The life cycle model and the rental housing expenditure share.[*]

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Abstract

Using Danish register data we observe that, for households living in rental housing, the housing expenditure share is negatively correlated with income, positively correlated with rent per squared meter, and displays significant cross sectional dispersion. We find that the life cycle model accounts for the average patterns in the data remarkably well, and that income uncertainty is not enough to generate the observed dispersion in the housing expenditure share.

JEL Classification: D12, R21.

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1 Introduction

Using Danish register data we observe that, for households living in rental housing, the housing expenditure share is negatively correlated with income, positively correlated with rent per squared meter, and displays significant cross sectional dispersion.

If households make expenditure decisions based on the present discounted value of income and assets, the variation of current income over the life cycle should generate a negative relationship between income and the expenditure share. The data shows that on average over the life cycle this does happen and in fact the life cycle model matches average data remarkably well.[1] The theory is less clear regarding the relationship between the rent and the expenditure share.

Non zero correlations between the expenditure share and income or prices are also present at any given age across households. This cross sectional dispersion is harder to replicate. Idiosyncratic income uncertainty goes some way towards generating dispersion of the expenditure share. Income path dispersion, reflecting educational differences among other factors, however, does not help in generating the observed dispersion in expenditure shares.

2 Income, Rent and the Housing Expenditure Share

In this section, we present properties of Income, Rents and Expenditures Shares for renters in Denmark. The overall average housing expenditure share for all the households ob-

[1] The result is similar to the result obtained by Ejrnaes and Browning (20**).

served in 2010 is 0.23.[2] Table 1 shows the mean, μ, standard deviation, σ, and the upper triangle of the correlation, ρ, matrix of: income, rent per squared meter, house size in squared meters, and the expenditure share.

Table 1: Data Moments 2010

	μ	σ	σ/μ	ρ Income	ρ H-area	ρ Rent
H-Share	0.230	0.143	0.624	-0.625	0.094	0.362
Income	296	173	0.585	1	0.283	-0.008
H-area	73.0	23.7	0.325		1	-0.260
Rent p/m^2	741	228	0.308			1

nobs 248710, ages 19 to 91 only.

As stated above, the housing expenditure share is negatively correlated with income (-0.625) and positively correlated with rent per squared meter (0.362). At first glance these facts are unsurprising since the expenditure share is given by rent per squared meter times size divided by income, $e = rs/y$. But an accouting identity is not the *only* way to evaluate data. For example, a static Cobb-Douglas utility function predicts constant expenditure shares, something which is thought to apply to the housing share by Davis and Ortalo-Magne (2009).

3 The life cycle

The life cycle may be at work. Over the life cycle, income and expenditure shares will be negatively correlated even with Cobb-Douglas utility.

Consider a frictionless perfect foresight life cycle model, with period utility given by

[2] The 1996, 2001, and 2006 samples display the same patterns discussed here. We have data on households where the oldest member is aged from 16 to 108 but we use only households where the oldest member is aged 19 to 91 inclusive. For these ages we have 1000 or more observations per age. The moments and data patterns are virtually identical if we use the entire sample.

$U = \alpha \log(c) + (1-\alpha)\, log(s)$, and period budget constraint $c_t = (1+i)\, a_t - a_{t+1} + y_t - r_t s_t$, where r_t is rent per squared meter (per unit) and s_t is the size of the house.[3] The discount factor is $1/(1+\rho)$. We obtain the Euler equation $c_{t+1} = c_t \delta$, where $\delta = (1+i)/\,(1+\rho)$, and the optimality condition for housing $r_t s_t = (1-\alpha)\, c_t/\alpha$. This implies $r_{t+1} s_{t+1} = \delta r_t s_t$.

We can now replace these relationships in the budget constraint to obtain:

$$r_t s_t \frac{\alpha}{(1-\alpha)} = c_t = (1+i)\, a_t - a_{t+1} + y_t - r_t s_t$$

and then compute the present discounted sum of all budget constraints over the life cycle:

$$\frac{r_j s_j}{(1+i)^{j-1}} = r_1 s_1 \left(\frac{\delta}{1+i}\right)^{j-1} = (1-\alpha)\, \frac{(1+i)\, a_j - a_{j+1} + y_j}{(1+i)^{j-1}}$$

for $j = 1 : T$. Define $\beta = 1/(1+i)$ and $\Delta = \beta\delta$, and summing over all j's we obtain

$$r_1 s_1 = (1-\alpha) \left[\frac{1-\Delta}{1-\Delta^T}\right] \left[\frac{a_1 - a_{T+1}\beta^T}{\beta} + Y\right]$$

where Y is the present discounted value (PDV) of all income flows and $r_1 s_1$ is total expenditure on housing in the first age of life.

