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Customer Poaching and Advertising

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Abstract

This paper is a first look at the dynamic effects of customer poaching in homogeneous product markets, where firms need to invest in advertising to generate awareness. When a firm is able to recognize customers with different purchasing histories, it may send them targeted advertisements with different prices. It is shown that only the firm which advertises the highest price in the first period will engage in price discrimination, a practice that clearly benefits the discriminating firm. This poaching gives rise to “the race for discrimination effect”, through which price discrimination may act to actually soften price competition rather than intensify it. As a result, all firms might become better off, even when only one of them can engage in price discrimination. This paper offers a first attempt to evaluate the effects of price discrimination on the efficiency properties of advertising. In markets with low or no advertising costs, allowing firms to price discriminate leads them to provide too little advertising, which is not good for consumers and overall welfare. Only in markets with high advertising costs, might firms over advertise. Regarding the welfare effects, price discrimination is generally bad for welfare and consumer surplus, though good for firms.

1 Introduction

Advancements in information and communication technologies have dramatically changed the way sellers and consumers interact. Although it is easier for consumers to obtain information, there is evidence that advertising performs an important informative task.¹ This is particularly the case in markets where firms are launching new products. In these markets, advertising is a key element in generating demand for a product.

The use of modern information technologies has also given firms the ability to track the behavior of individual customers and make them personalized offers. When a firm can recognize its loyal customers and their rival’s previous customers, it may want to send advertising messages (henceforth, ads) with differentiated content—like prices—to customers with different past behavior. In the e-marketplace, for instance, until recently, it was difficult for sellers to reconnect

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¹According to the Internet Advertising Revenue Report, conducted by the New Media Group of Pricewaterhouse Coopers, on behalf of the Internet Advertising Bureau, internet advertising revenue totalled \$4.06 billion in the second quarter of 2006. Consumer-related advertising was the top category in online advertising spending, accounting for 49% of total revenues for the second quarter in 2006. See www.iab.net.

and communicate with “lost” customers and entice them back.² However, particularly since the last quarter of 2005, advertising agencies that belong to advertising networks (e.g. DoubleClick, Tacoda, ValueClick Media, SpecificMedia, Revenue Science) have been offering their clients the possibility to identify those visitors that were in their websites but did not buy the first time and reconnect subsequently with those potential consumers in order to encourage them to return and purchase. This marketing practice, called *Retargeting* (sometimes also referred to as behavioral retargeting, remarketing or remessaging) is based on the following main idea.³ Once a potential customer is aware of a firm’s website (e.g. through normal advertising channels) and visits it, a cookie is passed to the consumer’s browser that records his behavior on the site and identifies him as either a nonpurchaser or a customer that bought from the firm. Then, at a determined time, loyal customers and potential consumers are retargeted with messages specific to them.⁴

A topic which has received much recent attention in the economics literature is “behavior-based price discrimination” (BBPD). When firms have information about consumers’ past behavior, they may use this information to offer different prices to consumers with different purchase histories.⁵ However, this literature has hitherto focused on the assumption that there is no role for advertising and that the market is fully covered. This paper departs from this hypothesis by assuming that advertising is needed to create awareness of a product and that it can be used by firms as a price discrimination device. In practice defining the number of consumers to reach and what price to advertise is at the heart of many business decisions. In contexts with repeated interactions, price discrimination is also a possibility. Yet, to date, little theoretical attention has been dedicated to the interaction between advertising decisions and price discrimination. To fill that gap, this paper proposes a model with a two-period interaction, where for example two online firms are launching a new homogenous product. All information about the existence of the product and price is channeled through advertising.⁶ Through advertising, ex-ante uninformed identical consumers become *endogenously* differentiated on an informational basis. Some consumers receive ads from both firms, are *selective* and will buy from the lowest-priced known firm. Others receive ads from only one firm, are *captive* to that firm and will buy from it provided that the price does not exceed the reservation value. Finally, some consumers receive no ads, remain uninformed and remain out of the market. When firms and consumers interact more than once, firms may be able to learn whether a previous reached consumer bought its product or not and try to poach the rival’s previous customers by sending them ads with special deals.

This paper aims to investigate the following questions: What are the effects of price discrimination on the firms’ first-period pricing and advertising decisions? Is there too little or too much advertising? Will the ability of firms to recognize customers and price accordingly increase the intensity of competition in the marketplace? Do firms benefit from being able to

²There are, of course, situations that motivate the present analysis where sellers have a way to communicate with current and potential consumers. There are for instance websites that ask consumers to register and their email may be one of the requirements, allowing the subsequent interactions. In the catalog industry, where firms rely on lists of names to advertise their products, it is also possible for sellers to identify different types of consumers and send them special offers.

³For more on retargeting see, for instance, www.retargeting.com or www.doubleclick.com/us/products/dfa.

⁴Boomerang, DoubleClick’s one-to-one targeting group, gives the following retargeting example. “A consumer goes to an online shoe retailer and leaves the site without making a purchase. Then by utilizing a retargeting technology, the shoe retailer can catch the consumer the next time (when he’s visiting a news site, perhaps). By visiting a site, a consumer has let that site know he is interested in the product and Retargeting helps the advertiser entice the consumer to return and buy its product (e.g. receive 10 percent off if you buy today)”. See <http://www.websmartamerica.com/behavioral-retargeting.php>.

⁵For a comprehensive survey on behavior-based price discrimination, see Fudenberg and Villas-Boas (2005).

⁶This assumption is in line with the stream of research that considers that in the absence of advertising potential buyers do not purchase the advertised good (e.g. Butters (1977), Stahl (1994)).

employ BBPD?

In order to evaluate the competitive effects of BBPD two settings are investigated. The first is the benchmark case where price discrimination is illegal. Here it is shown that, as in other models with imperfectly informed consumers, the "law of one price" does not hold in equilibrium (e.g. Varian (1980), Narasimhan (1988) and Baye and Morgan (2001)), and that firms fully advertise when advertising is costless. Secondly, the setting where price discrimination is permitted is analyzed. Here, the model yields a symmetric subgame perfect equilibrium in mixed strategies with the advertising component chosen deterministically. An interesting feature is that only the highest priced (or the lowest market share) firm in the initial period will have the required information to employ BBPD in the next period, and that this asymmetric knowledge about customers is good for subsequent profits. This is in contrast to the prisoner's dilemma result derived from extant models, where there is symmetric knowledge about consumers (e.g. Chen (1997), Fudenberg and Tirole (2000) and Vilas-Boas (1999)). Hence, the identification of the "*race for discrimination effect*", which gives firms a strategic incentive to set high first-period prices as a way to pursue price discrimination. This effect acts to soften competition in the initial period and explains why first-period prices with BBPD are on average above their non-discrimination counterparts. Regarding the profitability of BBPD, the main finding is that, at least when advertising costs are not too high, price discrimination boosts overall expected profits, regardless of the advertising technology in consideration (Corollary 4).

The firms' advertising decisions under BBPD are also examined. It is shown that when advertising is significantly expensive, there is more advertising with discrimination; when advertising is cheap, the reverse happens. Furthermore, unlike the no-discrimination benchmark, it is shown that the price discrimination model yields an equilibrium where *both* firms choose incomplete market coverage, even when advertising is costless (Proposition 7). Finally, this paper investigates, from a new perspective, the welfare issues of BBPD in a market where consumers are not fully informed and where advertising is needed to generate demand. Under no discrimination, it is shown that firms choose the social optimal level of advertising (Proposition 10). When discrimination is permitted, it is shown that firms may provide too little, too much or even the social optimal level of advertising (Proposition 11). In general, price discrimination is bad for consumers and welfare, though good for firms.

Related literature In broad terms, this paper is related to the following strands of the existing literature. First, the literature on informative advertising and price competition in homogeneous product markets with imperfectly informed consumers.⁷ And second, the stream of research on behavior-based price discrimination.

Following Butters (1977), rather than assuming that the information structure of consumers is exogenous, several papers have investigated how sellers can influence the consumers' information by investing in advertising. In a competitive market for a homogeneous product, where advertising is the sole source of information to uninformed customers, Butters shows that the equilibrium is characterized by price dispersion and that the equilibrium level of advertising is social optimal.⁸ The latter puzzling result was confirmed by Stahl (1994), who extended Butters' model to oligopolistic markets with more general demand curves and advertising technologies. Variations on Butters' model, such as the introduction of product differentiation (Grossman and Shapiro (1984)), or heterogeneity among buyers (Stegeman (1991)), were shown to easily offset this result. They also helped to establish the idea that increased competition stimulated

⁷For a comprehensive review of the advertising literature see Bagwell's (2005) survey on the "The Economic Analysis of Advertising."

⁸An equilibrium displaying price dispersion is a common result in the literature where some consumers are exogenously better informed than others (e.g. Varian (1980) and Narasimhan (1988)).

additional advertising—the *business stealing effect*—, while the incapability of the firm to appropriate the social surplus it generates acts as a deterrent to advertising —the *nonappropriability of social surplus effect* (Tirole (1988)).

This paper is also related to the stream of research looking at the strategic effects of advertising in sequential games where firms first invest in advertising and, then, compete in prices (e.g. Ireland (1993), McAfee (1994) and Roy (2000)).⁹ Here we propose a game, where firms compete simultaneously at advertising and prices in the initial period and, if permitted, engage in BBPD in the next stage of the game. The goal is to investigate the interaction between price discrimination and advertising choices.

Finally, this paper has important connections with recent economic research on behavior-based price discrimination in competitive markets. Basically, two approaches have been considered so far. In the switching costs approach, consumers initially view the two firms as perfect substitutes; but in the second period they face a switching cost if they change supplier. Here, purchase history discloses information about exogenous switching costs (e.g. Chen (1997) and Taylor (2003)). In the brand preferences approach (e.g. Villas-Boas (1999), Fudenberg and Tirole (2000)), purchase history discloses information about a consumer’s exogenous brand preference for a firm. Although the framework of competition differs in both approaches their predictions have some common features. First, when price discrimination is permitted, firms offer better deals to the competitor’s consumers than to its previous customers. Second, because both firms have *symmetric* information for price discrimination purposes and each firm regards its previous clientele as its strong market and the rival’s clientele as its weak market—in the terminology of Corts (1998) there is best-response asymmetry—firms find themselves in the classic prisoner’s dilemma. Third, there is socially excessive switching between firms. Nonetheless, important differences arise in both approaches when we take into account the effects of poaching on initial prices. While in the brand preferences approach when BBPD is permitted initial prices are high and then decrease (e.g. Fudenberg and Tirole (2000)), in the switching costs approach the reverse happens (e.g. Chen (1997)). Another closely related paper is Chen and Zhang (2004), who examine the profitability issue of BBPD using a discrete version of the Fudenberg-Tirole (2000) model. They assume that each firm has an exogenous “loyal” segment of the market, and that they compete for the remaining consumers, who are price-sensitive, with a reservation value lower than that of the loyal consumers’. As in the current model, they show that BBPD is only feasible if the first-period price of a firm is high enough, so that it is not accepted by all consumers. In this context they show that BBPD might benefit competing firms.

This paper proposes a new ground to evaluate the dynamic effects of BBPD in competitive markets. Unlike the extant literature where consumers are perfectly informed we assume that advertising is needed to transmit information to otherwise uninformed consumers. We also allow firms to *endogenously* segment the market into captive and selective customers by investing in advertising. Consumers are ex-ante identical regarding their preferences for the firms, however, in contrast to the switching costs approach, after advertising decisions have been made, some consumers are endogenously locked in with a certain firm, not due to the existence of switching costs, but rather because they ignore the other firm.

The rest of this paper is organized as follows. Section 2 sets out the model. Section 3 analyzes the benchmark case where price discrimination is illegal. Section 4 looks at the equilibrium advertising and pricing strategies, when firms are permitted to engage in behavior-based price discrimination. The competitive effects of customer poaching are discussed in Section 5. Issues such as the efficient properties of advertising and the welfare effects of price discrimination are

⁹An interesting contribution of Roy (2000) is to assume that firms can target consumers on the basis of their address (i.e. their location on a Hotelling framework). For other important contributions on targeted pricing in competitive settings see, for instance, Chen and Iyer (2002) and Iyer, *et al* (2005).

addressed in Section 6. Conclusions are provided in Section 7.

