Impossibility of market division with two-sided private information about production costs

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Abstract. In a market with several independent cities, two firms with private information about their production costs decide whether to open a store in each city or to restrict their activity to some cities. In cities where a single firm opens a store, this firm is a monopolist. In cities where both firms open stores, there is price competition. In equilibrium, both firms open stores in all the cities. Tacit collusion to divide the market is impeded because, by restraining from opening additional stores, a firm reveals its inefficiency, which triggers an attack from its rival.

Keywords: Collusion, Market division, Two-sided private information, Adverse selection, Compromise game.

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1 Introduction

The aim of this paper is to build a bridge between results on the impossibility of cooperation in games with two-sided adverse selection (Milgrom and Stokey, 1982; Tirole, 1982; Carrillo and Palfrey, 2009, 2011) and the theory of collusion with two-sided private information (Roberts, 1985; Cramton and Palfrey, 1990; Kihlstrom and Vives, 1992; Athey and Bagwell, 2001, 2008; Chakrabarti, 2010; Miller, 2012).

When there is two-sided adverse selection, cooperation may not resist the fact that restraining from competing is interpreted as a sign of weakness. Consider a candidate equilibrium in which firms agree to cooperate if and only if their privately observed strengths are below a certain threshold. Observing that the rival is willing to cooperate, a firm will know that the strength of the opponent is below that threshold. As a result, if its strength is sufficiently close to the threshold, the firm will prefer to compete.[1]

A similar phenomenon may occur when firms have the opportunity to divide the market, having private information about their costs. Consider a finite number of cities where a homogeneous good is demanded. To sell in a city, a firm needs to open a store there. After firms decide in which cities to open stores, there is price competition in the cities where both firms have stores and monopolies in the cities where there is a single store. In this setting, market division consists in firms not opening stores in all the cities.

Under perfect information, if firms have relatively similar production costs, there are mutually acceptable ways to divide the market. Full competition would imply that the low-cost firm captures all the market at a price that is equal to the marginal cost of the high-cost firm (or at the monopoly price, if it is lower). Therefore, both firms would be better off under any market sharing agreement that yields a higher payoff to the low-cost firm and a strictly positive payoff to the high-cost firm. For example: an agreement in which the low-cost firm would be a monopolist in all cities except one, and the high-cost firm would be a monopolist in that single city.

[1]This mechanism, explained in a simple setup by Carrillo and Palfrey (2009, 2011) is related to well-known *no trade* results. See Akerlof (1970), Aumann (1976), Milgrom and Stokey (1982), Tirole (1982), Myerson and Satterthwaite (1983), Chatterjee and Samuelson (1983), Cramton, Gibbons and Klemperer (1987) and Morris (1994).

But, if firms have private information about their production costs, a firm's willingness to divide the market partially reveals its inefficiency. This may lead the rival to act competitively. If the rival remains willing to cooperate, this reveals, to an even greater extent, the inefficiency of the rival. And so on. Until a point is reached at which some firm finds it profitable to trigger a fully competitive scenario. It is this failure to cooperate that is described in this paper.

The impossibility result that is obtained crucially depends on the assumption that the marginal cost of a firm can be as high as the market reservation price. This implies that, when both firms have costs that are as high as possible, profits with market division are null (as with full competition). Without this assumption, two-sided private information does not completely rule out the possibility of market sharing agreements. These will still take place whenever both firms have very high production costs.[2]

2 The model

Consider a market with two firms, $i \in \{A, B\}$, that potentially sell homogeneous goods in two cities, $j \in \{1, 2\}$. In each city, demand is $p_j = 1 - q_j$. At $\tau = 0$, nature draws the marginal costs of the firms, c_A and c_B, which are independently and uniformly distributed in the interval $[0, 1]$. The actual values of these parameters are private information of each firm. Then, at any $\tau \in (0, 1)$, each firm may open a store in one or two cities. These choices are observable. Once a store is open, it cannot be closed. A firm may start by opening a store in one city, and another store later (possibly as a response to the store-openings of the rival). At $\tau = 1$, private information about costs is publicly revealed and firms post prices in each of their stores. In cities with a single store, the firm that owns the store sets the monopoly price. In cities with two stores, the low-cost firm captures all the demand by setting a price equal to the marginal cost of the high-cost firm or at the monopoly price (whichever is the lowest).