We now make use of this expression in conjunction with the observed average expenditure share ($E = 0.23$) to find the value of α. In fact, we have a degree of freedom since we do not know initial or terminal assets. We make one consistency assumption that in

[3] House here stands for whatever type of home the household lives in.

a stationary equilibrium $a_1 = a_{T+1} = \bar{a}$. We then write

$$r_1 s_1 = (1-\alpha)\left[\frac{1-\Delta}{1-\Delta^T}\right] W(\bar{a})$$

where wealth is given by $W(\bar{a}) = \bar{a}\left(1/\beta - \beta^{T-1}\right) + Y$. Now, to match the average expenditure share, we use the number of households and average income by age to construct

$$\frac{r_1 s_1}{y_1}\frac{n_1}{n} + \frac{r_2 s_2}{y_2}\frac{n_2}{n} + ... + \frac{r_T s_T}{y_T}\frac{n_T}{n} = E$$

which turns into

$$\frac{(1-\alpha)\left[\frac{1-\Delta}{1-\Delta^T}\right] W(\bar{a})}{n}\left[\frac{n_1}{y_1} + \frac{n_2}{y_2}\delta + ... + \frac{n_T}{y_T}\delta^{T-1}\right] = E$$

and it is clear that any value we assume for $\bar{a}$ will only result in a compensating value of α. The size of $r_1 s_1$ is not affected by it, and the evolution of $r_j s_j$ over the life cycle depends only on δ. The life cycle path of the expenditure share, $r_j s_j / y_j$, is also not affected by it. The value of $\bar{a}$ is therefore set to a default value of zero.

Consider the case where $\rho = i$. Nothing much changes above except that $\delta = 1$ and $\Delta = \beta$. Consumption is then a constant over the life cycle and so is total housing expenditure. Expenditure shares out of current income will then necessarily be negatively correlated with income itself.

3.1 Data

To match life cycle facts we focus on age specific averages in the data. This creates a life cycle of averages rather than the average life cycle (as we do not follow households over lengths of time).[4] The average data correlate differently from the entire sample. Table 2 below shows the relationship between the averages where we see that the expenditure share is now almost perfectly correlated with income, while the correlation with rent drops down to 0.1392.

Table 2: Life cycle Correlation of age specific means 2010

		ρ	
	Income	H-area	Rent
H-Share	-0.9568	0.1514	0.1392
Income	1	0.0129	-0.1006
H-area	0.0129	1	-0.6963
Rent m^2	-0.1006	-0.6963	1

nobs 73, ages 19 to 91 only

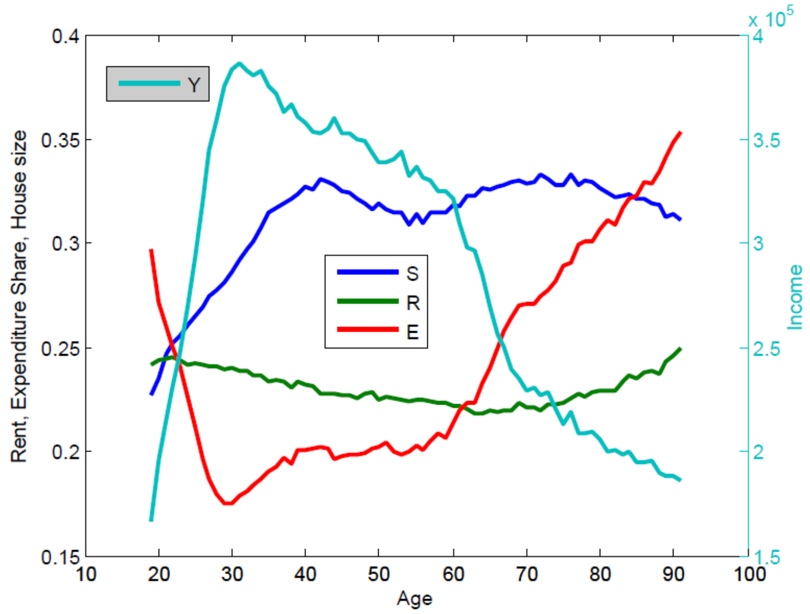
The implications of the life cycle model described above are clear in our data. Figure 1 shows the hump-shaped pattern of income, the slightly concave pattern of the house size, the mainly flat pattern of rents, and the convex curve for the housing expenditure share which strongly mirrors the income curve.[5] Note that in the model rents affect only house size, not the expenditure share.

[FIGURE 1 HERE]

[4]Since our data contains only renters it has a strong sample selection across different ages. We discuss this below.

[5]Figure 1 shows age specific sample *means*. Rents and house size are normalized to fit the scale of the expenditure share and income is measured in the right hand side axis.

