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Oil Dependence and Economic Instability*

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Abstract

We show that dependence on foreign energy can increase economic instability by raising the likelihood of equilibrium indeterminacy, hence making fluctuations driven by self-fulfilling expectations easier to occur. This is demonstrated in a standard neoclassical growth model. Calibration exercises, based on the estimated share of imported energy in production for several countries, show that the degree of reliance on foreign energy for many countries can easily make an otherwise determinate and stable economy indeterminate and unstable.

Keywords: Indeterminacy, Energy Imports, Externality, Returns to Scale, Sunspots, Self-Fulfilling Expectations.

JEL Classification: E13, E20, E30.

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1 Introduction

Sharp increases in the prices of oil have triggered two significant world-wide recessions since World War II: one in 1974-75, and another in 1979-81. The underling reason is that many industrial economies depend heavily on imported energy in production, making them vulnerable to changes in the prices and supply of oil in the world market. Although it is well known that increases in the prices of foreign energy can act like adverse productivity shocks to domestic economy, many economists also argue energy price shocks by themselves are not sufficient for causing a massive recession as large as we experienced in the 1970s and the early 1980s. For example, Hamilton (1988a, 1988b and 2003) argues that a sharp decrease of aggregate demand due to pessimistic expectation of the future at the time of oil shocks exacerbated the negative impact of higher energy prices. Bernanke et al. (1997), Barsky and Killian (2001) and Leduc and Sill (2004) argue that monetary policies significantly aggravated the negative impact of oil shocks. Aguiar-Conraria and Wen (2006), along the line of Hamilton, argue that the private sector's expectations played an important role to magnify the negative impact of oil shocks.

This paper shows reliance on foreign energy has another potentially important effect on economic activity – it destabilizes the economy by increasing its likelihood of indeterminacy, hence making fluctuations driven by self-fulfilling expectations more likely to occur.

The framework we adopt to make our point is Benhabib and Farmer (1994). In this influential paper, Benhabib and Farmer show the equilibrium of a standard RBC model can become indeterminate in the presence of externalities or increasing returns to scale, which makes possible endogenous fluctuations driven by self-fulfilling expectations.[1] Although this first-generation of indeterminate RBC model requires implausibly large degrees of externalities to generate indeterminacy (thereby casting doubt on their empirical relevance, see e.g., Schmitt-Grohé 1997), subsequent work by Benhabib and Farmer (1996), Benhabib and Nishimura (1997), Benhabib, Nishimura and Meng (2000), Perli (1998), Weder (1998 and 2001) and Wen (1998), among many others, show that adding other standard features of real economies into the model of Benhabib and Farmer (1994) can significantly reduce the degree of externalities required for inducing local indeterminacy.[2]

We add to this fast growing literature another mechanism relevant for indeterminacy: the dependence of production on imported goods. For some countries, energy imports account for a significant fraction of total costs in domestic production. For example, Table 1 shows the cost shares of imported energy can be as high as 16% of a country's GDP.[3] This paper shows

[1] See Farmer and Guo (1994) for a calibrated exercise based on this model. For the related literature, see Shell (1977, 1987), Cass and Shell (1983), Shell and Smith (1992), Azariadis (1981), Azariadis and Guesnerie (1986), and Woodford (1986a, 1986b, 1991).

[2] See Wen (2001) for an analysis of this class of models regarding mechanisms giving rise to local indeterminacy from the viewpoint of the permanent-income hypothesis. Also see Meng (2006) and Meng and Yip (2004) for other channels of generating indeterminacy.

[3] The data for all EU-25 countries are taken from Eurostat (2006). The energy share for the EU-15 countries is easy to estimate based on the database. But to estimate the energy share of the remaining 10 countries: Czech

heavy reliance on imported energy can have a significant effect on economic instability in the presence of increasing returns to scale: the larger the imported energy share in GDP, the easier it is for the economy to be subject to fluctuations driven by self-fulfilling expectations.