[2] Another crucial assumption is that the probability density over marginal costs is not too decreasing. Otherwise, the posterior probability distribution over the rival's cost would place a low weight on extreme inefficiency, and this would lead sufficiently inefficient firms to prefer market division relatively to full competition.

The consequence of considering an open interval of time during which firms open stores (together with the irreversibility of store openings) implies that firms are always able to respond to the rival's actions. A firm is not able to deviate unilaterally at the last moment, because there isn't a last moment. In equilibrium, firms' choices, besides being optimal responses, are common knowledge.

Since opening stores is costless, the only reason why a firm may not open stores in both cities is to sustain a tacit agreement to divide the market. To open zero stores is a dominated action, therefore, we can suppose that a firm opens one or two stores. There are two kinds of possibly optimal courses of action: (i) open a single store as long as the rival also opens a single store, and open two stores if the rival opens two stores; (ii) open two stores, independently of the actions of the rival.

We wish to investigate the impacts of two-sided private information about production costs on the incentives of the firms to tacitly collude by locating in a single city.

We start the analysis by calculating the payoffs of the firms under market division (each firm is a monopolist in one city) and under full competition (both firms open stores in the two cities). The profit of firm i when it is a monopolist in one city is:

$$\pi_i^m(c_i) = \frac{1}{4}(1 - c_i)^2.$$

When there is competition in the two cities, the profit of firm i is:

$$\pi_i^c(c_i, c_j) = \begin{cases} \frac{1}{2}(1 - c_i)^2, & \text{if } \ c_j > \frac{1+c_i}{2} \\ 2(c_j - c_i)(1 - c_j), & \text{if } \ c_j \in \left[c_i, \frac{1+c_i}{2}\right] \\ 0, & \text{if } \ c_j < c_i. \end{cases}$$

Proposition 1. *Firm i is better off with market division than with full competition if and only if:* $c_j < \frac{1}{4}\left[\left(2 - \sqrt{2}\right) + \left(2 + \sqrt{2}\right) c_i\right]$.

Proof. See Appendix. □

Under complete information, firms agree to divide the market if and only if the low-cost firm has higher profits by being a monopolist in a single city than by competing in both cities. The high-cost firm surely prefers to divide the market.

Proposition 2. *With perfect information, firms divide the market if and only if their costs are relatively similar. Precisely, letting $c_i \leq c_j$, market division is the unique equilibrium if and only if:* $c_j < \frac{1}{4}\left[\left(2-\sqrt{2}\right)+\left(2+\sqrt{2}\right)c_i\right]$.

Proof. See Appendix. □

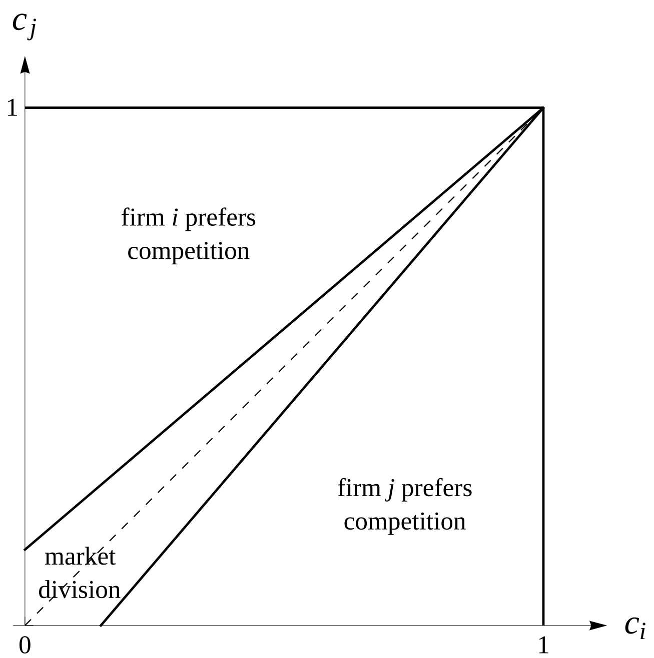

Figure 1: Under perfect information, firms divide the market when their costs are relatively similar.

This means that, with perfect information, firms divide the market whenever their costs are sufficiently similar. But, when firms have private information about their costs, is it still possible that they refrain from opening stores in the two cities? The answer is no.

Proposition 3. *With two-sided private information, firms open stores in both cities.*

Proof. See Appendix. □

This negative result regarding the possibility of market division can be extended to the case in which there is an arbitrary number of cities.