Figure 1:
Income, Size, Rent, and Expenditure Share
Age specific means, 2010



3.2 Matching the data.

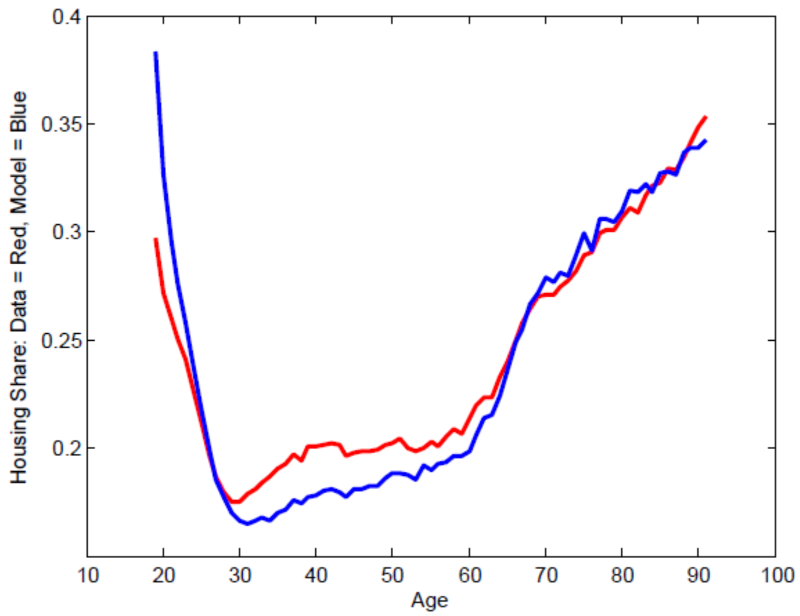
How well does the model described above match the data? Figure 2 shows what we get. The average *percentage absolute* deviation between the blue and the red line is 5.61% and the median deviation is 4.23%. The share in the model is off on average by 0.0126. The difference between the discount rate and the interest rate matters for the location of the expenditure share curve. It turns out that setting $\rho = i$ yields the best fit.[6]

[FIGURE 2 HERE]

The model fits the data well because the data so determines it. The data correlation coefficient between average income by age and the average expenditure share by age is -0.9568, nearly minus one, while this correlation in the model is -0.9861 (it is also the

[6] We minimize the average *percentage absolute* deviation between the life cycle expenditure share (ages 19 to 91) in the data and in the model.

Figure 2:
Housing Expenditure Share, Model and Data



correlation between Y and 1/Y).[7] Clearly, a value so close to minus one is only attainable if the expenditure on housing is all but constant over the life cycle.[8]

If we add a trend to the share with a factor 1.0043, we can hit the correlation exactly. That is, if we set $\rho = (1 + i)(1.0043) - 1$, the model generates exactly the correlation value -0.9568. The fit of the expenditure share curve is, however, worse, with an average *percentage absolute* deviation between the two curves of 10.56%

In the model the evolution of the rent is irrelevant since it is exogenous and only total housing expenditure matters. This implies rent in the model is identical to rent in the data (as is the case for income). Therefore the correlation between rent and the

[7] In logs these values are -0.9762 and -1, respectively.

[8] These averages do not yield the average life cycle because the "average" agent in the sample renting at age 20 is not the average agent renting at 40. The number of observations by age (see figure 3) sheds light on the quantity of sample selection taking place, although in itself this does not imply bias.

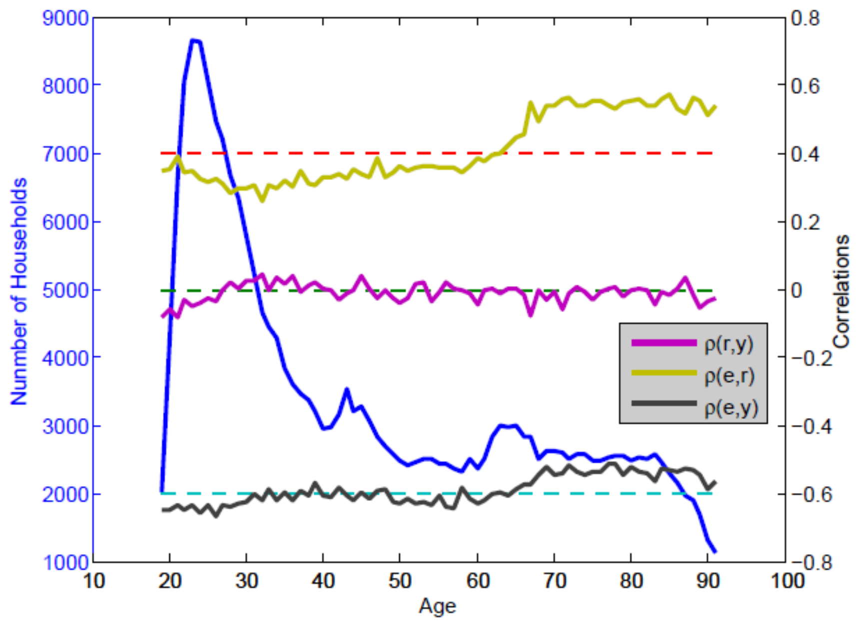
expenditure share will be also very close to the data, although it does not carry any additional information relative to the relationship between income and the expenditure share. Of course, the averages in the data do not have to be completely consistent in the sense that average size times average rent divided by average income should not necessarily equal the average expenditure share (due to Jensens's inequality).

4 The cross section

The life cycle, however, hides important features of the data. Figure 3 shows correlation coefficients computed *by age* for income, rents, and expenditure shares, for the year 2010.[9]

[FIGURE 3 HERE]

Figure 3:
ρ(e,y)<0, ρ(e,r)>0, ρ(r,y)=0, 2010 Data



[9]The figure shows five-age-averages (21 to 25, 26 to 30, etc) of age specific correlations, with the average assigned to the mid point marked in the graph (age 23, 28, 33, etc).

