Unspanned macroeconomic factors in the yield curve

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Abstract

We show that two macroeconomic factors have an important predictive content for government bond yields and excess returns. These factors are not spanned by the cross-section of yields and are well proxied by economic growth and real interest rates.

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1 Introduction

Government bond yields with different maturities and macroeconomic variables are both characterized by a high degree of comovement, indicating that the bulk of their dynamics is driven by a few common forces. Three common factors, usually interpreted as the level, slope and curvature of the yield curve, can explain changes and shift of the entire cross-section of yields, see Litterman and Scheinkman (1991). Although there is less consensus on the number and nature of macroeconomic factors, two factors, one nominal and one real, summarize well the dynamics of a large variety of macroeconomic indicators for the United States, see Sargent and Sims (1977), Giannone, Reichlin and Sala (2005) and Watson (2005).

Macroeconomic factors and yield curve factors are also characterized by a strong interaction. The short end of the yield curve moves closely to the policy instrument under the direct control of the central bank, which responds to changes in inflation, economic activity, or other economic conditions, see Taylor (1993). The average level of the yield curve is usually associated with the inflation rate and the spread between long and short rates with temporary business cycles conditions, see Diebold, Rudebusch and Aruoba (2006). For these reasons, macroeconomic information have been shown to help forecasting future interest rates and excess bond returns, see Ang and Piazzesi (2003), Mönch (2008), De Pooter, Ravazzolo and van Dijk (2007), Favero, Niu and Sala (2012) and Ludvigson and Ng (2009).

In this paper, we aim at identifying the factors summarizing macroeconomic information that is not spanned by the traditional yield curve factors. The economic literature so far has not addressed this problem since in existing studies macroeconomic factors are either extracted separately from a large set of macroeconomic indicators, see Ang and Piazzesi (2003), Mönch (2008), Favero et al. (2012) and Ludvigson and Ng (2009), or proxied by preselected observable variables, see Dewachter and Lyrio (2006), Diebold et al. (2006), Bianchi, Mumtaz and Surico (2009), Joslin, Priebsch and Singleton (2010) and Wright (2011).

We estimate a macro-yield model that treats macroeconomic factors as unobservable components that we extract simultaneously with the traditional yield curve factors. The latter are identified by constraining the loadings to follow the smooth pattern proposed by Nelson and Siegel (1987). More specifically, our empirical model is a Dynamic Factor Model (DFM) for Treasury zero-coupon yields and a representative set of macroeconomic variables with restrictions on the factor loadings. Estimation is performed using a Quasi-Maximum Likelihood approach, as proposed by Doz, Giannone and Reichlin (2012). This procedure is easily implementable using the Kalman smoother and the EM algorithm. The estimator has been shown to be feasible when the number of variables is large, and robust to non Gaussianity and to the presence of weak cross-sectional correlation among the idiosyncratic terms. We validate the model by assessing the forecasting ability for yields and excess returns of US government bonds.

Using monthly U.S. data from January 1970 to December 2008, we find that a significant component of macroeconomic information is not captured by the yield curve factors and, at the same time, is unspanned by the yield curve, in the sense that it does not affect contemporaneously the cross-section of yields. The unspanned macroeconomic information is driven by two factors that are well proxied by economic growth and real interest rates. These factors have substantial predictive information for bond yields and excess bond returns, in spite of the fact that they do not affect contemporaneously the shape of the yield curve. The macro-yields model explains up to 55% of the variation in excess bond returns and outperforms all existing models in forecasting bond yields and excess returns.

The paper is organized as follows. Section 2 presents the macro-yields model. Section 3 describes the data, the estimation procedure and the information criteria used for model selection. Section 4 contains empirical results about the estimated factors, the fit of the model for yields, macro variables and expected excess bond returns. Section 5 reports out of sample results for yields and excess bond returns and section 6 concludes. We report in Appendix details about the estimation procedure, the macroeconomic data, the out of sample tests used and results for the unrestricted macro-yields model.

2 The Macro-Yields Model

The macro-yields model that we propose is a dynamic factor model for the joint behavior of government bond yields and macroeconomic indicators. The cross-section of yields is described by the traditional level, slope and curvature factors. Macroeconomic variables load on both the yield curve factors as well as on some additional macro factors, that capture the information in macroeconomic variables over and above the yield curve factors. In addition, these additional macro factors are assumed to not provide any information about the contemporaneous shape of the yield curve. In practice, the level, slope and curvature implied by the Nelson and Siegel (1987) model are assumed to be spanned by both the bond yields and macroeconomic variables. The additional macro factors, instead, are contemporaneously loaded only by the macroeconomic variables and, thus, are unspanned by the cross-section of yields. The joint dynamics of the factors is an unrestricted Vector Autoregression and the idiosyncratic components follow independent univariate autoregressions. In what follows we detail on each of the points.

More specifically, we assume that yields on bonds with different maturities are driven by three common factors. Denoting by y_t the $N_y \times 1$ vector of yields with N_y different maturities at time t, we have:

$$y_t \;=\; a_y + \; \Gamma_{yy} \; F_t^y + v_t^y, \tag{1}$$

where F_t^y is a 3×1 vector containing the latent yield-curve factors at time t, Γ_{yy} is a $N_y \times 3$ matrix of factor loadings, and v_t^y is an $N_y \times 1$ vector of idiosyncratic components. The yield curve factors F_t^y are identified by constraining the factor loadings to follow the smooth pattern proposed by Nelson and Siegel (1987) (hereafter NS)

$$a_y = 0; \quad \Gamma_{yy}^{(\tau)} = \left[1 \;\; \frac{1 - e^{-\lambda\tau}}{\lambda\tau} \;\; \frac{1 - e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau} \right] \equiv \Gamma_{NS}^{(\tau)}, \tag{2}$$

where $\Gamma_{yy}^{(\tau)}$ is the row of the matrix of factor loadings corresponding to the yield with maturity τ and λ is a decay parameter of the factor loadings. Diebold and Li (2006) show that this functional form of the factor loadings, implies that the three yield curve factors can be interpreted as the level, slope, and curvature of the yield curve. Indeed, the loading equal to one on the first factor, for all maturities, implies that an increase in this factor increases all yields equally, shifting the level of the yield curve. The loadings on the second factor are high for short maturities, decaying to zero for the long ones. Accordingly, an increase in the second factor increases the slope of the yield curve. Loadings on the third factor are zero for the shortest and the longest maturities, reaching the maximum for medium maturities. Therefore, an increase in this factor augments the curvature of the yield curve. The specific shape of the loadings depends on the decay parameter λ, which we calibrate to the value that maximizes the loading on the curvature factor for the yields with maturity 30 months, as in Diebold and Li (2006).

Given these particular functional forms for the loadings on the three yield curve factors, one can disentangle movements in the term structure of interest rates into three factors which have a clear-cut interpretation. The NS factors are just linear combinations of yields. The level factor can be proxied by the long term yield, the slope by the spread between the long and short maturity yield (first derivative) and the curvature by sum of the spreads between a medium and a long term yield, and between a medium and the short term yield (second derivative), see Diebold and Li (2006).[1] The parameter λ governs the exponential decay rate: a small value of λ can better fit the yield curve at long maturities, while large values can better fit it at short maturities. This parameter determines the maturity at which the loadings on the curvature factor reaches the maximum. Due to its flexibility and parsimony, the NS model accurately fits the yield curve and performs well in out-of-sample forecasting exercises, as shown by Diebold and Li (2006) and De Pooter et al. (2007). For these reasons, fixed-income wealth managers in public organizations, investment banks and central banks rely heavily on NS type of models to fit and forecast yield curves, see BIS (2005), ECB (2008), Gürkaynak, Sack and Wright (2007) and Coroneo, Nyholm and Vidova-Koleva (2011).

Macroeconomic variables, are assumed to be potentially driven by two sources of co-movement, the yield curve factors F_t^y and macro specific factors. Denoting by x_t the $N_x \times 1$ vector of macroe-

[1] Similar proxies are used by Ang, Piazzesi and Wei (2006) and Duffee (2011).

conomic variables at time t, we have:

$$x_t \;=\; a_x + \Gamma_{xy}\, F_t^y + \Gamma_{xx}\, F_t^x + v_t^x, \tag{3}$$

where F_t^x is an $r \times 1$ vector of macroeconomic latent factors, Γ_{xy} is a $N_x \times 3$ matrix of factor loadings on the yield curve factors, Γ_{xx} is a $N_x \times r$ matrix of factor loadings on the macro factors, and v_t^x is an $N_x \times 1$ vector of idiosyncratic components.

