Uncertain Efficiency Gains and Merger Policy*

Mariana Cunha†

Paula Sarmento‡

Hélder Vasconcelos§

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Abstract

This paper studies the role of uncertainty in merger control and in merger decisions. In a Cournot setting, we consider that mergers may give rise to uncertain endogenous efficiency gains and that every merger has to be submitted for approval to the Antitrust Authority (AA). We assume that both the AA and the firms in the industry face the same uncertainty about the future efficiency gains induced by the merger. It is shown that an increase in the degree of uncertainty benefits both insider and outsider firms but also the consumers. Further, when uncertainty is high, there is a greater likelihood that firms propose a merger to the AA and that the AA accepts it. Interestingly, however, although uncertainty enhances merger approval chances, it also decreases merger's stability, by increasing outsiders' incentives to free-ride on it.

Keywords: Efficiency gains; Merger control; Uncertainty

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†FEP-UP, School of Economics and Management, University of Porto, Address: Rua Dr. Roberto Frias, 4200-464 Porto, Portugal; E-mail address: macunha.phd@fep.up.pt. The author thanks Fundação para a Ciência e Tecnologia for financial support (SFRH/BD/70000/2010).

‡FEP-UP, CEF-UP; E-mail address: sarmento@fep.up.pt.

§FEP-UP, CEF-UP and CEPR; E-mail address: hvasconcelos@fep.up.pt.

1 Introduction

The assessment of the efficiency gains resulting from a merger usually raises an information issue for antitrust authorities. Although some mergers can actually generate significant efficiency gains, these are usually difficult to measure and verify. In practice, it is often the case that both the firms and the antitrust authority (henceforth, AA) cannot predict exactly the post-merger efficiency gains, implying that they are not aware of all the conditions they are going to face after the merger. Sometimes, only after the merger firms and antitrust authorities will understand the true level of induced efficiency gains. For instance, some pharmaceutical firms may adopt merger decisions without knowing whether their R&D efforts will be successful or not. Also, any type of firms' investment could generate uncertainty about future costs and, sometimes, a merger could actually occur before the uncertainty is resolved.

The purpose of this paper is to contribute to the literature that studies the efficiency gains role in merger decisions, departing from a deterministic environment by considering a setting in which there is (symmetric) uncertainty. In particular, we assume that the decisions of the antitrust authority, when evaluating a merger case, crucially depend on the uncertainty regarding the efficiency gains realization. In the proposed model, firms and the AA are uncertain about the level of efficiency gains and, therefore, this uncertainty is going to influence the decision of the AA but also firms' incentives to merge. This analysis is useful to the AA in order to more properly evaluate merger proposals when there is uncertainty about the cost savings that mergers may induce or not.[1]

In the absence of uncertainty and in a context of symmetric Cournot oligopoly with linear demand and costs, a merger is profitable if it comprises a pre-merger market share of at least 80% (Salant et al., 1983; Perry and Porter, 1985). Also, when both firms and the AA are perfectly informed about the merger induced efficiency gains, the antitrust authority usually allows the merger when there are important efficiency gains that would lead to lower prices (Motta and Vasconcelos, 2005; Vasconcelos, 2010).

The present paper is related to two strands of literature on mergers in a Cournot framework: (i) the studies of antitrust authority's merger decisions, where the AA evaluates the welfare effects of mergers and allows for merger remedies under uncertainty, such as Cosnita and Tropeano (2009), Besanko and Spulber (1993), Corchón and Faulí-Oller (2004); and (ii) works that, investigating the impact of uncertainty on the incentives to merge, conclude that the incentives to merge depend on the information structure, such as Gal-Or (1988), Stenbacka (1991), Wong and Tse (1997), Stennek (2003), Qiu and Zhou (2006), Banal-Estañol (2007), Zhou (2008a,b), Amir et al. (2009).[2]

[1]See Morgan (2001) for the discussion about the significance of reviewing merger cases with uncertainty.

[2]Uncertainty can also be seen as an information sharing problem, for instance, firms can have more information about their own costs than the AA. Previous contributions to the literature on information sharing among oligopolists did not consider the possibility of mergers among firms nor AA intervention.

To our knowledge, however, none of the previous papers has addressed the role of efficiency gains which are uncertain for all players in the merger formation process (firms and the AA). The present paper then contributes to fill this gap in the extant literature by assuming that, when firms propose the merger to the AA, all the players (insider, outsiders and AA) are uncertain about the post-merger efficiency gains and therefore they decide by considering the expectations on those gains. Once the merger is consummated, both insider and outsider firms can observe the efficiency gains and compete *à la* Cournot. This is different from Amir et al. (2009)'s paper where, after the merger, only the insider firm observes its cost. Our model is also different from the literature cited before, where it is usually assumed that the merging firms have an informational advantage, knowing more about the merger induced efficiency gains than its rivals and the AA. Further, our analysis is close to Le Pape and Zhao (2013)'s paper. Le Pape and Zhao (2013) analyse Stackelberg mergers' decisions when there is uncertainty on productivity and informational asymmetry between firms. However, here we focus our analysis on both mergers and AA decisions, assuming that the firms compete in a Cournot setting and that all firms have the same degree of uncertainty.

We find that the increase in the degree of uncertainty about merger's efficiency gains benefits firms (in terms of higher expected profits) and consumers (with higher expected consumer surplus). Further, when the degree of uncertainty is high, there is a greater likelihood that firms propose a merger to the AA and the AA accepts it. Therefore, uncertainty enhances both the occurrence of merger proposals and the likelihood that those proposals are cleared by the AA. We also find that, higher degree of uncertainty hinders merger's stability, by increasing outsider firms' incentives to free-ride on the merger.

The remainder of the paper is organized as follows. In section 2 the basic framework of the model is described. Section 3 presents the pre-merger equilibrium results. Section 4 analyses the equilibrium of the proposed merger formation game. Also, in this section we study merger's stability. In section 5 we present and discuss the obtained numerical results. Finally, section 6 offers some concluding remarks. All the proofs are relegated to the appendix.

In most of the papers, it is assumed that there is market uncertainty because the marginal cost and/or the market demand are unknown to the firms, such as, Novshek and Sonnenschein (1982), Clarke (1983), Sakai (1985), Gal-Or (1985, 1986), Shapiro (1986), Vives (1984, 2002), Li (1985), Sakai and Yamato (1989), Raith (1996), Lagerlöf (2007), Jensen (1992), Elberfeld and O. Nti (2004), among others.

2 Basic Framework

We consider a homogeneous good industry with n firms that compete à la Cournot. The inverse demand function is given by $P = 1 - Q$, where $Q = \sum_{i=1}^{n} q_i$ is the aggregate output and q_i is the quantity produced by firm $i \in \{1, ..., n\}$.

Let k_i denote firm i's capital holdings, where $k_i \in \{1, 2, ..., K\}$. The cost function of a firm i, which owns k_i units of the industry capital and produces q_i units of output, is given by:[3]

$$C(\alpha_j, q_i, k_i) = \frac{\alpha_j K}{k_i} q_i,$$

where K is the total capital in the industry, $\Sigma_i k_i = K$, that is fixed and α_j measures the endogenous efficiency gains, with $\alpha_j \geq 0$.[4]

In the next sections we analyse the results before the merger, where the efficiency gains are known with certainty. We then discuss the results obtained after the merger, for both AA and merger decisions, in a context where there is uncertainty about merger induced efficiency gains.

3 Pre-Merger equilibrium

Before the merger (BM), we assume that firms are symmetric and that each firm owns one unit of capital, that is, $k_i = 1$. Also, each firm knows, with certainty, its level of efficiency gains and therefore $\alpha_j = \alpha$, where α is the common efficiency level of all firms before the merger.

The equilibrium profits and consumer surplus are then given by:[5]

$$\pi_i^{BM} = \frac{(1 - n\alpha)^2}{(n+1)^2}, \text{where } i = 1, .., n. \tag{1}$$

$$CS^{BM} = \frac{1}{2} \frac{n^2 (1 - n\alpha)^2}{(n+1)^2}. \tag{2}$$

Assumption 1. Assume that $\alpha < \frac{1}{n}$.

The previous assumption is imposed in order to exclude the case in which firms in the industry do not produce (are inactive) in the pre-merger equilibrium.

