Stability in price competition revisited

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Abstract. We define a game with incomplete information where firms compete in prices. The demand functions faced by the firms depends on the prices and "types" of all the firms. The variable "type" reflects the ability of firms to attract consumers and produces continuous demand functions, leading to stability in competition. In addition, we consider that the information about types is not complete and thus there is uncertainty on the residual demands. We show existence of equilibrium in distributional strategies and existence of approximate equilibria in pure strategies. Then, we analyze different specifications of the demands and the information structure which yield further results and interpretations, providing new insights to the phenomenon of price dispersion, Bertrand's paradox and monopolistic competition.

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1 Introduction

We state and analyze a game with incomplete information where firms compete in prices. In the model, the strategic opportunities which determine the market power are affected not only by prices but also by attributes of firms to attract costumers, leading to continuous residual demand functions. We will refer to this continuous, smooth kind of competition as "stability". Thus, this work adds to the literature on price competition which goes back to the classical Bertrand model that has originated numerous studies with alternative assumptions on economic primitives.

In this scenario of oligopolistic price competition, the formulations of the demands faced by firms play a key role. Different ways of defining these residual demands lead to different games with a variety of equilibrium notions and a wide range of results. For instance, the demand each firm faces may depend on the consumers' information. This is the case of the works by Salop and Stiglitz (1977) and Varian (1980), who considered only two kinds of consumers: informed and uniformed. Search theory is also broadly used to deal with the matter (see, for instance, Stahl ,1989, Jansen and Moraga-González, 2004 or Janssen, Moraga-González and Wildenbeest, 2005).

The majority of the models where firms select prices and the demands absorbed by firms depend on the ratio between informed and non informed consumers lack stability in competition due to the discontinuity of demands. We consider a market where neither all informed consumers choose the firm with the lowest price nor all uninformed costumers choose randomly among all firms that charge a price of the good below their reservation price. It is not easy to argue that arbitrarily small variations in the prices lead to significant changes in the demand functions. This was already pointed out by Hotelling (1929); if a seller gradually increases the price of a good while her rivals keep their prices fixed, sales will diminish continuously, rather than fall in an abrupt way. This line of arguments leads to the analysis of competition under stability.

The stability in competition in our model appears as the result of incorporating a variable into the residual demand functions which we refer to as "type" of the firm, allowing for gradual shiftings of consumers from one firm to another when they perceive differences in the price of the good. This variable represents certain attributes of the firms which may be perceived differently by consumers and may encompass many different features, like reputation, kind sellers, crowding effects, or even something as simple as having heating in winter time, or air conditioning in summer[1]. Overall, the type of the firms captures their ability to attract consumers, and goes in the line of Hotelling (1929) when he mentioned that some consumers buy the good in a certain place and others buy it in a different one, in spite of small differences in the price. On the other hand, these abilities of the firms are not necessarily perfectly known and, as we have already remarked, can be perceived in a different way by each consumer. Therefore, we introduce a game which allows for incomplete information in the type variable where firms compete in prices, and where such competition takes place in a stable way.

To be more precise, given a strategy profile (prices selected for each firm), the demand functions depend continuously on both prices and types of the firms which are underlain by their attributes or abilities to attract consumers and become an incomplete information issue. Therefore, the payoff functions depend on the prices chosen by all the firms and also on all their types . It is important to remark that such a type variable is not a strategy for any firm. Thus, in this game there is an exogenous information structure which is a probability distribution on the set of type vector and is common knowledge for the firms. When this probability distribution is degenerated we are in the complete information scenario.

Under standard assumptions, we show existence of equilibrium in distributional strategies.[2] We also prove that there exists an approximate equilibrium in pure strategies for this game. We emphasize that, in the incomplete information setting, a pure strategy of a firm is a function that assigns a price to each type. A game with demand functions which are linear on prices illustrates this point.

The model we consider opens up the possibility of testing it with different specifications of the information structure and the demands, which lead to many different games, explaining a variety of concerns in the light of a price competition

[1]Indeed, we can find somehow the idea of the type variable in Hotelling's work, when he mentions some reasons for which a costumer would prefer to buy a good in one shop than in another even if she pays more for the good, such as location, way of doing business, family relationship or friendship with the owner, etc. While Hotelling's work focuses on location, we intend to provide a framework where the type variable may encompass any of the aforementioned examples. In some sense, this work can be understood as a variant of Hotelling's work with incomplete information.

[2]We remark that Milgrom and Weber (1985) showed that distributional strategies are simply another way of representing mixed and/or behavioral strategies.

analysis under stability.