The yield curve and the macroeconomic factors are extracted by estimating (1) and (3) simultaneously:

$$\begin{pmatrix} y_t \\ x_t \end{pmatrix} = \begin{pmatrix} 0 \\ a_x \end{pmatrix} + \begin{bmatrix} \Gamma_{NS} & 0 \\ \Gamma_{xy} & \Gamma_{xx} \end{bmatrix} \begin{pmatrix} F_t^y \\ F_t^x \end{pmatrix} + \begin{pmatrix} v_t^y \\ v_t^x \end{pmatrix}. \tag{4}$$

where Γ_{NS} are the NS restrictions in (2). F_t^x captures the source of co-movement in the macroeconomic variables beyond the yield curve factors. In addition, the restriction $\Gamma_{yx} = 0$ ensures that, by construction, the macroeconomic factors F_t^x are unspanned by the cross-section of yields as they do not provide any information about the contemporaneous shape of the yield curve. In Appendix D we show that this assumption holds in the data as does it not affect neither the in sample nor the out of sample performance of the model for both yields and excess bond returns.

The joint dynamics of the yield curve and the macroeconomic factors follow a VAR(1)

$$\begin{pmatrix} F_t^y \\ F_t^x \end{pmatrix} = \begin{pmatrix} \mu_y \\ \mu_x \end{pmatrix} + \begin{bmatrix} A_{yy} & A_{yx} \\ A_{xy} & A_{xx} \end{bmatrix} \begin{pmatrix} F_{t-1}^y \\ F_{t-1}^x \end{pmatrix} + \begin{pmatrix} u_t^y \\ u_t^x \end{pmatrix}, \quad \begin{pmatrix} u_t^y \\ u_t^x \end{pmatrix} \sim N\left(0, \begin{bmatrix} Q_{yy} & Q_{yx} \\ Q_{xy} & Q_{xx} \end{bmatrix}\right) \tag{5}$$

The idiosyncratic components collected in $v_t = [v_t^y \quad v_t^x]'$ are modelled to follow independent autoregressive processes

$$v_t = B v_{t-1} + \xi_t, \quad \xi_t \sim N(0, R) \tag{6}$$

where B and R are diagonal matrices. These orthogonality conditions imply that the common factors fully account for the the joint correlation of the observations. This is a straightjacket and arguably unrealistic assumption, but recent results on approximate factor models show that these models are robust to the presence of limited correlations among idiosyncratic components, see Doz et al. (2012).

Furthermore, the shocks to the idiosyncratic components of the individual variables, ξ_t, and the innovations driving the common factors, u_t, are assumed to be mutually independent. This assumptions implies that the common factors are not allowed to react to variable specific shocks.

3 Estimation and Preliminary Results

3.1 Data

We use monthly U.S. Treasury zero-coupon yield curve data spanning the period January 1970 to December 2008. The bond yield data are taken from the Fama-Bliss dataset available from the Center for Research in Securities Prices (CRSP) and contain observations on three months and one through five-year zero coupon bond yields. The macroeconomic dataset consists of 14 macroeconomic variables, which include five inflation measures, seven real variables, the federal funds rate and a money indicator. Appendix B contains a complete list of the macroeconomic variables along with the transformation applied to ensure stationarity. Following Ang and Piazzesi (2003), De Pooter et al. (2007), Diebold et al. (2006) and Mönch (2008), we use annual growth rates for all variables, except for capacity utilization, the federal funds rate, the unemployment rate and the manufacturing index which we keep in levels. Notice that, in the spirit of the latent factor model literature, we include a larger dataset than the one considered in macro-yields models with observable macro factors, e.g. Ang and Piazzesi (2003), Diebold et al. (2006) and Joslin et al. (2010).[2]

[2]We focus on the main aggregate macroeconomic indicators since there have been extensive evidence that including disaggregated information does not provide any important gain in forecasting, as shown by De Mol, Giannone and Reichlin (2008), Bańbura, Giannone and Reichlin (2009) and Banbura and Modugno (2012). An alternative approach consists in selecting the variables using statistical criteria as suggested in Boivin and Ng (2006) and Bai and Ng (2008). However, because of co-linearity among predictors, variable selection is unstable, as the set of predictors selected is very sensitive to minor perturbation of the data-set, such as adding new variables or extending the sample length, see De Mol et al. (2008). For a discussion on the selection of variables see Banbura, Giannone, Modugno and Reichlin (2012).

3.2 Estimation

Equations (4)–(6) describe a restricted state-space model with autocorrelated idiosyncratic components for which maximum likelihood estimators of the parameters are not available in closed form. Conditionally on the factors, the model reduces in a set of linear regressions. As consequence estimation can be carried using the Expectation Maximization (EM) algorithm[3]. The procedure is computationally feasible for large cross-sections since the latent factors can be initialized by principal components, which provide a good approximation of the common factors in a large cross-section, see Doz et al. (2012). As anticipated above, this procedure is robust to miss-specification of the empirical model for the idiosyncratic component. The estimates are also robust to deviation from Gaussianity, see Doz et al. (2012).

For comparison, we also estimate an only-yields model, which uses only the information contained in the yields. This is a restricted version of the macro-yields model in equations (4)–(6) with $Q_{yx} = A_{yx} = \Gamma_{xy} = 0$ and can hence be estimated using the same procedure.

3.3 Model Selection

The macro-yields model decomposes variations in yields and macroeconomic variables into yield curve factors, unspanned macroeconomic factors and idiosyncratic noise. The yield curve factors are identified as the NS factors which have a clear interpretation as level, slope and curvature. However, the true number of unspanned macroeconomic factors is unknown. We select the optimal number of factors using an information criteria approach. The idea is to choose the number of factors that maximizes the general fit of the model using a penalty function to account for the loss in parsimony.

Bai and Ng (2002) derive information criteria to determine the number of factors in approximate factor models when the factors are estimated by principal components. They also show that their

[3]Using the Expectation Conditional Restricted Maximization (ECRM) algorithm is also possible to estimate λ, but, despite the increase in the computation burden, the empirical results remain qualitatively similar to those obtained by setting λ to the value that maximizes the loading of the the yields with maturity 30 months on the curvature factor.

Table 1: Model Selection

Number of factors	IC^*	V
3	0.02	0.44
4	-0.03	0.31
5	-0.11	0.22
6	0.01	0.18
7	0.23	0.17
8	0.43	0.16

This table reports the information criterion IC^*, as shown in (8) and (7), and the sum of the variance of the idiosyncratic components (divided by NT), V, when different numbers of factors are estimated.

IC_3 information criterion can be applied to any consistent estimator of the factors provided that the penalty function is derived from the correct convergence rate. For the quasi-maximum likelihood estimator, Doz et al. (2012) show that it converges to the true value at a rate equal to

$$C^{*2}_{NT} = \min\left\{\sqrt{T}, \frac{N}{\log N}\right\} \tag{7}$$

where N and T denote the cross-section and the time dimension, respectively. Thus, a modified Bai and Ng (2002) information criterion that can be used to select the optimal number of factor when estimation is performed by quasi-maximum likelihood is as follows

$$IC^*(s) = \log(V(s, \hat{F}^*_{(s)})) + s\, g(N,T), \quad g(N,T) = \frac{\log C^{*2}_{NT}}{C^{*2}_{NT}} \tag{8}$$

where s denotes the number of factors, $\hat{F}_{(s)}$ are the estimated factors and $V(s, \hat{F}^*_{(s)})$ is the sum of squared idiosyncratic components (divided by NT) when s factors are estimated. The penalty function $g(N,T)$ is a function of both N and T and depends on C^{*2}_{NT}, the convergence rate of the estimator, in our case given by (7).

To select the number of factors in the macro-yields model, we estimate the macro-yields model in equations (4)–(6) allowing from three, i.e. only the yield curve factors, up to a total of eight factors, where the first three are identified as the yield curve factors and the others are the unspanned macro factors. Table 1 reports the information criterion, as shown in Equation (8), and the sum of the variance of the idiosyncratic components for these different specifications of the macro-yields model. The information criterion selects the model with five factors, i.e. three yield curve factors plus two unspanned factors. This is also confirmed by the fact that the strongest reduction in the sum of the variances of the idiosyncratic components is obtained passing from the four to the five factors specification. Thus our macro-yields model is a latent factor model with three factors that explain the cross-section of yields and two unspanned macroeconomics factors.