[3]This is a simplified version of the cost structure proposed by Motta and Vasconcelos (2005) and captures also the specific case studied in Horn and Persson (2001). This cost function is based on the one proposed by Perry and Porter (1985). In their framework firms' marginal cost is linear in output and mergers reduce variable costs.

[4]Efficiency gains may result from firm's combined ability to exploit economies of scale or raise larger amounts of capital, but also from complementarity between technological or administrative capabilities of firms (Röller et al., 2001).

[5]For a detailed description of the results before the merger see Appendix A1.

4 Merger Analysis

In this section we analyse both the AA's and firms' merger decision, in a setting where all firms in the industry and the AA are uncertain over the merger induced efficiency gains.

Assume that, at the status quo industry, one firm in the industry is randomly selected and has the opportunity to propose, to the AA, a merger involving $m \geq 2$ firms. This firm may propose a merger with all or a subset of its rivals. Since each firm operates with a constant marginal cost of production, but the level of its marginal cost is a decreasing function of its capital holdings, the resultant merged firm becomes more efficient than outsiders by having $k_i = m$ units of assets. We assume that m is given exogenously. Hence, the merger brings the capital of merging parties into a single larger firm and, therefore, gives rise to endogenous efficiency gains by decreasing the marginal cost. The level of these potential efficiency gains is captured by the parameter α_j.

When the level of efficiency gains is the same both before and after the merger ($\alpha_j = \alpha$), all firms and the AA know with certainty what will be the merger's cost savings. Hence, the higher is the value of α, the stronger the efficiency gains induced by a merger are (Motta and Vasconcelos, 2005; Vasconcelos, 2010).

However, in our model, the merger also brings uncertainty about the induced efficiency gains, that is, all firms (insiders and outsiders) and the AA cannot predict precisely the level of the future merger-induced efficiency gains. Thus, we assume that all players are uncertain on what will be the exact value of the efficiency gains level, as measured by $\alpha_j = \alpha_u$. This level of future efficiency gains, α_u, could be higher, lower or equal than the level of efficiency gains before the merger, given by α. If α_u is equal to α then, after the merger, the merged firm's cost decreases and the outsider firm's cost does not change. If α_u is lower than α, the costs of both insider and outsider firms decrease after the merger. Different results are obtained when α_u is higher than α: if $\alpha_u = k\alpha$, then the insider firm's cost remains the same as before the merger however, the outsider firms' costs increase; if $\alpha_u > k\alpha$ both insider and outsider firms' costs increase after the merger; and if $\alpha_u < k\alpha$ then, after the merger, the insider firm's cost decrease and the outsider firms' costs increase.

Assumption 2. Let α_u be a random variable distributed over $[0, \frac{1}{2n}]$.

By assuming that $\alpha_u < \frac{1}{2n}$, we are excluding the case where outsiders exit the market after the merger takes place. The expected value of the efficiency gains in the future is denoted by $E(\alpha_u) = \mu$, where $\mu \in [0, \frac{1}{2n}]$, and the variance is denoted by $V(\alpha_u) = \sigma^2 > 0$. The σ^2 represents the degree of uncertainty and captures the efficiency gains fluctuation. The higher is σ^2, the greater is the uncertainty about the merger's efficiency gains.

Firm i has to decide whether or not to merge and the AA has to decide whether or not to accept the proposed merged, both without knowing the actual cost of the merged firm in the future. We assume that firms set output decisions after uncertainty is solved, given that we want to study the effects of uncertainty on the decisions of both merger firms and the AA.

We assume that firms are risk neutral and therefore firms' decisions regarding the merger are based on the expected values of their profits. We also assume that the AA is risk neutral, and that its decision is based on the expected value of consumer surplus. We consider that the AA's decision is based on expected consumer surplus instead on social welfare and, thus, the AA approves the merger if the expected consumer surplus increases.[6] Thus, the private incentives to merge and the AA decisions are based on expected values.

We model the interactions between the antitrust authority and the merging firms as a four-stage game, with the following timing:

- **Stage 1:** Firms have to decide whether or not to propose a merger to the AA.
- **Stage 2:** The AA decides whether or not to accept the proposed merger.
- **Stage 3:** Nature chooses α_u and reveals it to all players.
- **Stage 4:** Insider and outsider firms choose quantities competing in the usual Cournot fashion.

In what follows, we will solve the model by following the usual backward induction procedure.

4.1 Product Market Competition

The results presented below refer to the Cournot-Nash equilibrium after the merger (AM), where firms know the level of efficiency gains (since uncertainty has been resolved in the previous stage of the game).

In the fourth-stage of the game, firms have already observed the actual value of α_u so, they choose to produce the quantities that maximize their profits. The Cournot equilibrium profits of insider firm (I) and outsider firms (O) and the consumer surplus (CS) are, respectively, given by:[7]

$$\pi_I^{AM} = \frac{[\, m - n\alpha_u \left((m-1)(m-n) + 1 \right)]^2}{m^2 (n-m+2)^2} \tag{3}$$

$$\pi_O^{AM} = \frac{[\, m - n\alpha_u (2m-1)]^2}{m^2 (n-m+2)^2} \tag{4}$$

[6]By assuming that the AA evaluates mergers according to a consumer surplus standard this does not mean that this is always better than the total welfare standard. However, as Lyons (2002) argued, the consumer surplus standard is applied in most antitrust jurisdictions. Other papers also study how the AA should apply the consumer surplus standard when challenging a merger, such as Besanko and Spulber (1993), Neven and Röller (2005), Vasconcelos (2010), Nocke and Whinston (2010), Jovanovic and Wey (2012), among others.

[7]For a detailed description of the results obtained after the merger see Appendix A2.

$$CS^{AM} = \frac{\left[\, m\,(n-m+1) - n\alpha_u\,(m\,(n-m)+1)\right]^2}{2m^2\,(n-m+2)^2}. \tag{5}$$

Under uncertainty, outsider firms would exit the market if they expected to produce zero in equilibrium, that is, if $\mu \geq \frac{m}{n(2m-1)}$. If the expected efficiency gains from merger are very high, outsider firms would not be able to make positive expected profits and may exit the market. However, since we assume that $\mu < \frac{1}{2n}$, we exclude this possibility.

The next subsection presents the expected insider and outsider profits, the expected consumer surplus and discusses the effects of uncertainty on these outcomes. These results are based on the results obtained after the uncertainty about the efficiency gains has been resolved.

4.2 Expected Profits and Consumer Surplus

The expected profits of both insiders and outsider firms, respectively, $E\left[\pi_I^{AM}\right]$ and $E\left[\pi_O^{AM}\right]$, and the expected consumer surplus, $E\left[CS^{AM}\right]$, are then given by:[8]

$$E\left[\pi_I^{AM}\right] = \frac{\left[m - n\mu\,((m-1)\,(m-n)+1)\right]^2}{m^2\,(n-m+2)^2} + \sigma^2 \frac{n^2\left[(m-1)\,(m-n)+1\right]^2}{m^2\,(n-m+2)^2} \tag{6}$$

$$E\left[\pi_O^{AM}\right] = \frac{\left[m - n\mu\,(2m-1)\right]^2}{m^2\,(n-m+2)^2} + \sigma^2 \frac{n^2\,(2m-1)^2}{m^2\,(n-m+2)^2} \tag{7}$$

$$E\left[CS^{AM}\right] = \frac{\left[m\,(n-m+1) - n\mu\,(m\,(n-m)+1)\right]^2}{2m^2\,(n-m+2)^2} + \sigma^2 \frac{n^2\left[m\,(n-m)+1\right]^2}{2m^2\,(n-m+2)^2} \tag{8}$$

As we can see, uncertainty affects both the expected profits and the expected consumer surplus.

Lemma 1: An increase in the level of efficiency gains' uncertainty benefits both insider and outsider firms but also the consumers, i.e., both $E\left[\pi_I^{AM}\right]$, $E\left[\pi_O^{AM}\right]$ and $E\left[CS^{AM}\right]$ are increasing in σ^2. Moreover, uncertainty benefits more the insider firm than the outsider firms, if:

$$\frac{\partial E\left[\pi_I^{AM}\right]}{\partial \sigma^2} > \frac{\partial E\left[\pi_O^{AM}\right]}{\partial \sigma^2} \Longleftrightarrow n > \frac{m(m+1)}{m-1} \equiv \overline{n} \quad \wedge \quad m > 1. \tag{9}$$

Proof. See Appendix B. □

A higher degree of uncertainty improves firms' expected profits, because profit functions are convex in α_u. Since a firm's profit is convex in its own cost, uncertainty increases the firm's expected profit. Both insider and outsider firms will choose their quantities without knowing the exact value of the costs.[9] The insider firm expects that, after the merger, the cost is going to decrease due to the synergies generated, that is, due to an

[8] For a detailed description of the results obtained with expectations see Appendix A3.