First, taking residual demand functions which are linear in prices, and in a context of incomplete information, price dispersion arises as an equilibrium in pure strategies, in contrast with other approaches that explain this phenomenon by means of a mixed strategy equilibrium (see, for instance, Varian, 1980). Then, we also explore whether the incomplete information framework might give some firms advantages over the complete information situation. For it, using the same game, we show that in the equilibrium, the expected payoffs of a firm are higher under uncertainty on its type than in a complete information scenario. Therefore, an analysis of the expected payoffs under complete or incomplete information would be an interesting exercise for the firms in order to apply it to their advertising policy: depending on the result, firms may prefer to advertise in such a way their type becomes public information, or on the contrary, to do it trying to keep their type "hidden".

Next, we consider a particular formulation of the residual demands by separating the effects of prices and types. Within this setting, we state a game which not only provides a different way to overcome the Bertrand paradox, but also shows that its equilibrium results in a monopolistic competition situation, in accordance with the view of Chamberlin (1933, 1937), when the number of firms increases.

Finally, we observe that consumers sometimes choose a certain shop even though they are aware that the price of the good is slightly more expensive than in another. We argue that in this case, what is happening is that the type variable has almost all the effect in the distribution of demand. In other words, types become a relevant variable only when the difference in prices is small enough. On the other hand, when the differences among prices are sufficiently large, the effect of types becomes negligible and the firm charging the lowest price faces all the demand. We remark that this can be carried out in a continuous way, preserving stability in competition. We state specific demands highlighting this fact. In this case, we point out that our analysis allows for a better explanation of the degree of price dispersion.

The remainder of the paper is organized as follows. In Section 2, we present an incomplete information game where a finite number firms compete in prices and there is stability in competition. We also show existence results for different notions of equilibrium. In Section 3, we analyze a more particular situation where the demands faced by the firms depend separately on types and prices. In Section 4, we study a game in which the type becomes the relevant variable only when prices do not differ too much. Each section includes different specifications of the residual demands which illustrate our general approach and give rise to a variety of further results and interpretations.

2 The game

Let us consider a continuum of consumers[3] represented by the interval $[0,1]$, who desire to buy, at most, one unit of a commodity. Every consumer has the same reservation price r, which is the maximum price they are willing to pay for the good.

There are n firms or stores that produce the commodity and each one has a continuous cost function $C_i : [0,1] \to \mathbb{R}_+$ defined on the measures of customers, $i = 1, \ldots, n$. Firms have market power and compete in prices. When there is price competition, the demand each firm faces (residual demands) becomes crucial. We consider a scenario with stability in competition in the sense that small changes in prices do not lead to abrupt modifications in the residual demands. This relies on the fact that firms have relevant abilities or attributes to attract customers. That is, each firm has a set of possible types corresponding to the values its attributes can take and different type vectors may lead to different assignments of buyers among firms. Then, the residual demands depend not only on prices but also on the profile of types.

The type variable encompasses several features. For instance, it may be interpreted in terms of reputation, crowding types, transmission of the degree of satisfaction by previous clients, skills of each firm's employees or any other characteristic of the firm itself which affects the number of customers it is able to get.

Let t_i denote the type variable of firm i whose values lie in a set $T \subset \mathbb{R}_+$. The prior probability distributions on the types is assumed to be common knowledge and is given by an information structure η which is a probability measure on T^n. Let $\eta_i,\ i = 1, \ldots, n$, be the marginal distributions of η.

[3]Note that the consideration of a continuum of consumers allows us to provide reasons for their non-strategic behavior. We might also consider a large number of consumers as in Varian (1980).

In this incomplete information setting, a strategy is a complete plan of actions that covers every contingency of the game. That is, a pure strategy for firm i is a measurable function from T to K_i, being K_i a closed real subinterval of $[0, r]$, where firm i selects a price.[4] Moreover, a distributional strategy[5] for the firm i is given by a probability measure on $T \times K_i$ for which the marginal distribution on T is η_i, that is, the one specified by the information structure. Note that pure strategies are in one-to-one correspondence with distributional strategies whose conditional distributions are Dirac measures for each type.

To define the payoff functions of the game, given a vector of types $t \in T^n$ and a vector of prices $p \in \mathcal{K} = K_1 \times \ldots \times K_n$, let us define the function π_i as follows:

$$\pi_i(t, p) = d_i(t, p)p_i - C_i(d_i(t, p)),$$

where $d_i(t, p)$ is the demand that firm i faces whenever the vector of types is t and firms chooses prices p. We point out that, given a strategy profile of prices p and the vector of types t, the aggregate demand equals 1, that is, $\sum_{i=1}^{n} d_i(t, p) = 1$ for every $(t, p) \in T^n \times \mathcal{K}$.

We state the following regularity assumptions on the game. These hypothesis allow us to express the players' expected payoff (firms' expected profits) in a convenient manner and, moreover, to get existence results for both distributional strategy equilibria and pure strategy approximate equilibria, which we define later on.

(A.1) The informational variable t_i belongs to a compact set $T \subset \mathbb{R}_+$ for every firm $i = 1, ..., n$.