2 The model

Two firms, A and B, are launching a new homogeneous good. The good is produced at a constant marginal cost, which is assumed to be zero without loss of generality. There are two periods, 1 and 2, and the firms act to maximize the expected discounted value of their profits, using a common discount factor $\delta \in [0, 1]$. On the demand side, there are a large number of consumers, with mass normalized to one, who desire to buy at most one unit of the good in each period. Consumers have a common reservation price v and they are initially uninformed about the existence and the price of the good. As in Stahl (1994) a potential consumer cannot be an actual buyer unless firms invest in advertising.¹⁰ After being exposed to advertising, each consumer acquires one unit of the product from the firm that advertises the lowest price, whenever that price is below v . When a consumer is indifferent between the two firms, he assigns an equal chance to any of the firms. Further, we assume that consumers are *naive* in the sense that they do not take into account that second-period prices may be affected by their initial buying decisions. This assumption is especially relevant in a new product market where consumers have not yet learned the firm's pricing behavior.

The game proceeds as follows. In the first-period, firms choose advertising intensities and prices simultaneously and non-cooperatively. The advertising messages of each firm contain truthful¹¹ and complete information about the existence of its product and price. After firms have sent their ads independently (i.e., advertising reach is independent for each firm), there are, in principle, four different mutually exclusive and exhaustive market segments. Some consumers are *captive* to a given firm, because they are only aware of that firm. Other consumers receive ads from both firms and are *selective*. This latter group of consumers has complete information and will buy from the firm that offers them the highest surplus. Finally, the remaining consumers receive no ad from either firm, are *uninformed* and excluded from the market.

Advertising in this model is used as a variable with a long-run nature. The decision of how many consumers to reach is made in period 1. After that we assume that in the subsequent period firms can identify and reach the same consumers. Further, we assume that firms can reach those consumers again with no additional cost, which means that advertising costs sank in period 1.¹²

In the first-period, firms quote the same price for all consumers. (In period 1 firms have no way to know whether consumers are captive or selective.) However, in a repeated interaction, firms might be able to learn whether a previous contacted consumer is an actual buyer or rather a selective consumer that bought from the rival before. Note that the fact that an informed consumer did not buy from the firm in the past reveals to that firm that he must be a selective

¹⁰Implicitly we are assuming that for new products search costs are prohibitively high.

¹¹This is guaranteed by the FTC regulation that prohibits advertisers from making false and deceptive statements about their products (see www.ftc.gov/bcp/online/pubs/buspubs/ad-faqs.htm).

¹²There are several examples where firms can identify consumers that received their initial ads. First, firms have access to syndicated vendors of information about potential consumers; thus, they may decide to buy external databases with certain market coverage for advertising purposes. In the catalog industry, for instance, firms buy lists of names to advertise their products. In this case firms can easily identify those customers that received their ads in the first period and to communicate again with them. Second, as explained in the Introduction, when firms advertise their products through an advertising network they can have access to a retargeting technology through which it is easy for them to communicate with those customers that received their ads in the first period. Finally, firms may identify consumers at subsequent moments because in a first interaction they ask consumers to register. In this case, their email may be one of the requirements allowing sellers to next time communicate on a one-to-one basis. In all cases we can think of the advertising cost being entirely borne in the beginning of the game, since thereafter firms can contact the same consumers with almost no additional cost.

consumer who received an ad from the rival quoting a lower price. Likewise, this allows the firm to infer that all consumers that bought its product must be captive. When a firm achieves this type of learning, it may have incentives to poach the selective customers that previously bought from its competitor. Conversely, when the firm sells its product to all consumers that received one of its first-period ads, it learns nothing; thereby, it cannot employ price discrimination in the next period.

In the second-period, firms observe each other's choice of first-period advertising intensity and price. In the no discrimination benchmark, it is assumed that somehow public policies prohibit price discrimination. Thus, in the second period, firms are constrained to choosing the same advertising intensities and prices. In the price discrimination analysis, the firms are constrained to reach the same consumers but they can choose different prices to consumers with different past behavior.¹³

Before proceeding, we look next at different advertising cost functions that have been used in the literature and that will be used throughout the analysis.

Advertising technologies Advertising is a costly activity for firms. The cost of reaching a fraction ϕ of consumers is given by the function $A(\phi) = \alpha\eta(\phi)$. Following the extant literature on informative advertising (e.g. Grossman and Shapiro (1984) and Stahl (1994)), it is assumed that the cost of reaching consumers increases at an increasing rate, which formally can be written $\frac{\partial A(\phi)}{\partial \phi} = A_\phi > 0$ and $\frac{\partial^2 A(\phi)}{\partial \phi^2} = A_{\phi\phi} \geq 0$. The latter condition means that it is increasingly more expensive to inform an additional customer or likewise, to reach a higher proportion of customers.¹⁴ It is also assumed that there are no fixed costs in advertising, i.e. $A(0) = 0$. Finally, in order to make advertising viable, it is assumed that $A_\phi(0) < v$. In a two-period game, the latter assumption translates to $A_\phi(0) < v(1 + \delta)$.

In online markets, for instance, different types of advertising (e.g. banner ads, sponsorship advertising, keyword-related search advertising) may be reasonably well represented by the Butters' technology or the quadratic technology proposed in Tirole (1988). Although this latter technology is not based upon an underlying technology of message production, as happens with the Butters' technology, it has the advantage of being extremely simple to manipulate algebraically. It is given by $A(\phi) = \alpha\eta(\phi)$, where $\eta(\phi) = \phi^2$. The Butters' technology is given by $A(\phi) = \alpha\eta(\phi)$ where $\eta(\phi) = \ln\left(\frac{1}{1-\phi}\right)$. Butters (1977) notes that messages are sent out in a purely random fashion at a fixed cost per unit. If L messages are sent to M buyers and both L and M are large, then the fraction of buyers who do not receive any ad is $1 - \phi = (1 - \frac{1}{M})^L \simeq e^{-\frac{L}{M}}$. If each ad has a fixed cost of λ , then the total advertising cost is λL , or in terms of ϕ , $\lambda M \ln\left(\frac{1}{1-\phi}\right)$. If we let α equal λM , the cost of informing ϕ consumers is $A(\phi) = \alpha \ln\left(\frac{1}{1-\phi}\right)$. As in our model there is a large number of buyers, which we normalized to one, α can be identified with the cost per ad. In what follows, whenever a functional form is needed, we will make use of one of these technologies.

¹³We can think of second-period prices being quoted via private and personalized offers (e.g. retargeted ads, direct mail, email, creation of targeted websites, and so on).

¹⁴Several justifications support this assumption. One justification has to do with the advertising technology itself. In other words, if ads are sent randomly at a fixed cost per ad, then the probability of reaching a consumer not yet informed decreases with the amount already advertised (e.g. the Butters' urn technology and the Constant Reach Independent Readership (CRIR) technology proposed by Grossman and Shapiro (1984).) Other justifications are the existence of different predispositions to view ads on the part of the target population and the possibility of media saturation.

3 No discrimination benchmark

Consider next that somehow public policies prohibit any form of price discrimination. In the second period, firms are forced to set the same prices. This means that, once prices are publicly announced through advertising in period 1, they must remain for the entire duration of the game (i.e., $p_1 = p_2$). Throughout the paper, we will use this benchmark case to evaluate the competitive and welfare effects of price discrimination. To solve for the equilibrium without discrimination, we start looking at the behavior of firms in a static game.

There are two components to a firm's strategy: Firm i must choose its advertising level (denoted by ϕ_i), as well as its price (denoted by p_i). After firms have sent their ads independently, a proportion ϕ_i and ϕ_j of customers is reached, respectively, by firm i and j advertising. Therefore, the potential demand of firm i is made of a group of captive (locked-in) customers, namely $\phi_i(1 - \phi_j)$, and a group of selective customers, namely $\phi_i\phi_j$. Since the product is homogeneous, buyers care only about price. When consumers receive ads from both firms, they are indifferent between firms if these quote the same price; otherwise, they purchase at the lowest advertised price—provided it does not exceed v . When consumers are captive, they only purchase from the only known firm, as long as the advertised price is below the reservation price v . In the price setting game, each firm faces a trade-off between quoting a low price and compete for the segment of selective customers; or setting a high price and extract surplus from its captive segment.

Proposition 1. *There is no symmetric Nash equilibrium in pure strategies in prices.*¹⁵

Proof. See the Appendix.

There is, however, a symmetric Nash equilibrium in prices involving mixed strategies.¹⁶ Suppose that firm i selects a price randomly from the distribution function $F_i(p)$ with support $[p_{\min}, v]$. Since the marginal cost of production is assumed null, no ads will be sent specifying prices below the marginal cost of advertising or above v . Given the price and advertising strategies of say firm B, the expected profit of firm A is:

$$E\pi_A = p\phi_A(1 - \phi_B) + p\phi_A\phi_B[1 - F_B(p)] - A(\phi_A). \quad (1)$$

In equilibrium, firm A should be indifferent between quoting any price that belongs to the equilibrium support. When it chooses price v , it knows that this price is only accepted by a consumer who is not aware of any other price. In this case its expected profit is equal to $v\phi_A(1 - \phi_B)$. This implies that in equilibrium, the following condition must be satisfied:

$$p(1 - \phi_B) + p\phi_B[1 - F_B(p)] = v(1 - \phi_B).$$

Solving for $F_B(p)$, we get

$$F_B(p) = 1 - \frac{(v - p)(1 - \phi_B)}{p\phi_B}. \quad (2)$$

From $F_B(p_{\max}) = 1$ and $F_B(p_{\min}) = 0$ it follows that $p_{\max} = v$ and $p_{\min} = v(1 - \phi_B)$.

The equilibrium level of advertising maximizes (1) with respect to ϕ_A . Plugging the expression that defines $F_B(p)$, the first-order condition is equal to $v(1 - \phi_B) = A_{\phi}$.¹⁷ Under symmetry

¹⁵More precisely, a price equilibrium in pure strategies fails to exist unless firms' advertising gives rise to complete market coverage and perfect information, that is to say, unless $\phi_i = \phi_j = 1$.

¹⁶For a similar derivation of the mixed strategy equilibrium in prices, but with exogenous captive and selective consumers, see for example Varian (1980) and Narasimhan (1988).

¹⁷Second-order conditions also hold since $-A_{\phi\phi} < 0$.

we obtain that $v(1 - \phi) = A_\phi$ or, equivalently, $p_{\min} = A_\phi$.¹⁸ Basically, in equilibrium, firms should advertise up to the point where the cost of the last ad sent equals the expected revenue of a sale at the highest price to an uninformed consumer.

Turn now to the two-period game without discrimination. As we assume that in the second period firms are constrained to quote the same prices, the overall expected profit say for firm A is

$$p\phi_A(1 - \phi_B) + p\phi_A\phi_B[1 - F_B(p)] - A(\phi_A) + \delta v\phi_A(1 - \phi_B). \quad (3)$$

Again, in equilibrium, firm A should be indifferent between quoting any price that belongs to the equilibrium support. When it chooses price v , its expected profit is now equal to $v(1 + \delta)\phi_A(1 - \phi_B) - A(\phi_A)$. In equilibrium, we must observe

$$p\phi_A(1 - \phi_B) + p\phi_A\phi_B[1 - F_B(p)] - A(\phi_A) + \delta v\phi_A(1 - \phi_B) = v(1 + \delta)\phi_A(1 - \phi_B) - A(\phi_A)$$

from which it follows that

$$F_B(p) = 1 - \frac{(v - p)(1 - \phi_B)}{p\phi_B}, \quad (4)$$

which is the same expression obtained in the static game. The equilibrium level of advertising maximizes (3) with respect to ϕ_A . From the first-order condition, it follows that the equilibrium level of advertising is implicitly given by,¹⁹

$$v(1 + \delta)(1 - \phi_B) = A_\phi. \quad (5)$$

Equation (5) gives firm A 's best response function for advertising intensity when price discrimination is not permitted. Let the superscript nd identify the no-discrimination case.