[9] This is a similar result as obtained by Zhou (2008a)'s paper.

increase in the capital stock of the merged firm. However, the insider firm does not know how much is this cost saving, since the insider firm's cost decreases in three different scenarios: (i) $\alpha_u < \alpha$, (ii) $\alpha_u = \alpha$ and (iii) $\alpha < \alpha_u < k\alpha$. Since the insider firm expects a lower cost, it knows that, even without uncertainty, the resultant firm is going to produce a large quantity. The uncertainty about how much the cost reduces also affects the outsider firms. Although each outsider firm still has one unit of capital, if the cost saving is very high, we know that, without uncertainty, outsiders will respond by reducing quantity. However, under uncertainty, only if α_u is smaller than α, there will also be significant cost savings for outsider firms and this will increase both quantities and profits of all outsider firms. If this is the case, the expected net effect is an increase in the total quantity and a decrease in the total price (which also happens without uncertainty).

Without uncertainty and cost synergies, mergers usually lead to a sharp reduction of the total output. As consequence, in deterministic models, mergers usually reduce the consumer surplus. However, when there are cost synergies, the reverse can actually occur even without uncertainty. In our model, the higher is the uncertainty about the cost synergies generated by the merger, the larger the value of σ^2 and the greater the expected consumer surplus will be. After the merger, both the insider and outsider firms expect to benefit from cost synergies if α_u is smaller than α and, therefore, expect to increase the quantities produced. However, it may happen that, after the merger, α_u is greater than α and, therefore, the insider firm's cost decreases but the outsider firms' cost increase. In this case, the insider firm still produces more quantity but the outsider firms' will respond by producing a lower quantity. Nevertheless, under uncertainty, both insider and outsider firms do not adjust so sharply their production. Consequently, the net effect is an increase in the total quantity in the market, which affects positively the consumer surplus. Hence, both profits and consumer surplus are increasing functions with respect to the uncertainty parameter, σ^2.

Additionally, under uncertainty, the profits of the insider and outsider firms are affected differently. More precisely, if the number of firms in the market is higher than a threshold $\overline{n}$, uncertainty has a higher effect on insider firm's profits than on outsiders. However, if the number of firms in the market is lower than $\overline{n}$, the reverse occurs.

Further, the extent of the uncertainty effect on both expected profits and consumer surplus is shown to depend on the number of firms in the market (n) and on the number of firms involved in the merger (m). The higher is the number of firms in the market, the higher the impact of uncertainty on consumer surplus and on insider profits is, but the lower the impact on outsider profits is. Also, the higher is the number of insider firms, the larger the effect of uncertainty on (insider and outsider) profits and on consumer surplus is. However, from numerical simulation, we have seen that as the number of insider firms increases, that is, when insiders involve more than 50% of the firms in the market, uncertainty begins to have a negative effect on both consumer surplus and insider profits since these start to decrease.

Lemma 2: If:

(i) $m = n$ (full merger), both expected consumer surplus and insider profits always decrease with the expected mean over the efficiency gains level;

(ii) $m < n$ (partial merger), the expected outsider profits and the expected consumer surplus decrease with the level of expectations over the efficiency gains, however the expected insider profits increase if those expectations satisfy the following threshold: $\mu > \frac{m}{n[(m-n)(m-1)+1]}$.

Proof. See Appendix B. □

Lemma 2 states that, after the merger, the expected outsider firms' profits and the expected consumer surplus decrease with the expected mean over the efficiency gains (μ). Also, the expected insider firms' profits increase if the expectations over the efficiency gains are not too high and if the merger does not involve all the firms in the market.

Before looking at the results obtained to the AA decision, we assume the following:[10]

Assumption 3. Assume that $\mu = \alpha$.

Assumption 3 states that the expected efficiency gains level are equal to the benchmark firm's efficiency gains level, that is, the efficiency gains level of both insider and outsider firms at the status quo industry. After the merger, the insider firm expects that its cost is going to reduce, since now only one firm encompasses all capital assets of the merging firms. However, the insider firm does not know how much is this reduction. Therefore, for simplicity, we assume that the expectations on the level of efficiency gains are rational and equal to the efficiency gains before the merger. Hence, the merged firm expects the cost reduction to only depend on the level of capital of the merging parties. This assumption allows us isolate the effects of uncertainty on both AA's and firms' merger decisions. By assuming $\mu = \alpha$ the interpretation of the results is the same as before: the higher is α, the stronger the efficiency gains induced from the merger are.

Recall that we are only considering the scenario in which outsider firms do not exit the market after the merger. Hence, the merger induced market structure consists of $n - m + 1$ firms: $n - m$ outsiders with one unit of capital and one firm with $k = m$ units of capital.[11]

In what follows, both AA and merging firms decisions are made under uncertainty.

[10]This assumption is not crucial. Actually, we obtain our main results without assuming it, but, in order to present clear expressions we need to impose it. See Appendix C for further information on the results obtained without Assumption 3.

[11]However, we do not exclude the case where the merger involves all the firms in the industry. See Appendix A4 for further information on full merger results.

4.3 Antitrust Authority's Decision

The AA decides whether to allow or block any proposed merger. The AA accepts the merger if the expected consumer surplus after the merger is greater or equal than the consumer surplus before the merger ($E\left[CS^{AM}\right] \geq CS^{BM}$).

Proposition 1. The AA will accept the merger if $E\left[CS^{AM}\right] - CS^{BM} > 0$, i.e. iff:

$$\sigma^2 \geq \sigma^2_{AA\{m,n-m\}} \equiv \frac{m^2\left(n-m+2\right)^2 n^2\left(1-n\alpha\right)^2 - \left[m\left(n-m+1\right) - n\alpha\left(m\left(n-m\right)+1\right)\right]^2\left(n+1\right)^2}{\left(n+1\right)^2 n^2\left[m\left(n-m\right)+1\right]^2}. \tag{10}$$

Proof. See Appendix B. □

Proposition 1 states that when uncertainty is high, there is a greater likelihood that the AA accepts the merger. Since the consumer surplus before the merger (whose expression is given in (2)) does not depend on the uncertainty parameter, the higher is the uncertainty, σ^2, the greater is the expected consumer surplus after the merger. Consequently, when uncertainty is high, the expected consumer surplus variation also increases.

When there are cost synergies, mergers can improve consumer surplus by increasing output. This improvement could actually be higher when there is uncertainty about cost synergies. Therefore, under uncertainty there is a greater likelihood that the merger is accepted by the AA.

4.4 Merger Decision

In this section, we examine firms incentives to merge. Firms decisions on whether to merge or not result from the comparison between the expected profits of the merged firm with the profits before the merger. Hence, the merger profitability condition depends on the expected profits of the merged firm, that is, firms will propose the merger if $E\left[\pi_I^{AM}\right] \geq m\pi^{BM}$.

Proposition 2. If the level of uncertainty is sufficiently high, there is a greater likelihood that firms propose a merger. Firms will propose the merger if they anticipate that it will be profitable, i.e. iff:[12]

$$\sigma^2 \geq \sigma^2_{MP\{m,n-m\}} \equiv \frac{\left(1-n\alpha\right)^2 m^3\left(n-m+2\right)^2 - \left[m - n\alpha\left(\left(m-1\right)\left(m-n\right)+1\right)\right]^2\left(n+1\right)^2}{\left(n+1\right)^2 n^2\left[\left(m-1\right)\left(m-n\right)+1\right]^2}. \tag{11}$$

Proof. See Appendix B. □

From Proposition 2, we conclude that uncertainty promotes the expected merger profitability. As the extent of the variance exceeds a certain threshold ($\sigma^2_{MP\{m,n-m\}}$) the expected profit of the merged firm becomes larger than the sum of the firm's profits in the benchmark case, and firms that face cost uncertainty choose to merge. In the deterministic model, without efficiency gains, unless the merger involves a sufficient number of insiders, most of the horizontal mergers are unprofitable (Salant et al., 1983). However, in our model, as the uncertainty increases, the

[12]Note that $n > 3$, otherwise insider firm's profit would not depend on the σ^2 parameter and therefore this expression would not exist.

expected profit also increases because the gain of the optimal quantity adjustment increases, and therefore it is possible that the expected profit of insider firm (as measured by (6)) exceeds the sum of profits of the pre-merger firms (whose expression is given in (1)). Hence, the insider firm has a higher profit both directly from the cost advantage and indirectly from the favourable responses from outsider firms. Since the pre-merger profits are not affected by uncertainty, it is expected that the merger becomes at least more profitable than in the deterministic models.