(A.2) The measure η is absolutely continuous with respect $\hat{\eta} = \eta_1 \times \ldots \times \eta_n$. Let f be the density of η with respect to $\hat{\eta}$.

(A.3) The residual demand $d_i : T^n \times \mathcal{K} \to [0, 1]$ is a continuous function for every firm $i = 1, \ldots, n$.

Note that the continuity requirement in (A.3) guarantees that the functions $\pi_i, i = 1, \ldots, n$, are continuous, which is a standard condition in games with incomplete information. We remark that, this assumption (A.3) bears the stability

[4] For instance, if we consider a technology resulting in strictly decreasing average costs, we may consider $K_i = [\delta_i, r]$, where δ_i is the minimum average cost $C_i(1)$.

[5] See Milgrom and Weber (1985) for a discussion on distributional, mixed and behavioral strategies.

in competition property. We also observe that the assumption (A.1) guarantees the equicontinuity property of the functions $(t, p) \to \pi_i(t, p)$ which is the usual hypothesis to obtain purification results.

Given a profile $(\nu_1, \ldots, \nu_n)$ of distributional strategies, assumption (A.2) allows us to write the expected payoff Π_i to firm i as follows:

$$\Pi_i(\nu_1, \ldots, \nu_n) = \int_{T^n \times \mathcal{K}} \pi_i(t, p)\ f(t)\ d\nu_1 \ldots d\nu_1$$

The price competition game $\mathcal{G}$ with incomplete information is defined by the informational structure η, the strategy set K_i for each firm $i = 1, ..., n$ and the payoff functions Π_i, $i = 1, ..., n$.

A profile $(\nu_1, \ldots, \nu_n)$ of distributional strategies is an equilibrium of the game $\mathcal{G}$ if $\Pi_i(\nu_1, \ldots, \nu_n) \geq \Pi_i(\nu_1, \ldots, \nu_i', \ldots, \nu_n)$ for every firm i and every alternative distributional strategy ν_i'.

Theorem 2.1 *The set of equilibrium points in distributional strategies for the game $\mathcal{G}$ is non empty.*

Proof. First, assumption (A.3) guarantees that the functions $\pi_i, 1 = 1, \ldots, n$ are continuous. By assumptions (A.1) the type set is compact and therefore the continuity of $\pi_i(t, \cdot)$ is uniform over types t.

Second, by assumption (A.2) the game $\mathcal{G}$ has absolutely continuous information. Therefore, we conclude that there exists a distributional strategy equilibrium for the game $\mathcal{G}$ (see theorem 1 in Milgrom and Weber, 1985).[6]

Q.E.D.

An ε-equilibrium point of the game $\mathcal{G}$ is an n-tuple $(\nu_1, \ldots, \nu_n)$ such that $\Pi_i(\nu_1, \ldots, \nu_n) + \varepsilon \geq \Pi_i(\nu_1, \ldots, \nu_i', \ldots, \nu_n)$ for every firm i and every alternative strategy ν_i'. That is, we have an approximate equilibrium (i.e., ε-equilibrium) whenever every firm is not able to increase its expected profit more than ε by deviating unilaterally.

Theorem 2.2 *If each η_i is atomless, then for every $\varepsilon > 0$ there exists a pure strategy ε-equilibrium point for the game $\mathcal{G}$.*

[6] We remark that this existence result by Milgrom and Weber (1985) was extended by Balder (1988) to a setting with abstract type spaces under weaker assumptions in the payoff functions and the proofs are based it the theory of weak convergence for transition probabilities. See also Balder (2004) for more recent developments.

Proof. Since each η_i is atomless, it follows that for every player, the set of degenerated distributional strategies (those which are in one-to-one correspondence with pure strategies) is dense in her set of distributional strategies (see theorem 3 in Milgrom and Weber, 1985). This denseness property together with the equicontinuity properties of the functions π_i allow us to conclude, as in Milgrom and Weber (1985), that for any mixed strategy equilibrium we can find an ε-equilibrium in pure strategies which is actually arbitrarily close to the former (for the weak* topology).

Q.E.D.

For the special situation where the payoff function for each firm depends only on her own type and each firm's strategy set is restricted to a finite subset, the Theorem 4 in Milgrom and Weber (1985) allows us to conclude that the game with incomplete information has an equilibrium point in pure strategies.

Representing stability in competition by means of an additional variable allows us to consider and analyze different scenarios to illustrate special economic situations. In other words, the generality of the model allows us to specify different residual demands, which lead to different games, each of them shedding light on a precise issue in an industrial economics framework. This is the aim in the reminder of the paper.