Proposition 2. *In the benchmark case without price discrimination, there is a symmetric subgame perfect Nash equilibrium in which,*

(i) *each firm chooses a price randomly from the distribution*

$$F^{nd}(p) = \left\{ \begin{array}{ll} 0 & \text{if } p < v(1 - \phi^{nd}) \\ 1 - \frac{(v-p)(1-\phi^{nd})}{p\phi^{nd}} & \text{if } v(1 - \phi^{nd}) \leq p \leq v \\ 1 & \text{if } p > v \end{array} \right\} \quad (6)$$

with minimum equilibrium price equal to

$$p_{\min}^{nd} = v(1 - \phi^{nd}). \quad (7)$$

(ii) *each firm chooses an advertising reach, denoted $\phi^{nd} \in (0, 1)$ implicitly given by*

$$v(1 + \delta)(1 - \phi^{nd}) = A_\phi(\phi^{nd}) \quad (8)$$

where $A_\phi(0) \leq v(1 + \delta)$.

(iii) *Each firm earns overall expected equilibrium profits equal to*

$$E\Pi^{nd} = v(1 + \delta)\phi^{nd}(1 - \phi^{nd}) - A(\phi^{nd}) = \phi^{nd}A_\phi - A(\phi^{nd}) \quad (9)$$

Regarding the price-setting solution, the equilibrium identified above shares features present in Butters (1977), Varian (1980), Narasimhan (1988) and Baye and Morgan (2001), where firms use a price strategy that prevents their opponents from predicting their price setting behavior.

¹⁸The condition that defines the equilibrium level of advertising is simply the same we would obtain by solving the Butters static model with only two firms.

¹⁹Second-order conditions also hold since $-A_{\phi\phi} < 0$.

Corollary 1. (Effects of advertising costs)

(i) As advertising becomes costless, i.e., as α approaches 0, $\phi^{nd} \rightarrow 1$, p_{\min}^{nd} and expected profits approach zero and F^{nd} converges to unit mass at zero;

(ii) As $A_\phi(0)$ approaches $v(1 + \delta)$, ϕ^{nd} and expected profits approach zero, p_{\min}^{nd} approaches v and F^{nd} converges to unit mass at v .

4 Equilibrium analysis of poaching with advertising

In the first period, firms cannot know whether informed consumers are captive or selective. Thus, price discrimination is unfeasible. However, after consumers have made their buying decisions in period 1, a seller might be able to recognize those customers that previously bought its product and non-purchasers (i.e., those customers that received one of its ads, but decided not to buy its product in period 1). When a firm achieves that type of learning, it may have incentives to send targeted ads with better deals to the latter group of consumers, in an effort to poach them from the rival firm. Note that in period 2, while a seller might be able to identify those selective customers that bought from the rival firm before, he cannot identify those consumers that belong to its opponent's captive segment. Considering that firms are forward-looking ($\delta > 0$), the next step is to solve the game working back from the second period.

4.1 Second-period pricing game

Depending on first period pricing and advertising decisions, two scenarios are possible, in period 2, for each firm. In one scenario, firm i ($i = A, B$) is the lowest-priced firm in period 1, thereby it sells to its captive group of customers as well as to the entire group of selective customers. In this situation, first-period demand for the highest-priced firm—in this case, firm j ($i \neq j$)—comes exclusively from its captive segment. Since the lowest-priced firm sells to all consumers that received one of its first period ads, it learns nothing and, consequently, it cannot engage in price discrimination in period 2. In opposition, the highest-priced (or the lowest market share) firm in period 1 learns that some consumers that received one of its ads did not buy its product. It also infers that those customers that bought its product did not receive any of the rival's ads and, for that reason they must be captive. Note that in the present model the smaller firm learns more about consumers and becomes the discriminating firm. The larger firm learns nothing and cannot discriminate. Based on this reasoning, the highest-priced firm in period 1 is able to sort out customers into different segments: its own captive customers and the selective customers that bought previously from the rival. Because this firm can, in the second period, reach the same customers again, it will tailor ads with different prices to its own previous customers and to selective customers that bought from the competitor before.

For a given price p_i chosen by firm i in period 1, firm i is the lowest-priced firm in period 1 (or the non-discriminating firm in period 2) with a probability equal to $1 - F_j(p_i)$. It is the highest-priced firm in period 1 (or the discriminating firm in period 2) with a probability equal to $F_j(p_i)$, where $F_j(p_i)$ denotes the probability that firm j 's price in period 1 is less than or equal to p_i .

Subgame 1: Firm i is the discriminating firm Consider next that firm i is the highest priced firm in period 1. It is the firm with smaller market share since its demand is equal to $\phi_i(1 - \phi_j)$. Firm j quotes the lowest first-period price, serves all consumers that received one of its ads and its demand is given by $\phi_j(1 - \phi_i) + \phi_i\phi_j$. In this case, in the second stage of the game, firm i is able to recognize old customers and selective customers that bought from j before, and

to entice the latter customers to switch. Let us denote by p_i^o and p_i^r firm i 's second-period price, respectively, to its old customers and to the rival's previous customers.

Corollary 2. *The discriminating firm will charge its old captive customers the highest possible price, namely $p_i^o = v$, regardless of the price it charges to the rival's customers.*

The proof of this result is quite simple. The ability of firm i to fully separate its captive customers from selective customers that bought from the rival before, together with the incapability of firm j to reach any of firm i 's captive customers, allows firm i to charge its captive customers their reservation price, without fearing any poaching attempt by its rival. Then, firm i 's profit from its old captive customers, denoted by π_i^o , is equal to $\pi_i^o = v\phi_i(1 - \phi_j)$.

Since consumers remain anonymous to firm j , it is forced to advertise the same price to all consumers. We denote by \hat{p}_j firm j 's non-discrimination second-period price.

Proposition 3. *There is no pure strategy equilibrium in the price setting game for the group of the non-discriminating firm's previous customers.*

Proof. See the Appendix.

The intuition is that even though firm j can always guarantee itself a profit equal to $v\phi_j(1 - \phi_i)$, the presence of a positive fraction of selective consumers creates a tension between its incentives to price low, in order to attract the selective customers, and to price high, in order to extract rents from its captive customers. This tension results in an equilibrium displaying price dispersion. The following proposition, which is proved in the Appendix, characterizes the mixed strategy equilibrium.

Let $G_i^r(\hat{p}_j)$ denote the probability that firm i 's price to the rival's customers is no higher than \hat{p}_j and $\hat{G}_j(p_i^r)$ denote the probability that firm j 's price is less than or equal to p_i^r .

Proposition 4. *When one firm can engage in price discrimination, whilst the other cannot, price competition over the group of consumers previously buying from the non-discriminating firm gives rise to an asymmetric mixed strategy equilibrium in which:*

(i) *The non-discriminating firm chooses a price randomly from the distribution*

$$\hat{G}_j(p_i^r) = \begin{cases} 0 & \text{for } p_i^r < \hat{p}_j \min \\ 1 - \frac{v(1-\phi_i)}{p_i^r} & \text{for } \hat{p}_j \min \leq p_i^r \leq v \\ 1 & \text{for } p_i^r > v \end{cases} \quad (10)$$

with support $[\hat{p}_j \min, v]$ and has a mass point at v equal to

$$m_j = (1 - \phi_i). \quad (11)$$

(ii) *The discriminating firm chooses a price randomly from the distribution*

$$G_i^r(\hat{p}_j) = \begin{cases} 0 & \text{for } \hat{p}_j < \hat{p}_j \min \\ 1 - \frac{(v-\hat{p}_j)(1-\phi_i)}{\hat{p}_j\phi_i} & \text{for } \hat{p}_j \min \leq \hat{p}_j \leq v \\ 1 & \text{for } \hat{p}_j \geq v \end{cases} \quad (12)$$

with support $[\hat{p}_j \min, v]$, where

$$\hat{p}_j \min = v(1 - \phi_i). \quad (13)$$

(iii) The expected profit for the discriminating firm equals

$$\pi_i^r = v(1 - \phi_i)\phi_i\phi_j, \quad (14)$$

and the expected profit for the non-discriminating firm equals

$$\hat{\pi}_j = v\phi_j(1 - \phi_i). \quad (15)$$

Proof. See the Appendix.

Therefore, total second-period expected profit for the discriminating firm equals

$$\pi_i = \pi_i^o + \pi_i^r = v\phi_i(1 - \phi_j) + v(1 - \phi_i)\phi_i\phi_j. \quad (16)$$

In the equilibrium derived above it can be said that the non-discriminating firm uses a “Hi-Lo” pricing strategy. To squeeze more surplus from its captive customers, it charges the highest price v , with probability m_j . However, in order to avoid being poached, it quotes occasionally a low price.

Corollary 3. From the equilibrium distribution functions defined by (10) and (12) it follows that:

- (i) $\hat{G}_j(p) < G_i^r(p)$, that is, $\hat{G}_j(p)$ first-order stochastically dominates $G_i^r(p)$;
- (ii) $E(\hat{p}) > E(p^r)$; and
- (iii) the mass point m_j is decreasing in ϕ_i .

Proof. See the Appendix.

In other words, part (i) states that \hat{p}_j is stochastically larger than p_i^r , because it assumes large values with higher probability. Consequently, regarding price competition for the group of selective consumers, (ii) says that on average the non-discriminating firm charges higher prices than its competitor. The discriminating firm has an advantage over its rival. It sets lower prices on average, because it is able to entice some of the rival’s customers to switch, without damaging the profit from its locked in segment. Conversely, the non-discriminating firm cannot protect its captive market from price cuts. Thus, when it charges low prices, in order to avoid poaching, it damages the profit that comes from its captive segment as well. The discriminating seller has less to lose, is more aggressive and, therefore, charges, on average, lower prices.

Not surprisingly, part (iii) states that the greater is the size of the non-discriminating firm’s captive group, the higher is the probability of this firm charging the monopoly price v . Note that there is a negative relation between the captive segment size of a firm and the rival’s advertising intensity. Clearly, when, say, firm i increases its advertising reach, the size of firm j ’s group of captive customers falls, as more consumers will be aware of both firms. By decreasing its advertising effort in period 1, firm i expects firm j to play less aggressively in period 2. We will see that this strategic reasoning will be important to understanding the advertising choice of firms in the first stage of the game.

Subgame 2: Firm i is the non-discriminating firm Here, in comparison to the previous subgame, firms interchange positions with each other. Thus, we may now compute the expected profit for a representative firm, say firm i , in the second-stage of the game, denoted by π_i^2 :

$$\pi_i^2 = v\phi_i(1 - \phi_j) + F_j(p_i)v(1 - \phi_i)\phi_i\phi_j. \quad (17)$$

Note that with symmetric first-period advertising intensities, the firm that becomes the discriminating firm in period 2 earns higher profits than the non-discriminating firm (i.e. $\pi_i > \hat{\pi}_j$). This is not, of course, a surprising result. A single oligopolistic firm is always better off when it can price discriminate, provided that none of the other firms have this ability (e.g. Thisse and Vives (1988)). In this way, we have a measure of the benefit of price discrimination. This benefit will give a firm an incentive to pursue price discrimination, when choosing its pricing and advertising behavior in the first-stage of the game. While each firm has a dynamic incentive to learn more about customers in order to become the discriminating firm, it also has an incentive to reduce the rival's information and its ability to price discriminate. Interestingly, a firm will have a strategic incentive to be the highest priced firm in period 1.

4.2 First-period pricing and advertising

In the initial period, firms make their advertising and pricing decisions rationally anticipating how such decisions will affect their profits in the subsequent period. We have seen that the benefit of engaging in price discrimination in period 2 may give firms an incentive to price high in period 1. Before proceeding, it is appropriate to investigate whether or not the monopoly price v is a pure strategy equilibrium of this game.