As the degree of uncertainty grows larger, firms have more incentives to merge and therefore, we conclude that cost uncertainty is able to induce the firms to merge. This relationship between merger profitability and cost uncertainty is also investigated by Banal-Estañol (2007) and Zhou (2008a,b). In a different framework from ours, Banal-Estañol (2007) considers that firms face idiosyncratic uncertainty about costs and that uncertainty generates an informational advantage only to the merging firms, increasing merger profitability. Also, Zhou (2008a,b) argues that when costs are uncertain and firms choose quantities before the uncertainty is resolved, a merger is more profitable the greater the uncertainty. However, differently from our paper, the author assume that after the merger, costs are realized and each firm learns its own costs but not the cots of its rivals. Zhou (2008a,b) shows that firm's incentives to merge are enhanced by production rationalization.

4.5 Free-riding Problem

In this subsection we study the effects of uncertain efficiency gains on merger's stability.

In the deterministic models, mergers are usually not stable, given that outsider firms are the ones that benefit most from the merger (free-riding problem). In order to assess if there is a free-riding problem in our model, we compare the expected profit of the merged firm with the expected profits of outsiders.

Proposition 3. There is a free-riding problem if $E\left[\pi_I^{AM}\right] < mE\left[\pi_O^{AM}\right]$, i.e., iff:

$$\sigma^2 > \sigma_{fr}^2 \equiv \frac{m(m-1)[2n\alpha(m+n+1)-m]}{mn^2(2m-1)^2 - n^2[(m-1)(m-n)+1]^2} - \alpha^2$$

Proof. See Appendix B. □

Proposition 3 states that, if the uncertainty over merger's efficiency gains is high, outsider firms benefit more from the merger than the insider firms. Hence, as the uncertainty increases, mergers become less stable. After the merger, the merged firm expects to produce more quantity than each outsider firm, due to the synergies generated. The outsider firms' production depends on whether α_u is greater, equal or lower than α. If the level of expected efficiency gains increases and becomes higher than α after the merger, the insider firm still produces more quantity but the outsider firms' will respond by producing a lower quantity. While the profit of the insider firm always increases with the uncertainty, that of an excluded rival also increases as a result of the merger. For high degree of uncertainty, firms would prefer to wait for their rivals to merge and, thus, benefit from the merger.

Lemma 3. Keeping the number of firms constant (n), as the number of insider increases (m), the region of uncertainty level below which there is no free-riding decreases, i.e., $\frac{\partial \sigma^2_{fr}}{\partial m} > 0$. Also, keeping the number of insiders constant, an increase in the number of firms in the market, increases the region of uncertainty level below which there is no free-riding, i.e., $\frac{\partial \sigma^2_{fr}}{\partial n} < 0$.

We find that, under uncertainty about cost synergies and keeping the number of firms constant, as the number of insider firms increases, more difficult is for the merger to be stable. Firms have more incentives to deviate and to compete against its rivals as an outsider. Also, keeping the number of insiders constant, we find that the merger becomes more stable, as the number of firms in the market increases.

5 Numerical Application

In what follows, assume that the total quantity of capital available in the industry is equal to four units ($K = 4$) and that this capital is equally distributed amongst four firms ($n = 4$) in the status quo industry structure.

We analyse the results obtained for both firms and AA decisions for three cases:

1) Merger involving two firms ($m = 2$);
2) Merger involving three firms ($m = 3$); and
3) Merger to monopoly ($m = 4$).

Again, we restrict our attention to the case in which, after the merger, outsiders do not exit the market ($\alpha_u < \frac{1}{8}$). Further, we also analyse if there is a free-riding problem for both cases 1 and 2.

The following proposition sums up the AA decisions for the three merger cases.

Proposition 4.

- The AA accepts the merger involving two firms (case 1) if: $\sigma^2 \geq \sigma^2_{AA\{2,1,1\}}$;
- The AA accepts the merger involving three firms (case 2) if: $\sigma^2 \geq \sigma^2_{AA\{3,1\}}$;
- The AA accepts the merger to monopoly (case 3) if: $\sigma^2 \geq \sigma^2_{AA\{4\}}$.

Proof. See Appendix B. □

The following proposition sums up firms decisions for the three merger cases.

Proposition 5.

- Firms will propose a merger involving two firms (case 1) if: $\sigma^2 \geq \sigma^2_{MP\{2,1,1\}}$;
- Firms will propose a merger involving three firms (case 2) if: $\sigma^2 \geq \sigma^2_{MP\{3,1\}}$;
- Firms will propose a merger to monopoly (case 3) if: $\sigma^2 \geq \sigma^2_{MP\{4\}}$.

Proof. See Appendix B. □

The following proposition sums up the results obtained for the free-riding problem for the first two merger cases.

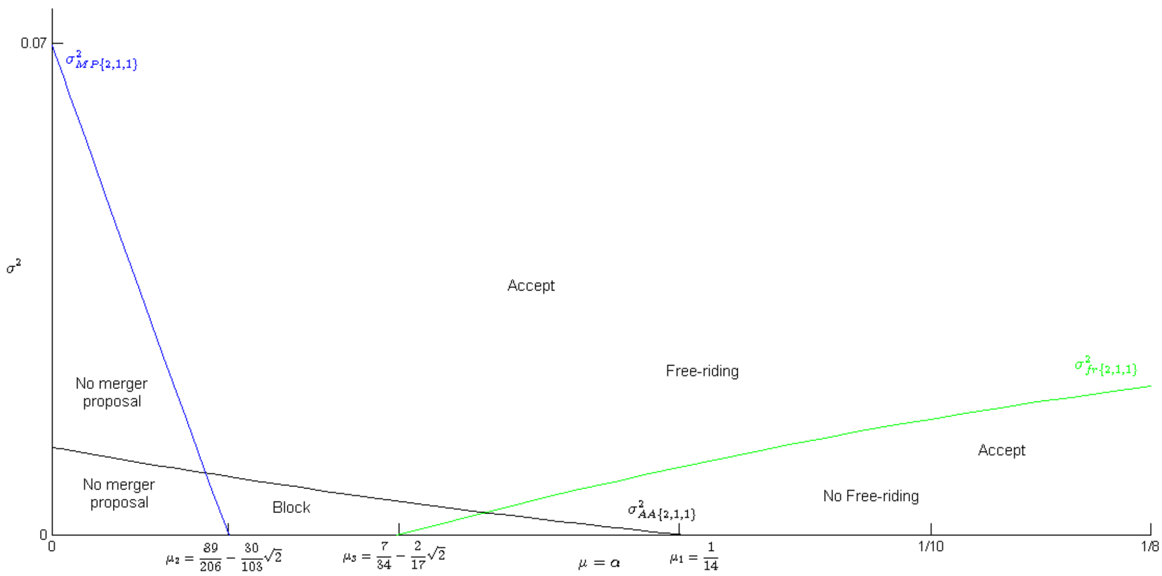
Proposition 6.

- There is a free-riding problem (case 1) if: $\sigma^2 \geq \sigma^2_{fr\{2,1,1\}}$;
- There is a free-riding problem (case 2) if: $\sigma^2 \geq \sigma^2_{fr\{3,1\}}$.

Proof. See Appendix B. $\square$

The next three figures represent graphically the results obtained in Propositions 3, 4 and 5.