Price dispersion as pure strategy equilibrium Consider two firms competing in prices and facing residual demands that depend continuously on prices and types. Both have just fixed costs and choose prices in $[0, r]$, where r is the reservation price of their customers represented by the unit interval $[0, 1]$. The type of each firm takes values in the closed interval $[1, 2]$. [7] The demands associated to firms are given by

$$d_1(p_1, p_2, t_1, t_2) = -\frac{t_2}{t_1 + t_2} p_1 + \frac{t_1}{t_1 + t_2} p_2 + \frac{1}{2} \quad \text{and}$$
$$d_2(p_1, p_2, t_1, t_2) = -\frac{t_1}{t_1 + t_2} p_2 + \frac{t_2}{t_1 + t_2} p_1 + \frac{1}{2}.$$

Consider the informational structure such that the type of firm 1 is known, namely, $t_1 = c$, while the type of firm 2 is uniformly distributed in $[1, 2]$. Then, some calculations show that there is an equilibrium in pure strategies given by

$$\bar{p} = \frac{1}{\int_1^2 \frac{t}{t+c} d\mu(t)} = \frac{1}{1 + c \ln \frac{1+c}{2+c}} \quad \text{and } p(t) = \frac{2tp_1 + t + c}{4c},$$

[7] We consider $T = [1, 2]$ for simplicity.

where $\bar{p}$ is the price charged by firm 1 and $p : [1, 2] \to [0, r]$ is the pure strategy for firm 2 defining the equilibrium. Note that since there is incomplete information regarding the type of firm 2, a pure strategy for this firm assigns a price to each type. We remark that in this case price dispersion arises as pure strategy equilibrium of a game with incomplete information.

Observe that when the informational structure η is given by a Dirac measure on the set of vector of types T^n, there exists just one possible profile of types $t \in T^n$. Therefore, in this situation, the uncertainty disappears and then we recast a complete information game, where the payoff functions are $\pi_i(t, \cdot) : \mathcal{K} \to \mathbb{R},\ i = 1, \ldots, n$. The continuity of these functions ensures existence of Nash equilibrium in mixed strategies.[8] Furthermore, if for every i, the profit $\pi_i(t, \cdot)$ is also quasi-concave in the strategy (price) selected by firm i, there is Nash equilibrium in pure strategies. Therefore, our framework paves the way to compare equilibria with complete and incomplete information.

Complete vs. incomplete information. Consider again the previous game but with complete information. For every vector of types (t_1, t_2), there is an equilibrium in pure strategies, given by $p_1^* = \frac{t_1+t_2}{2t_2}$ and $p_2^* = \frac{t_1+t_2}{2t_1}$, which leads to profits $\pi_1^* = \frac{t_1+t_2}{4t_2}$ and $\pi_2^* = \frac{t_1+t_2}{4t_1}$ for firms 1 and 2, respectively. Then, we find price dispersion provided that firms with different attributes charge different prices whereas in the incomplete information setting price dispersion appears as an equilibrium in pure strategies (where each firm sets a price for every type in T). We remark that, in this case, the ratio of prices is given by the ratio of types, which determines the degree of dispersion of prices.

To compare, let us return to this example with incomplete information. Computing the expected payoffs at the equilibrium, we have $\Pi_1^* = \frac{\bar{p}}{4} = \frac{1}{4\left(1+c\ln\frac{1+c}{2+c}\right)}$ and $\Pi_2^* = \frac{3\bar{p}^2}{8c} + \frac{\bar{p}(3-2c)}{8c} + \frac{3}{32c} + \frac{1}{16}$ for firms 1 and 2, respectively. It is not hard to show that the equilibrium payoff Π_1^* of firm 1 is increasing in its own type, whereas the equilibrium profits Π_2^* of firm 2 decrease as the type of their opponent increases instead. Moreover, the equilibrium expected payoff of firm 2, when $c = 2$ is higher that the maximum payoff that firm 2 can obtain at equilibrium with complete information, which is attained when $t_1 = 1$ and $t_2 = 2$. A conclusion from this fact is that it would affect the advertising policy of firm 2 since it prefers to keep incomplete information.

[8] We recall that in this normal form game a mixed strategy for firm i is a probability measure on the set of pure strategies K_i which is compact and convex.

3 Separating the effects of types and prices

Let us consider a particular game where the demand that each firm gets depends separately on the two kinds of variables that we have considered, namely, prices and types. The way in which the prices affect the demands is captured by a function denoted by α while the implication of the types is expressed by a function β. The overall effect is then stated by a convex combination of these functions, α and β, depending on the profiles of prices and types, respectively.

To be precise, α is a continuous function from $\mathcal{K}$ to $[0,1]^n$, where the parameter $\alpha_i(p)$ defines the part of demand faced by firm i that is determined by the profile of prices selected by the stores. On the other hand, given a vector of types t, $\beta_i(t)$ reflects the proportion of consumers that firm i gets coming from such types or abilities. That is, a realization of types t determines $\beta(t) \in [0,1]^n$ which joint with the previous function α affect the residual demands.