Suppose, now, that there is an arbitrary number of cities, $n \in \mathbb{N}$. May there exist an equilibrium in which, when $c_A \in [c_A^*, 1]$ and $c_B \in [c_B^*, 1]$, firm A opens stores in $n_A < n$ cities and firm B opens stores in $n_B < n$ cities? The answer is, again, no.

Proposition 4. *If there is a finite number of cities, with two-sided private information, firms always open stores in all the cities.*

Proof. See Appendix. □

If the upper bound of the marginal costs is strictly lower than the reservation price, then market division occurs whenever both firms are sufficiently inefficient. To understand why cooperation becomes possible if $c_H < 1$, notice that, when both firms have the maximal marginal costs ($c_A = c_B = c_H$), competitive payoffs are null while the cooperation payoffs are now strictly positive.

Proposition 5. *If $c_H < 1$, there exists a threshold, $c^* = 3c_H - 2$, such that firms divide the market if and only if $c_A \in [c^*, c_H]$ and $c_B \in [c^*, c_H]$.*

Proof. See Appendix. □

3 Concluding remarks

There is a striking similarity between the mechanisms that generate the colapse of the market for lemons (Akerlof, 1970), the impossibility of agreeing to disagree (Aumann, 1976), the absence of trade based on private information alone (Milgrom and Stokey, 1982), the inefficiency of trade with two-sided private information (Myerson and Satterthwaite, 1983), the inability to cooperate in the compromise game (Carrillo and Palfrey, 2009), and, in the setting of this paper, the non-sustainability of tacit collusive agreements to divide the market.

All these theoretical results are related to the fact that the willingness to accept some kind of agreement reveals information that induces the other party to reject that agreement. To study how mechanisms of this kind operate when firms with two-sided private information are incapable of reaching a collusive agreement, it was considered that firms' actions take place in an open interval of time. This uncommon structure for the strategic interaction rules out unilateral deviations, as there is always time for the rival to respond. Relatively to standard models with instantaneous and simultaneous decisions, this setting seems to favor cooperation. In spite of that, in the model presented in this paper, firms still deviate, being unable to settle on mutually beneficial market-sharing arrangements.

Appendix

Proof of Proposition 1

It is clear that firm i prefers to compete if $c_j > \frac{1+c_i}{2}$ (it becomes a monopolist in both cities instead of a single one) and that it prefers to divide the market if $c_j < c_i$ (otherwise it has zero profits). When $c_j \in \left[c_i, \frac{1+c_i}{2}\right]$, there is a threshold, c^*, such that firm i prefers to divide the market if and only if $c_j > c^*$. This threshold can be calculated as follows:

$$\frac{1}{4}(1-c_i)^2 \; > \; 2(c_j - c_i)(1-c_j) \; \Leftrightarrow \; 8c_j^2 - 8(1+c_i)c_j + 1 + 6c_i + c_i^2 \; > \; 0.$$

Using $c_j \in \left[c_i, \frac{1+c_i}{2}\right]$ to select the relevant root, we obtain:

$$\begin{aligned} c_j \; &> \; \frac{1+c_i}{2} - \frac{1}{2}\sqrt{(1+c_i)^2 - \frac{1}{2}(1+6c_i+c_i^2)} \\ \Leftrightarrow \; c_j \; &> \; \frac{1+c_i}{2} - \frac{1-c_i}{2\sqrt{2}} \\ \Leftrightarrow \; c_j \; &> \; \frac{1}{4}\left[\left(2-\sqrt{2}\right) + \left(2+\sqrt{2}\right)c_i\right]. \end{aligned}$$

□

Proof of Proposition 2

($\Leftarrow$) Let $c_j < \frac{1}{4}\left[\left(2-\sqrt{2}\right)+\left(2+\sqrt{2}\right)c_i\right]$. There are strategy profiles that sustain an equilibrium in which firm i opens a store in one city and firm j opens a store in the other city. An example of such an equilibrium profile of strategies is the following: firm i opens a store in city 1 at $\tau = \frac{1}{3}$ and opens a store in city 2 at $\tau = \frac{1-t}{2}$ if and only if firm j opens a store in city 1 at $\tau = t$; firm j opens a store in city 2 at $\tau = \frac{2}{3}$ and opens a store in city 2 at $\tau = \frac{1-t}{2}$ if and only if firm i opens a store in city 2 at $\tau = t$. Each firm refrains from opening a store in the other city to avoid retaliation by its rival.