Figure 1: AA's decision ($m = 2$)



Analysing Figure 1, we conclude that, when uncertainty and expected mean about the future efficiency gains level are low, $\sigma^2 < \sigma^2_{MP\{2,1,1\}}$ and $\mu < \mu_2 = \frac{89}{206} - \frac{30}{103}\sqrt{2}$, any two-firm merger will never be proposed to the AA. As the expected efficiency gains increase, $\sigma^2_{MP\{2,1,1\}} < \sigma^2 < \sigma^2_{AA\{2,1,1\}}$ and $\mu_2 < \mu < \mu_1 = \frac{1}{14}$, the two firm merger will be proposed to the AA, however it will be blocked since it is expected to reduce the consumer surplus. Further, if both expectations and uncertainty are high, $\sigma^2 > \sigma^2_{AA\{2,1,1\}}$, any two firm merger will be proposed and accepted by the AA. Additionally, there exist a free-riding problem when $\sigma^2 > \sigma^2_{fr\{2,1,1\}}$, that is, outsider firms earn more from the merger than the insider firms and this could decrease merger's stability. Also, if the expected efficiency gains are higher than $\mu_3 = \frac{7}{34} - \frac{2}{17}\sqrt{2}$ and if uncertainty is not very high $\sigma^2 < \sigma^2_{fr\{2,1,1\}}$ there is a possibility that the merger is accepted by the AA and there is no free-riding problem.

Figure 2: AA's decision $(m = 3)$

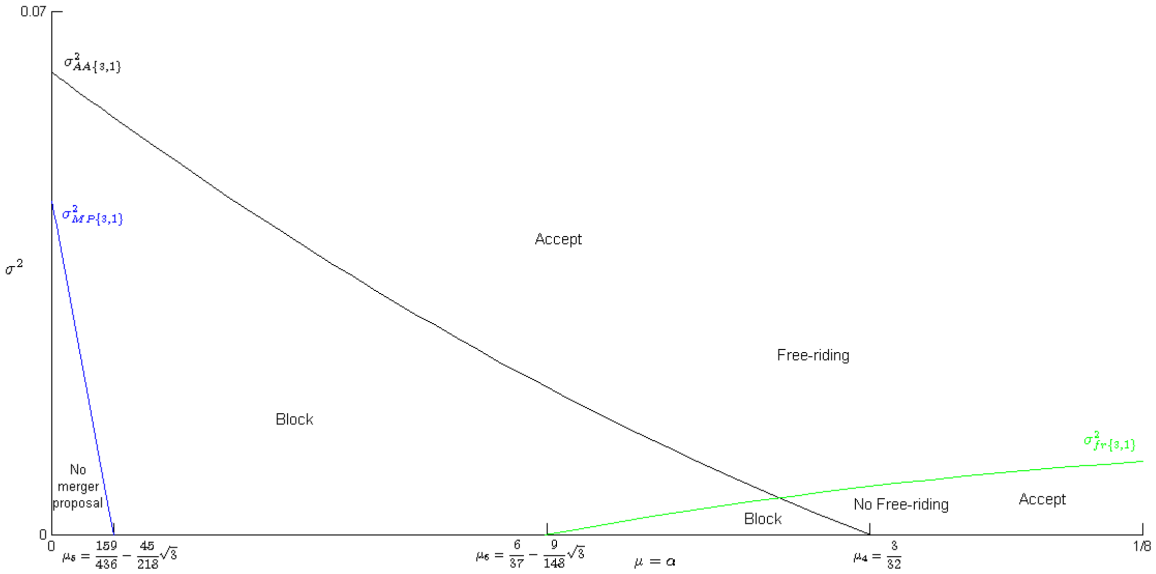


Figure 3: AA's decision $(m = 4)$

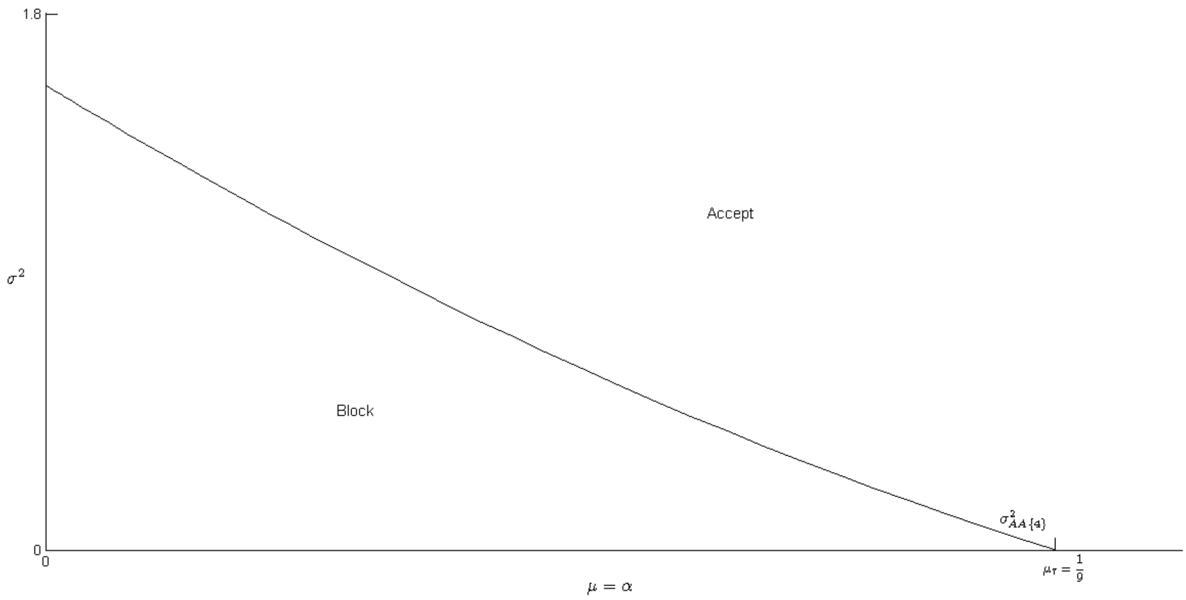
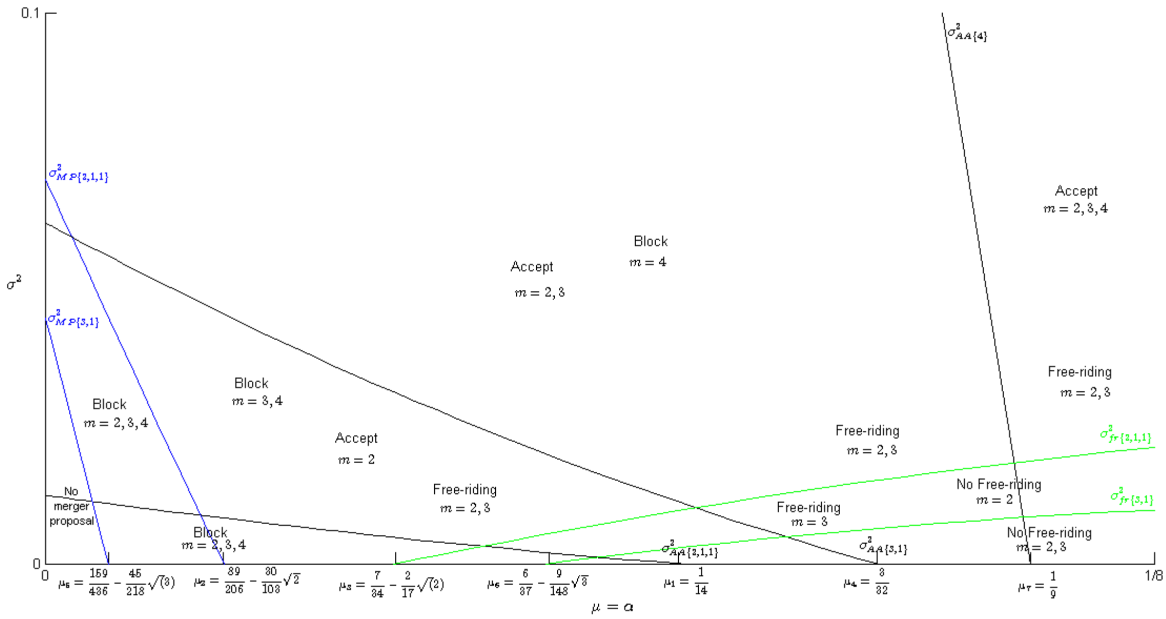


Figure 2 summarizes the results obtained when the merger involves three firms. We can observe that when both expected efficiency gains and uncertainty are low, $\sigma^2 < \sigma^2_{MP\{3,1\}}$, $\mu < \mu_5 = \frac{159}{436} - \frac{45}{218}\sqrt{3}$, any three-firm merger will never be proposed to the AA. As the expected efficiency gains increase, $\sigma^2_{MP\{3,1\}} < \sigma^2 < \sigma^2_{AA\{3,1\}}$ and $\mu < \mu_4 = \frac{3}{32}$, any three-firm merger proposal will be proposed and blocked by the AA. However, when both the expected efficiency gains and the uncertainty increase, $\sigma^2 > \sigma^2_{AA\{3,1\}}$ and $\mu_4 < \mu < \frac{3}{32}$, the three-firm merger will always be proposed and accepted by the AA. Further, in this case there is also a free-riding problem when $\sigma^2 > \sigma^2_{fr\{3,1\}}$, that is, the only outsider firm earns more from the merger than the insider firms and this could also decrease merger's stability. Also, if the expected efficiency gains are higher than $\mu_6 = \frac{6}{37} - \frac{9}{148}\sqrt{3}$ and if uncertainty is not very high $\sigma^2 < \sigma^2_{fr\{3,1\}}$ there is a greater likelihood that the merger of three firms is accepted by the AA and outsiders have no incentives to free ride it.