4 In Sample Results

4.1 Model Fit

Table 2 reports the share of variance of the macroeconomic variables explained by the macro-yields factors. Results shows that the yield curve factors explain most of the variance of the yields and federal funds rate. They also explain the part of the variance of price indices, unemployment, nominal earnings, nominal consumption and money, in line with previous studies (see Diebold et al. (2006)). The first unspanned macro factor captures the dynamics of industrial production and other real variables, while the second unspanned factor mainly explains inflation and other nominal variables.[4]

Figure 1 displays the estimated factors of the macro-yields model. The top three plots report the yield curve factors, while the bottom two refer to the unspanned factors. The estimated yield curve factors of the macro-yields model are highly correlated with the NS factors, which we estimate by ordinary least squares as in Diebold and Li (2006) and report in dashed red lines in the top plots. The differences between the NS factors and the first three macro-yields factors are due to the fact that, in the macro-yields model, the yield curve factors are common to both yield curve and

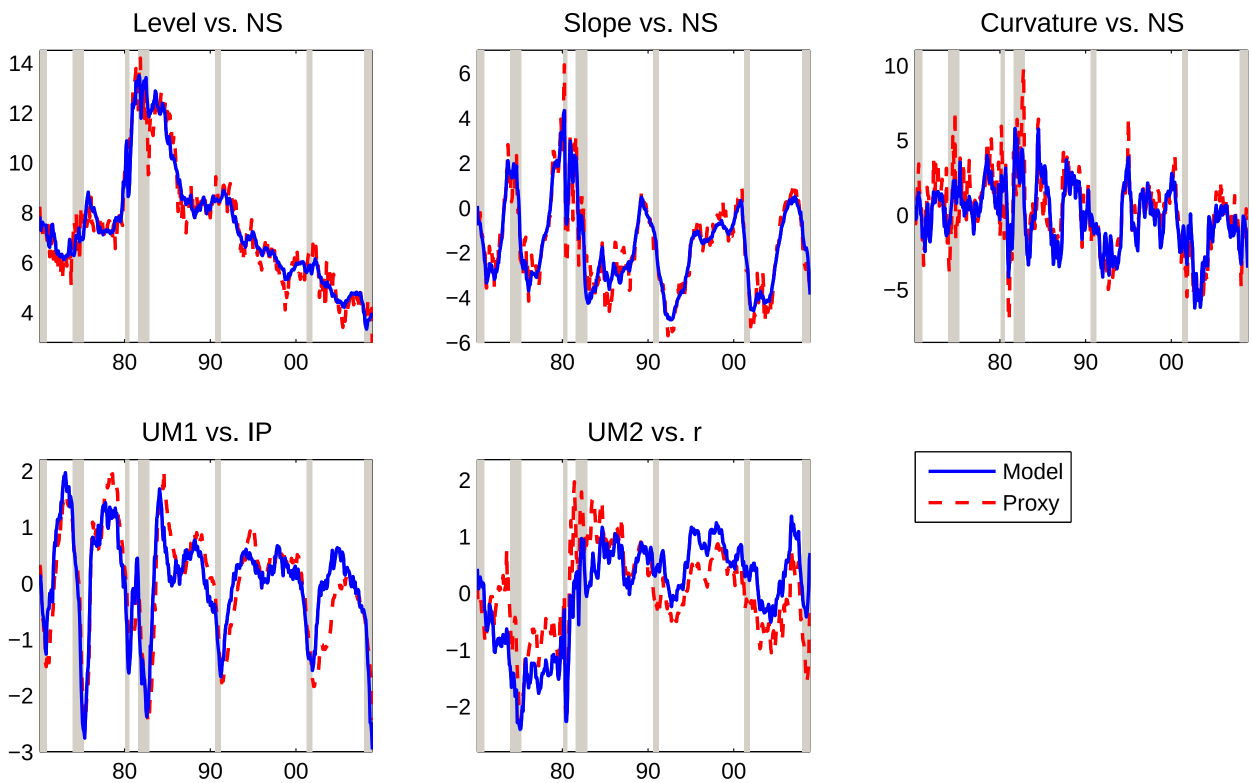
[4]For comparison, in Appendix D we report in-sample results for an unrestricted macro-yields model, which does not impose the zero restrictions on the factor loadings. Results show that these restrictions do not have any effect on the fit of the model.

Table 2: Cumulative variance of yields and macro variables explained by the macro-yields factors

	Level	Slope	Curv	UM1	UM2
Government bond yield with maturity 3 months	0.59	0.94	1.00	1.00	1.00
Government bond yield with maturity 1 year	0.61	0.83	0.99	0.99	0.99
Government bond yield with maturity 2 years	0.65	0.78	0.99	0.99	0.99
Government bond yield with maturity 3 years	0.70	0.79	1.00	1.00	1.00
Government bond yield with maturity 4 years	0.74	0.80	0.99	0.99	0.99
Government bond yield with maturity 5 years	0.78	0.82	0.99	0.99	0.99
Average Hourly Earnings: Total Private	0.07	0.29	0.33	0.33	0.67
Consumer Price Index: All Items	0.19	0.48	0.48	0.50	0.85
Real Disposable Personal Income	0.00	0.02	0.03	0.34	0.36
Effective Federal Funds Rate	0.53	0.93	0.96	0.96	0.97
House Sales - New One Family Houses	0.00	0.19	0.19	0.23	0.23
Industrial Production Index	0.02	0.02	0.03	0.69	0.69
M1 Money Stock	0.17	0.25	0.25	0.25	0.31
ISM Manufacturing: PMI Composite Index (NAPM)	0.03	0.05	0.05	0.61	0.65
Payments All Employees: Total nonfarm	0.00	0.02	0.10	0.70	0.70
Personal Consumption Expenditures	0.16	0.23	0.33	0.46	0.78
Producer Price Index: Crude Materials	0.03	0.14	0.14	0.20	0.43
Producer Price Index: Finished Goods	0.03	0.32	0.32	0.33	0.80
Capacity Utilization: Total Industry	0.02	0.16	0.21	0.63	0.64
Civilian Unemployment Rate	0.44	0.54	0.55	0.65	0.68

This table reports the cumulative share of variance of yields and macro variables explained by the macro-yields factors. The first three columns refer to the yield curve factors (level, slope and curvature) and the last two to the unspanned macroeconomic factors ($UM1$ and $UM2$).

Figure 1: Macro-yields factors



This figure displays the estimated factors of the macro-yields model. The dashed red lines in the three top graphs refer to the NS yield curve factors estimated by ordinary least squares as in Diebold and Li (2006). The red dashed line in the bottom left plot refers to the industrial production index (IP), while the red dashed line in the bottom plot refers to the real interest rate (FFR-CPI). The grey-shaded areas indicate the recessions as defined by the NBER.

macroeconomic variables. In fact, in the macro-yields model, we extract the yield curve factors from both yields and macroeconomic variables and impose the NS restrictions on the factors loadings of the yields to identify them as yield curve factors. The two bottom plots of Figure 1 show the unspanned macro factors. The bottom left plot reports the first unspanned macro factor along with the industrial production index, while the bottom right plot reports the second unspanned macroeconomic factor along with the real interest rate (computed as the difference between the federal funds rate and the consumer price index). As it is clear from the plots, the first unspanned macroeconomic factor closely tracks the industrial production index, with a correlation of 90%, and the second unspanned macroeconomic factor proxies the real interest, with a correlation of 74%. This is in line with the fact that, as reported in Table 2, the first unspanned macroeconomic factor explains mainly measures of real economic activity, while nominal variables are explained partly by the yield curve factors and partly by the second unspanned factor. We can thus conclude that the macro-yields models identifies two unspanned macroeconomic factors: real economic activity and real interest rate. To understand whether these unspanned macroeconomic factors affect the bond premium, in the next section, we analyze the ability of the macro-yields factors to explain expected excess returns.