A situation in which firm i is a monopolist in the two cities cannot be sustained as an equilibrium. To understand why, suppose that the (candidate) equilibrium is such that firm i opens a store in city 1 at $\tau = t_1$ and a store in city 2 at $\tau = t_2$ (where, w.l.o.g., $t_1 \leq t_2$), while firm j does not open stores. This cannot be an equilibrium, because firm j would be better off with the following strategy: open a store in city 2 at $\tau = \frac{t_1}{2}$ and open a store in city 1 at $\tau = \frac{1-t}{2}$ if and only if firm i opens a store in city 2 at $\tau = t$. This would prevent firm i from opening a store in city 2.

($\Leftarrow$) If $c_j > \frac{1}{4}\left[\left(2-\sqrt{2}\right)+\left(2+\sqrt{2}\right)c_i\right]$, it is a dominant strategy for firm i to open stores in both cities. Therefore, regardless of any potential retaliation by firm j, a situation in which firm i has opened a store in one city and firm j has opened a store in the other city cannot be sustained as an equilibrium.

It follows, from the reasonings described above, that in the borderline case in which $c_j = \frac{1}{4}\left[\left(2-\sqrt{2}\right)+\left(2+\sqrt{2}\right)c_i\right]$, market division can be sustained as an equilibrium, but there also exists an equilibrium in which firms compete in both cities. $\square$

Proof of Proposition 3

(i) Start by considering that firms use the following threshold strategies:

- firm i opens a store in city 1 at $\tau = t_1$ if its rival has opened a store in city 2 or has opened no stores; firm i also opens a store in city 2 if $c_i < c^*$ or if its rival opens a store in city 1;

- firm j opens a store in city 2 at $\tau = t_2$ if its rival has opened a store in city 1 or has opened no stores; firm j also opens a store in city 1 if $c_j < c^*$ or if its rival opens a store in city 2.

This means that firms divide the market whenever $c_i \geq c^*$ and $c_j \geq c^*$. We will rule out the possibility that this is an equilibrium if $c^* < 1$.

For this strategy profile to constitute an equilibrium, it is necessary that, when $c_i \geq c^*$, firm i perceives a higher expected value (conditionally on $c_j \geq c^*$) with market division than with full competition. Formally:

$$\pi_i^m(c_i) \ \geq \ \frac{1}{1-c^*} \int_{c^*}^{1} \pi_i^c(c_i, c_j) \ dc_j \ , \quad \forall c_i \geq c^*.$$

Replacing the expressions for profits and considering (the critical case) $c_i = c^*$, we obtain:

$$\begin{aligned}
\frac{(1-c^*)^2}{4} \ &\geq \ \frac{1}{1-c^*} \int_{c^*}^{\frac{1+c^*}{2}} 2(c_j - c^*)(1-c_j) \ dc_j + \frac{1}{1-c^*} \int_{\frac{1+c^*}{2}}^{1} \frac{(1-c^*)^2}{2} \ dc_j \ \Leftrightarrow \\
\frac{(1-c^*)^2}{4} \ &\geq \ \frac{1}{1-c^*} \int_{c^*}^{\frac{1+c^*}{2}} 2(c_j - c^*)(1-c_j) \ dc_j + \frac{(1-c^*)^2}{4} \ \Leftrightarrow \\
0 \ &\geq \ \frac{1}{1-c^*} \int_{c^*}^{\frac{1+c^*}{2}} (c_j - c^*)(1-c_j) \ dc_j,
\end{aligned}$$

which clearly cannot hold for $c^* < 1$.

(ii) We now check that only threshold strategies can be optimal.

Let $A \subseteq [0,1]$ denote the values of c_j for which firm j opens a single store (if and only if firm i also opens a single store).

The expected increase of the profit of firm i if it engages in full competition instead of dividing the market is given by:

$$\Delta(c_i) \equiv \int_A \pi_i^c(c_i, c_j) - \pi_i^m(c_i) \ dc_j.$$

Suppose that $\Delta(c_i) > 0$. We want to show that $c_i' < c_i$ implies that $\Delta(c_i') > 0$. This being true, only threshold strategies can be used in equilibrium.