While we see some variation over the life cycle, the main feature of this figure is the relative stability of all correlations across age. Income and the expenditure share are negatively correlated, the rent and the expenditure share are positively correlated, and Income and rents are uncorrelated. The life cycle is not the driving force behind these correlations.

This figure also helps give some perspective of the impact of sample selection. One possible type of selection is that young agents in this data are credit constrained (they eat too little, but still cannot borrow to buy houses), while the surviving old are the impatient (they eat too much so they never buy houses). Somewhere in this selection is an idea that adult patient unconstrained households who by chance are renting have some correlation between income and rent while impatient ones do not. It is unclear how that comes out of our model.

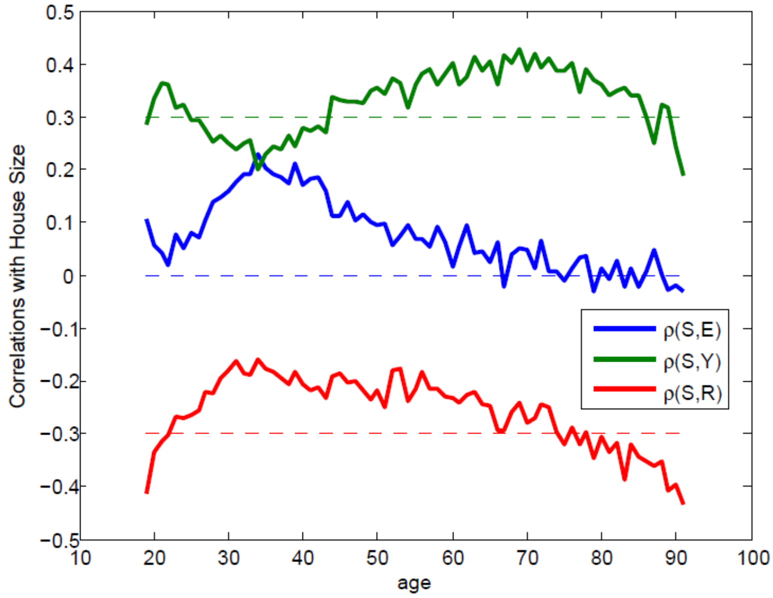
The house size is also correlated with the other 3 variables. We see that $\rho(s, y) > 0$ and $\rho(s, r) > 0$ at every age as expected from a model with a downward sloping demand. The correlation $\rho(s, e)$ is more interesting. It is not significant from around age 60 onwards and at early ages. It mirrors the correlation with income because income and the expenditure share mirror each other.[10]

Finally, the data shows substantial within-age dispersion in the expenditure share. This is the endogenous variable we are mostly interested and its dispersion is one fact we aim to explain.

Can we extend the previous model so that we can track the properties of the cross

[10] It does have, however, a positive hump shaped life cycle profile for middle ages, perhaps reflecting family composition....

Figure 4:
Correlation of House Size with Income, Rent, and the Expenditure Share, 2010



section as well as the properties of the mean over the life cycle?

4.1 Income Heterogeneity

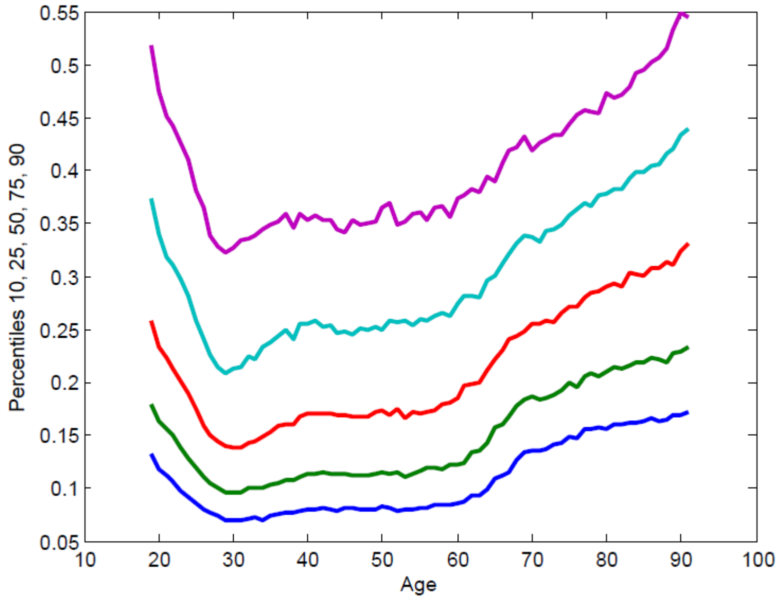
The obvious place to start is with income. Households of different educational levels have markedly different life cycle average paths. In our data we have

If we go back to our algebra above we have

$$r_1 s_1 = (1-\alpha)\left[\frac{1-\Delta}{1-\Delta^T}\right] W(\bar{a})$$

and keeping $\bar{a} = 0$ for all agents, we have that $W(\bar{a}) = \bar{a}\left(1/\beta - \beta^{T-1}\right) + Y = Y$. changes. So, if there are permanent differences in the net present value of income they