Finally, the demands' dependence on prices and types is given by a convex combination of the aforementioned functions α and β. The balance of the corresponding effects is given by the weights defining such a convex combination which may depend on the prevailing prices and types. Thus, for each $(t,p) \in T^n \times \mathcal{K}$ let us consider the parameter $\lambda(t,p) \in [0,1]$. Then, the residual demand for each store i is

$$d_i(t,p) = \lambda(t,p)\alpha_i(p) + (1 - \lambda(t,p))\beta_i(t).$$

Let us consider this particular formulation of the residual demand in the game with incomplete information described in the previous section. Applying Theorems 2.1 and 2.2 we have existence of equilibrium in distributional strategies and also of approximate equilibrium in pure strategies.

Note that taking $\lambda(t,p) = 1$ for every (t,p) there is no effect of types and therefore we have a game with no incomplete information. In this case, if we consider a duopoly we can obtain as a particular situation the classical Bertrand's model resulting in the Bertrand paradox. In addition, as the next example points out, this way of separating types and prices effects gives light to some other interesting features of price competition even when one considers complete information. Actually, the following price competition game highlights a way of overcoming the Bertrand paradox that differs from those that have already been considered in the literature. Furthermore, the same game allows us to illustrate a monopolistic competition situation à la Chamberlin (1933,1937).

Take $n > 1$ firms with the same technology and consumers in $[0, 1]$ For a set of consumers of measure m, the cost function for every firm is $C(m) = cm + F$, where F denotes the fixed costs and $c < r$. Firms choose prices in the compact set $[c, r]$.

Let $t = (t_1, \ldots, t_n)$ be a vector of types of firms. For each profile of prices $p = (p_1, \ldots, p_n)$, the demand faced by firm i is $d_i(t, p) = \lambda\, \alpha_i(p) + (1 - \lambda)\, \beta_i(t)$, where $\alpha_i(p) = \frac{1}{n-1}\left(1 - \frac{p_i}{\sum_{j \in N} p_j}\right)$ and $\beta_i(t) = \frac{t_i}{\sum_{j \in N} t_j}$. Note that $\lambda(\cdot)$ is constant and equals $\lambda \in (0, 1)$. Then, the profit function for each firm $i \in N = \{1, \ldots, n\}$ is given by

$$\pi_i(t, p) = p_i \left(\frac{\lambda}{n-1}\left(1 - \frac{p_i}{\hat{p}}\right) + \frac{(1-\lambda)t_i}{\hat{t}}\right) - C\left(\frac{\lambda}{n-1}\left(1 - \frac{p_i}{\hat{p}}\right) + \frac{(1-\lambda)t_i}{\hat{t}}\right),$$

where $\hat{p} = \sum_{j \in N} p_j$ and $\hat{t} = \sum_{j \in N} t_j$.

Some calculations show that the payoff function π_i of firm i is strictly increasing in the price selected by firm i. That is, $\frac{\partial \pi_i}{\partial p_i}$ is positive for every i. Therefore, independently of the number n of firms and of their types, the unique Nash equilibrium in pure strategies is the profile where all the firms charge the reservation price r.

In this situation, for the case of a duopoly, Bertrand paradox is overcome. Moreover, when the number of firms is enlarged, the equilibrium is also attained when all the firms chose the reservation price $r > c$, although the profits of every i tend to zero when n increases. Therefore, this example also illustrates a situation of monopolistic competition in the sense that arbitrarily small firms have market power. It is not an explicit commodity differentiation, but the approach leading to the definition of the residual demands that underlies this imperfect competition situation, even though the variables defining these demands may be interpreted as a degree of "differentiation of firms". Moreover, the result is the same when there are no fixed costs and then we have constant returns to scale. Thus, our remark is in accordance with Chamberlin (1933, 1937), who pointed out, in contrast to Kaldor (1935), that what marks the contrast between monopolistic competition and perfect competition is the shape of demand curve and not the shape of the cost curve.

4 Types affect demand only if prices are similar

In many markets the type variable becomes really effective or shapes the demand functions only when prices belong to a certain threshold. In other words, when prices differ too much the type is not a relevant variable and the firms that offer the lowest price face all the demand. However, when prices are close, individuals are more vulnerable to the abilities of firms to attract costumers.

To address this feature, let us consider again the game presented in Section 2 where demand d_i that firm i faces is a continuous function on types and prices (t, p). Given a vector of prices $p \in \mathcal{K}$ that firms select, let $m(p)$ denote the minimum price. Now, let us consider a threshold $\varepsilon > 0$ and determine the new residual demands as follows:

$$d_i^{\varepsilon}(t,p) = \begin{cases} d_i(t,p) & \text{if } p_i \leq m(p) + \varepsilon \\ 0 & \text{if } p_i > m(p) + \varepsilon \end{cases}$$

If the above demands are continuous, the additional assumptions stated in Section 2 allow us to apply Theorems 2.1 and 2.2 obtaining existence of equilibrium.