Proposition 5 *There is no subgame perfect Nash equilibrium in which both firms set the monopoly price. Moreover, there is no subgame perfect Nash equilibrium in pure strategies.*

Proof. See the Appendix.

If there were such a pure strategy equilibrium in period 1, both firms would quote the same first-period price and, therefore, they would share equally the group of selective consumers. In period 2, within the group of old customers, no firm would be able to distinguish the fraction of selective and captive customers. However, each firm would be able to recognize those selective customers that bought from the rival before. As a result, in the second period, both firms could engage in price discrimination, as they could advertise a different price to their old clients and their rival's previous (selective) customers. As can be seen in the appendix provided, when both firms advertise price v in period 1, it is always profitable for a given firm to deviate and slightly reduce its price to $v - \varepsilon$, with $\varepsilon > 0$ and arbitrarily small, and capture the remaining selective customers.

There is, however, a mixed strategy equilibrium, the existence of which is proved by construction. The overall expected profit for firm i , when it charges first-period price p_i , uses a discount factor equal to δ , and its competitor charges a first-period price equal to p_j according to $F_j(p_i)$, is equal to:

$$E\Pi_i = p_i\phi_i(1 - \phi_j) + p_i\phi_i\phi_j[1 - F_j(p_i)] - A(\phi_i) + \delta[v\phi_i(1 - \phi_j) + F_j(p_i)v(1 - \phi_i)\phi_i\phi_j].$$

Again, in equilibrium, firm i must be indifferent between quoting any price that belongs to its equilibrium support, where $p_i \in [p_{i\min}, v]$. This allows us to obtain:

$$F_j(p_i) = 1 - \frac{(v - p_i)(1 - \phi_j)}{\phi_j(p_i - \delta v(1 - \phi_i))}. \quad (18)$$

Note that if the discount factor is zero, we get the same distribution function as in the static case (see equation (2)). From the conditions which establish that $F_j(p_{i\min}) = 0$ and $F_j(p_{i\max}) = 1$, it follows that:

$$p_{i\min} = v(1 - \phi_j) + \delta v\phi_j(1 - \phi_i) \quad \text{and} \quad p_{i\max} = v. \quad (19)$$

Now consider the equilibrium choice of advertising intensity. Plugging $F_j(p_i)$ into the overall expected profit of firm i , it is straightforward to find that the first-order condition with respect to ϕ_i is given by²⁰

$$v(1 - \phi_j) + v\delta(1 - 2\phi_j\phi_i) = A_\phi. \quad (20)$$

The following proposition characterizes the behavior of firms in the subgame perfect nash equilibrium.

Proposition 6. *There is a symmetric mixed strategy subgame perfect Nash equilibrium in which,*

(i) *each firm advertises a first-period price randomly chosen from the distribution*

$$F^*(p) = \left\{ \begin{array}{ll} 0 & \text{for } p \leq p_{\min}^* \\ 1 - \frac{(v-p)(1-\phi^*)}{\phi^*(p-\delta v(1-\phi^*))} & \text{for } p_{\min}^* \leq p \leq v \\ 1 & \text{for } p \geq v \end{array} \right\} \quad (21)$$

with minimum equilibrium price equal to

$$p_{\min}^* = v(1 - \phi^*) + \delta v\phi^*(1 - \phi^*); \quad (22)$$

(ii) *each firm selects an advertising reach, denoted ϕ^* , implicitly defined by*

$$v(1 - \phi^*) + v\delta(1 - 2(\phi^*)^2) = A_\phi(\phi^*) \quad (23)$$

where $A_\phi(0) \leq v(1 + \delta)$.

(iii) *Each firm earns expected overall equilibrium profits equal to*

$$E\Pi^* = v\phi^*(1 - \phi^*)(1 + \delta(1 + \phi^*)) - A(\phi^*) = \phi^*A_\phi - A(\phi^*) + \delta v(\phi^*)^3. \quad (24)$$

Equation (20) gives firm i 's best response function for advertising intensity. It shows that advertising choices are strategic substitutes. It also shows that, when there are no costs to advertising, the best response of firm i to firm j choosing an advertising intensity of 1 (i.e., complete market coverage), is to choose an advertising intensity of 0.5. The intuition is that, in doing so, firm i is able to strategically reduce the size of the selective group of consumers, thereby avoiding the head-to-head price competition that would otherwise arise under full market coverage. Based on these two facts, we will see below that in the symmetric subgame perfect equilibrium with price discrimination, both firms select incomplete market coverage, even when advertising is costless.

Proposition 7. *When advertising is costless, forward-looking firms do not select full market coverage. To be exact, in this case, the symmetric equilibrium level of advertising is given by $\phi^* = \frac{1}{4\delta}(\sqrt{1 + 8\delta + 8\delta^2} - 1) < 1$.*

The proof of the above result is trivial. Given any general advertising technology, with form $A(\phi) = \alpha\eta(\phi)$, when α goes to zero, A_ϕ goes to zero as well. In that way, the right-hand side of (23) equals zero; thus, solving for ϕ , we obtain the result.

²⁰Second order condition also holds, since $\frac{\partial^2 E\Pi_i}{\partial \phi_i^2} = -2v\delta\phi_j - A_{\phi\phi} < 0$.

Unlike the behavior of firms in the no-discrimination benchmark (see corollary 1), we find that, being price discrimination permitted, firms do not fully advertise, even when advertising is costless. Considering, for instance $\delta = 1$ and that advertising costs are null, our findings show that there is a *symmetric* equilibrium where each firm provides information to 78% of the market.²¹

These findings shed some light on the dynamic effects of price discrimination on firms' advertising decisions. In this model firms are allowed to compete in prices in period 2 and to price discriminate if possible. Thus, we need to take these two facts into account when explaining the above result. First, when advertising is costless, each firm realizes that, by reducing its market coverage in period 1, it strategically reduces the size of the selective group of consumers, thereby avoiding the head-to-head price competition that would otherwise arise under full market coverage. (This would be the only strategic effect in a model with no discrimination, but where firms could set different prices in period 2.) Second, when price discrimination is permitted in the second stage, besides the previous reasoning, each firm acknowledges that the benefit of price discrimination will be greater, when its market coverage is not too high, as in this way it induces the rival to play less aggressively in the subsequent period. Altogether, each firm can commit to play less aggressively in the next stage of the game by not selecting very high market coverage in the initial period.

5 Competitive implications of price discrimination

In this section, we examine how the permission of price discrimination affects the equilibrium outcomes—i.e., advertising intensity, prices and profits—in comparison to the situation in which price discrimination is banned. Figure 1 illustrates the solutions to equations (8) and (23), and to equations (7) and (22), for the special case where $\delta = 1$ and using, without loss of generality, the Butters' technology.

The downward-sloping curves MRA D and MRA ND are, respectively, the marginal revenue of advertising with discrimination and with no discrimination. The upward-sloping curves MCA', MCA and MCA'' are the marginal cost of advertising for the Butters' technology when α is respectively equal to $\frac{3v}{4}$, $\frac{v}{2}$ and $\frac{v}{4}$. The intersection between MRA D and a MCA curve provides the equilibrium level of advertising with price discrimination and, similarly, the intersection between MRA ND and a MCA curve provides the equilibrium level of advertising with no discrimination. Additionally, the curves Pmin D and Pmin ND are, respectively, the minimum price selected by firms in period 1 when discrimination is permitted and when it is banned. In this manner, given the advertising equilibrium level in each case, we may determine the support of equilibrium prices, with and without discrimination. This figure is helpful to see mainly: (i) the effects of moving from non-discrimination to discrimination, (ii) the effects of advertising intensity on the equilibrium outcomes and (iii) the effects of changes in the exogenous parameters, namely α , v and δ .

5.1 Advertising decisions

Figure 1 shows that the left-hand side of equation (23)—i.e., the marginal revenue of advertising—is strictly decreasing, and that shifts in that curve arise when there is a change in v or in δ .

²¹It is important to stress that full market coverage for both firms, even when advertising is costless, does not arise in a simpler model, where advertising choices are committed to in period one and firms, then, choose prices in a second period, after observing the opponent's advertising levels. To be precise, when advertising is costless and firms compete in prices after advertising decisions have been made, the subgame perfect equilibrium is asymmetric; while one firm fully advertises, the other covers 50% of the market. (See, for instance, Ireland (1993), McAfee (1994) and Chen and Iyer (2002)).

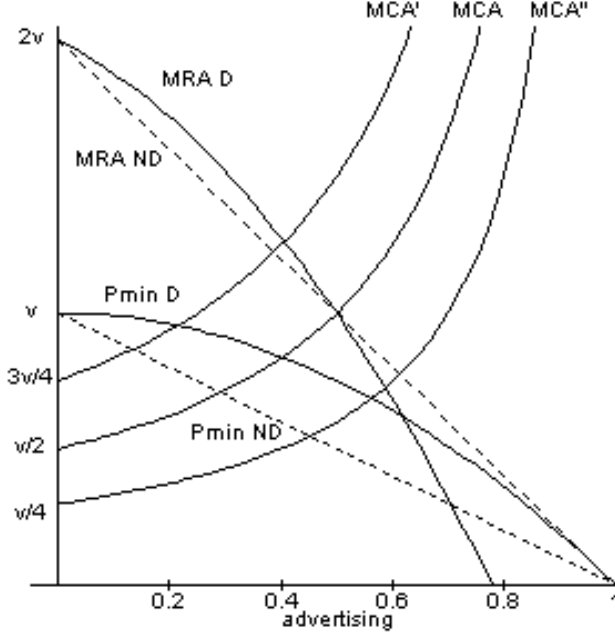


Figure 1: Equilibrium Solutions

Concretely, the effect of an increase in v is the upward shift of the curve, whilst the effect of a decrease in δ , from one to zero, is the downward shift of the curve until overlapping the Pmin ND, which is exactly equal to the MRA in the static game. (Recall that in the static case, the equilibrium level of advertising is given by $v(1 - \phi) = A_{\phi}$.) The right-hand side of equations (8) and (23)—i.e., the marginal cost of advertising—is the same with and without discrimination, and whatever the advertising technology considered. Shifts in the latter curve are only due to changes in α . Thus, we can see that an increase (decrease) in marginal advertising costs makes the curve move upwards (downwards) and gives rise to less (more) advertising in equilibrium (see, respectively, MCA' and MCA''). To be precise, because $A_{\phi\alpha} > 0$ and $A_{\phi\phi} \geq 0$, static comparative analysis shows that $\frac{\partial\phi^*}{\partial\alpha} = -\frac{A_{\phi\alpha}}{v+4\delta v\phi+A_{\phi\phi}} < 0$ and $\frac{\partial\phi^{nd}}{\partial\alpha} = -\frac{A_{\phi\alpha}}{v+\delta v+A_{\phi\phi}} < 0$.

Turn now to the comparison between the equilibrium advertising decisions with and without discrimination. We can observe in Figure 1 that firms advertise less under discrimination for not too high marginal advertising costs, whilst they advertise more for high advertising costs. Clearly, price discrimination has no effect on advertising decisions when $\phi^* = \phi^{nd}$. From Figure 1, it is clearly seen that both advertising intensities coincide when the marginal revenues of advertising with and without discrimination cross. Indeed, using (8) and (23), it can easily be checked that for $\delta > 0$, regardless of v and δ , it follows that the latter condition is satisfied whenever $\phi^* = \phi^{nd} = 0.5$ (which obviously occurs for a specific α). Otherwise, when α is such that $\phi^* > 0.5$, it follows that $\phi^* < \phi^{nd}$. Likewise, when α is such that $\phi^* < 0.5$, it ensues that $\phi^* > \phi^{nd}$.²² Thus, the comparison is ambiguous. When the advertising costs are high, there is more advertising with discrimination; when advertising is cheap, the reverse happens.

