite and established fact. However, we can define analytically and numerically one can advance the following proposition:

Proposition 4: Land fertility has a positive impact on elites decision to educate population. The higher the fertility during the process of industrialization the more elites are prone to support education.

Proof: It follows from the numerical simulation of the model and can be derived from Lemma 1, Lemma 4, Lemma 5, (22), elites' decision after structural break, (35) and (36).

4. Concluding Remarks

The results presented in the previous sections show how interest groups can have a role on determining the pace of the economy in a society. The relevance of these results are on one side the contribution given to Unified Growth Theory and the study of the Industrial Revolution and on the other side the contribution for today's analyses on the way developing economies may face delays on their processes of development due to these political forces which are present in their societies. Much research has been made on the interconnections and willingness of land elites to be interested in promoting education. Much of these studies show they are against it. Nevertheless, this paper shows that with the right incentives even land elites ultimately agree with the promotion of education. This does not dismiss the other theories but instead complements them. It is important to show that during all the political process that opposed capitalists and landlords in the nineteenth century the rising power of the former and the increasing willingness of the latter to allow education may have reached a confluent point where capitalists demanded education and landlords did not oppose. As for today, the lesson to take is that it important to be aware of how interest groups react and which incentives they have, in order to intervene in the best way possible and reach an agreement all groups are happy and population in general benefit from the gains of education or any other element that may be a source of conflict between groups in the same society.

Further contributes to the historical literature are given by the insights on the role of Agricultural Revolution and whether it was a continuous sequence of technology shocks during the eighteenth century. This is a novelty and an important contribution to the literature. The conclusion that the Agricultural Revolution may have contributed to the early onset of the Industrial Revolution and, more important, to a quicker process of education of the masses is a new highlight to the debate going on in the literature. It may then represent a reason why England developed first than other countries from continental Europe. As for land fertility, it was possible to find this positive force underlying land endowments which also confirms that there are positive effects deriving from land fertility rather than the referred in the literature where it tends to indicate a reversal of fortunes relationship between better endowed countries and worse endowed countries.

Finally, the numerical simulation presents the main insights of the model and shows the main conclusions referred above. In line with Unified Growth Theory it is possible to conclude that interest groups had a role on the main events during the period of industrialization. Given their decisions, the rise of education was initially halted until it was allowed and, by consequence, the process of demographic transition occurred later in the nineteenth century. Furthermore, the event of Agricultural Revolution in the previous century positively influenced the onset of industrialization and of education. It contributed to the divergence between countries that witnessed it more intensely being the ones that also first witnessed industrialization and education.

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Appendix

Proof Lemma 1:

If we equalize (6) and (7), using the assumption of perfect labor mobility, we know that workers are employed also in the industrial sector if the marginal productivity in the industrial sector A_t^I is equal to or exceeds the marginal productivity in the agricultural sector.

Proof Lemma 2:

Take (16), (A1) and the properties of $g(\tau^e, T_t)$. We know that $E(0, T_t)$ is increasing in T_t. Also, the $\lim_{T\to\infty} E(0, T_t)$ is higher than $E(0,0)$ so that from (A1) it is positive. Therefore, for $T_t > 0$, and from (6), $e_{t+1} > 0$. Also, from the Implicit Function Theorem and (6), we can show that e_{t+1} is a single valued function of T_t and $e_{t+1} = e_{t+1}(T_t)$ so that $e'_{t+1}(T_t) = -\frac{\partial E/\partial e}{\partial E/\partial T} > 0$.

Proof Lemma 3:

Since we are maximizing the utility we want the values of t_t that for the interval [0,1] yield that maximum. It must be added also that from the numerical simulations the function $G(.)$ is always decreasing in t_t. So,

When $\frac{du}{dt_t} \neq 0$ for the interval of $t_t \in [0,1]$;

$$If\ \frac{du}{dt_t} > 0\ \Rightarrow G(.) = \bar{x}^i \frac{d\rho_{t+1}}{dt_t} - b_t > 0\ \Rightarrow t_t = 1$$

$$If\ \frac{du}{dt_t} < 0\ \Rightarrow G(.) = \bar{x}^i \frac{d\rho_{t+1}}{dt_t} - b_t < 0\ \Rightarrow t_t = 0$$

Since $\frac{du}{dt_t}$ is a decreasing function, from numerical simulations, these are the only valid cases, and $\frac{du}{dt_t}|t_t = 0 < 0$ and $\frac{du}{d\iota_t}|\iota_t = 1 > 0$ does not apply.