Table 1. Cost of Imported Energy as Percentage of GDP

Lithuania	16.0%	Luxembourg	3.6%
Ukraine	15.7%	Austria	3.4%
Slovakia	12.1%	Portugal	3.5%
Latvia	8.3%	Greece	3.2%
Belgium	7.7%	Finland	2.9%
South Korea	7.6%	Sweden	2.8%
Holland	5.8%	Spain	2.8%
Estonia	5.5%	Germany	2.4%
Hungary	5.4%	France	2.3%
Czech Republic	5.2%	Italy	2.0%
Malta	4.9%	United States	1.8%
Cyprus	4.5%	Ireland	1.8%
Slovenia	4.0%	United Kingdom	1.4%
Poland	3.9%	Denmark	1.3%

Our model is based on the standard DSGE models that incorporate foreign energy as a third production factor. This class of models have been used widely to study the business-cycle effects of oil price shocks. This literature includes Kim and Loungani (1992), Finn (1995 and 2000), Rotemberg and Woodford (1996), Atkeson and Kehoe (1993), Wei (2003), Leduc and Sill (2004) and Aguiar-Conraria and Wen (2006). We show that having a foreign produced energy input in domestic production as an additional production factor can significantly increase the parameter region of indeterminacy of the Benhabib-Farmer type models. For example, the required returns to scale for indeterminacy in this class of models can be reduced by 50% when the share of imported energy reaches 15% of GDP.

The rest of the paper is organized as follows. Section 2 uses the Benhabib-Farmer model as a benchmark model to demonstrate our point. Section 3 uses a more realistic model with variable capacity utilization to conduct a calibrated exercise. Section 4 concludes the paper.

Republic, Estónia, Cyprus, Latvia, Lithuania, Hungary, Malta, Poland, Slovenia, and Slovakia, we assume the unit import cost of oil or oil equivalent is the same as for the other EU-15 countries. The figure for Ukraine is from Davis et al. (2005). For the United States, we use data from the Energy Information Administration. Finally, information for South Korea was found in Rabobank (2006). All data refers to the year of 2004, except for South Korea, which refers to 2005.

2 The Benchmark Model

This is a slightly modified version of the Benhabib-Farmer (1994) model. We introduce imported energy as a third production factor into this model in the same way as in Kim and Loungani (1992), Finn (1995 and 2000) and Rotemberg and Woodford (1996), among others. There are two production sectors in the economy, the final goods sector and the intermediate goods sector. The final goods sector is competitive and it uses a continuum of intermediate goods to produce final output according to the production technology,

$$Y = \left(\int_{i=0}^{1} y_i^{\lambda} di \right)^{\frac{1}{\lambda}}$$

where $\lambda \in (0,1)$ measures the degree of factor substitution among intermediate goods. Let p_i be the relative price of the ith intermediate goods in terms of the final good, the profits of the final good producers are given by

$$\Pi = Y - \int_{i=0}^{1} p_i y_i di.$$

First order conditions for profit maximization lead to the following inverse demand functions for intermediate goods:

$$p_i = Y^{1-\lambda} y_i^{\lambda-1}.$$

The technology for producing intermediate goods is given by

$$y_i = k_i^{a_k} n_i^{a_n} o_i^{a_o},$$

where k and n represent labor and capital, as usual, o is the third factor, say imported oil, used in production, and $(a_k + a_n + a_o) \geq 1$ measures returns to scale at the firm level. Assuming that firms are price takers in the factor markets, the profits of the ith intermediate good producer are given by

$$\Pi_i = p_i y_i - (r + \delta) k_i - w n_i - p^o o_i,$$

where $(r + \delta)$ denotes the user cost of renting capital, w denotes real wage, and p^o denotes the real price of imported oil. The intermediate goods producers are monopolists facing downward sloping demand curves for intermediate goods, hence the profit functions can be rewritten as