²²For the Butters' technology, we find that $\phi^* \geq \phi^{nd}$, when $\alpha \geq \frac{v(1+\delta)}{4}$ (see the intersection between a MRA curve with MCA'). Otherwise, it follows that $\phi^* < \phi^{nd}$ (see the intersection between a MRA curve with MCA''). Similarly, if $\eta(\phi) = \phi^2$, then $\phi^* \geq \phi^{nd}$ when $\alpha \geq \frac{v(1+\delta)}{2}$; otherwise $\phi^* < \phi^{nd}$.

5.2 Prices

This subsection examines the impact of behavior-based price discrimination on second and first-period prices.

Second-period prices Being price discrimination permitted, the discriminating firm increases (or at least does not reduce) the price to its captive group of consumers. Thus, those consumers that bought from the highest priced firm in period 1, are expected to pay higher first and second-period prices. From a comparison between F^* and G^r , we observe that $F^* < G^r$, which means that the first-period price is stochastically larger than p^r , as it assumes large values with higher probability.²³ From the latter result, it follows that, if poaching occurs, selective consumers will pay, on average, a lower price in period 2. Finally, regarding the group of captive consumers that bought from the lowest priced firm before, the conclusion is less clear-cut. We have seen that the non-discriminating firm uses a “Hi-Lo” pricing strategy in the second period. With a probability equal to m_j , its locked-in customers will pay a higher second-period price, namely v . Otherwise, because it is not possible to establish a general stochastic order between F^* and \widehat{G} , this set of consumers may end up paying a higher or lower second-period price.

As with the extant literature on BBPD, our model shares the feature that being price discrimination permitted, the discriminating firm will price high to their own previous customers and will price low in order to poach its rival’s previous customers. In models where both firms have the required information to price discriminate and the market exhibits best-response asymmetry, second-period prices are *all* lower than if BBPD were not permitted (e.g. Thisse and Vives (1988), Chen (1997) and Fudenberg and Tirole (2000)). Here, in contrast, only a single firm has information to discriminate, as its opponent learns nothing. Therefore, some consumers may end up paying higher second-period prices.

First-period prices The benefit of price discrimination gives rise to what we designate as the “*race for discrimination effect*”. This effect is related to the idea that the benefit of price discrimination motivates a firm to advertise high first-period prices, in order to secure the discriminating position in the subsequent period. Furthermore, because prices are strategic complements, when one firm raises its price, it induces the rival firm to do the same. In other words, the “*race for discrimination effect*” acts to soften first-period price competition, suggesting that, price discrimination being permitted, firms tend to charge higher first-period prices than if discrimination were banned.²⁴

Besides the previous effect, there is also the “*advertising effect*” on first-period prices. If price discrimination had no effect on advertising, the advertising effect would play no role; in which case, F^* would dominate stochastically F^{nd} , and first-period prices would move upwards (exclusively due to the former effect).²⁵ Because there is a negative relation between the advertising reach and prices,²⁶ the advertising effect is expected to reinforce the “*race for discrimination effect*”, when price discrimination gives rise to less advertising and, so, to higher prices. This conclusion is less clear cut when price discrimination gives rise to more advertising (this is particularly the case when α is too high).

²³Because ϕ is the same in period one and two, F^* first-order stochastically dominates G^r whenever $1 - \frac{(v-p)(1-\phi)}{\phi(p-\delta v(1-\phi))} \leq 1 - \frac{(v-p)(1-\phi)}{p\phi}$; i.e., whenever $-\delta v(1-\phi) \leq 0$, which is always true.

²⁴A similar result is obtained in Chen and Zhang (2004).

²⁵In this case we would have $\phi^* = \phi^{nd}$. Then F^* first-order stochastically dominates F^{nd} if $1 - \frac{(v-p)(1-\phi)}{\phi(p-\delta v(1-\phi))} \leq 1 - \frac{(v-p)(1-\phi)}{p\phi}$, i.e., if $-\delta v(1-\phi) \leq 0$, which is always true.

²⁶Note that $\frac{\partial p_{\min}}{\partial \phi^*} < 0$ and $\frac{\partial F^*}{\partial \phi^*} > 0$.

Even though it is not possible to establish a general stochastic ordering between $F^*(p)$ and $F^{nd}(p)$, we can see in Figure 1 that, when marginal advertising costs are not too high, price discrimination leads firms to select a lower advertising reach and higher first-period prices than under non-discrimination. To confirm this, we carried out a numerical analysis for the Butters' technology, as well as for the quadratic technology, and for different values of α . For both technologies considered, we found that first-period prices with discrimination are, on average, above their non-discrimination counterparts, even when α is extremely high. Thus, our numerical investigation suggests that, even when advertising costs are high, first-period prices tend to be, on average, above their non-discrimination counterparts.²⁷

In sum, it can be said that, for marginal advertising costs such that $\phi^* < \phi^{nd}$, first-period prices are, on average, above their non-discrimination counterparts. As a general rule, when the “*race for discrimination effect*” dominates, we expect first-period prices to be on average above the non-discrimination levels; otherwise, the reverse might happen.

Our findings show that, at least for not too high advertising costs, first-period prices are, on average, above their non-discrimination counterparts. A similar result is obtained by Fudenberg and Tirole (2000), who show that when BBPD is employed, first-period prices are above the non-discrimination levels. In their model, firms raise their first-period prices above their non-discrimination counterparts because consumers anticipate poaching and become less price sensitive in period 1.²⁸ Here, each firm may strategically price above the non-discrimination level initially because in doing so it increases the likelihood of becoming the single discriminating firm in the subsequent period. Thus, first period prices are on average above their non-discrimination counterparts due to a strategic behavior of forward looking firms.

Finally, the next proposition summarizes some of the effects of changes in α on first-period prices. Advertising costs affect the distribution of prices through ϕ^* . Since increases in α make ϕ^* fall, the price distribution shifts towards higher prices.²⁹

Proposition 8. *Regardless of the advertising technology considered, as advertising becomes cheaper, the minimum price in the equilibrium support is smaller and lower prices are more likely. However, as advertising becomes costless,*

$$p_{\min}^* \rightarrow v \left[1 - \frac{1}{4\delta} \left(\sqrt{1 + 8\delta + 8\delta^2} - 1 \right) \right] \left(\frac{3}{4} + \frac{1}{4} \sqrt{1 + 8\delta + 8\delta^2} \right) > 0.$$

Note also that price dispersion, measured by the range of prices, tends to increase with decreases in advertising costs. Propositions 7 and 8 suggest that we should observe higher levels of price dispersion in markets characterized by low costs of information provision. A similar finding is obtained by Baye and Morgan (2001), who show that price dispersion is greater, as it becomes less costly for firms to list prices at price-comparison sites. In our framework, as it becomes less costly to provide information to customers, more consumers will be aware of both firms. This reduces the lower bound of the distribution of prices, which gives rise to a wider range of advertised prices. Hence, this result challenges the view that price dispersion should decrease as more consumers become informed.³⁰

²⁷Details are available from author on request.

²⁸In comparison to the no discrimination framework, Fudenberg and Tirole show that if consumers are naive, forward-looking firms do not distort first-period prices when discrimination is permitted.

²⁹More precisely, it is easy to see that $\frac{\partial F^*}{\partial \alpha} = \frac{\partial F^*}{\partial \phi^*} \frac{\partial \phi^*}{\partial \alpha} < 0$ and $\frac{\partial p_{\min}}{\partial \alpha} = \frac{\partial p_{\min}}{\partial \phi^*} \frac{\partial \phi^*}{\partial \alpha} > 0$.

³⁰Brynjolfsson and Smith (2000), for instance, argue that the observed price dispersion in online markets is largely explained by awareness, branding and trust—factors clearly affected by promotion activities of online sellers, such as advertising.

5.3 Profits

Consider next the effect of price discrimination on second-period profits. We have already seen that the discriminating firm's second period profits rise under discrimination. More interestingly, we will see that the non-discriminating firm's second-period profit might also increase when BBPD is permitted. Under symmetry, from the comparison between $v\phi^{nd}(1 - \phi^{nd})$ and $v\phi^*(1 - \phi^*)$ (see equation (15)), it is straightforward to see that the non-discriminating firm might also gain when its rival discriminates.

On the one hand, when $\phi^{nd} < \phi^* < 0.5$, the size of the captive segment is greater than the size of the selective segment, and it is even higher if price discrimination is permitted. With discrimination, the non-discriminating firm faces a greater captive segment in period 2 and, consequently, its second-period expected profit moves upwards. On the other hand, the non-discriminating firm might also become better off when $0.5 < \phi^* < \phi^{nd}$. In this case, although the size of the captive segment is smaller than the size of the selective segment, moving from no discrimination to discrimination increases the size of the former segment at the expense of the selective group's size. Clearly, this benefits the non-discriminating firm. A key finding is that whenever price discrimination increases the number of captive customers, the non-discriminating firm will also become better off in period 2 when its rival discriminates. Since the two conditions hold in equilibrium for any advertising cost function, it immediately follows that the non-discriminating firm might also benefit when its competitor discriminates.

Consider next the effect of price discrimination on overall expected profit. Intuitively, as first-period prices tend to be above their non-discrimination counterparts, we may expect first-period profits to also be above the non-discrimination levels. Obviously, this seems to suggest that overall expected profits would move upwards when price discrimination is introduced. It is worth recalling that from (9) when price discrimination is banned, the overall expected profit is equal to

$$E\Pi^{nd} = v(1 + \delta)\phi^{nd}(1 - \phi^{nd}) - A(\phi^{nd}), \quad (25)$$

whereas when price discrimination is allowed, the overall expected profit is equal to

$$E\Pi^* = v(1 + \delta)\phi^*(1 - \phi^*) + \delta v\phi^*(1 - \phi^*) - A(\phi^*). \quad (26)$$

It thus follows that whenever $\phi^* = \phi^{nd}$, $E\Pi^* > E\Pi^{nd}$. Additionally, it is easily verified that the first term in (25) and (26) has an inverted-U relationship with the advertising intensity, reaching its maximum value at $\phi^{nd} = 0.5$ and $\phi^* = 0.5$, respectively. From our previous discussion, it has become clear that, in equilibrium, depending on α , we may obtain: (i) $\phi^* = \phi^{nd} = 0.5$, (ii) $0.5 < \phi^* < \phi^{nd}$ or (iii) $\phi^{nd} < \phi^* < 0.5$. The case in (i) was already discussed. Compare now overall equilibrium profits when $0.5 < \phi^* < \phi^{nd}$. In this case, it follows that the first term in (26) is greater than the first term in (25). On the other hand, because $\phi^* < \phi^{nd}$, it follows that $A(\phi^*) < A(\phi^{nd})$. If we further take into account that the second term in (26) is positive, it ensues that $E\Pi^* > E\Pi^{nd}$.

As previously mentioned, the more complex situation occurs when price discrimination leads firms to increased advertising. This happens when α is such that $\phi^{nd} < \phi^* < 0.5$. Again, the first term in (26) is larger than the first term in (25), and the second term in (26) is positive. However, since $\phi^* > \phi^{nd}$, it follows that $A(\phi^*) > A(\phi^{nd})$. This implies that overall expected profits under discrimination will be above their non-discrimination counterparts, as long as the increase in advertising costs does not offset the increase in sales revenue.³¹

³¹In order to verify whether profits increase with discrimination in this case, we have carried out a numerical analysis using the quadratic and Butters' technologies for different values of α . Our numerical simulations confirmed our intuition that the increase in advertising costs is not enough to offset the increase in sales revenue that results from departing from non-discrimination to discrimination. Details are available from author on request.