4.2 Excess Bond Returns

The one-year holding period excess bond return for a bond with maturity τ is defined as the return of buying a bond with τ years to maturity at time t, selling it one year later, at time $t+12$, as a bond with $\tau-1$ years to maturity, and financing this strategy borrowing a bond with one year to maturity at time t, i.e.

$$rx_{t+12}^{(\tau)} = -(\tau-1)y_{t+12}^{(\tau-1)} + \tau y_t^{(\tau)} - y_t^{(1)}. \tag{9}$$

Collecting the excess returns for bonds with different maturities at time $t+12$ in the vector rx_{t+12}, we get

$$rx_{t+12} = \Pi_1 y_{t+12} + \Pi_2 y_t \tag{10}$$

where $\Pi_1 = \begin{bmatrix} D_{[-1:-K]} & 0_{[K\times1]} \end{bmatrix}$, $\Pi_2 = \begin{bmatrix} 1_{[K\times1]} & D_{[2:K+1]} \end{bmatrix}$, $D_{[-1:-K]}$ denotes a diagonal matrix with elements $-1, -2, \ldots, -K$ in the diagonal and $K+1$ denotes the total number of maturities.[5]

The compensation required by risk averse investors to hold long-term government bonds for facing capital loss risk, if the bond is sold before maturity, is the bond term premium which can be measured as the expected excess return

$$E_t(rx_{t+12}) = \Pi_1 E_t(y_{t+12}) + \Pi_2 y_t. \tag{11}$$

The Expectations Hypothesis of the term structure of interest rates states that the long-term yields are determined by market expectations for the short-term rates over the holding period of the long-term asset, plus a constant risk premium. This implies that expected excess returns are time invariant and, thus, excess bond returns should not be predictable with variables in the information set at time t.

The Expectations Hypothesis has been empirically rejected since Fama and Bliss (1987) and Campbell and Shiller (1991), that find that excess returns can be predicted by forward rate spreads and by yield spreads, respectively. More recent evidence by Cochrane and Piazzesi (2005) shows that a linear combination of forward rates (the CP factor) explains between 30% and 35% of the variation in expected excess bond returns. Moreover, Ludvigson and Ng (2009) find that macroeconomic factors constructed as linear and non-linear combinations of principal components extracted from a large data-set of macroeconomic variables (the LN factor) have important forecasting power for future excess returns on U.S. government bonds, above and beyond the predictive power contained in forward rates and yield spreads. Cooper and Priestley (2009) also find that the output gap has in-sample and out-of-sample predictive power for U.S. excess bond returns.

As a preliminary analysis of the predictive ability of the macro-yields factors for one year holding period excess bond returns, we compare the model-implied expected excess returns of the macro-yields and the only-yields models obtained using equation (11). To compare with the previous

[5] Notice that to compute one-year holding period excess bond returns we use only five maturities, i.e. the one-year riskless bond and the $K = 4$ risky bonds with maturities from two to five years.

literature, we also report the predictive regressions that use the CP factor, the LN factor and the CP and LN factors combined. In addition, in Appendix D, we show that the zero restrictions on the factor loadings do not play any role when fitting excess bond returns.

We implement predictive regressions for the CP and LN factors by regressing excess bond returns on the predictive factors $X_t = \{CP_t, LN_t\}$, as follows

$$rx_{t+12}^{(\tau)} = \beta X_t + \varepsilon_{t+12}^{(\tau)}, \tag{12}$$

where we construct the predictive factors by pooling the predictive regression for the individual maturities as follows

$$\overline{rx}_{t+12} = \gamma x_t + \overline{\varepsilon}_{t+12}, \tag{13}$$

where $\overline{rx}_{t+12} = \frac{1}{4}\sum_{\tau=2}^{5} rx_{t+12}^{(\tau)}$ and x_t contains the predictor variables. To construct the CP factor we use the following predictor variables $x_t^{CP} = [1, y_t^{(1)}, f_t^{(2)}, \ldots, f_t^{(5)}]$, where $f_t^{(\tau)}$ denotes the τ-years forward rate.[6] We estimate equation (13) using x_t^{CP} as predictor variables and construct the CP factor as $CP_t = \hat{\gamma}^{CP} x_t^{CP}$. To construct the LN factor, we use as predictor variables $x_t^{LN} = [1, PC1_t, \ldots, PC8_t, PC1_t^3]$, where PC denotes principal components extracted from a large dataset of 131 macroeconomic data series.[7] We then estimate equation (13) using x_t^{LN} as predictor variables and construct the LN factor as $LN_t = \hat{\gamma}^{LN} x_t^{LN}$.

Results in Table 3 show that the macro-yields models explain about 46-55% of the variation of the expected one year holding period excess returns, while the only-yields model can explain only the 12-15% of the variation of expected excess returns. Table 3 reports also the R-squared from the predictive regressions of excess bond returns on the CP and the LN factors. Results show that the CP factor explains 22-27% of the variation in one-year ahead excess returns, slightly lower than

[6] The τ-years forward rate for loans between time $t + 12\tau - 12$ and $t + 12\tau$ is defined as

$$f_t^{(\tau)} = -(\tau - 1)y_t^{(\tau-1)} + \tau y_t^{(\tau)}.$$

[7] The 131 macroeconomic data series used to construct the LN factor have been downloaded from Sydney C. Ludvigson's website at http://www.econ.nyu.edu/user/ludvigsons/Data&ReplicationFiles.zip.

Table 3: In-sample fit of excess bond returns

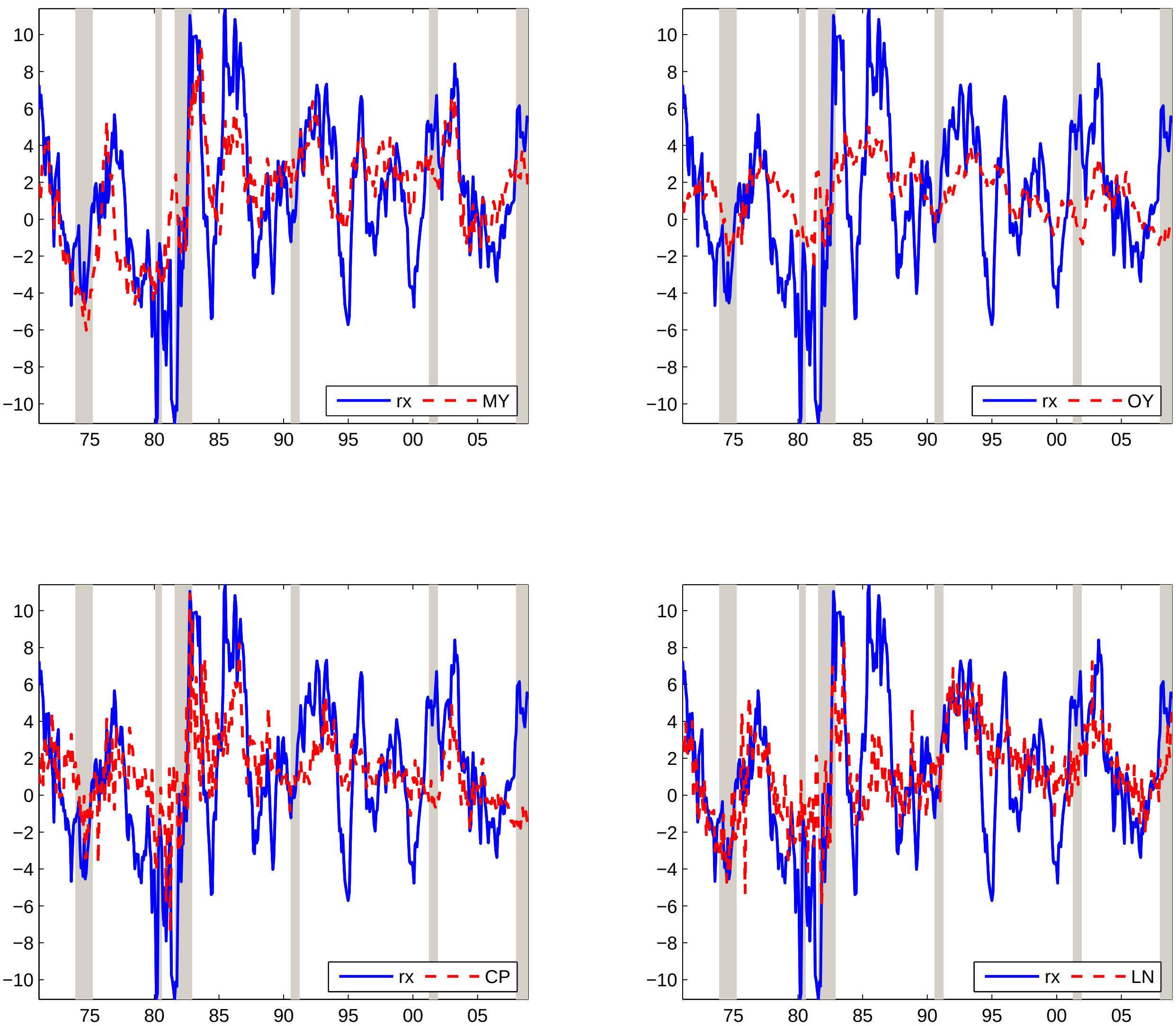
Maturity	MY	OY	CP	LN	LN+CP
2y	0.55	0.12	0.22	0.33	0.41
3y	0.53	0.12	0.24	0.33	0.43
4y	0.50	0.14	0.27	0.32	0.43
5y	0.46	0.15	0.24	0.30	0.40