Figure 5:
Age Specific Percentiles of the Expenditure Share, 2010



will show up in wealth. Nevertheless

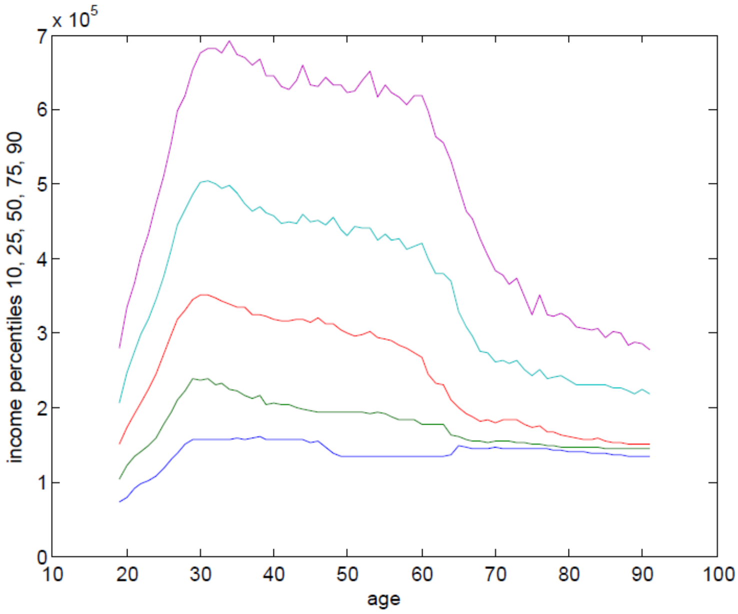
$$\frac{r_1 s_1}{W(\bar{a})} = (1-\alpha)\left[\frac{1-\Delta}{1-\Delta^T}\right]$$

is still invariant. Therefore income *path* heterogeneity will not succeed in generating cross sectional dispersion in the expenditure share.

4.2 Taste Heterogeneity

In our model rents are exogenous, so the extension of the life cycle model into a general equilibrium environment would yield a single price. Yet we see cross sectional variation in the price. Not only that, this cross sectional variation is reasonably stable over the life cycle.

Figure 6:
Percentiles of the income distribution by age

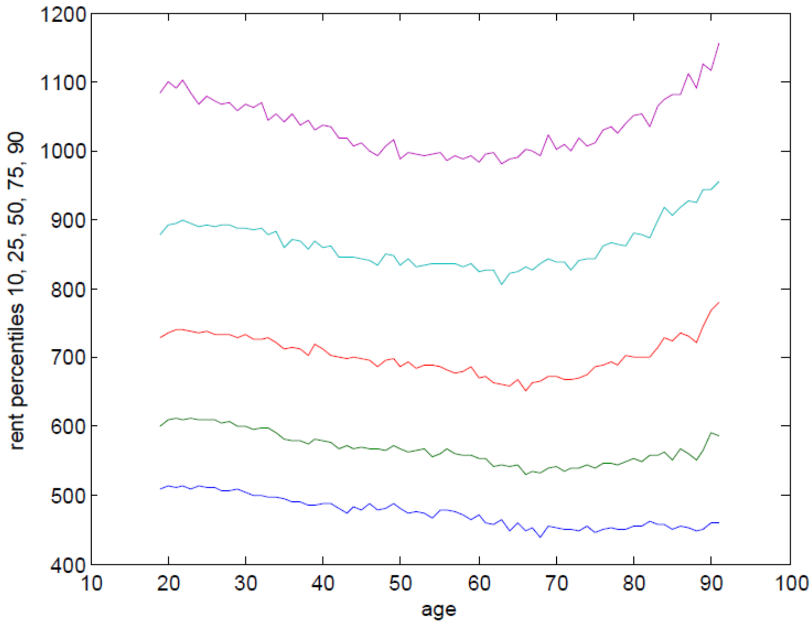


No doubt this cross sectional variation occurs because not all houses are the same. We now introduce this characteristic extending the previous model to include housing quality.

Households have an idiosyncratic preference for housing quality, $\mu(q)$, which has a counterpart in the rental rate, which now becomes also a function of quality, $r(q)$. Specifically we write $U = \alpha \log(c) + (1-\alpha)\, log(\mu s)$, and period budget constraint $c_t = (1+i)\, a_t - a_{t+1} + y_t - r_t(q) s_t$, where $r_t(q)$ is rent per squared meter as a function of quality.

We specify the functions $\mu = e^{\gamma q}$, and $r = r_0(1+q)$. In the function μ, the parameter γ is household specific, and has an exogenous constant distribution over the population. The first order condition for quality then yields $q = (1-\gamma)/\gamma$. Replacing q one obtains $r = r_0/\gamma$, and $\mu = e^{1-\gamma}$, where r_0 is the market clearing component of price, and where in order for the rent to be positive we must have that $\gamma > 0$. Note also that in this solution the rent paid is independent of income and assets as suggested by the lack of correlation

Figure 7:
Percentiles of the rent distribution by age



between these two variables we observe in the data.