Corollary 4. *Regardless the advertising technology considered, the overall equilibrium profit with discrimination is above its non-discrimination counterpart, at least for not too high advertising costs (i.e., when α is such that $\phi^* \leq \phi^{nd}$).*

Two important common features of models where firms are worse off by being able to price discriminate are: (i) *both* firms have information that is obtained by observing the customers' past behavior, which means that *both* firms engage in price discrimination, and (ii) the market displays *best-response asymmetry*. In this set of models, as each firm tries to poach the rival's previous customers, price discrimination intensifies competition, and subsequent prices and profits fall (e.g. Chen (1997), Villas-Boas (1999), Fudenberg and Tirole (2000)). In our model, in contrast, *only* the firm with the lowest market share (or the highest priced firm) in the initial period has information to engage in BBPD in period 2, as its opponent learns nothing. This kind of asymmetric knowledge about consumers—the smaller firm knows who its captive customers are while the other does not—acts to soften price competition in period 2, permitting second-period profit to move upwards. Armstrong (2006) points out that, in the context of the Fudenberg-Tirole model, where there is best-response asymmetry, equal market shares in the first-period give rise to the most informative outcome and to the lowest second-period profit. Similarly, Esteves (2005) shows that when initial market shares are very asymmetric, little is learned about consumers and subsequent profits are high.³² The present model shows that being price discrimination permitted, symmetric firms may benefit when the initial market is not shared equally. This gives rise to the “*race for discrimination effect*”, through which price discrimination acts to *soften* price competition in the initial period as well, permitting first-period profits to rise.

Next we investigate the response of firms' profits to variations in advertising costs.

Proposition 9. (Advertising costs and profits)

For the quadratic technology, it follows that, when $\delta = 1$:

- (i) *under non-discrimination, sellers benefit from advertising costs increases if advertising costs are low;*
- (ii) *under discrimination, sellers are always better off with decreases in advertising costs.*

Proof. See the Appendix.

Part (i) confirms a well-known result in the literature on informative advertising: firms' profits may increase with advertising costs (e.g. Grossman and Shapiro (1984), Stahl (1994), to name a few). In general, whilst an increase in advertising costs has a negative *direct effect* on profits, there is, as well, a *strategic effect*: as advertising costs increase, firms respond with less advertising, permitting prices to rise. When the strategic effect dominates, profits may increase with advertising costs. In particular, in the appendix provided, it is shown that, for the quadratic technology, firms benefit from advertising costs increases, if advertising costs are relatively low (i.e., if $\alpha < \frac{\nu}{2}(1 + \delta)$). If we depart from a situation where ϕ^{nd} is low (α is high), we observe that additional advertising is more likely to increase the fraction of captive customers than the fraction of selective customers. It turns out that the probability of reaching

³² Esteves (2005) proposes a variant of the Fudenberg-Tirole model, by assuming that consumers' preferences follow a binary distribution: 50% prefer firm A by a fixed amount and the remaining consumers prefer firm B by the same fixed amount. In this setting, if firms quote extremely different first-period prices, the market is entirely served by the same firm, and nothing is learned from period one. When this happens, price discrimination is not feasible and, so, the second period profit is high. On the other hand, when firms share the initial market equally, both have the same information for price discrimination, and second period profit decreases.

an uninformed buyer is high and, then, firms have more incentives to focus on the group of captive consumers, thereby quoting high prices. However, as α becomes increasingly smaller and advertising becomes increasingly higher, the reverse happens.

Proposition 9 claims that (at least for the quadratic technology) when price discrimination is allowed, profits and advertising costs move in opposite directions. That is, an increase in advertising costs is always bad for profits. Following the theory, this suggests that the direct effect is stronger than the strategic effect. When α is high (ϕ is low), the arguments presented above are valid to explain why profits decrease with the increases in α . However, when α is low (ϕ is high) and we move from non-discrimination to discrimination, we find that the relationship between profits and advertising costs is reversed—i.e., profits no longer increase with the increases in advertising costs. The reason is that under no discrimination, lower and lower advertising costs tend to push firms to the Bertrand outcome, while with price discrimination the Bertrand result is far from being reached. Expressed differently, without discrimination, an increase in α , when α is low, has the strategic effect of avoiding a more aggressive behavior, thereby increasing the firms' profits; in contrast, with price discrimination, the strategic effect plays a very small role, as full market coverage is never provided in equilibrium. The direct effect dominates, and profits are clearly negatively affected by increases in advertising costs.

6 Welfare issues

This section evaluates the welfare effects of price discrimination, enabled by informative advertising. First, we examine the conventional question of whether there is too little or too much informative advertising. With that in mind, we draw a comparison between the level of advertising that would be selected by an advertising regulator, whose aim would be to maximize total welfare, and compare that level with the market equilibrium. Second, we discuss how the transition from non-discrimination to discrimination affects aggregate welfare, as well as the sellers' and consumers' surplus. Although prices play no welfare role here—due to the unit demand assumption, no dropping out of consumers and no loyalty costs—price discrimination being permitted affects advertising decisions and thereby, the efficiency properties of the equilibrium advertising level.³³ In this way, given that the regulator would select the same level of advertising, regardless of price discrimination, we also evaluate whether price discrimination does or does not enhance the efficiency properties of the sellers' advertising decisions. To simplify the analysis, throughout this section, it is assumed that $\delta = 1$.

Since production costs are assumed to be zero, total welfare is equal to the value of the good for all buyers that enter the market in both periods minus total advertising costs, that is

$$W = 2v \left[1 - (1 - \phi)^2 \right] - 2A(\phi). \quad (27)$$

6.1 Do firms under or over advertise?

In a homogeneous product market with competition, there are mainly two effects behind the divergences between the equilibrium and the social optimal level of advertising. Firstly, an ad which reaches any consumer that would be otherwise uninformed generates a sale whatever the price. From a social point of view, this action is desirable, because more consumers can enter the market, thereby increasing total welfare. Nonetheless, due to price competition, firms cannot extract all the surplus generated by advertising, because they are not able to charge the

³³ Obviously, we should be wary of drawing policy conclusions on the basis of the unit demand assumption. Note that variations in prices in both periods due to the permission of price discrimination would have a welfare impact under the elastic demand assumption.

highest price. As a result, due to the “*nonappropriability of social surplus effect*”, firms tend to underadvertise. Secondly, when a firm decides to invest in more advertising, it does not take into account the profit reduction of the rival firm, as it increases its own advertising level and poaches some of the rival’s customers. When this latter effect—the “*business stealing effect*”—is the dominant one, firms tend to over advertise. In this case, more advertising may be socially undesirable, if it merely leads to a switching of consumers between firms.

In what follows, we assume that the regulator has access to the same advertising technology as firms and that maximizes W , given by (27), with respect to ϕ . From the first-order condition, the social optimal level of advertising, denoted by ϕ^w , is implicitly given by³⁴

$$2v(1 - \phi^w) = A_\phi. \quad (28)$$

Recall that, in the overall game with price discrimination, the equilibrium level of advertising, ϕ^* , is implicitly given by

$$v(1 - \phi^*) + v(1 - 2\phi^{*2}) = A_\phi; \quad (29)$$

whilst, without discrimination, the equilibrium level of advertising, ϕ^{nd} , is implicitly given by

$$2v(1 - \phi^{nd}) = A_\phi. \quad (30)$$

It immediately follows that the expression that defines the social optimal level of advertising is equal to the expression that defines the equilibrium level of advertising under non-discrimination.

Figure 2 is based on Figure 1 except that now the left-hand side of equation (28) is introduced, which is the social marginal value of advertising net of advertising costs, namely SMVA, and the equation that defines the minimum equilibrium price is removed. In addition, the MRA with and without discrimination, for $\delta = 0$, given by MRA St, is also represented. We can see below that as δ decreases, the marginal revenue of advertising with and without discrimination, as well as the social marginal value of advertising, move downwards until reaching the static marginal revenue.

We may now establish the following propositions:

Proposition 10. (Efficiency properties with no-discrimination) *Suppose there is a ban on price discrimination. Then, firms select the social optimal level of advertising.*

Proposition 11. (Efficiency properties with discrimination) *Regardless the advertising technology considered, when price discrimination is permitted and $\delta = 1$, firms select the social optimal level of advertising if α is such that $\phi^* = \phi^w = 0.5$; under advertise if α is such that $0.5 < \phi^* < \phi^w$; and over advertise if α is such that $\phi^w < \phi^* < 0.5$.*

Proof. See the Appendix.

Although we illustrate these results using the Butters’ technology, both propositions are robust for any advertising technology, because we only need to look at MRA and SMVA. Proposition 10 can be simply proved using (28) and (30). Since the left-hand side of both equations coincides, one gets the result. A similar result was first obtained in Butters’ seminal paper. The intuition for this result runs as follows.³⁵ Since a consumer that receives an ad today will enter the market in both periods, welfare only increases if an additional ad is received by an uninformed consumer. That is, the social benefit of an additional ad is equal to $v(1 + \delta)$ times the

³⁴Second order conditions also hold.

³⁵The intuition presented is partially based on Bagwell (2005)

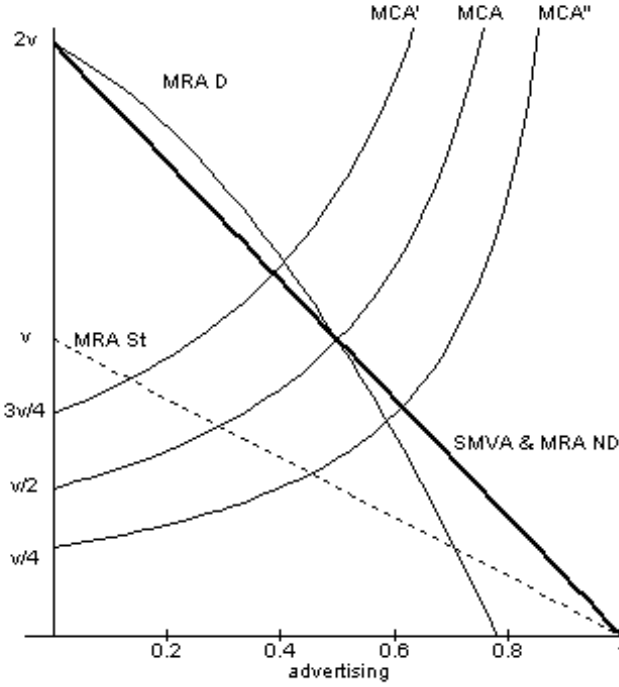


Figure 2: Efficiency of advertising decisions

probability of that consumer receiving no other ad. In a mixed strategy equilibrium in prices, the overall private benefit to a firm of sending an ad at any price $p \in [p_{\min}, v]$ is constant and equal to $v(1 + \delta)$ times the probability of the consumer receiving no other ad. It is, therefore, evident that in the benchmark game without discrimination, private and social benefits coincide.

New theoretical findings arise when the model is extended to allow price discrimination. To the best of our knowledge, this is the first attempt to evaluate the advertising behavior of firms in a market where price discrimination may occur. Interestingly, when $\delta = 1$, we obtain that, from the social point of view, firms may supply too little, too much or even the efficient level of advertising. To understand this outcome, next we examine separately the effect of advertising on firms' and consumers' surplus. From the sellers overall expected profit, given by (24), we obtain that, when $\delta = 1$, $\frac{\partial E\Pi^*}{\partial \phi} = 2v(1 - \phi) - 3v\phi^2 - A_\phi$. Evaluating at the non-cooperative equilibrium level of advertising, where $A_\phi = v(1 - \phi) + v(1 - 2\phi^2)$, it follows that at $\phi = \phi^*$

$$\frac{\partial E\Pi^*}{\partial \phi} = -\phi^*v(1 + \phi^*) < 0.$$

Hence, from the firms' point of view, there is too much advertising. When a firm decides to invest in more advertising, it does not take into account the profit reduction at the rival firm, as it increases its own advertising level and poaches some of the rival's customers.

Conversely, from the consumers' point of view, we will see below that there is under advertising. Expected consumer surplus, denoted by ECS , is given by total welfare minus industry profits. In the price discrimination game, the expected consumer surplus, when $\delta = 1$, is equal to

$$ECS^* = W^* - 2E\Pi^* = 2v(\phi^*)^3.$$

Thus, at $\phi = \phi^*$,

$$\frac{\partial ECS^*}{\partial \phi} = 6v(\phi^*)^2 > 0.$$

It is obvious that, from the consumers' point of view, for $\phi = \phi^* < 1$, there is always too little advertising. When firms are not able to appropriate the entire surplus generated by advertising, they tend to under advertise.