This table reports the R^2 for expected one year holding period excess bond returns from different models. The columns MY and OY refer to the model-implied expected excess bond returns from the macro-yields model (MY) and the only-yields model (OY) respectively. The columns CP, LN and CP+LN refer to the predictive regression using the Cochrane and Piazzesi (2005) factor (CP), the Ludvigson and Ng (2009) factor (LN), and both the Cochrane and Piazzesi (2005) and the Ludvigson and Ng (2009) factors jointly.

the value reported in Cochrane and Piazzesi (2005). This is due to the fact that our predictive regressions use more updated data and the performance of the CP factor has deteriorated over time, as also shown by Thornton and Valente (2012). The LN factor explains a third of the variation of future excess bond returns, while the CP and LN factors jointly explain 40-43% of the variation in one-year ahead excess bond returns, lower than what is explained by the macro-yields model. We can thus conclude that, in-sample, the macro-yields model outperforms the CP and the LN factors even combined.

Figure 2 shows the predicted and the realized average excess bond returns from the macro-yields and the only-yields model, and also from the predictive regressions using the CP and the LN factors. The figure shows that the predictive excess bond returns from the only-yields model are quite flat, indicating that the yield curve factors poorly predict excess bond returns. The CP factor seems doing a better job than the only-yields model, but does not improve over the macro-yields model, which is able to better predict the average expected excess return, also with respect to the LN factor.

Figure 2: Average 1-year holding period excess return: realized and predicted



This figure displays the average excess return $\overline{rx}_{t+12}$ (blue continuous line) and the corresponding predicted values from different models (dashed red line). The dashed red line in the top plots refer to the model-implied predicted values from the macro-yields MY model (top right) and only-yields OY model (top left). The dashed red line in the bottom plots refer to the predicted values from the predictive regressions using the CP factors (bottom left) and the LN factor (bottom right). The grey-shaded areas indicate the recessions as defined by the NBER.

5 Out of Sample Forecast

To evaluate the predictive ability of the macro-yields model, we generate out-of-sample iterative forecasts of the factors, as follows

$$E_t(F^*_{t+h}) \equiv \hat{F}^*_{t+h|t} = (\hat{A}^*_{|t})^h \hat{F}^*_{t|t},$$

where h denotes the forecast horizon and $\hat{A}^*_{|t}$ is estimated using the information available till time t.[8] We then compute out-of-sample forecasts of the yields given the projected factors, as follows

$$E_t(z_{t+h}) \equiv \hat{z}_{t+h|t} = \hat{\Gamma}^*_{|t} \hat{F}^*_{t+h|t}.$$

where $\hat{\Gamma}^*_{|t}$ is estimated using data up to time t. We also compute out-of-sample predictions of excess bond returns as follows

$$E_t(rx_{t+12}) \equiv rx_{t+12|t} = \Pi_1(\hat{\Gamma}^*_{|t} F^*_{t+12|t}) + \Pi_2 y_t$$

where Π_1 and Π_2 are defined in (10).

We forecast yields and excess returns recursively using data from January 1970 until the time that the forecast is made, beginning in January 1990 to December 2008.

5.1 Yields

To evaluate the prediction accuracy of the macro yields model for out of sample forecasts of yields, we use the Mean Squared Forecast Error (MSFE), i.e. the average squared error in the evaluation period for the h-months ahead forecast of the yield (or excess return) with maturity τ

$$\text{MSFE}^{t_1}_{t_0}(\tau, h, M) = \frac{1}{t_1 - t_0 + 1} \sum_{t=t_0}^{t_1} \left(\hat{y}^{(\tau)}_{t+h|t}(M) - y^{(\tau)}_{t+h} \right)^2, \tag{14}$$

[8]See Appendix A for the definitions of F^*_t, Γ^* and A^*.

Table 4: Out-of-sample performance for yields

Macro-Yields

Maturity	3m	1y	2y	3y	4y	5y
h=1	1.17	1.05	1.06	1.00	1.05	1.14
h=3	0.79*	0.93	0.99	0.96	0.99	1.02
h=6	0.78**	0.89	0.94	0.93	0.93	0.94
h=12	0.69**	0.74**	0.79**	0.80***	0.80***	0.80***
h=24	0.62***	0.66***	0.74**	0.82**	0.88*	0.97

Only-Yields

Maturity	3m	1y	2y	3y	4y	5y
h=1	0.93	1.09	1.17	1.11	1.07	1.11
h=3	0.96	1.13	1.20	1.14	1.10	1.13
h=6	0.99	1.18	1.25	1.21	1.15	1.16
h=12	1.04	1.16	1.26	1.27	1.25	1.26
h=24	1.06	1.12	1.27	1.39	1.49	1.62

This table reports the relative MSFE of the macro-yields model and the only-yields model over the MSFE of the random walk for multi-step predictions of the yields. The first column reports the forecast horizon h. The sample starts on January 1970 and the evaluation period is January 1990 to December 2008. *, ** and *** denote significant outperformance at 10%, 5% and 1% level with respect to the random walk according to the White (2000) reality check test with 1,000 bootstrap replications using an average block size of 12 observations.

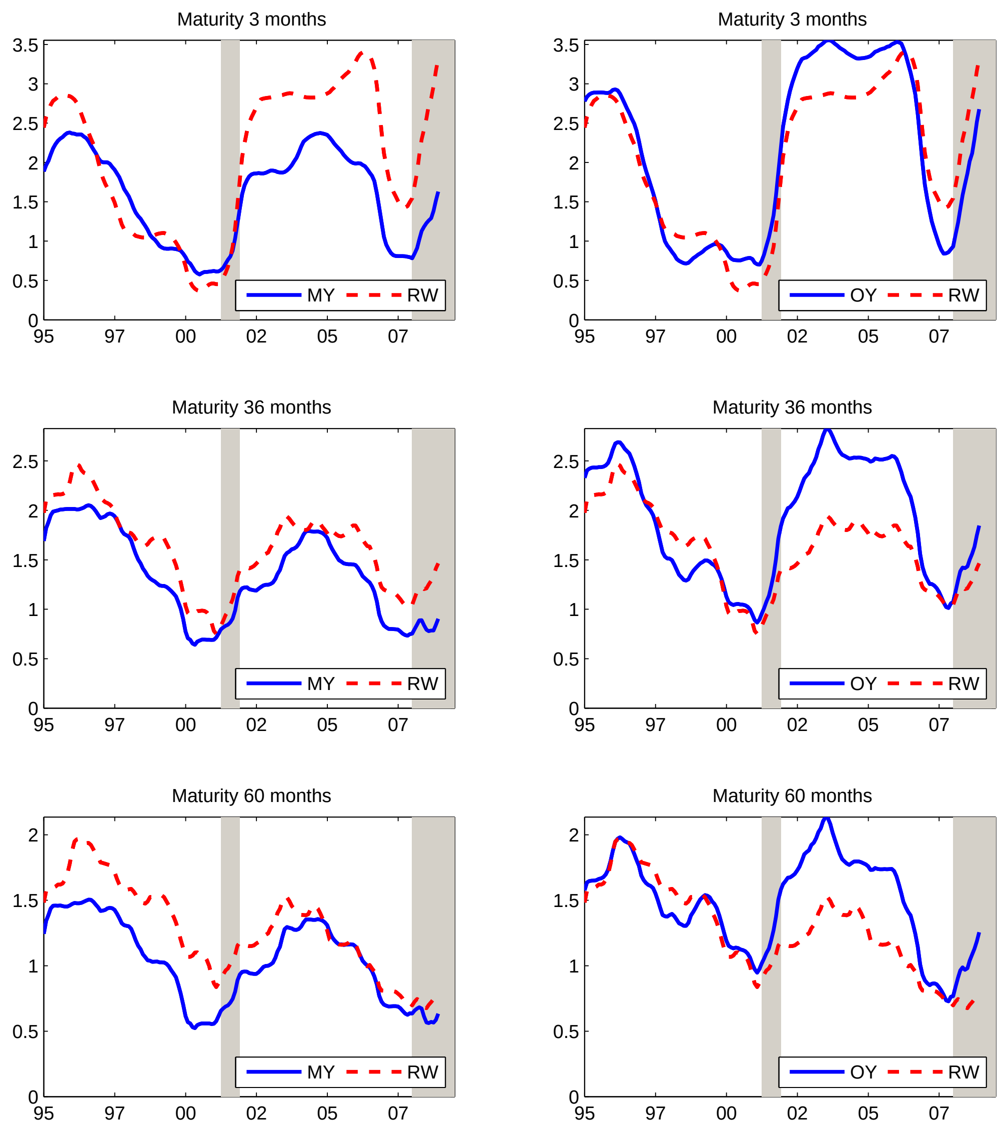
where t_0 and t_1 denote, respectively, the start and the end of the evaluation period, $y_{t+h}^{(\tau)}$ is the realized yield with maturity τ_i at time $t+h$ and $\hat{y}_{t+h|t}^{(\tau)}(M)$ is the h-step ahead forecast of the yield with maturity τ from model M using the information available up to t.