To illustrate how types affect demands only if prices are similar, we consider the next two different scenarios where the demands satisfy the aforementioned continuity property.

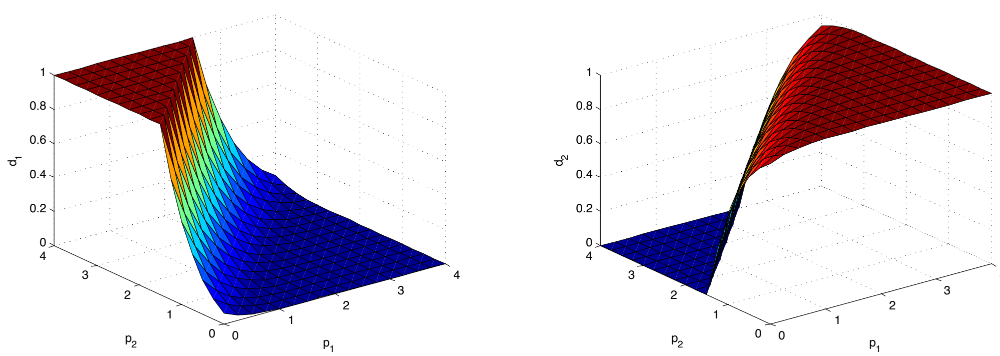
Consider two firms with types t_1 and t_2 for which the demand functions are given as follows: If the difference between the prices is large (i.e., higher than a given threshold) then the firm charging the lowest price gets all the demand, but when this difference is small (i.e., below the given threshold) then the demand is shared by both firms in such a way that the firm with the better type will face a higher demand.

Without loss of generality we assume $t_1 \geq t_2$. Let us fix a threshold $\varepsilon > 0$ and consider the following residual demands:

$$d_1(t_1, t_2, p_1, p_2) = \begin{cases} 1 & \text{if} \quad p_1 < p_2 - \varepsilon \\ 1 - \left(\frac{p_1 - p_2 + \varepsilon}{2\varepsilon}\right)^{1+t_1-t_2} & \text{if} \quad \mid p_2 - p_1 \mid \leq \varepsilon \\ 0 & \text{if} \quad p_1 > p_2 + \varepsilon \end{cases}$$

$$d_2(t_1,t_2,p_1,p_2)=\begin{cases} 0 & \text{if} \quad p_1 < p_2-\varepsilon \\ \left(\frac{p_1-p_2+\varepsilon}{2\varepsilon}\right)^{1+t_1-t_2} & \text{if} \quad \mid p_2-p_1 \mid \leq \varepsilon \\ 1 & \text{if} \quad p_1 > p_2+\varepsilon \end{cases}$$

Note that the above demands are continuous functions and when there is a tie they are determined by the types vector which avoids instability. We observe that when the prices differ in less than ε, the firm with a higher type absorbs a larger part of demand. In addition, when $p_1 < p_2-\varepsilon$ firm 1 gets all the demand (i.e., firms 2 sells nothing) but once it increases the prices so that $\mid p_1-p_2 \mid \leq \varepsilon$, its residual demand decreases very quickly, equivalently, the demand faced by firm 2 increases promptly. Thus, the type variable not only provides stability in the demands but also determine the shape of such demands. (See next figures).



In the figure on the left (resp. right) d_1 (resp. d_2) is represented taking $\varepsilon = 3/2, t_1 = 1$ and $t_2 = 4$.

When $t_1-t_2 = 1$, we obtain the following Nash equilibrium: $p_1 = \left(\frac{3\sqrt{17}-5}{8}\right)\varepsilon = 0.92\varepsilon$ and $p_2 = \frac{p_1+\varepsilon}{3} = 0.64\varepsilon$. However, when both firms have the same type, we have that the equilibrium is $p_1 = p_2 = \varepsilon$. Note that when types are different $(t_1 - t_2 = 1)$ we obtain price dispersion. Moreover, if we understand price dispersion as the difference between both prices, such a dispersion is increasing with ε. Note also that when types are equal, we find no dispersion of prices at equilibrium.

We conclude that at equilibrium the difference between prices is less than the threshold. Moreover, when types are distinct, then the prices can actually be different and as we have obtained in the example, the difference depends on the threshold. Thus, the type variable can explain not only price dispersion in the sense that at equilibrium different prices can arise, but also the degree of price dispersion, that is, how different the equilibrium prices can be.

Let us state a final specification of the residual demands which separate prices and types effects (as in the previous section) and, in addition, types affect demands only if prices are similar.