Given that consumers desire more advertising and firms desire less advertising, the net social effect is not immediately obvious. Adding the two previous effects we find that, evaluated at $\phi = \phi^*$,

$$\frac{\partial ECS^*}{\partial \phi} + 2\frac{\partial E\Pi^*}{\partial \phi} = 2\phi^*v(2\phi^* - 1).$$

Therefore, the net social effect is equal to zero if and only if $\phi^* = 0.5$. This advertising intensity is efficient from the social point of view. However, the net effect is positive, when α is such that $\phi^* > 0.5$, which means that firms advertise too little. (Indeed, Figure 2 shows that, when α is such that $\phi^* > 0.5$, it follows that $\phi^* < \phi^w$.) In this case, the “*nonappropriability of social surplus effect*” dominates. From our previous discussion, recall that firms charge, on average, lower prices, when ϕ is high (α is low). Finally, when $\phi^* < 0.5$, it follows that $\phi^* > \phi^w$ (see Figure 2), so firms over advertise.³⁶

6.2 Welfare effects of price discrimination

A recurrent policy question in the price discrimination literature is whether to restrict or encourage the practice of setting discriminatory prices. We have already noted that in this model, there is a unique source of social inefficiency, the advertising intensity selected by firms, which endogenously determines the number of consumers that will enter the market. In this setting, it follows that a key ingredient to understanding the welfare effects of BBPD is the impact of price discrimination on the firms' advertising choices. When firms can engage in price discrimination, welfare is equal to

$$W^* = 2v \left[1 - (1 - \phi^*)^2 \right] - 2A(\phi^*),$$

whilst when there is a ban on price discrimination it is equal to

$$W^{nd} = 2v \left[1 - (1 - \phi^{nd})^2 \right] - 2A(\phi^{nd}).$$

Proposition 12. *Regardless the advertising technology considered, when α is such that $\phi^* \neq 0.5$, welfare falls with price discrimination.*

Proof. See the Appendix.

The intuition for this result is straightforward if we take into account our previous findings. We have seen that a ban on price discrimination leads firms to *always* select the social optimal level of advertising. Proposition 12 provides a simple criterion for predicting whether price discrimination will be desirable or not in a market where advertising is the consumers' sole source of information. Because advertising choices are ultimately dependent on advertising costs, any policy drawn in favor or against discrimination should also take into account the costs' structure of each advertising-product market. As a whole we observe that, as advertising costs rise, welfare falls ($\frac{\partial W}{\partial \alpha} < 0$). If price discrimination is permitted, when advertising is cheap

³⁶Considering, for instance, the Butters' technology, we can see in Figure 2 that, when $\delta = 1$ and $\alpha = \frac{v}{2}$, $\phi^* = \phi^w = 0.5$. For higher values of α , firms over advertise; whilst for smaller values, they under advertise.

firms advertise too little, and thereby, from the social point of view, too many consumers are left out of the market. On the contrary, when advertising costs are high, firms provide too much advertising from the social point of view, and less advertising would be welfare enhancing. Only in the special case where the advertising cost is such that $\phi^* = 0.5$, will price discrimination introduce no welfare distortion.

We have seen that, at least in markets where advertising costs are not too high, profits rise when firms can add price discrimination to their pricing strategies in period 2. Together, the profit and the welfare effects of price discrimination, suggest that consumer surplus falls when price discrimination is introduced.

Therefore, this paper highlights the importance of taking into account different forms of market competition when public policy tries to evaluate the welfare effects of behavior-based price discrimination. In broad terms, in the two approaches considered in the literature so far, BBPD has been proved to be welfare reducing, due to excessive inefficient switching (e.g. Chen (1997), Fudenberg and Tirole (2000)).³⁷ Here, in contrast, price discrimination based on purchase history causes welfare to fall, not due to excessive switching, but because firms do not advertise in a socially efficient manner. Further, in contrast with Fudenberg and Tirole, our analysis shows that price discrimination does not benefit consumers, as at least some of them are expected to pay higher prices.

7 Conclusions

This paper has provided a first look at the dynamic effects of customer poaching in homogeneous product markets, where advertising plays two major roles: it is used by firms as a way to transmit relevant information to otherwise uninformed consumers, and it is used as a price discrimination device. When a firm can recognize customers with different purchasing histories, it may send them targeted ads with different prices. It was shown that the highest priced firm in the first period has an information advantage over its competitor and it is the single discriminating firm in the second period. The “*race for discrimination effect*” was identified, through which each firm has an incentive to price high in the initial period. Mainly on the basis of this strategic effect, it was proved that price discrimination may act to *soften* price competition rather than to intensify it. As a result of that, it was shown that, regardless the advertising technology considered, *all* firms might become better off, even when only one of them can engage in price discrimination.

By proposing a framework in which advertising is the consumers’ sole source of information, this paper has allowed us to investigate the interaction between advertising decisions and price discrimination. For the special case where advertising is costless and discrimination is not permitted, it was found that the market provides full market coverage and firms realize no economic profit. Conversely, departing from non-discrimination to discrimination has proved to change that result—i.e., both firms choose incomplete market coverage—, which in turn translates into positive profits. Further, it was shown that when advertising is costly, there is more advertising with discrimination than with no discrimination; when advertising is cheap, the reverse happens.

The welfare effects of informative advertising and price discrimination were also analyzed with a new perspective. In the no-discrimination benchmark it was shown that the market provides the social optimal level of advertising. In contrast, when firms can engage in price discrimination, it was found that firms might provide too little, too much or even the social

³⁷Considering a discrete framework for consumer types, Esteves (2005) shows that the static and the first-period equilibrium of the two period price discrimination game are in mixed strategies. Thus, under certain conditions, she shows that behavior-based price discrimination may increase efficiency.

optimal level of advertising. Obviously, the effect of price discrimination on the efficiency properties of advertising is strongly related to the costs structure of each advertising-product market. In this way, this paper highlighted that, in markets with low advertising costs, allowing firms to price discriminate, leads them to provide too little advertising. Only in markets with very high marginal advertising costs can firms over advertise from a social point of view. Based on the effects of discrimination on the efficiency properties of informative advertising, it was possible to establish a simple criterion for predicting whether price discrimination would be desirable or not. In this regard, it was found that BBPD is generally good for firms (at least when advertising costs are not too high), but bad for overall welfare and consumer surplus.

In light of the above, this paper has tried to contribute to the ongoing debate on the implications of new forms of price discrimination, only made possible in the context of new media markets. It is obvious that the specificity of each market plays an important role in the conclusions derived. A special limitation of the stylized model addressed in this paper is the assumption that advertising is the consumers' sole source of information. Although this assumption may at first sight seem odd in the context of online markets, it helped us to isolate the effects of price discrimination on the advertising decisions of firms. Evidently, while in new product markets this assumption might not be very restrictive, in other markets it might be inadequate. Allowing consumers to obtain information through advertising and costly search could be a natural extension, bringing new insights to the analysis. Another worthwhile extension would be to depart from the unit demand assumption. Extending the present analysis to elastic demands would bring new interesting insights, improving the economic understanding of BBPD in competitive markets.

Appendix

This appendix collects the proofs that were omitted from the text.

Proof of Proposition 1: Suppose (p_A^*, p_B^*) is an equilibrium in pure strategies. Then, by definition, there is no such p_i , $i = A, B$, such that $\pi_i(p_i, p_j^*) > \pi_i(p_i^*, p_j^*)$. The proof proceeds by contradiction.

(i) If $p_A^* = p_B^*$,

$$\pi_A(p_A^*, p_B^*) = \phi_A(1 - \phi_B)p_A^* + \frac{1}{2}\phi_A\phi_B p_A^*. \quad (31)$$

If firm A deviates and charges $p_A = p_A^* - \varepsilon$, with $\varepsilon > 0$, then, its profit from deviation is

$$\pi_A(p_A, p_B^*) = \phi_A(1 - \phi_B)(p_A^* - \varepsilon) + \phi_A\phi_B(p_A^* - \varepsilon). \quad (32)$$

From (31) and (32), $\pi_A(p_A, p_B^*) > \pi_A(p_A^*, p_B^*)$ if $\varepsilon < \frac{1}{2}\phi_B p_A^*$. Since $\phi_B > 0$ such an ε always exists. A contradiction. *Q.E.D.*

(ii) Without loss of generality, let $p_A^* < p_B^*$. Then $\pi_A(p_A^*, p_B^*) = \phi_A(1 - \phi_B)p_A^* + \phi_A\phi_B p_A^*$. Let $p_A = p_A^* + \varepsilon < p_B^*$, then, firm A 's profit from deviation is $\pi_A(p_A, p_B^*) = \phi_A(1 - \phi_B)(p_A^* + \varepsilon) + \phi_A\phi_B(p_A^* + \varepsilon)$, from which it is easy to see that there is an ε such that the deviation is profitable. A contradiction. *Q.E.D.*

(iii) If $p_A^* > p_B^*$, then $p_B^* < p_A^*$ and, as firm A does in (ii), there is a profitable deviation for firm B . A contradiction. *Q.E.D.*

Proof of Proposition 3: Suppose $(\hat{p}_i^*, \hat{p}_j^*)$ is an equilibrium in pure strategies. Then, by definition, there is no such p_i^r , $i = A, B$, such that $\pi_i^r(p_i^r, \hat{p}_j^*) > \pi_i^r(\hat{p}_i^*, \hat{p}_j^*)$. The proof proceeds by contradiction.

(i) If $p_i^{r*} = \widehat{p}_j^*$, then

$$\pi_i^{r*} = \frac{1}{2} \phi_i \phi_j p_i^{r*}. \quad (33)$$

If firm i deviates and quotes $p_i^r = p_i^{r*} - \varepsilon$, with $\varepsilon > 0$, its profit from deviation is $\pi_i^r = \phi_i \phi_j (p_i^{r*} - \varepsilon)$. It is then trivial to see that there exists such an ε that makes the deviation profitable. A contradiction. *Q.E.D.*

(ii) Let $p_i^{r*} < \widehat{p}_j^*$ then

$$\pi_i^{r*} = \phi_i \phi_j p_i^{r*}. \quad (34)$$

Let $p_i^r = p_i^{r*} + \varepsilon < \widehat{p}_j^*$, then, firm i 's profit from deviation is $\pi_i^r = \phi_i \phi_j (p_i^{r*} + \varepsilon)$, from which it is straightforward to see that the deviation is profitable. A contradiction. *Q.E.D.*

Proof of Proposition 4: The existence of such an equilibrium is proved by construction. It is a dominated strategy for each firm to set a price above v . Additionally, firm j can guarantee itself a profit of $v \phi_j (1 - \phi_i)$, charging v to its captive customers. It thus follows that at price \widehat{p}_j the best it can do is to attract all selective customers as well as its captive customers. This means that a necessary condition for it to be willing to charge \widehat{p}_j is:

$$\widehat{p}_j (\phi_j (1 - \phi_i) + \phi_i \phi_j) \geq v \phi_j (1 - \phi_i).$$

In other words, any $\widehat{p}_j < \widehat{p}_{j \min} = v(1 - \phi_i)$ is a dominated strategy for the non-discriminating firm. As firm j would never want to price below $\widehat{p}_{j \min}$, by quoting a price p_j^r arbitrarily close to $\widehat{p}_{j \min}$, the discriminating firm poaches all the selective customers that bought previously from the rival, guaranteeing itself a profit of $\widehat{p}_{j \min} \phi_i \phi_j = v(1 - \phi_i) \phi_i \phi_j$. Thus, any price $p_i^r < \widehat{p}_{j \min}$ is a dominated strategy for firm i .