Will housing quality solve our problem? Yes and no. We still have

$$\frac{r_1 s_1}{W(\bar{a})} = (1 - \alpha) \left[\frac{1 - \Delta}{1 - \Delta^T} \right]$$

which means we are still not able to generate cross sectional dispersion in the expenditure share. But, because the price depends on γ, size will depend on γ too. So, housing quality and price dispersion will help us with quantity dispersion. But the housing share is still not affected.

4.3 Uncertainty

So how can we generate dispersion in the expenditure share? An immediate look at the previous expression suggests we need to affect its right hand side. This means either heterogeneity in (ρ, α, i) or the need for more general utility such as the CES function used in Ejarque and Christensen (2013). Rather than tampering with preferences we look at uncertainty in income and/or rents.

Utility is given by $U = \alpha \log(c) + (1-\alpha)\log(\mu s)$, and the period budget constraint is $c_t = (1+i)\,a_t - a_{t+1} + y_t - r_t s_t$, where r_t is rent per squared meter (per unit) as a function of quality and s_t is the size of the house.

The optimality condition for housing size yields $r_t s_t = (1-\alpha)\,c_t/\alpha$. This implies we can eliminate housing and rewrite utility as

$$\begin{aligned} U &= \alpha \log(\alpha) + (1-\alpha)\log\left(\frac{1-\alpha}{r_t}\right) \\ &\quad + (1-\alpha)\log(\mu) + \log(\psi_t) \end{aligned}$$

where $\psi_t = (1+i)\,a_t - a_{t+1} + y_t$. We can then write $c_t = \alpha\psi_t$ and $r_t s_t = (1-\alpha)\,\psi_t$, so that total current expenditure is given by $\psi_t = r_t s_t + c_t$.

Given that the current rent appears only in an autonomous term, and is exogenous, savings and assets will move to accomodate income variations only. The Euler equation is

$$\frac{1}{\psi_t} = \frac{1+i}{1+\rho} E_{y_{t+1}}\left(\frac{1}{\psi_{t+1}}\right)$$

If income does not vary too much expenditure should be almost flat over the life cycle (if

$\rho = i$).[11]

The same is then true for $r_t s_t$. Therefore, variations in stochastic rent will be absorbed by variations in housing size only. Optimal savings are independent of both the shock in r_t and the taste heterogeneity γ. However, exactly because $r_t s_t$ is flat, income variations will translate into variations *of the expenditure share.*

Note finally that while the left hand side of $r_t s_t = (1-\alpha)\,\psi_t$ depends on the quality index, the right hand side, $(1-\alpha)\,\psi_t$, does not. Taste heterogeneity does not add to the cross sectional dispersion of the expenditure share.[12] Neither does heterogeneity of the income path (reflecting educational differences for example). But the observed taste heterogeneity does generate - through this model - almost all observed size heterogeneity.[13] However, we do not require taste heterogeneity to generate house size dispersion. The dispersion of income given a single rent implies dispersion in expenditure which translates one for one into dispersion in house size. In fact, given the way the values of γ are obtained - by normalizing all rents by the maximum rent observed - tastes heterogeneity actually compresses the dispersion in house sizes generated by income, bringing it more in line with the data.

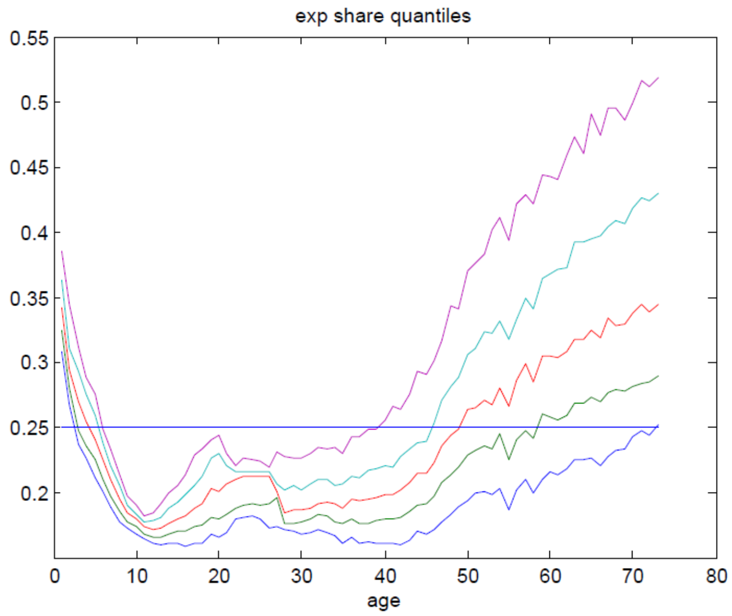
How much dispersion in the expenditure share can income uncertainty generate? The pattern w eobtain depends on the peristence of shocks. With a serial correlation coefficient of 0.95, if we introduce enough uncertainty to match the income dispersion between the 90th and 10th percentiles in the age interval 20 to 60, we obtain around one third of

[11] In case $\rho = i$ and dividing both sides by α we obtain $1/c_t = E_{y_{t+1}}\left(1/c_{t+1}\right)$.