Consider two firms which produce with null variable costs. Given types $t = (t_1, t_2)$ and prices $p = (p_1, p_2)$, the residual demands are $d_i(t, p) = \lambda(p)\alpha_i(p) + (1 - \lambda(p))\beta_i(t),\ i = 1, 2,$ where

$$\lambda(p_1, p_2) = \begin{cases} 1 & \text{if} \quad |p_1 - p_2| > \varepsilon \\ \frac{p_2 - p_1}{\varepsilon} & \text{if} \quad -\varepsilon \leq p_1 - p_2 < 0 \\ \frac{p_1 - p_2}{\varepsilon} & \text{if} \quad 0 \leq p_1 - p_2 \leq \varepsilon \end{cases}$$

$$\alpha_i(p_1, p_2) = \begin{cases} 1 & \text{if} \quad p_i < p_j \\ 1/2 & \text{if} \quad p_i = p_j \\ 0 & if \quad p_i > p_j \end{cases} \qquad \text{and} \qquad \beta_i(t_1, t_2) = \frac{t_i}{t_1 + t_2}$$

That is, the demands faced by firm 1 and 2 are

$$d_1(t_1, t_2, p_1, p_2) = \begin{cases} 1 & \text{if} \quad p_1 - p_2 < -\varepsilon \\ \frac{p_2 - p_1}{\varepsilon} + \left(1 - \frac{p_2 - p_1}{\varepsilon}\right) \frac{t_1}{t_1 + t_2} & \text{if} \quad -\varepsilon \leq p_1 - p_2 < 0 \\ \left(1 - \frac{p_1 - p_2}{\varepsilon}\right) \frac{t_1}{t_1 + t_2} & \text{if} \quad 0 \leq p_1 - p_2 \leq \varepsilon \\ 0 & \text{if} \quad p_1 - p_2 > \varepsilon \end{cases}$$

$$d_2(t_1, t_2, p_1, p_2) = 1 - d_1(t_1, t_2, p_1, p_2)$$

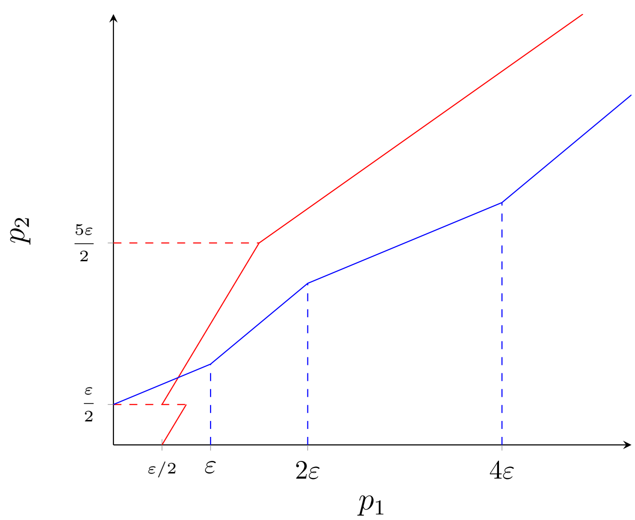
As we show below, the equilibrium solution depends basically on the ratio $\tau = t_1/t_2$. Thus, first we calculate the equilibrium when types are different and then when they are equal.

To obtain the best response functions when types differ, two cases are considered: $t_1 > t_2$ and $t_2 > t_1$, respectively.

For the case $t_2 > t_1$, some calculations allow us to write the best response functions as follows:

Reaction function for firm 1: $$\begin{cases} p_1 = \frac{p_2}{2} + \frac{\varepsilon}{2} & if \quad 0 < p_2 < \varepsilon\frac{t_1}{t_2} \\ p_1 = \frac{p_2}{2} + \frac{\varepsilon}{2}\frac{t_1}{t_2} & if \quad \varepsilon\frac{t_1}{t_2} \leq p_2 < \varepsilon(2+\frac{t_1}{t_2}) \\ p_1 = p_2 - \varepsilon & if \quad p_2 > \varepsilon(2+\frac{t_1}{t_2}) \end{cases}$$

Reaction function for firm 2: $$\begin{cases} p_2 = \frac{p_1}{2} + \frac{\varepsilon}{2} & if \quad 0 < p_1 < \varepsilon \\ p_2 = p_1 & if \quad \varepsilon \leq p_1 < \varepsilon\frac{t_2}{t_1} \\ p_2 = \frac{p_1}{2} + \frac{\varepsilon}{2}\frac{t_2}{t_1} & if \quad \varepsilon\frac{t_2}{t_1} < p_1 < \varepsilon(2+\frac{t_2}{t_1}) \\ p_2 = p_1 - \varepsilon & if \quad p_1 > \varepsilon(2+\frac{t_2}{t_1}) \end{cases}$$