$$\pi_i = Y^{1-\lambda} y_i^{\lambda} - (r + \delta) k_i - w n_i - p^o o_i,$$

which is concave as long as $\lambda(a_k + a_n + a_o) \leq 1$. Profit maximization by each intermediate

goods producing firm leads to the following first order conditions:

$$r + \delta = \lambda a_k \frac{p_i y_i}{k_i}$$

$$w = \lambda a_n \frac{p_i y_i}{n_i}$$

$$p^o = \lambda a_o \frac{p_i y_i}{o_i}.$$

In a symmetric equilibrium, we have $n_i = n, k_i = k, o_i = o, y_i = y = Y, \pi_i = \pi, p_i = 1$, and

$$\Pi = Y - \left(\int_{i=0}^{1} y_i^{\lambda} di \right)^{\frac{1}{\lambda}} = 0$$

$$\pi = \left(1 - \lambda(a_k + a_n + a_o)\right) Y.$$

In words, perfect competition in the final goods sector leads to zero profit and imperfect competition in the intermediate goods sector leads to positive profit if $\lambda(a_k + a_n + a_o) < 1$.

A representative consumer in the economy maximizes utility,

$$\sum_{t=0}^{\infty} \beta^t \left(\log c_t - b \frac{n_t^{1+\gamma}}{1+\gamma} \right)$$

subject to

$$c_t + s_{t+1} = (1 + r_t) s_t + w_t n_t + \pi_t,$$

where s is aggregate saving. Since the aggregate factor payment, $p^o o$, goes to the foreigners, it is not included in the consumer's income. The first order conditions for utility maximization with respect to labor supply and savings are given respectively by

$$b n_t^{\gamma} = \frac{1}{c_t} w_t,$$

$$\frac{1}{c_t} = \beta \frac{1}{c_{t+1}} \left(1 + r_{t+1}\right).$$

In equilibrium, $s_t = k_t$, and factor prices equal marginal products, the first order conditions and the budget constraint then become

$$b n_t^{1+\gamma} = \frac{1}{c_t} \lambda a_n y_t \tag{1.1}$$

$$\frac{1}{c_t} = \beta \frac{1}{c_{t+1}} \left(1 - \delta + \lambda a_k \frac{y_{t+1}}{k_{t+1}} \right) \tag{1.2}$$

$$c_t + k_{t+1} = (1 - \delta) k_t + (1 - \lambda a_o) y_t \tag{1.3}$$

$$y_t = k_t^{a_k} n_t^{a_n} o_t^{a_o}. \tag{1.4}$$

Note that the international trade balance is always zero. Foreigners are paid in goods. This is clear in equation (1.3), according to which domestic production is divided between consumption, investment and imports ($c_t + i_t + p_t^o o_t = y_t$). So part of what is produced domestically is used to pay for the imports ($p^o o$). This is the interpretation of Finn (2000), Barski and Killian (2001), Wei (2003), Leduc and Sill (2004) and Aguiar-Conraria and Wen (2006) in similar models. Alternatively, one could consider $p^o o$ as the value added of an exogenous input production sector. This latter possibility is adopted by Kim and Loungani (1992), Finn (1995) and Rotemberg and Woodford (1996).

3 Conditions for Indeterminacy

Assuming the imported energy price is exogenous, p^o, we can substitute out o_t in the production function using

$$o_t = \lambda a_o \frac{y_t}{p^o},$$

to obtain the following reduced-form production function:

$$y_t = A k_t^{\frac{a_k}{1-a_o}} n_t^{\frac{a_n}{1-a_o}}, \tag{1.5}$$

where $A = \left(\frac{\lambda a_o}{p^o}\right)^{\frac{a_o}{1-a_o}}$ is a Solow residual, which is inversely related to the oil price. In this reduced-form production function, the effective returns to scale is measured by

$$\frac{a_k + a_n}{1 - a_o},$$

which exceeds the true returns to scale, $(a_k + a_n + a_o)$, provided that $(a_k + a_n + a_o) > 1$. Hence, the reliance on foreign oil amplifies the true returns to scale when there are increasing returns to scale in the economy.