[12] Rent shocks also do not have an effect here, but that is because there are no frictions. With frictions (moving costs) rent shocks will add to the dispersion of the expenditure share.

[13] We measure this by killing the income shock. In this case there is a single path of expenditure. The heterogeneity in γ is constructed from the quantiles of the data on rent/max(rent). The five values of gama associated with the 10, 25, 50, 75, and 90th percentiles then generate 5 curves for house sizes.

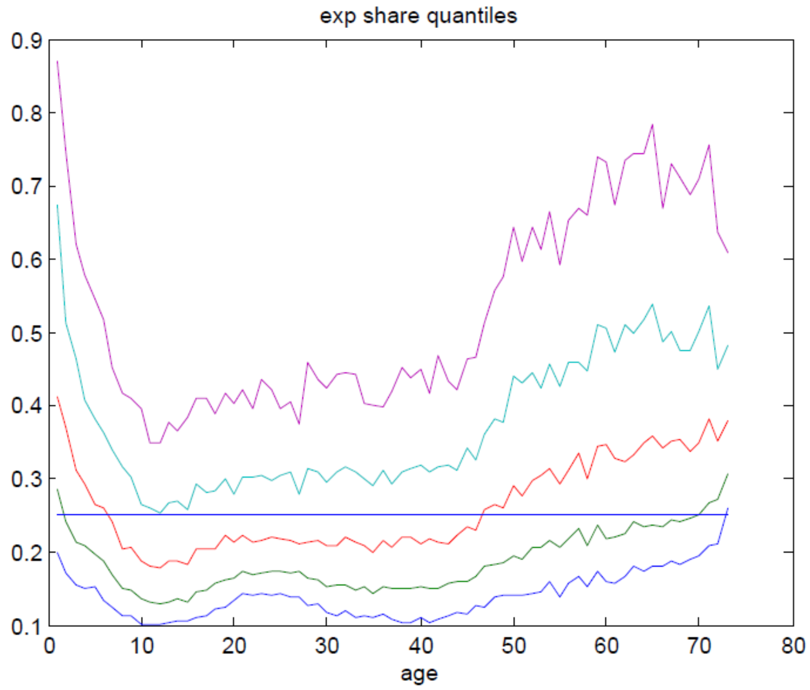
the observed dispersion (90/10 ratio) in the expenditure share.[14] This dispersion also increases with age as the possibilities to smooth income shocks also fall with age.



This pattern changes with the level of serial correlation. If we have zero serial correlation in the shock process we do not generate the same dramatic increase in dispersion with age. Figure XX shows the results with iid shocks. They actually match the data

[14]But this amount of dispersion is the result of unrecognizable individual life cycle income paths. It is clear that the dispersion in observed income is largely path (education) dispersion rather than the result of idiosyncratic uncertainty over life in a given path.

better.



5 Conclusion

The life cycle model is able to reproduce well the average patterns of rental housing expenditure observed in the data. With income uncertainty it is also able to generate some of the dispersion observed in the housing expenditure share, particularly with iid shocks.

Heterogeneity in income paths does not affect the housing expenditure share. Neither does heterogeneity in preferences for housing quality, although it does generate the observed dispersion in house sizes.

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5.1 Life cycle Model 2, CES

Would a more general model allow us to obtain a more meaningfull relationship between rent, income, and the expenditure share? WHAT DOES MORE MEANINGFULL MEAN?

Consider now the following period utility with minimum housing and housing quality.

$$U = \left[\alpha c^{\theta} + (1-\alpha)\left(\mu\left(q\right)s - x_0\right)^{\theta}\right]^{\frac{\eta}{\theta}} = M^{\frac{\eta}{\theta}}$$

and budget constraint $c_t = (1+i)\,a_t - a_{t+1} + y_t - r_t\,(q)\,s_t$. We specify the functions $\mu = e^{\gamma q}$, and $r = r_0(1+q)$, where q denotes quality, and γ is a potentially household specific parameter that defines preference for quality.

First order conditions are

$$M_t^{\frac{\eta}{\theta}-1} c_t^{\theta-1} = \frac{1+i}{1+\rho} M_{t+1}^{\frac{\eta}{\theta}-1} c_{t+1}^{\theta-1}$$

$$\frac{r_q}{r} = \frac{\mu_q}{\mu} \Rightarrow q = \frac{1-\gamma}{\gamma}$$

and

$$\left[\frac{\alpha}{1-\alpha}\frac{r_t}{\mu_t}\right]^{\frac{1}{1-\theta}} \equiv Z_t \equiv Z = \frac{c_t}{\mu_t s_t - x_0}$$