Forecast results for yields are usually expressed as relative performance with respect to the random walk, which is a naïve benchmark for yield curve forecasting very difficult to outperform, given the high persistency of the yields. The random walk h-steps ahead prediction at time t of the yield with maturity τ is

$$E_t(y_{t+h}^{(\tau)}) \equiv \hat{y}_{t+h|t}^{(\tau)} = y_t^{(\tau)},$$

where the optimal predictor does not change regardless of the forecast horizon. To measure the relative performance of the macro-yields model with respect to the random walk, we use the relative

Figure 3: 12-months ahead smoothed squared forecast errors for yields



This figure displays the 5-years rolling 12-months ahead squared forecast error for the yields with 3, 36 and 60 months to maturity. The blue continuous line refers to the 5-years rolling squared forecast error of the macro-yields MY model (left plots) and of the only-yields OY model (right plots). The dashed red line refers to 5-years rolling squared forecast error of the random walk. The dates on the horizontal axis refer to the end of the rolling window period. The grey-shaded areas indicate the recessions as defined by the NBER.

MSFE computed as

$$\text{rMSFE}_{t_0}^{t_1}(\tau, h, M) = \frac{\text{MSFE}_{t_0}^{t_1}(\tau, h, M)}{\text{MSFE}_{t_0}^{t_1}(\tau, h, RW)}.$$

Table 4 reports the rMSFE with respect to the random walk for the macro-yields model the only-yields model. Results in Table 4 show that the only-yields model is outperformed by the macro-yields model for all but the 1-month horizon. Moreover, the macro-yields model outperforms the random walk at 3, 6, 12 and 24 steps ahead for all the maturities, with significant outperformance according to the White (2000) reality check test for the 12 and 24 steps ahead forecasts.[9] This evidence is corroborated by Figure 3, which reports the 12-months ahead smoothed squared forecast errors of the macro-yields, the only-yields and the random walk models for yields with 3, 36 and 60 months to maturity. The figure highlights how the macro-yields model has been systematically outperforming the random walk, especially in the last part of the evaluation sample for the short maturity and the first part of the sample for the long maturities. The only-yield model, instead has been performing as the random walk in the first part of the evaluation sample but its performance deteriorated in the last part of the evaluation sample, significantly underperforming the random walk. This suggests that the unspanned macroeconomic factors, while not important for explaining the contemporaneous yields curve, contain useful information to predict the future yield curve factors and, thus, the future evolution of the yield curve.

5.2 Excess Bond Returns

Out of sample forecast results for excess bond returns are reported in Table 5, which contains the relative MSFE of the macro-yields model (MY) with respect to the constant excess return benchmark, where one-year holding period excess returns are unforecastable at one year horizon, as in the expectation hypothesis. The macro-yields model outperforms the constant excess return benchmark for all maturities and the outperformance is significant for all maturities according to the White (2000) reality check test. In Appendix D we show that the same results hold also when the zero restrictions on the factor loadings are removed, confirming that the macro factors help to

[9] For more details about the reality check test see Appendix C.

Table 5: Out-of-sample predictive performance for excess returns

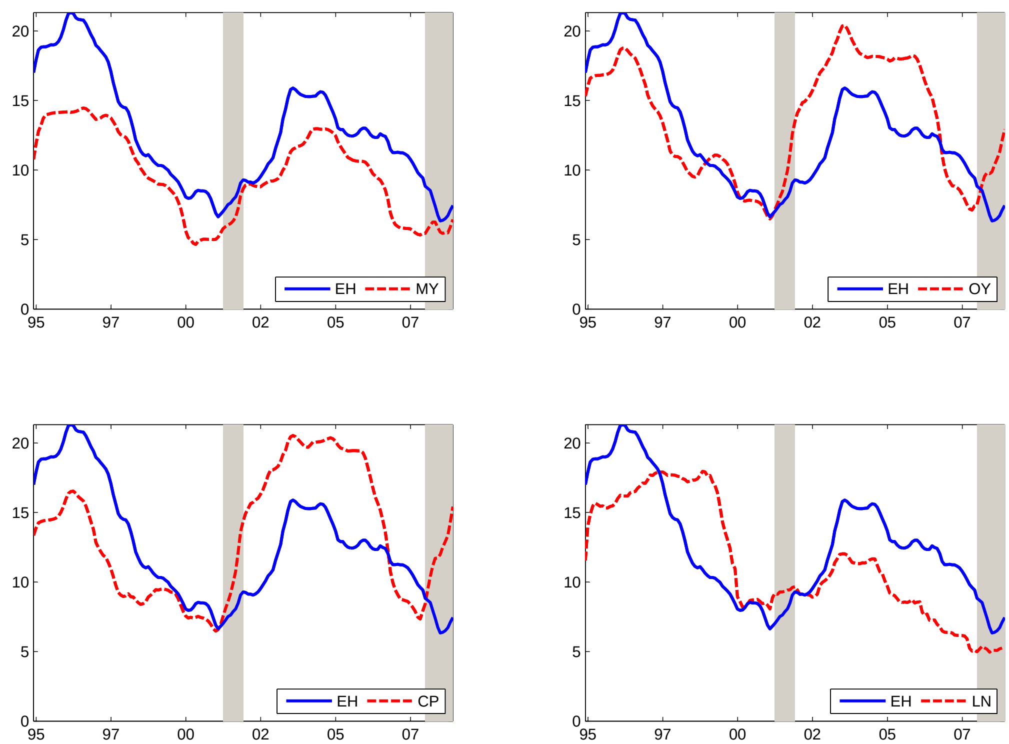
Maturity	MY	OY	CP	LN	LN+CP
2y	0.76**	1.20	1.17	0.80	0.80
3y	0.75**	1.20	1.21	0.79	0.83
4y	0.74**	1.18	1.21	0.78	0.83
5y	0.75**	1.18	1.18	0.81	0.83

This table reports the relative MSFE of the macro-yields model (MY), the only-yields model (OY), the Cochrane and Piazzesi (2005) factor (CP), the Ludvigson and Ng (2009) (LN) factor, the Cochrane and Piazzesi (2005) and the Ludvigson and Ng (2009) factors combined ($LN+CP$) with respect to the expectation hypothesis for excess returns. The sample starts on January 1970 and the evaluation period is January 1990 to December 2008. * and ** denote significant outperformance at 10% and 5% level with respect to the expectation hypothesis according the White (2000) reality check test with 1,000 bootstrap replications using an average block size of 12 observations.

predict the yield curve factors but do not explain the cross-section of yields. As a comparison, we report relative MSFE of the only-yields model (OY) that shows that unspanned macro-factors are indeed important in order to predict excess bond returns. Table 5 also reports the out-of-sample relative MSFEs of the excess bond returns forecasts using the CP factor, the LN factor, and the CP and LN factors combined obtained from the predictive regressions in (12). The worst performing models are the ones that do not use macroeconomic variable, i.e. the only-yield model and the CP factors. In line with the predictive regressions of excess bond returns and with the 12 steps ahead out-of-sample forecast performance of the macro-yields model for the yields, results in Table 5 show that the macro-yields model is the best performing model for the prediction of the one-year excess bond returns for all maturities.