Noticing that:

$$\begin{aligned}\Delta(c_i) \;=\; &-\int_{A\cap[0,c_i]} \frac{1}{4}(1-c_i)^2 \; dc_j + \int_{A\cap\left[c_i,\frac{1+c_i}{2}\right]} 2(c_j-c_i)(1-c_j) - \frac{1}{4}(1-c_i)^2 \; dc_j \\ &+ \int_{A\cap\left[\frac{1+c_i}{2},1\right]} \frac{1}{4}(1-c_i)^2 \; dc_j.\end{aligned}$$

Multiplying $\Delta(c_i)$ by $(1-c_i')^2$ and dividing by $(1-c_i)^2$ preserves its signal:

$$\begin{aligned}&-\int_{A\cap[0,c_i]} \frac{1}{4}(1-c_i')^2 \; dc_j + \int_{A\cap\left[c_i,\frac{1+c_i}{2}\right]} 2(c_j-c_i)(1-c_j)\frac{(1-c_i')^2}{(1-c_i)^2} - \frac{1}{4}(1-c_i')^2 \; dc_j \\ &+ \int_{A\cap\left[\frac{1+c_i}{2},1\right]} \frac{1}{4}(1-c_i')^2 \; dc_j \;>\; 0.\end{aligned}$$

The above expression was built to coincide with that of $\Delta(c_i')$ for some values of c_j. Observe that $\forall c_j \in [0, c_i']$:

$$\frac{(1-c_i')^2}{(1-c_i)^2}\left[\pi_i^c(c_i,c_j) - \pi_i^m(c_i)\right] \;=\; \pi_i^c(c_i',c_j) - \pi_i^m(c_i') \;=\; -\frac{1}{4}(1-c_i')^2,$$

and, similarly, that $\forall c_j \in [0, c_i'] \cup \left[\frac{1+c_i}{2}, 1\right]$:

$$\frac{(1-c_i')^2}{(1-c_i)^2}\left[\pi_i^c(c_i,c_j) - \pi_i^m(c_i)\right] \;=\; \pi_i^c(c_i',c_j) - \pi_i^m(c_i') \;=\; \frac{1}{4}(1-c_i')^2.$$

It can also be verified that for the remaining possible values of c_j, i.e., $\forall c_j \in \left[c_i', \frac{1+c_i}{2}\right]$:

$$\frac{(1-c_i')^2}{(1-c_i)^2}\left[\pi_i^c(c_i,c_j) - \pi_i^m(c_i)\right] \;>\; \pi_i^c(c_i',c_j) - \pi_i^m(c_i').$$

This implies that $\Delta(c_i') > 0$ (only threshold strategies can be used in equilibrium).

Figure 2 illustrates the strategy of this part of the proof. For an arbitrary domain of integration, if the integral of the function $\pi_i^c(c_i,c_j) - \pi_i^m(c_i)$ (dashed line) is positive, the integral of $\left[\pi_i^c(c_i,c_j) - \pi_i^m(c_i)\right]\frac{(1-c_i')^2}{(1-c_i)^2}$ (dotted line) is also positive because this function only differs by the multiplication of a positive constant. This, in turn, implies that the integral of $\pi_i^c(c_i', c_j) - \pi_i^m(c_i')$ (solid line) is also positive, as this function in everywhere greater or equal than $[\pi_i^c(c_i, c_j) - \pi_i^m(c_i)]\,\frac{(1-c_i')^2}{(1-c_i)^2}$.