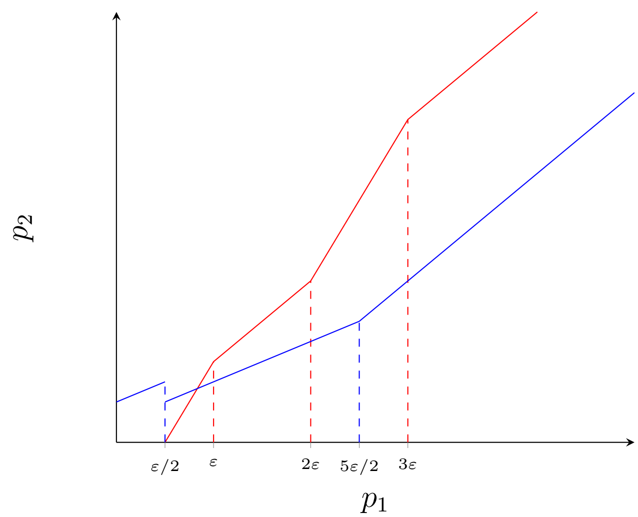
The figure above represents the reaction functions for firm 1(red line) and for firm 2 (blue line), taking t_1=1, $t_2 = 2$, and $\varepsilon = 1.5$

For the case $t_1 > t_2$, the best response functions are as follows:

Reaction function for firm 1: $$\begin{cases} p_2 = 2p_1 - \varepsilon & if \quad \frac{\varepsilon}{2} < p_1 < \varepsilon \\ p_2 = p_1 & if \quad \varepsilon \leq p_1 < \varepsilon\frac{t_1}{t_2} \\ p_2 = 2p_1 - \varepsilon\frac{t_1}{t_2} & if \quad \varepsilon\frac{t_1}{t_2} < p_1 < \varepsilon(1+\frac{t_1}{t_2}) \\ p_2 = p_1 + \varepsilon & if \quad p_1 > \varepsilon(1+\frac{t_1}{t_2}) \end{cases}$$

$$\text{Reaction function for firm 2: } \begin{cases} p_2 = \frac{p_1}{2} + \frac{\varepsilon}{2} & if \quad 0 < p_1 < \varepsilon \frac{t_2}{t_1} \\ p_2 = \frac{p_1}{2} + \frac{\varepsilon}{2}\frac{t_2}{t_1} & if \quad \varepsilon\frac{t_2}{t_1} < p_1 < \varepsilon(2 + \frac{t_2}{t_1}) \\ p_2 = p_1 - \varepsilon & if \quad p_1 > \varepsilon(2 + \frac{t_2}{t_1}) \end{cases}$$

Note that in this second situation, to simplify the expressions, the reaction function of firm 1 is written with p_2 in terms of p_1.



Reaction functions for firm 1(red line) and for firm 2 (blue line),

taking in this case t_2=1, $t_2 = 1$, and $\varepsilon = 1.5$

Let $\tau = t_1/t_2$. When $\tau > 1$ the equilibrium of this game is $p_1 = \frac{\varepsilon}{3}(1 + 2\tau)$ and $p_2 = \frac{\varepsilon}{3}(2 + \tau)$. However, when $\tau < 1$, the equilibrium is $p_1 = \frac{\varepsilon}{3}\left(1 + \frac{2}{\tau}\right)$ and $p_2 = \frac{\varepsilon}{3}\left(2 + \frac{1}{\tau}\right)$. Then, at equilibrium, the firm with higher type selects a higher price. Moreover, the ratio of prices $\theta = p_1/p_2$ does not depend on the threshold ε and just depends on the ratio of types τ. To be precise, $\theta = \frac{1+2\tau}{2+\tau}$ if $\tau > 1$ whereas $\theta = \frac{2+\tau}{1+2\tau}$ if $\tau < 1$ instead. Therefore, τ and in turn the dispersion of prices is uniformly bounded on types. As in the example 3, when both firms are of the same type (i.e., $\tau = 1$), they charge a same price equals to ε.

In the setting addressed in this section, when $\varepsilon = 0$ we have the classical Bertrand's price competition model. Moreover, when ε goes to zero, the continuous demands d_i^ε converge to the discontinuous residual demands which lead to the Bertrand paradox. Thus, our approach provides a way of solving smoothly such a paradox. This is due to the presence of stability in competition, jointly with the consideration that types matter only when prices are similar.

5 Concluding remarks

We have provided a game with incomplete information, where firms compete in prices, for which we have shown existence of different kinds of equilibrium. We have also specified residual demand functions, as particular cases of our model, that lead to different games which are used to explain several topics in industrial economics. We have considered the case when the effects of types and prices enter separately in the residual demands, and also the situation where the types matter only if the prices belong to a certain threshold. We have also drawn conclusions regarding each case. For instance, we have provided alternative explanations to those already present in the literature for the phenomenon of price dispersion and also for the Bertrand's paradox. In addition, our approach gives room to compare the equilibria with complete and incomplete information.

The different specifications of the residual demands that we have considered lead to a decrease in the competition in the sense that the equilibrium of the game deviates from the competitive equilibrium and therefore allows for the analysis of several issues where market power is strengthened through the type variable, like the case of monopolistic competition.

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