To further understand the performance of the macro-yields model to predict one-year holding period excess bond returns, Figure 4 plots the 5-years rolling mean squared forecast error of the macro-yields model, the only-yields model, the CP and LN factors along with the 5-years rolling mean squared forecast error under the expectation hypothesis (EH). The figure shows that the performance of the only-yield model and the CP factors are similar: both models outperform the expectation hypothesis in the first part of the evaluation sample but display large forecast errors in

Figure 4: Smoothed mean squared forecast errors for excess bond returns



This figure displays the 5-years rolling mean squared forecast error for one-year holding period excess bond returns from the expectation hypothesis EH (blue continuous line) and the corresponding values from different models (dashed red line). The dashed red line in the top plots refer to 5-years rolling mean squared forecast error of the macro-yields MY model (top right) and only-yields OY model (top left). The dashed red line in the bottom plots refer to the 5-years rolling mean squared forecast error from the predictive regressions using the CP factors (bottom left) and the LN factor (bottom right). The dates on the horizontal axis refer to the end of the rolling window period. The grey-shaded areas indicate the recessions as defined by the NBER.

the second part. Also the performance of the macro-yields model and the LN factors are similar, they both provide more accurate predictions than the expectation hypothesis, in particular in the last part of the evaluation period. However, it is clear from the figure that the macro-yields model, apart from being the best performing model on average as seen in Table 5, is the best performing model for the whole evaluation period. This is a clear evidence that the unspanned macroeconomic factors identified by the proposed macro-yields model as related with economic growth and real interest rates have predictive ability for the yield curve factors and, thus, for excess bond returns.

6 Conclusions

In this paper we analyze the predictive content of macroeconomic information for the yield curve of interest rates and excess bond returns in the United States. We find that two macroeconomic factors characterizing economic growth and real interest are unspanned by the cross-section of government bond yields and have significant predictive power for the bond yields and excess returns.

In future research, we plan to extend our empirical specification to allow for the zero lower bound of interest rates, non-synchronicity of macroeconomic data releases and mixed frequencies. The macro-yields model presented in this paper cannot be estimated on a sample that includes the great recession, as it does not ensure a zero lower bound on interest rates. Our model model can, however, be easily extended to deal with this issue by anchoring the shorter end of the yield curve using market expectation, along the lines of Altavilla, Costantini, Giacomini and Ragusa (2012).

Data revisions and jagged edges due to the non-synchronicity of macroeconomic data releases are important characteristics to be taken into account when extracting macroeconomic information, see Giannone, Reichlin and Small (2008). In addition, bond yields are available at higher frequencies than macroeconomic variables. These features can be easily incorporated into our empirical model along the line described in Banbura et al. (2012).

A Estimation Procedure

We can rewrite the macro-yields model in equations (4)–(6) in compact form as

$$z_t = a + \Gamma F_t + v_t, \tag{15}$$

$$F_t = \mu + AF_{t-1} + u_t, \quad u_t \sim N(0, Q) \tag{16}$$

$$v_t = Bv_{t-1} + \xi_t, \quad \xi_t \sim N(0, R) \tag{17}$$

where $z_t = \begin{pmatrix} y_t \\ x_t \end{pmatrix}$, $F_t = \begin{pmatrix} F_t^y \\ F_t^x \end{pmatrix}$, $a = \begin{bmatrix} 0 \\ a_x \end{bmatrix}$, $\Gamma = \begin{bmatrix} \Gamma_{yy} & \Gamma_{yx} \\ \Gamma_{xy} & \Gamma_{xx} \end{bmatrix}$, $A = \begin{bmatrix} A_{yy} & A_{yx} \\ A_{xy} & A_{xx} \end{bmatrix}$, $Q = \begin{bmatrix} Q_{yy} & Q_{yx} \\ Q_{xy} & Q_{xx} \end{bmatrix}$, $\mu = \begin{bmatrix} \mu_y \\ \mu_x \end{bmatrix}$ and $\Gamma_{yy} = \Gamma_{NS}$ is the matrix whose rows correspond to the smooth patterns proposed by Nelson and Siegel (1987) and shown in equation (2). In addition $\Gamma_{yx} = 0$, as the macroeconomic factors F_t^x are unspanned by the cross-section of yields $\Gamma_{yx} = 0$.[10]

The macro-yields model in (15)–(16) can be put in a state-space form augmenting the states F_t with the idiosyncratic components v_t and a constant c_t as follows

$$z_t = \Gamma^* F_t^* + v_t^*, \quad v_t^* \sim N(0, R^*)$$

$$F_t^* = A^* F_{t-1}^* + u_t^*, \quad u_t^* \sim N(0, Q^*)$$

where $\Gamma^* = \begin{bmatrix} \Gamma & a & I_N \end{bmatrix}$, $F_t^* = \begin{bmatrix} F_t \\ c_t \\ v_t \end{bmatrix}$, $A^* = \left[\begin{array}{cc|c|c} A & \mu & \dots & 0 \\ \hline \vdots & \iddots & 1 & \vdots \\ \hline 0 & \dots & \dots & B \end{array}\right]$, $u_t^* = \begin{bmatrix} u_t \\ \nu_t \\ \xi_t \end{bmatrix}$, $Q^* = \left[\begin{array}{c|c|c} Q & \dots & 0 \\ \hline \vdots & \varepsilon & \vdots \\ \hline 0 & \dots & R \end{array}\right]$

and $R = \varepsilon I_n$, with ε a very small fixed coefficient. In this state-space form, c_t an additional state variable restricted to one at every time t.

[10] In Appendix D, we report results for an unrestricted macro-yields model that allows all factor loadings to be different from zero. We also estimate the only-yields model using the same procedure, as it implies the following restrictions in (15)–(16): $z_t = y_t$, $F_t = F_t^y$, $a = 0$, $\Gamma = \Gamma_{NS}$, $\mu = \mu_y$.

The restrictions on the factor loadings Γ^* and on the transition matrix A^* can be written as

$$H_1 \ \mathrm{vec}(\Gamma^*) = q_1, \qquad H_2 \ \mathrm{vec}(A^*) = q_2,$$

where H_1 and H_2 are selection matrices, and q_1 and q_2 contain the restrictions.

We assume that $F_1^* \sim N(\pi_1, V_1)$, and define $y = [y_1, \ldots, y_T]$ and $F^* = [F_1^*, \ldots, F_T^*]$. Then denoting the parameters by $\theta = \{\Gamma^*, A^*, Q^*, \pi_1, V_1\}$, we can write the joint loglikelihood of z_t and F_t, for $t = 1, \ldots, T$, as

$$\begin{aligned}
L(z, F^*; \theta) \quad = \quad & -\sum_{t=1}^{T} \left(\frac{1}{2} \left[z_t - \Gamma^* F_t^* \right]' (R^*)^{-1} \left[z_t - \Gamma^* F_t^* \right] \right) + \\
& -\frac{T}{2} \log |R^*| - \sum_{t=2}^{T} \left(\frac{1}{2} [F_t^* - A^* F_{t-1}^*]' (Q^*)^{-1} [F_t^* - A^* F_{t-1}^*] \right) + \\
& -\frac{T-1}{2} \log |Q^*| + \frac{1}{2} [F_1^* - \pi_1]' V_1^{-1} [F_1^* - \pi_1] + \\
& -\frac{1}{2} \log |V_1| - \frac{T(p+k)}{2} \log 2\pi + \lambda_1' \left(H_1 \, \mathrm{vec}(\Gamma^*) - q_1 \right) + \lambda_2' \left(H_2 \, \mathrm{vec}(A^*) - q_2 \right)
\end{aligned}$$

where λ_1 contains the lagrangian multipliers associate with the constraints on the factor loadings Γ^* and λ_2 contains the lagrangian multipliers associated with the constraints on the transition matrix A^*.

The computation of the Maximum Likelihood estimates is performed using the EM algorithm. Broadly speaking, the algorithm consists in a sequence of simple steps, each of which uses the Kalman smoother to extract the common factors for a given set of parameters and multivariate regressions to estimate the parameters given the factors. We initialize the yield curve factors with the NS factors using the two-steps OLS procedure introduced by Diebold and Li (2006). We then project the macroeconomic variables on the NS factors and use the principal components of the residuals of this regression to initialize the unspanned macroeconomic factors. These estimated factor are then treated as if they were the true observed factors. The initial parameters are hence estimated by OLS. After the parameters are estimated, a new set of factors is obtained by using the Kalman smoother. If we stop at this stage, we have the two-step procedure of Doz, Giannone and Reichlin (2011). The quasi-maximum likelihood estimate consists essentially in iterating these steps until convergence, see Doz et al. (2012).