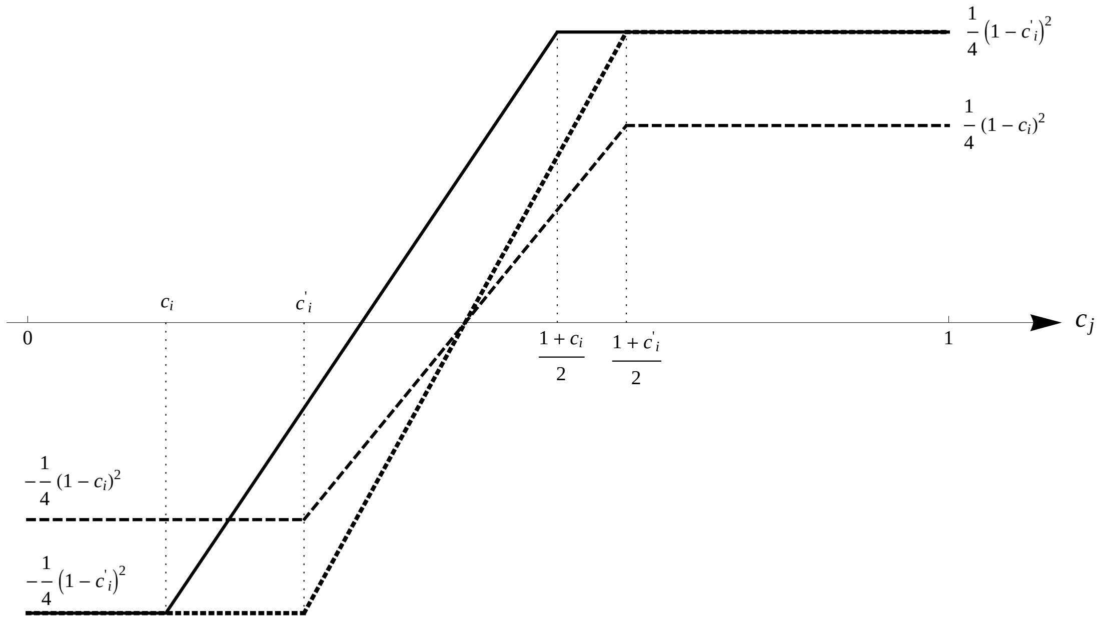

Figure 2: Difference between the profit of firm i under full competition relatively to market division, as a function of the cost of firm j. solid line: $\pi_i^c(c_i', c_j) - \pi_i^m(c_i')$; dashed line: $\pi_i^c(c_i, c_j) - \pi_i^m(c_i)$; dotted line: $[\pi_i^c(c_i, c_j) - \pi_i^m(c_i)]\,\frac{(1-c_i')^2}{(1-c_i)^2}$.

(iii) We now check that firms must choose the same threshold in equilibrium.

Suppose, by way of contradiction, that, in equilibrium, firm i opens a single store if $c_i > c_i^*$ while firm j opens a single store if $c_j > c_j^*$, with $c_i^* < c_j^*$. Define $c^* \equiv \frac{c_i^* + c_j^*}{2}$.

When there is market division ($c_i > c_i^*$ and $c_j > c_j^*$), firm j reveals a greater inefficiency than firm i. Therefore, if firm j finds it optimal to engage in full competition, for example, when $c_j = c^*$, then firm i should also find it optimal to engage in full competition when

$c_i = c^*$. Formally:

$$\begin{aligned}
&\int_{c_i^*}^{1} \pi_j^c(c^*, c_i) - \pi_j^m(c^*)\ dc_i\ \geq 0 \ \Leftrightarrow\ \int_{c_i^*}^{1} \pi_i^c(c^*, c_j) - \pi_i^m(c^*)\ dc_j\ \geq 0 \\
&\Leftrightarrow\ \int_{c_i^*}^{c_j^*} \pi_i^c(c^*, c_j) - \pi_i^m(c^*)\ dc_j + \int_{c_j^*}^{1} \pi_i^c(c^*, c_j) - \pi_i^m(c^*)\ dc_j\ \geq 0 \\
&\Rightarrow\ \int_{c_j^*}^{1} \pi_i^c(c^*, c_j) - \pi_i^m(c^*)\ dc_j\ \geq 0.
\end{aligned}$$

The last implication is a straightforward consequence of the fact that the argument of the integral is increasing in c_j. □

Proof of Proposition 4

In equilibrium, threshold strategies are optimal as shown in the proof of Proposition 3. We want to rule out the possibility of an equilibrium in which firm A opens stores in $n_A < n$ cities and firm B opens stores in $n_B < n$ cities when $c_A \in [c_A^*, 1]$ and $c_B \in [c_B^*, 1]$

Suppose, w.l.o.g., that $c_A^* \leq c_B^*$. If $c_B^* \geq \frac{1+c_A^*}{2}$, then firm A surely deviates in order to become a monopolist in n cities instead of n_A. We can consider, therefore, that $c_B^* < \frac{1+c_A^*}{2}$.

The ICC condition for firm A when $c_A = c_A^*$ is:

$$\begin{aligned}
&n_A \frac{(1-c_A^*)^2}{4}\ \geq\ \frac{n}{1-c_B^*}\int_{c_B^*}^{\frac{1+c_A^*}{2}} (c_B - c_A^*)(1-c_B)\ dc_B + \frac{n}{1-c_B^*}\int_{\frac{1+c_A^*}{2}}^{1} \frac{(1-c_A^*)^2}{4}\ dc_B\ \Leftrightarrow \\
&\left[n_A - \frac{n(1-c_A^*)}{2(1-c_B^*)}\right]\frac{(1-c_A^*)^2}{4}\ \geq\ \frac{n}{1-c_B^*}\int_{c_B^*}^{\frac{1+c_A^*}{2}} (c_B - c_A^*)(1-c_B)\ dc_B,
\end{aligned}$$

which implies that $\frac{1-c_A^*}{1-c_B^*} < \frac{2n_A}{n}$.