After-structural-break implicit function and lemma 5 explanation:

$$G(.)=\frac{d\rho_{t+1}}{d\iota_t}=\frac{\bar{x}^i\alpha\Upsilon(1-\alpha)^{\frac{1-\alpha}{\alpha}}(L_t)^{1+\varepsilon-\alpha}(A_t^A)^\delta}{\left[(1+e_{t+1}(A_t^I)^b)(L_t)^\varepsilon(A_t^A)^\delta\Upsilon X_{t+1}(1-\alpha)^{\frac{1}{\alpha}}+\theta^{\frac{1}{\alpha}}(1+h_tL_t^A)^\zeta A_t^I(1+e_{t+1})^{\beta\frac{\theta}{\alpha}}\right]^{1-\alpha}}$$

$$\left(\frac{de_{t+1}}{d\iota_t}n_t^{1-\alpha}(A_t^I)^b\left(1-(1-\alpha)\frac{\Upsilon(L_t)^\varepsilon(A_t^A)^\delta X_{t+1}(1-\alpha)^{\frac{1}{\alpha}}+\theta^{\frac{1}{\alpha}}(1+h_tL_t^A)^\zeta A_t^I(1+e_{t+1})^{\beta\frac{\theta}{\alpha}}\left(\beta\frac{\theta}{\alpha}\frac{1}{(1+e_{t+1})(A_t^I)^b}\right)}{\Upsilon(L_t)^\varepsilon(A_t^A)^\delta X_{t+1}(1-\alpha)^{\frac{1}{\alpha}}+\theta^{\frac{1}{\alpha}}(1+h_tL_t^A)^\zeta A_t^I(1+e_{t+1})^{\beta\frac{\theta}{\alpha}}\left(\frac{1}{(1+e_{t+1}(A_t^I)^b)}\right)}\right)\right.$$

$$\left.-(1-\alpha)(1+e_{t+1}(A_t^I)^b)n_t^{-\alpha}(1-\gamma)(\tau^r+g(.)e_{t+1})^{-2}\left(\frac{dg(.)}{d\iota_t}e_{t+1}+\frac{de_{t+1}}{d\iota_t}g(.)\right)\right)-b_t$$

Since b_t is always positive, and the derivative with respect to n_t is also always positive - these are the two latter parts of $\mathrm{G}(.)$ – then, if Lemma 5's condition is negative, $\mathrm{G}(.)<0$. So, only when Lemma 5's condition is positive will we have at some point $\mathrm{G}(.)\geq 0$.

More explicitly, as $\beta\frac{\theta}{\alpha}$ is constant, $\frac{1}{(1+e_{t+1})\left(A_t^I\right)^b}>\frac{1}{\left(1+e_{t+1}\left(A_t^I\right)^b\right)}\Leftrightarrow 1>(A_t^I)^b$ which happens only when the economy is almost rural. When the economy starts to industrialize and $(A_t^I)^b>1$ then, for sure we can guarantee that:

$$0<(1-\alpha)\frac{\Upsilon(L_t)^\varepsilon(A_t^A)^\delta X_{t+1}(1-\alpha)^{\frac{1}{\alpha}}+\theta^{\frac{1}{\alpha}}(1+h_tL_t^A)^\zeta A_t^I(1+e_{t+1})^{\beta\frac{\theta}{\alpha}}\left(\beta\frac{\theta}{\alpha}\frac{1}{(1+e_{t+1})(A_t^I)^b}\right)}{\Upsilon(L_t)^\varepsilon(A_t^A)^\delta X_{t+1}(1-\alpha)^{\frac{1}{\alpha}}+\theta^{\frac{1}{\alpha}}(1+h_tL_t^A)^\zeta A_t^I(1+e_{t+1})^{\beta\frac{\theta}{\alpha}}\left(\frac{1}{(1+e_{t+1}(A_t^I)^b)}\right)}<1$$

And, hence,

$$\left(1-(1-\alpha)\frac{\Upsilon(L_t)^\varepsilon(A_t^A)^\delta X_{t+1}(1-\alpha)^{\frac{1}{\alpha}}+\theta^{\frac{1}{\alpha}}(1+h_tL_t^A)^\zeta A_t^I(1+e_{t+1})^{\beta\frac{\theta}{\alpha}}\left(\beta\frac{\theta}{\alpha}\frac{1}{(1+e_{t+1})(A_t^I)^b}\right)}{\Upsilon(L_t)^\varepsilon(A_t^A)^\delta X_{t+1}(1-\alpha)^{\frac{1}{\alpha}}+\theta^{\frac{1}{\alpha}}(1+h_tL_t^A)^\zeta A_t^I(1+e_{t+1})^{\beta\frac{\theta}{\alpha}}\left(\frac{1}{(1+e_{t+1}(A_t^I)^b)}\right)}\right)>1$$

So, for A_t^I sufficiently big we will have a positive condition and at some point in time $\mathrm{G}(.)\geq 0$ - we will observe it in the numerical simulations.