The results obtained for the full merger case are illustrated in Figure 3. Since the $\sigma^2_{MP\{4\}}$ is always negative, for any level of uncertainty, the firms have always incentive to propose a merger to monopoly. However, if the expected efficiency gains and uncertainty are low, $\sigma^2 < \sigma^2_{AA\{4\}}$ and $\mu < \mu_7 = \frac{1}{9}$, any merger proposal to monopoly will be blocked by the AA. As both uncertainty and expected efficiency gains increase, $\mu > \mu_7 = \frac{1}{9}$, and $\sigma^2 > \sigma^2_{AA\{4\}}$, the merger to monopoly will always be accepted by the AA.

Figure 4 sums up the results obtained for the three merger cases.

Figure 4: AA's decision ($m = 2, 3, 4$)



We conclude that when the level of uncertainty and expected efficiency gains are high, the firms have incentives to propose any type of merger and this merger has a greater likelihood of being accepted by the AA. When both levels of uncertainty and expected efficiency gains are low, the firms have less incentives to propose the merger. As the expected efficiency gains increase, the firms usually propose the merger in each case, however the AA rejects it for all or some particular cases. Usually in the models without uncertainty about the future synergies generated by the merger, the AA only accepts the merger if these efficiency gains are sufficiently high (Motta and Vasconcelos, 2005; Vasconcelos, 2010). In contrast, the present paper shows that, with uncertainty, the probability of the merger being accepted is higher than without uncertainty and, for some cases, mergers could be accepted even if the expected efficiency gains are intermediate. Finally, as the number of insider firms increases it is more difficult for the merger to be approved by any AA, even with uncertainty, unless the expected efficiency gains are very high.

6 Concluding Remarks

In this paper we investigate the effects of mergers in a context wherein, both firms and the AA are uncertain about the level of efficiency gains and, therefore, this uncertainty is going to influence the decision of AA but also firms' incentives to merge.

In the absence of uncertainty and when firms are symmetric, the merger will not be profitable unless 80% of the firms in the industry are part of the merger. However, we find that, under uncertainty and asymmetric firms, even when the merged firm is not composed of at least 80% of the firms in the industry, when the uncertainty increases, the expected profit of the merged firm exceeds insiders' pre-merger profits, and therefore firms have incentives to merge. Moreover, we find that the higher the level of uncertainty in the market regarding merger induced efficiencies, the higher the expected consumer surplus after the merger. This then implies that, given the AAs usually base their merger policy decisions on a consumer welfare standard, higher uncertainty also increases the likelihood that a merger proposal ends up being approved by the AA. Further, we find that a higher degree of uncertainty decreases merger stability, by increasing the likelihood that outsider firms benefit more from the merger than the insider firm.

The framework and the assumptions we have assumed are of a particular kind. It would be interesting to extend our model to assess the effects on the decisions of all players, when both firms (insider and outsiders) and the AA face different degrees of uncertainty over the merger's cost savings. We think that this is a very important subject for further research.

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Appendix

Appendix A - Model derivations

A1. Pre-Merger equilibrium

Before the merger, firms decide individually their quantity. Knowing that firms are symmetric and that each firm owns one unit of asset, the quantity produced by each firm is given by: $q_i = \frac{1-n\alpha}{(n+1)}$, with $i = 1, .., n$. Therefore, the total quantity and price are respectively given by $Q = n\frac{1-n\alpha}{(n+1)}$ and $P^{BM} = \frac{1+n^2\alpha}{n+1}$. The profit and the consumer surplus are then given by: $\pi_i^{BM} = \frac{(1-n\alpha)^2}{(n+1)^2}$ and $CS^{BM} = \frac{1}{2}\frac{n^2(1-n\alpha)^2}{(n+1)^2}$.

A2. Post-Merger equilibrium

After the merger of m firms, insider firm now owns m units of capital while the outsiders still own 1 unit of capital. Then the merged firm will produce, in equilibrium, $Q_I = \frac{m-n\alpha_u((m-1)(m-n)+1)}{m(n-m+2)}$ and the outsiders will produce $q_o = \frac{m-n\alpha_u(2m-1)}{m(n-m+2)}$.

The equilibrium profits are then given by:
$\pi_I^{AM} = \frac{[\ m-n\alpha_u((m-1)(m-n)+1)]^2}{m^2(n-m+2)^2}$ and $\pi_O^{AM} = \frac{[\ m-n\alpha_u(2m-1)]^2}{m^2(n-m+2)^2}$.

The consumer surplus (CS) is then given by:
$CS^{AM} = \frac{[\ m(n-m+1)-n\alpha_u(m(n-m)+1)]^2}{2m^2(n-m+2)^2}$.

A3. Expected Insider and Outsider Profits and Expected Consumer Surplus

Applying the expectations on equation (3), we obtain equation (6):

$$E\left[\pi_I^{AM}\right] = E\left[\frac{m^2 - 2mn\alpha_u\left((m-1)(m-n)+1\right) + n^2\alpha_u^2\left((m-1)(m-n)+1\right)^2}{m^2(n-m+2)^2}\right]$$
$$= \frac{m^2 - 2mnE(\alpha_u)\left((m-1)(m-n)+1\right) + n^2E\left(\alpha_u^2\right)\left((m-1)(m-n)+1\right)^2}{m^2(n-m+2)^2}.$$

Knowing that $V(\alpha_u) = E\left(\alpha_u^2\right) - E^2(\alpha_u)$ and that $E(\alpha_u) = \mu$ and $V(\alpha_u) = \sigma^2$ we get that:

$$E\left[\pi_I^{AM}\right] = \frac{m^2 - 2mn\mu\left((m-1)(m-n)+1\right) + n^2\left(\sigma^2+\mu^2\right)\left((m-1)(m-n)+1\right)^2}{m^2(n-m+2)^2}$$
$$= \frac{\left[m - n\mu\left((m-1)(m-n)+1\right)\right]^2}{m^2(n-m+2)^2} + \sigma^2\frac{n^2\left[(m-1)(m-n)+1\right]^2}{m^2(n-m+2)^2}.$$

Applying the expectations on equation (4), we get equation (7):

$$E\left[\pi_O^{AM}\right] = E\left[\frac{\left[\ m - n\alpha_u(2m-1)\right]^2}{m^2(n-m+2)^2}\right] = E\left[\frac{m^2 - 2mn\alpha_u(2m-1) + n^2\alpha_{iu}^2(2m-1)^2}{m^2(n-m+2)^2}\right]$$
$$= \frac{\left[m - n\mu(2m-1)\right]^2}{m^2(n-m+2)^2} + \sigma^2\frac{n^2(2m-1)^2}{m^2(n-m+2)^2}.$$

Applying the expectations on equation (5) we get equation (8):

$$E\left[CS^{AM}\right] = E\left[\frac{\left[\, m\left(n-m+1\right) - n\alpha_u\left(m\left(n-m\right)+1\right)\right]^2}{2m^2\left(n-m+2\right)^2}\right]$$
$$= \frac{\left(m\left(n-m+1\right) - n\mu\left(m\left(n-m\right)+1\right)\right)^2}{2m^2\left(n-m+2\right)^2} + \sigma^2\frac{n^2(m\left(n-m\right)+1)^2}{2m^2\left(n-m+2\right)^2}.$$