Note now that t indexes for age, and that we do not have time variation. Furthermore, $r = r(q)$ and $\mu = \mu(q)$ have no time (or age) variation either. Therefore Z is a constant over the life cycle. We can then rewrite M

$$M = c^{\theta}\left[\alpha + (1-\alpha)\,Z^{-\theta}\right]$$

and replace it in the Euler equation

$$\begin{aligned}
\left[c_t^{\theta}\left[\alpha + (1-\alpha)\,Z^{-\theta}\right]\right]^{\frac{\eta}{\theta}-1} c_t^{\theta-1} &= \frac{1+i}{1+\rho}\left[c_{t+1}^{\theta}\left[\alpha + (1-\alpha)\,Z^{-\theta}\right]\right]^{\frac{\eta}{\theta}-1} c_{t+1}^{\theta-1} \\
c_t^{\eta-1} &= \frac{1+i}{1+\rho} c_{t+1}^{\eta-1}
\end{aligned}$$

and again

$$c_{t+1} = \left[\delta^{\frac{-1}{1-\eta}}\right] c_t = \delta_0 c_t$$

Now we go back to the budget constraint. This is now a bit more involved. Define $\beta = 1/(1+i)$ and $\Delta_0 = \beta\delta_0$. The PDV of consumption is given by

$$c_1 + \beta c_2 + ... = c_1 + c_1\Delta_0 + ... + c_1\Delta_0^{T-1} = c_1\frac{1-\Delta_0^T}{1-\Delta_0} = c_1\Omega$$

But now the budget constraint is harder to rewrite because

$$\frac{c_2}{c_1} = \delta_0 = \frac{\mu s_2 - x_0}{\mu s_1 - x_0}$$

and we need a recursion

$$
\begin{aligned}
rs_2 &= \delta_0 rs_1 + (1-\delta_0)\frac{r}{\mu}x_0 \\
rs_3 &= \delta_0^2 rs_1 + (1+\delta_0)(1-\delta_0)\frac{r}{\mu}x_0 \\
rs_4 &= \delta_0^3 rs_1 + \left(1+\delta_0+\delta_0^2\right)(1-\delta_0)\frac{r}{\mu}x_0
\end{aligned}
$$

so that the NPDV of housing expenditures is a bit more involved. The first set of terms adds to $rs_1\Omega$. The second set of terms adds to a less elegant object which we rename here:

$$
\Gamma = \left[(1-\delta_0)\frac{r}{\mu}x_0\right]\left[\begin{array}{c} \beta+\beta^2(1+\delta_0)+\beta^3\left(1+\delta_0+\delta_0^2\right) \\ +...+\beta^{T-1}\left(1+\delta_0+..+\delta_0^{T-2}\right) \end{array}\right]
$$

Note that the sum does not involve the parameter θ. The path of expenditures will not depend on substitutability within the utility function. The relative level of these expenditures will.

This recursion does highlight one point. Assume consumption grows with age. Then, if x_0 is small, housing expenditures will also grow with age. But if x_0 is big they will fall with age. This is because a big value of x_0 may imply too much housing expenditure at young ages so that the otherwise optimally increasing pattern of housing expenditure is no longer feasible.

$$
c_t = \left[Z\frac{\mu}{r}\right] r_t s_t - Zx_0 = (1+i)\,a_t - a_{t+1} + y_t - r_t s_t
$$

or again

$$r_t s_t = \frac{Zx_0 + (1+i)\,a_t - a_{t+1} + y_t}{Z\frac{\mu}{r} + 1}$$

and finally we are ready to compute present discounted values.

$$rs_1\Omega + \Gamma = \left[\frac{Zx_0}{Z\frac{\mu}{r} + 1}\right]\frac{1-\beta^T}{1-\beta} + \frac{1}{Z\frac{\mu}{r} + 1}W(\bar{a})$$

and obtain

$$rs_1 = \frac{\left[\frac{Zx_0}{Z\frac{\mu}{r}+1}\right]\frac{1-\beta^T}{1-\beta} - \Gamma + \frac{1}{Z\frac{\mu}{r}+1}W(\bar{a})}{\Omega}$$

If we set $x_0 = 0$ this expression simplifies to:

$$rs_1 = \frac{\Omega}{Z\frac{\mu}{r} + 1}W(\bar{a})$$

and for now this is what we look at.

Note that we have three utility parameters (α, θ, η), and one observed value of the expenditure share to work with. The parameters (α, θ) are only present inside of Z.

Now that we have an expression for rs_1, we can again compute the calibration expression for the average expenditure share in the data, which will pin down one parameter.

6 Tables and Figures

Table 3: Distribution of House Sizes

	1996		2001		2006		2010	
	R	U	R	U	R	U	R	U
00 to 39 m^2	10583	315	9782	680	9097	813	8630	887
40 to 59 m^2	72576	759	63025	1434	60940	2666	60626	3843
60 to 79 m^2	97657	4089	87654	7239	84847	10773	83494	12813
80 to 99 m^2	51252	2542	46986	4900	43663	8242	42559	11371
100 to 119 m^2	17415	377	15832	861	15469	2174	14855	4101
120 to 159 m^2	8225	47	7526	164	7155	399	7189	983
160 to 199 m^2	1600	3	1439	19	1277	11	1294	37
200 plus m^2	497	1	441	2	368	3	371	7
Total N	259805	8133	232685	15299	222816	25081	219018	34042
%>$99m^2$	10.7	5.3	10.8	6.8	10.9	10.3	10.8	15.1

Number of observations in regulated (R) and unregulated (U) housing, by size.
Average size is around $75m^2$. Around 4% of observations under $40m^2$.