Proof Lemma 10:

Given the Jacobian matrix:

$$J(A_{ss}^A, L_{ss}) = \begin{bmatrix} \frac{dA^A(A_{ss}^A, L_{ss})}{dA_t^A} & \frac{dA^A(A_{ss}^A, L_{ss})}{dL_t} \\ \frac{dL(A_{ss}^A, L_{ss})}{dA_t^A} & \frac{dL(A_{ss}^A, L_{ss})}{dL_t} \end{bmatrix} =$$

$$= \begin{bmatrix} \delta & \varepsilon \left[\frac{(1-\tau^r)(1-\alpha)}{\tilde{c}}\right]^{-\frac{1}{\alpha}} \\ \bar{L}\frac{1 + \frac{\alpha\tilde{c}}{(1-\alpha)}\left[\frac{(1-\tau^r)(1-\alpha)}{\tilde{c}}\right]^{1-\frac{\varepsilon}{\alpha(1-\delta-\varepsilon)}}}{\tau^r} & \frac{1-(1+\alpha)(1-\tau^r)}{\tau^r} \end{bmatrix}$$

The eigenvalues are given by $\{\lambda_1, \lambda_2\}$. We know that: $det(A_{ss}^A, L_{ss}) = \lambda_1\lambda_2$ and $tr(A_{ss}^A, L_{ss}) = \lambda_1 + \lambda_2$

$tr(A_{ss}^A, L_{ss}) = \delta + \frac{1-(1+\alpha)(1-\tau^r)}{\tau^r} > 0$ for $(1+\alpha)(1-\tau^r) < 1 \Leftrightarrow \tau^r > \frac{\alpha}{1+\alpha}$

$$det(A_{ss}^A, L_{ss}) = \delta\frac{1-(1+\alpha)(1-\tau^r)}{\tau^r} - \varepsilon\frac{1}{\tau^r}\left(\left[\frac{(1-\tau^r)(1-\alpha)}{\tilde{c}}\right]^{\frac{\varepsilon}{\alpha(1-\delta-\varepsilon)}} + \alpha(1-\tau^r)\right)$$

so that the equilibrium is globally stable if: $\lambda_1, \lambda_2 \epsilon (-1,1)$

(1) To guarantee that the convergence to the steady state is monotonically stable:

i. $Det(A_{ss}^A, L_{ss}) > 0$;

ii. and $Tr(A_{ss}^A, L_{ss}) > 0$.

For (i):

$$\delta\frac{1-(1+\alpha)(1-\tau^r)}{\tau^r} - \varepsilon\frac{1}{\tau^r}\left(\left[\frac{(1-\tau^r)(1-\alpha)}{\tilde{c}}\right]^{\frac{\varepsilon}{\alpha(1-\delta-\varepsilon)}} + \alpha(1-\tau^r)\right) > 0$$

$$\Leftrightarrow \delta\big(1-(1+\alpha)(1-\tau^r)\big) > \varepsilon\left(\left[\frac{(1-\tau^r)(1-\alpha)}{\tilde{c}}\right]^{\frac{\varepsilon}{\alpha(1-\delta-\varepsilon)}} + \alpha(1-\tau^r)\right)$$

$(\boldsymbol{condition}\ \mathbf{1})$

For (ii): $Tr(A_{ss}^A, L_{ss})$ always higher than zero from the above inequality

(2) To guarantee that the equilibrium is globally stable:

i. $-2 < Tr(A_{ss}^A, L_{ss}) < 2$;

ii. $-1 < Det(A_{ss}^A, L_{ss}) < 1$;

iii. $Det(A_{ss}^A, L_{ss}) - Tr(A_{ss}^A, L_{ss}) \geq -1$;

iv. and $Det(A_{ss}^A, L_{ss}) + Tr(A_{ss}^A, L_{ss}) \geq -1$.