A4. Full Merger Results

The monopoly (M) profit, price and consumer surplus for Stages 3 and 4 are given by:

$$\pi^M = \left(\frac{m-n\alpha_u}{2m}\right)^2$$
$$P^M = \frac{m+n\alpha_u}{2m}$$
$$CS^M = \frac{1}{2}\frac{\left(m-n\alpha_u\right)^2}{\left(2m\right)^2}$$
$$E\left[\pi^M\right] = \frac{m^2-2mn\mu+n^2\left(\sigma^2+\mu^2\right)}{4m^2}$$
$$E\left[CS^M\right] = \frac{m^2-2mn\mu+n^2\left(\sigma^2+\mu^2\right)}{8m^2}.$$

Appendix B - Proofs

B1. Proof of Lemma 1

Deriving $E\left[\pi_I^{AM}\right]$, $E\left[\pi_O^{AM}\right]$ and $E\left[CS^{AM}\right]$ with respect to the variance, σ^2, we obtain:

$\frac{\partial E\left[\pi_I^{AM}\right]}{\partial\sigma^2} = \frac{n^2((m-1)(m-n)+1)^2}{m^2(n-m+2)^2} > 0$

$\frac{\partial E\left[\pi_O^{AM}\right]}{\partial\sigma^2} = \frac{n^2(2m-1)^2}{m^2(n-m+2)^2} > 0$

$\frac{\partial E\left[CS^{AM}\right]}{\partial\sigma^2} = \frac{n^2(m(n-m)+1)^2}{2m^2(n-m+2)^2} > 0$

$\frac{\partial E\left[\pi_I^{AM}\right]}{\partial\sigma^2} - \frac{\partial E\left[\pi_O^{AM}\right]}{\partial\sigma^2} = \frac{n^2(m-1)[n(m-1)-m(m+1)]}{m^2(n-m+2)}$

$\frac{\partial E\left[\pi_I^{AM}\right]}{\partial\sigma^2} > \frac{\partial E\left[\pi_O^{AM}\right]}{\partial\sigma^2} \iff n > \frac{m(m+1)}{m-1} \equiv \overline{n} \wedge m > 1 \wedge n > m-2.$

Both $E\left[\pi_I^{AM}\right]$, $E\left[\pi_O^{AM}\right]$ and $E\left[CS^{AM}\right]$ are increasing with respect to the variance, σ^2. Thus the profits and the consumer surplus increase as the uncertainty grows.

■

B2. Proof of Lemma 2

Differentiating $E\left[\pi_I^{AM}\right]$, $E\left[\pi_O^{AM}\right]$ and $E\left[CS^{AM}\right]$ with respect to μ we find that the derivatives are positive if and only if:

$$\frac{\partial E\left[\pi_I^{AM}\right]}{\partial\mu} = \frac{-2mn((m-1)(m-n)+1)+2n^2\mu((m-1)(m-n)+1)^2}{m^2(n-m+2)^2} > 0 \Leftrightarrow \mu > \frac{m}{n((m-n)(m-1)+1)}.$$

$$\frac{\partial E\left[\pi_O^{AM}\right]}{\partial\mu} = \frac{-2mn(2m-1)+2n^2\mu(2m-1)^2}{m^2(n-m+2)^2} > 0 \Leftrightarrow \mu > \frac{m}{n(2m-1)}.$$

In the region of parameter values wherein the outsiders are active after the merger, $\mu < \frac{m}{n(2m-1)}$, their expected profits always decrease in the level of expected efficiency gains. Otherwise, they will exit the market (and here we assume that all firms are active after the merger takes place).Hence, when the merger does not involve all the firms in the industry $(n > m)$, outsiders' profits could increase with the level of expectations if $\mu > \frac{m}{n(2m-1)}$. However, in this region outsiders would exit the market. Since we have excluded this scenario, outsiders' profits always decrease with the μ.

$$\frac{\partial E\left[CS^{AM}\right]}{\partial\mu} = \frac{-2nm(n-m+1)(m(n-m)+1)+2n^2\mu(m(n-m)+1)^2}{2m^2(n-m+2)^2} > 0 \Leftrightarrow \mu > \frac{m(n-m+1)}{n(m(n-m)+1)}.$$

Different results are obtained when the level of expectations is not too high and the merger does not involve all the firms in the industry $(n > m)$. Both expected insider profits and expected consumer surplus increase if the expectations satisfy the following thresholds: $\mu > \frac{m}{n((m-n)(m-1)+1)}$ and $\mu > \frac{m(n-m+1)}{n(m(n-m)+1)}$, respectively. From the numerical simulation we know that $\mu > \frac{m}{n((m-n)(m-1)+1)}$ is always negative and since we assume that $\mu > 0$, hence when the merger does not involve all the firms in the industry, the expected insiders' profits increase with the expected efficiency gains level. However, we also know that $\mu > \frac{m(n-m+1)}{n(m(n-m)+1)}$ is always greater than $\frac{1}{2n}$. Hence, when the merger does not involve all the firms in the industry, the expected consumer surplus decreases with the expected efficiency gains level.

Additionally, if the merger involves all the firms in the industry $(n = m)$ both expected consumer surplus and expected insider profits increase if $\mu > 1$. Since we assume that $\mu < \frac{1}{2n} < 1$, therefore both expected consumer surplus and insider profits always decrease with the level of expectations, when the merger is to monopoly.

■

B3. Proof of Proposition 1

Equation (9) is obtained from the expression for the expected variation of the consumer surplus. It is straightforward to show that $\Delta ECS > 0$ holds for $\sigma^2 > \sigma^2_{\{m,n-m\}}$. Hence, the AA will accept the merger if it anticipates it will enhance expected consumer surplus, i.e., iff: $E\left[CS^{AM}\right] \geq CS^{BM}$. This happens when:

$$\Delta E[CS] = \frac{[m(n-m+1)-n\mu(m(n-m)+1)]^2}{2m^2(n-m+2)^2} + \sigma^2\frac{n^2[m(n-m)+1]^2}{2m^2(n-m+2)^2} - \frac{1}{2}\frac{n^2(1-n\alpha)^2}{(n+1)^2} \geq 0$$

From Assumption 3, we know that $\alpha = \mu$:

$$\frac{[m(n-m+1)-n\alpha(m(n-m)+1)]^2}{2m^2(n-m+2)^2} + \sigma^2\frac{n^2[m(n-m)+1]^2}{2m^2(n-m+2)^2} - \frac{1}{2}\frac{n^2(1-n\alpha)^2}{(n+1)^2} \geq 0$$

Solving with respect to σ^2, we get:

$$\sigma^2 \geq \frac{m^2(n-m+2)^2n^2(1-n\alpha)^2-[m(n-m+1)-n\alpha(m(n-m)+1)]^2(n+1)^2}{(n+1)^2n^2[m(n-m)+1]^2}.$$

■

B4. Proof of Proposition 2

Firms will propose the merger if: $E\left[\pi_I^{AM}\right] \geq m\pi^{BM}$

$$E[MP] \equiv \frac{[m-n\mu((m-1)(m-n)+1)]^2}{m^2(n-m+2)^2} + \sigma^2\frac{n^2[(m-1)(m-n)+1]^2}{m^2(n-m+2)^2} - m\frac{(1-n\mu)^2}{(n+1)^2} \geq 0$$

Solving with respect to σ^2 and knowing that $\mu = \alpha$ get:

$$\sigma^2 \geq \frac{(1-n\alpha)^2 m^3(n-m+2)^2 - [m-n\alpha((m-1)(m-n)+1)]^2(n+1)^2}{(n+1)^2 n^2[(m-1)(m-n)+1]^2}.$$

■

B5. Proof of Proposition 3

There is a free-riding problem when $E\left[\pi_I^{AM}\right] < mE\left[\pi_O^{AM}\right]$, that is,

$$m\left(\frac{[m-n\mu(2m-1)]^2}{m^2(n-m+2)^2} + \sigma^2\frac{n^2(2m-1)^2}{m^2(n-m+2)^2}\right) - \left(\frac{[m-n\mu((m-1)(m-n)+1)]^2}{m^2(n-m+2)^2} + \sigma^2\frac{n^2[(m-1)(m-n)+1]^2}{m^2(n-m+2)^2}\right) > 0$$

Solving with respect to σ^2 :

$$\sigma^2 > \frac{m(m-1)[2n\alpha(m+n+1)-m]}{mn^2(2m-1)^2 - n^2[(m-1)(m-n)+1]^2} - \alpha^2.$$

■

B6. Proof of Proposition 4

Replacing $n = 4$ and $m = 2, 3, 4$, for each merger case, we get the results for the AA's decision in Proposition 4:

- The AA accepts the merger of two firms (case 1) if: $\sigma^2 \geq \sigma^2_{AA\{2,1,1\}} \equiv \frac{(14\mu-1)(114\mu-31)}{2500}$;
- The AA accepts the merger of three firms (case 2) if: $\sigma^2 \geq \sigma^2_{AA\{3,1\}} \equiv \frac{(32\mu-3)(112\mu-33)}{1600}$;
- The AA accepts the merger to monopoly (case 3) if: $\sigma^2 \geq \sigma^2_{AA\{4\}} \equiv \frac{3(9\mu-1)(37\mu-13)}{25}$.