In practice, we use the restricted version of the EM algorithm, the Expectation Restricted Maximization, since we need to impose the smooth pattern on the factor loadings of the yields on the NS factors. The ERM algorithm alternates Kalman filter extraction of the factors to the restricted maximization of the likelihood. At the j-th iteration the ERM algorithm performs two steps:

1. In the Expectation-step, we compute the expected log-likelihood conditional on the data and the estimates from the previous iteration, i.e.

$$\mathcal{L}(\theta) = E[L(z, F^*; \theta^{(j-1)})|z]$$

   which depends on three expectations

$$\begin{aligned} \hat{F}_t^* &\equiv E[F_t^*; \theta^{(j-1)}|z] \\ P_t &\equiv E[F_t^*(F_t^*)'; \theta^{(j-1)}|z] \\ P_{t,t-1} &\equiv E[F_t^*(F_{t-1}^*)'; \theta^{(j-1)}|z] \end{aligned}$$

   These expectations can be computed, for given parameters of the model, using the Kalman filter.

2. In the Restricted Maximization-step, we update the parameters maximizing the expected log-likelihood with respect to θ:

$$\theta^{(j)} = \arg\max_{\theta} \mathcal{L}(\theta)$$

   This can be implemented taking the corresponding partial derivative of the expected log likelihood, setting to zero, and solving.

The procedure outlined above can be extended to estimate also the decay parameter λ controlling for the shape of the loadings of the yields on the slope and curvature factors. Since the factor loadings are a non-linear function λ, an additional step consisting in the numerical maximization of the conditional likelihood with respect to λ is required. The procedure is know as Expectation Conditional Restricted Maximization (ECRM) algorithm.

B Data

Table 6: Macroeconomic Variables

Series N.	Mnemonic	Description	Transformation
1	AHE	Average Hourly Earnings: Total Private	1
2	CPI	Consumer Price Index: All Items	1
3	INC	Real Disposable Personal Income	1
4	FFR	Effective Federal Funds Rate	0
5	HSal	House Sales - New One Family Houses	1
6	IP	Industrial Production Index	1
7	M1	M1 Money Stock	1
8	Manf	ISM Manufacturing: PMI Composite Index (NAPM)	0
9	Paym	All Employees: Total nonfarm	1
10	PCE	Personal Consumption Expenditures	1
11	PPIc	Producer Price Index: Crude Materials	1
12	PPIf	Producer Price Index: Finished Goods	1
13	CU	Capacity Utilization: Total Industry	0
14	Unem	Civilian Unemployment Rate	0

This table lists the 14 macro variables used to estimate the macro-yields. Most series have been transformed prior to the estimation, as reported in the last column of the table. The transformation codes are: 0 = no transformation and 1 = annual growth rate.

C Reality Check Test

To compare the out of sample predictive ability of a model with respect to the benchmark, we use the reality check test of White (2000), as we compare only non-nested models.

If we denote by $e_t(b)$ the forecast errors of the benchmark and by $e_t(M)$ the forecast errors of the model under consideration. Then we can define the null hypothesis of no predictive superiority over the benchmark as

$$H_0 : f = E(f_t) \equiv E(e_t(b)^2 - e_t(M)^2) \leq 0 \tag{18}$$

The test is then based on the statistic

$$\overline{f} = \frac{1}{t_1 - t_0} \sum_{t=t_0}^{t_1} \hat{f}_t \tag{19}$$

where t_0 and t_1 denote, respectively, the start and the end of the evaluation period, and hats denote estimated statistics.

To approximate the asymptotic distribution of the test statistic, we use block-bootstrap as follows:

1. We generate bootstrapped forecast errors $\hat{e}_t^*(b)$ and $\hat{e}_t^*(M)$ using the stationary block-bootstrap of Politis and Romano (1994) with average block size of 12. This procedure is analogous to the moving blocks bootstrap, but, instead of using blocks of fixed length uses blocks of random length, distributed according to the geometric distribution with mean block length 12. Also to give the same probability of resampling to all observations, we use a circular scheme.

2. Construct the bootstrapped test statistic as

$$\overline{f}^* = \frac{1}{t_1 - t_0} \sum_{t=t_0}^{t_1} (\hat{e}_t^*(b)^2 - \hat{e}_t^*(M)^2)$$

3. Repeat steps 1 and 2 for 1,000 times to obtain an estimate of the distribution of the test statistic $\overline{f}^* = [\overline{f}^*_{(1)}, \ldots, \overline{f}^*_{(1,000)}]$.

4. Compare $V = (t_1 - t_0)^{1/2}\overline{f}$ with the quantiles of $V^* = (t_1 - t_0)^{1/2}(\overline{f}^* - \overline{f})$ to obtain the p-value.

D Unrestricted Macro-Yields Model

In this Appendix, we report results for an unrestricted macro-yields model, which does not impose the zero restrictions on the factor loadings of the yields on the macro factors. In practice, this model allows the macro factors F_t^x to directly affect the cross-section of yields. Results in Tables 7–9, show that the in sample and out of sample performance for yields and excess bond returns of the unrestricted macro-yields model are equal to the ones of the macro-yields model in Tables 2–9. This provides evidence that the zero restrictions on the factor loadings are satisfied by the data, implying that the macro factors F_t^x are unspanned by the cross-section of yields.

Table 7: Unrestricted Macro-Yields Model: Fit

	Level	Slope	Curv	M1	M2
Government bond yield with maturity 3 months	0.59	0.95	1.00	1.00	1.00
Government bond yield with maturity 1 year	0.61	0.83	0.99	0.99	0.99
Government bond yield with maturity 2 years	0.65	0.79	0.99	0.99	0.99
Government bond yield with maturity 3 years	0.70	0.79	1.00	1.00	1.00
Government bond yield with maturity 4 years	0.74	0.80	0.99	0.99	0.99
Government bond yield with maturity 5 years	0.78	0.82	0.99	0.99	0.99
Average Hourly Earnings:Total Private	0.07	0.29	0.33	0.33	0.67
Consumer Price Index: All Items	0.19	0.48	0.48	0.50	0.86
Real Disposable Personal Income	0.00	0.02	0.03	0.34	0.36
Effective Federal Funds Rate	0.54	0.93	0.96	0.96	0.97
Hose Sales - New One Family Houses	0.00	0.19	0.19	0.22	0.22
Industrial Production Index	0.02	0.02	0.03	0.70	0.70
M1 Money Stock	0.17	0.25	0.25	0.25	0.31
ISM Manufacturing: PMI Composite Index (NAPM)	0.03	0.05	0.05	0.61	0.65
Payments All Employees: Total nonfarm	0.00	0.02	0.10	0.71	0.71
Personal Consumption Expenditures	0.16	0.23	0.33	0.47	0.79
Producer Price Index: Crude Materials	0.03	0.13	0.13	0.20	0.43
Producer Price Index: Finished Goods	0.03	0.31	0.31	0.32	0.81
Capacity Utilization: Total Industry	0.02	0.16	0.20	0.62	0.63
Civilian Unemployment Rate	0.44	0.53	0.55	0.64	0.67

This table reports the cumulative share of variance of the yields and macro variables explained by the yield curve factors (level, slope and curvature) and the macroeconomic factors in an unrestricted macro-yields model.

Table 8: Unrestricted Macro-Yields Model: Out-of-sample performance for yields

Maturity	3m	1y	2y	3y	4y	5y
h=1	1.21	1.04	1.05	1.01	1.05	1.12
h=3	0.81*	0.93	0.99	0.97	0.99	1.02
h=6	0.79**	0.89	0.95	0.93	0.93	0.94
h=12	0.67**	0.73**	0.78**	0.79***	0.79***	0.79***
h=24	0.59***	0.63***	0.71***	0.78***	0.84**	0.92

This table reports the relative MSFE of the unrestricted macro-yields model over the MSFE of the random walk for multi-step predictions of the yields. The first column reports the forecast horizon h. The sample starts on January 1970 and the evaluation period is January 1990 to December 2008. *, ** and *** denote significant outperformance at 10%, 5% and 1% level with respect to the random walk according the White (2000) reality check test with 1,000 bootstrap replications using an average block size of 12 observations.

Table 9: Unrestricted Macro-Yields Model: Excess Bond Returns

Maturity	In Sample	Out of Sample
2y	0.57	0.76**
3y	0.55	0.75**
4y	0.50	0.74**
5y	0.47	0.75**

The column In Sample reports the R^2 of the unrestricted macro-yields model for excess bond returns. The column Out of Sample reports the out of sample relative MSFE of the unrestricted macro-yields model with respect to the expectation hypothesis. * and ** denote significant outperformance at 10% and 5% level with respect to the expectation hypothesis according the White (2000) reality check test with 1,000 bootstrap replications using an average block size of 12 observations.

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