For (i): from before we know that $Tr(A_{ss}^A, L_{ss}) > 0 > -2$

$Tr(A_{ss}^A, L_{ss}) < 2$

$\Rightarrow \delta + \frac{1-(1+\alpha)(1-\tau^r)}{\tau^r} < 2 \Leftrightarrow 1-(1+\alpha)(1-\tau^r) < (2-\delta)\tau^r \Leftrightarrow \tau^r(1-\delta-\alpha) + 1 + \alpha > 1$ P.V.

$\Rightarrow Tr(A_{ss}^A, L_{ss}) \in (-2,2)$

For (ii):

$Det(A_{ss}^A, L_{ss}) > -1$:

$$\delta\frac{1-(1+\alpha)(1-\tau^r)}{\tau^r} - \varepsilon\frac{1}{\tau^r}\left(\left[\frac{(1-\tau^r)(1-\alpha)}{\tilde{c}}\right]^{\frac{\varepsilon}{\alpha(1-\delta-\varepsilon)}} + \alpha(1-\tau^r)\right) > -1 \text{ P.V.}$$

From condition 1 we know this inequality holds.

$Det(A_{ss}^A, L_{ss}) < 1$: (by contradiction)

$$\delta\frac{1-(1+\alpha)(1-\tau^r)}{\tau^r}-\varepsilon\frac{1}{\tau^r}\left(\left[\frac{(1-\tau^r)(1-\alpha)}{\tilde{c}}\right]^{\frac{\varepsilon}{\alpha(1-\delta-\varepsilon)}}+\alpha(1-\tau^r)\right)>1$$

$$\Leftrightarrow(\delta-1)\tau^r-\alpha(1-\tau^r)>\varepsilon\left(\left[\frac{(1-\tau^r)(1-\alpha)}{\tilde{c}}\right]^{\frac{\varepsilon}{\alpha(1-\delta-\varepsilon)}}+\alpha(1-\tau^r)\right)\qquad\Rightarrow\qquad \text{P.F.}$$

$$\Rightarrow\ Det(A^A_{ss},L_{ss})<1$$

For (iii):

$$\delta\frac{1-(1+\alpha)(1-\tau^r)}{\tau^r}-\varepsilon\frac{1}{\tau^r}\left(\left[\frac{(1-\tau^r)(1-\alpha)}{\tilde{c}}\right]^{\frac{\varepsilon}{\alpha(1-\delta-\varepsilon)}}+\alpha(1-\tau^r)\right)-\delta-\frac{1-(1+\alpha)(1-\tau^r)}{\tau^r}\geq -1$$

$$\Leftrightarrow\alpha(1-\delta)(1-\tau^r)\geq\varepsilon\left(\left[\frac{(1-\tau^r)(1-\alpha)}{\tilde{c}}\right]^{\frac{\varepsilon}{\alpha(1-\delta-\varepsilon)}}+\alpha(1-\tau^r)\right)\qquad\qquad(\boldsymbol{condition}\ \mathbf{2})$$

For (iv):

$$\delta\frac{1-(1+\alpha)(1-\tau^r)}{\tau^r}-\varepsilon\frac{1}{\tau^r}\left(\left[\frac{(1-\tau^r)(1-\alpha)}{\tilde{c}}\right]^{\frac{\varepsilon}{\alpha(1-\delta-\varepsilon)}}+\alpha(1-\tau^r)\right)+\delta+\frac{1-(1+\alpha)(1-\tau^r)}{\tau^r}\geq -1$$

If condition 2 holds, then, since the trace is positive, this inequality will also hold.

Proof proposition 3:

To know the impact of agricultural technology on elites' decisions we need to apply the Implicit Function Theorem on (21) so that we can derive the impact of A^A_t on t_t. Computing $\frac{dG}{dt_t}$ and $\frac{dG}{dA^A_t}$ and since it is not possible to reach analytically a definite signal we must use a numerical simulation. What it is found is that $\frac{dG}{dt_t}<0$ for all the period of time whereas $\frac{dG}{dA^A_t}$ is oscillatory. Nevertheless, for the period previous to the rise of education $\frac{dG}{dA^A_t}<0$. Therefore:

$$\frac{dt_t}{dA^A_t}=-\frac{\frac{dG}{dt_t}}{\frac{dG}{dA^A_t}}\begin{cases}<0 & \text{before onset of Industrial Revolution}\\ >0 & \text{after onset of Industrial Revolution}\end{cases}$$

So, it is inferable that the higher the value of A_t^A on the onset of industrialization the more likely in that the onset of education will follow sooner. Therefore, previous improvements on agriculture (historically, during the eighteenth century) influence positively the early rise of education.