Next, we prove that neither firm has a mass point p^* , such that $\widehat{p}_{j \min} < p^* < v$. By way of contradiction, assume that p^* is chosen with positive probability by firm i . Then by choosing $\widehat{p}_j = p^* - \varepsilon$, where ε is arbitrarily small, firm j becomes the low priced firm and can increase its profits. There is a profitable deviation. A contradiction. Assume now that that p^* is chosen with positive probability by firm j . Then by choosing $p_i^r = p^* - \varepsilon$, where ε is arbitrarily small, firm i has a profitable deviation. A contradiction. By similar arguments it is also straightforward to show that neither firm has a mass point at $\widehat{p}_{j \min}$. It remains to prove that only the non-discriminating firm has a mass point at the highest price v . If the non-discriminating firm has a mass point at v , the discriminating firm is always better off not charging that price but coming arbitrarily close to it. Following Narasimahn (1988) it is also straightforward to prove that both distribution functions are strictly increasing and continuous over the interval with lower bound $\widehat{p}_{j \min}$ and upper bound v . In equilibrium, for the non-discriminating firm the following condition must be satisfied:

$$\widehat{p}_j \phi_j (1 - \phi_i) + \widehat{p}_j \phi_i \phi_j [1 - G_i^r(\widehat{p}_j)] = v \phi_j (1 - \phi_i)$$

It follows that

$$G_i^r(\widehat{p}_j) = 1 - \frac{(v - \widehat{p}_j)(1 - \phi_i)}{\widehat{p}_j \phi_i}, \quad (35)$$

with $G_i^r(\widehat{p}_{j \min}) = 0$ and $G_i^r(v) = 1$. This proves part (ii).

Similarly, in equilibrium, the discriminating firm i must be indifferent between prices that belong to the half open interval $[\widehat{p}_{j \min}, v)$, i.e.:

$$p_i^r \phi_i \phi_j \left[1 - \widehat{G}_j(p_i^r) \right] = \widehat{p}_{j \min} \phi_i \phi_j$$

from which it follows that:

$$\widehat{G}_j(p_i^r) = 1 - \frac{\widehat{p}_{j \min}}{p_i^r} = 1 - \frac{v(1 - \phi_i)}{p_i^r}, \quad (36)$$

with $\widehat{G}_j(p_i^r = \widehat{p}_{j \min}) = 0$ and $\widehat{G}_j(v) = 1 - (1 - \phi_i) < 1$. This implies that firm j has a mass point at v . This completes the proof. *Q.E.D.*

Proof of Corollary 3: To prove part (i) take as given the level of advertising selected by firms in period 1. Thus, $G^r(p) - \widehat{G}(p) = (1 - \phi) \frac{v\phi - v + p}{p\phi} > 0$. This condition is satisfied if and only if $p \geq v(1 - \phi)$. Since $v(1 - \phi) = \widehat{p}_{\min}$ we get that $G^r(p) - \widehat{G}(p) > 0$, as long as $p \geq \widehat{p}_{\min}$, which is true for the equilibrium support of prices. When (i) holds, result (ii) follows. *Q.E.D.*

Proof of Proposition 5: This result is proved by contradiction. Suppose that $(p_A, p_B) = (v, v)$ is a subgame perfect equilibrium of this game. In this case, in period 1, firms share equally the selective group of consumers. In period 2, within the group of old customers, each firm is not able to distinguish the fraction of selective and captive customers. However, each firm is able to recognize those selective customers that bought from the rival before and price accordingly. Let p_i^o and p_i^r be respectively firm i 's price to its old customers and to the rival's customers. Again it is straightforward to show that there is no pure strategy equilibrium. Next we prove the existence of a mixed strategy equilibrium. Suppose that each firm's prices are selected according to the distribution functions $G_i^r(p_j^o)$ and $G_i^o(p_j^r)$.

When say firm A decides what price to charge to the rival's previous customers it takes into account that firm B is not willing to charge a price yielding an expected profit lower than $v\phi_B(1 - \phi_A)$ (even in the case where it is able to sell to both groups of old customers). Therefore, it is easy to see that, the minimum price firm B is willing to quote, $p_{B \min}^o$, is $p_{B \min}^o(\phi_B(1 - \phi_A) + \frac{1}{2}\phi_A\phi_B) = v\phi_B(1 - \phi_A)$ or $p_{B \min}^o = \frac{2v(1 - \phi_A)}{2 - \phi_A}$. It is also straightforward to see that it is a dominated strategy for firm A to set a price below $p_{B \min}^o$. Quoting $p_A^r = p_{B \min}^o$ allows firm A to grab the entire group of selective customers that bought from B before, whilst makes firm B indifferent. Therefore, firm A's minimum price to the rival's customers is $p_{B \min}^o$, and its expected profit, from that group of customers, is equal to $p_{B \min}^o(\frac{1}{2}\phi_A\phi_B)$. In equilibrium we must observe $p_A^r(\frac{1}{2}\phi_A\phi_B)[1 - G_B^o(p_A^r)] = 2v(\frac{1 - \phi_A}{2 - \phi_A})(\frac{1}{2}\phi_A\phi_B)$ which yields

$$G_B^o(p_A^r) = 1 - \frac{2v(1 - \phi_A)}{p_A^r(2 - \phi_A)}, \quad (37)$$

with $G_B^o(p_A^r = p_{B \min}^o) = 0$ and $G_B^o(p_A^r = v) = 1 - \frac{2v(1 - \phi_A)}{v(2 - \phi_A)}$. Thus, firm B has a mass point at v within the group of its old customers.

Looking now at firm A's pricing strategy for its old customers, we know that firm A's expected profit from its old customers group is equal to $v\phi_A(1 - \phi_B)$. In equilibrium we must observe $p_A^o\phi_A(1 - \phi_B) + p_A^o(\frac{1}{2}\phi_A\phi_B)[1 - G_B^r(p_A^o)] = v\phi_A(1 - \phi_B)$ which yields,

$$G_B^r(p_A^o) = 1 - \frac{2(v - p_A^o)(1 - \phi_B)}{p_A^o\phi_B}. \quad (38)$$

From $G_B^r(p_A^o) = 0$ we obtain $p_{A \min}^o = \frac{2v(1 - \phi_B)}{2 - \phi_B}$ and from $G_B^r(p_A^o) = 1$ we obtain $p_{A \max}^o = v$. Second-period expected profit for firm A is, therefore,

$$\pi_A^2 = \pi_A^o + \pi_A^r = v\phi_A \left(\frac{2 - \phi_A - \phi_B}{2 - \phi_A} \right).$$

In sum, overall profit with symmetric advertising decisions when both firms quote price v in period 1 is equal to:

$$\Pi = v \left(\phi(1 - \phi) + \frac{1}{2}\phi^2 \right) + \delta 2v\phi \left(\frac{1 - \phi}{2 - \phi} \right) - A(\phi).$$

Now we need to prove that no firm has an incentive to deviate from the equilibrium proposed. Taking as given its rival pricing and advertising strategy in period 1, if, say, firm A decides to undercut its rival, by quoting a price slightly below v , it is able to sell to the entire group of selective consumers in that period, but it is not allowed to discriminate in period 2. In this situation, firm A's first-period profit from deviation is $(v - \varepsilon)(\phi_A(1 - \phi_B) + \phi_A\phi_B) \approx v(\phi_A(1 - \phi_B) + \phi_A\phi_B)$, where $\varepsilon > 0$ and infinitely small. In period 2, firm A is the non-discriminating firm and its expected profit is $v\phi_A(1 - \phi_B)$. Summing up, firm A's profit from deviation is equal to $\Pi^d = v(\phi_A(1 - \phi_B) + \phi_A\phi_B) + \delta v\phi_A(1 - \phi_B) - A(\phi_A)$, which under symmetry is equal to:

$$\Pi^d = v(\phi(1 - \phi) + \phi^2) + \delta v\phi(1 - \phi) - A(\phi).$$

Since, $\Pi^d - \Pi = \frac{1}{2}v\phi^2 \frac{2 - 2\delta + 2\delta\phi - \phi}{2 - \phi}$ it is straightforward to see that, since $\phi \in (0, 1)$, $\Pi^d - \Pi > 0$ as long as $2 - 2\delta + 2\delta\phi - \phi > 0$. For $\delta \in (0, 1)$ the previous condition is always positive. A contradiction. *Q.E.D.*

Proof of Proposition 9: To prove (i) from (9) the derivative of overall profit, with respect to α , is

$$\frac{\partial E\Pi^{nd}}{\partial \alpha} = \phi^{nd} A_{\phi\phi} \phi_\alpha^{nd} + \phi^{nd} A_{\phi\alpha} - A_\alpha$$

where $\phi_\alpha^{nd} = \frac{\partial \phi^{nd}}{\partial \alpha} = -\frac{A_{\phi\alpha}}{v(1+\delta) + A_{\phi\phi}} < 0$. For the quadratic technology, $A(\phi) = \alpha\phi^2$, it follows that $\frac{\partial \phi^{nd}}{\partial \alpha} = \left(-\frac{2\phi}{v(1+\delta) + 2\alpha} \right)$ and $\phi^{nd} = \frac{v}{v+2\alpha}$. It ensues that

$$\frac{\partial E\Pi^{nd}}{\partial \alpha} = \phi^2 \frac{(-2\alpha + v(1 + \delta))}{v + v\delta + 2\alpha}.$$

Therefore, $\frac{\partial E\Pi^{nd}}{\partial \alpha} > 0$ whenever $\alpha < \frac{v}{2}(1 + \delta)$. Otherwise, $\frac{\partial E\Pi^{nd}}{\partial \alpha} < 0$. *Q.E.D.*

To prove part (ii), from (24), the derivative of overall profit, with respect to α , is

$$\frac{\partial E\Pi^*}{\partial \alpha} = \phi^* A_{\phi\phi} \phi_\alpha^* + \phi^* A_{\phi\alpha} - A_\alpha + 3\delta v(\phi^*)^2 \phi_\alpha^*$$

where $\phi_\alpha^* = \frac{\partial \phi^*}{\partial \alpha} = -\frac{A_{\phi\alpha}}{v(1+4\delta\phi) + A_{\phi\phi}} < 0$. When $A(\phi) = \alpha\phi^2$, it follows that,

$$\frac{\partial E\Pi^*}{\partial \alpha} = -\phi \left(\frac{v\phi - 2\delta v\phi^2 - 2\alpha\phi}{v + 4\delta v\phi + 2\alpha} \right).$$

Thus, $\frac{\partial E\Pi^*}{\partial \alpha} < 0$ if $v\phi - 2\delta v\phi^2 - 2\alpha\phi > 0$. Using (23) it follows that $\frac{\partial E\Pi^*}{\partial \alpha} < 0$ whenever $v(1 + \delta - 2\phi) > 0$, which, for $\delta = 1$, is always true. *Q.E.D.*

Proof of Proposition 11: Evaluating the regulator's first order condition, given by (28), at $\phi = \phi^*$, it follows that $\frac{\partial W}{\partial \phi} = 2v(1 - \phi^*) - A_\phi(\phi^*)$. Using the fact that, when $\delta = 1$, $A_\phi(\phi^*) = v(1 - \phi^*) + v(1 - 2\phi^{*2})$, and plugging this expression into the FOC, we obtain that $\frac{\partial W}{\partial \phi} = v\phi^*(2\phi^* - 1)$. This implies that ϕ^* maximizes welfare, when $\phi^* = 0.5$. Otherwise, when $\phi^* > 0.5$, it follows that $\frac{\partial W}{\partial \phi} > 0$ and, so, welfare increases if ϕ^* becomes increasingly higher. This implies that $\phi^* < \phi^w$. Similarly, when $\phi^w < \phi^* < 0.5$, welfare increases if ϕ^* becomes smaller. This completes the proof. *Q.E.D.*

Proof of Proposition 12: From Proposition 10, it follows that welfare is maximized at $\phi = \phi^{nd}$. In contrast, from Proposition 11, it follows that, when $\delta = 1$, $\frac{\partial W}{\partial \phi} = 0$ at $\phi^* = 0.5$; while $\frac{\partial W}{\partial \phi} \geq 0$ when $\phi^* \geq 0.5$. *Q.E.D.*

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