Similarly, the ICC condition for firm B when $c_B = c_B^*$ is:

$$\begin{aligned}
n_B \frac{(1-c_B^*)^2}{4} &\geq \frac{n}{1-c_A^*} \int_{c_B^*}^{\frac{1+c_B^*}{2}} (c_A - c_B^*)(1-c_A)\ dc_A + \frac{n}{1-c_A^*} \int_{\frac{1+c_B^*}{2}}^{1} \frac{(1-c_B^*)^2}{4}\ dc_A \Leftrightarrow \\
\left[n_B - \frac{n(1-c_B^*)}{2(1-c_A^*)}\right] \frac{(1-c_B^*)^2}{4} &\geq \frac{n}{1-c_A^*} \int_{c_B^*}^{\frac{1+c_B^*}{2}} (c_A - c_B^*)(1-c_A)\ dc_A,
\end{aligned}$$

which implies that $\frac{1-c_A^*}{1-c_B^*} > \frac{n}{2n_B}$.

The two ICCs imply, therefore, that $\frac{2n_A}{n} > \frac{n}{2n_B}$. But this is impossible, as:

$$\frac{2n_A}{n} > \frac{n}{2n_B} \;\Rightarrow\; n_A^2 + 2n_A n_B + n_B^2 < 4n_A n_B \;\Leftrightarrow\; (n_A - n_B)^2 < 0. \qquad \square$$

Proof of Proposition 5

Suppose that $c_H < 1$ and that firms choose market division when their costs are above $c^* = c_H - \epsilon$, for some ϵ that is small enough for $c_H \leq \frac{1+c^*}{2}$.

This choice is optimal for firm i, when $c_i = c^*$, if and only if:

$$\begin{aligned}
\frac{(1-c^*)^2}{4} &\geq \frac{1}{c_H - c^*} \int_{c^*}^{c_H} (c_j - c^*)(1-c_j)\ dc_j \Leftrightarrow \\
(1-c^*)^2 &\geq \frac{4}{c_H - c^*} \int_{c^*}^{c_H} -c_j^2 + (1+c^*)c_j - c^*\ dc_j \Leftrightarrow \\
1 + 2c^* + c^{*2} &\geq \frac{4}{3\,(c_H - c^*)}(c^{*3} - c_H^3) + \frac{2}{(c_H - c^*)}(1+c^*)(c_H^2 - c^{*2}) \Leftrightarrow \\
(1 + c_H - \epsilon)^2 &\geq \frac{4}{3}(-3c_H^2 + 3\epsilon c_H - \epsilon^2) + 2(1 + c_H - \epsilon)(2c_H - \epsilon) \Leftrightarrow \\
1 - 2c_H + c_H^2 + \frac{\epsilon^2}{3} &\geq 0.
\end{aligned}$$

The above condition is always true. Firms always prefer to cooperate if it is common knowledge that their costs are above any given c^* such that $c_H \leq \frac{1+c^*}{2}$.

The threshold at which firms become indifferent between cooperation and competition must be low enough so that $c_H > \frac{1+c^*}{2}$, i.e., $c^* < 2c_H - 1$. It is implicitly defined by:

$$\frac{(1-c^*)^2}{4} \geq \frac{1}{c_H - c^*}\int_{c^*}^{\frac{1+c^*}{2}} (c_j - c^*)(1-c_j)\, dc_j + \frac{1}{c_H - c^*}\int_{\frac{1+c^*}{2}}^{c_H} \frac{(1-c^*)^2}{2}\, dc_j.$$

Simplifying, we obtain:

$$\begin{aligned}(1-c_H)\frac{(1-c^*)^2}{4} &\geq \int_{c^*}^{\frac{1+c^*}{2}} (c_j - c^*)(1-c_j)\, dc_j \Leftrightarrow \\ (1-c_H)\frac{(1-c^*)^2}{4} &\geq \frac{1}{12}(1-c^*)^3 \Leftrightarrow \\ c^* &\geq 3c_H - 2. \end{aligned}$$

□

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