■

B7. Proof of Proposition 5

Replacing $n = 4$ and $m = 2, 3, 4$, for each merger case, we get the results for the merger profitability in Proposition 5:

- Firms will propose a merger of two firms (case 1) if: $\sigma^2 \geq \sigma^2_{MP\{2,1,1\}} \equiv \frac{-356\mu+412\mu^2+7}{100}$;
- Firms will propose a merger of three firms (case 2) if: $\sigma^2 \geq \sigma^2_{MP\{3,1\}} \equiv \frac{-1272\mu+1744\mu^2+9}{200}$;
- Firms will propose a merger to monopoly (case 3) if: $\sigma^2 \geq \sigma^2_{MP\{4\}} \equiv \frac{3(11\mu+1)(7\mu-3)}{25}$.

■

B8. Proof of Proposition 6

Replacing $n = 4$ and $m = 2, 3, 4$, for each merger case, we get the results for the free-riding problem in Proposition 6:

- There is a free-riding problem (case 1) if: $\sigma^2 \geq \sigma^2_{fr\{2,1,1\}} \equiv -\mu^2 + \frac{7}{17}\mu - \frac{1}{68}$;

- There is a free-riding problem (case 2) if: $\sigma^2 \geq \sigma^2_{fr\{3,1\}} \equiv -\mu^2 + \frac{12}{37}\mu - \frac{9}{592}$.

$\blacksquare$

Appendix C - Results without Assumption 3

- If $\mu < \frac{m}{n(2m-1)}$, outsiders do not exit (NE) the market, hence the AA will accept the merger if $E\left[CS^{AM}\right] \geq CS^{BM}$, that is, if:

$$\Delta CS_{NE} = \frac{m^2(n-m+1)^2 - 2\mu nm(n-m+1)(m(n-m)+1) + n^2\left(\sigma^2+\mu^2\right)(m(n-m)+1)^2}{2m^2(n-m+2)^2} - \frac{n^2(n\alpha-1)^2}{2(n+1)^2} \geq 0;$$

$$\sigma^2 > \frac{-n^2\mu^2(m(n-m)+1)^2(n+1)^2 + 2\mu nm(n-m+1)(m(n-m)+1)(n+1)^2}{n^2(m(n-m)+1)^2(n+1)^2} + \\ + \frac{m^2\left(n^2(n\alpha-1)^2(n-m+2)^2 - (n-m+1)^2(n+1)^2\right)}{n^2(m(n-m)+1)^2(n+1)^2}.$$

$\frac{\partial \Delta CS_{NE}}{\partial \mu} < 0 \Leftrightarrow \mu < \frac{m(n-m+1)}{n(m(n-m)+1)}$;

$\frac{\partial \Delta CS_{NE}}{\partial \sigma^2} = \frac{1}{2}n^2 \frac{(m(m-n)-1)^2}{m^2(n-m+2)^2}$.

The higher is the σ^2 the greater is the CS^{AM}, the greater the ΔCS_{NE} is.

$\frac{\partial \Delta CS_{NE}}{\partial \alpha} = n^3 \frac{1-n\alpha}{(n+1)^2}$;

$\frac{\partial \Delta CS_{NE}}{\partial \alpha} > 0 \Leftrightarrow \alpha > \frac{1}{n}$, however α must be lower than $\frac{1}{n}$, otherwise firms do not produce before the merger.

- If $\mu > \frac{m}{n(2m-1)}$, then outsider firms will exit the market, hence the AA will accept the merger if $E\left[CS^{AM}\right] \geq CS^{BM}$, that is, if:

$$\Delta CS_E \equiv \frac{(n+1)^2 m^2 - mn\mu + n^2\left(\sigma^2+\mu^2\right)(n+1)^2 - 4m^2n^2(n\alpha-1)^2}{8(n+1)^2 m^2} \geq 0.$$

$\frac{\partial \Delta CS_E}{\partial \mu} > 0 \Leftrightarrow \mu > \frac{m}{2n(n+1)^2}$;

$\frac{\partial \Delta CS_E}{\partial \alpha} = \frac{-16m^2n^4\alpha + 16m^2n^3}{2(n+1)^2 8m^2} = n^3 \frac{1-n\alpha}{(n+1)^2}$;

$\frac{\partial \Delta CS_E}{\partial \alpha} > 0 \Leftrightarrow \alpha < \frac{1}{n}$, which is always true.

$\frac{\partial \Delta CS_E}{\partial \sigma^2} = \frac{n^2(n+1)^2}{8(n+1)^2 m^2} > 0$.

The higher is the σ^2 the greater the CS^M is and, thus, the greater is ΔCS_E. Hence, with high uncertainty on the level of efficiency gains, the greater is the likelihood that the AA accepts the merger to monopoly.

- If $\mu < \frac{m}{n(2m-1)}$, outsiders do not leave the market. Hence, firms will propose the merger if it is profitable, i.e., $E\left[\pi_I^{AM}\right] \geq m\pi^{BM}$:

$$\frac{m^2 - 2mn\mu((m-1)(m-n)+1) + n^2\left(\sigma^2+\mu^2\right)((m-1)(m-n)+1)^2}{m^2(n-m+2)^2} \geq m\frac{(1-n\alpha)^2}{(n+1)^2}.$$

$$MP_{NE} \equiv \frac{m^2 - 2mn\mu\left((m-1)(m-n)+1\right) + n^2\left(\sigma^2+\mu^2\right)\left((m-1)(m-n)+1\right)^2}{m^2(n-m+2)^2} - m\frac{(1-n\alpha)^2}{(n+1)^2} \geq 0.$$

$\frac{\partial MP_{NE}}{\partial \mu} = \frac{-2mn((m-1)(m-n)+1)+2n^2\mu((m-1)(m-n)+1)^2}{m^2(n-m+2)^2}$.

$\frac{\partial MP_{NE}}{\partial \mu} > 0 \Leftrightarrow \mu > \frac{m}{n((m-n)(m-1)+1)}$ (always true).

$\frac{\partial MP_{NE}}{\partial \alpha} = 2nm\frac{1-n\alpha}{(n+1)^2}$.

$\frac{\partial MP_{NE}}{\partial \alpha} > 0 \Leftrightarrow \alpha < \frac{1}{n}$ (always true).

$\frac{\partial MP_1}{\partial \sigma^2} = \frac{n^2((m-1)(m-n)+1)^2}{m^2(n-m+2)^2} > 0$.

- If instead, $\mu > \frac{m}{n(2m-1)}$, outsider firms exit the market. Hence, firms will propose the merger to monopoly if it is profitable:

$$MP_E \equiv \frac{m^2 - 2mn\mu + n^2\left(\sigma^2 + \mu^2\right)}{4m^2} - m\frac{(1-n\alpha)^2}{(n+1)^2} \geq 0.$$

$\frac{\partial MP_E}{\partial \mu} = \frac{-2mn+2n^2}{4m^2}$.

$\frac{\partial MP_E}{\partial \mu} > 0 \Leftrightarrow \mu > \frac{m}{n}$.

$\frac{\partial MP_E}{\partial \alpha} = 2mn\frac{1-n\alpha}{(n+1)^2}$.

$\frac{\partial MP_E}{\partial \alpha} > 0 \Leftrightarrow \alpha > \frac{1}{n}$ this is not true since $\alpha < \frac{1}{n}$.

$\frac{\partial MP_E}{\partial \sigma^2} = \frac{n^2}{4m^2} > 0$.