It can be easily shown that a unique steady state exists in this economy. To study indeterminacy, we substitute y by utilizing equation 1.5 and log linearize equations 1.1-1.3 around the steady state. This gives

$$\left(1 + \gamma - \frac{a_n}{1 - a_o}\right) \hat{n}_t = \frac{a_k}{1 - a_o} \hat{k}_t - \hat{c}_t$$

$$-\hat{c}_t = -\hat{c}_{t+1} + (1 - \beta(1 - \delta)) \left(\left(\frac{a_k}{1 - a_o} - 1 \right) \hat{k}_{t+1} + \frac{a_n}{1 - a_o} \hat{n}_{t+1} \right)$$

$$(1 - s)\hat{c}_t + \frac{s}{\delta} \hat{k}_{t+1} = \left(\frac{a_k}{1 - a_o} + s \frac{1 - \delta}{\delta} \right) \hat{k}_t + \frac{a_n}{1 - a_o} \hat{n}_t$$

where s is the adjusted steady-state saving rate (investment-to-national income ratio) given

by

$$s = \frac{\delta k}{(1-\lambda a_o)y} = \frac{\delta\beta\lambda a_k}{(1-\lambda a_o)(1-\beta(1-\delta)}.$$

The above system of linear equations can be reduced to

$$M_1 \begin{bmatrix} k_{t+1} \\ \lambda_{t+1} \end{bmatrix} = M_2 \begin{bmatrix} k_t \\ \lambda_t \end{bmatrix}$$

where

$$M_1 = \begin{bmatrix} \frac{1-\beta(1-\delta)}{1-a_o}\left(a_k + a_o - 1 + \frac{a_n a_k}{(1+\gamma)(1-a_o)-a_n}\right) & 1 + \frac{(1-\beta(1-\delta))a_n}{(1+\gamma)(1-a_o)-a_n} \\ s\frac{1}{\delta} & 0 \end{bmatrix}$$

$$M_2 = \begin{bmatrix} 0 & 1 \\ \frac{a_k}{1-a_o}\left(1 + \frac{a_n}{(1+\gamma)(1-a_o)-a_n}\right) + s\frac{1-\delta}{\delta} & \frac{(1+\gamma)(1-a_o)}{(1+\gamma)(1-a_o)-a_n} - s \end{bmatrix}$$

Denote $B = M_1^{-1}M_2$, a necessary and sufficient condition for indeterminacy is that both eigenvalues of B are less than one in modulus. This is true if and only if the determinate and the trace of B satisfy

$$\begin{aligned} -1 &< \det(B) < 1 \\ -(1+\det(B)) &< \mathrm{tr}(B) < 1 + \det(B) \end{aligned}$$

The determinate and the trace of B are given by (see Appendix 1):

$$\det(B) = \frac{1}{\beta}\left[1 + \frac{(1+\gamma)(1-\beta(1-\delta))\frac{(1-\lambda)}{\lambda(1-a_o)}}{1+\gamma-\beta(1-\delta)\frac{a_n}{1-a_o}}\right] \tag{1.6}$$

$$\mathrm{tr}(B) = 1 + \det(B) + \frac{(1-\beta(1-\delta))(1+\gamma)\left(\frac{1-a_o-a_k}{1-a_o}\right)\delta\frac{1-s}{s}}{1+\gamma-\beta(1-\delta)\frac{a_n}{1-a_o}} \tag{1.7}$$

Notice that when $\lambda = 1$, then $\det(B) = 1/\beta > 1$, indicating saddle-path-stability as in a standard RBC model. Hence, what is crucial for indeterminacy is not increasing returns to scale *per se*, but also the degree of market power or imperfect competition.

The common denominator in the second term in expression (1.6) and the third term in (1.7) suggests that when the labor's elasticity of output in the reduced-form production function, $\frac{a_n}{1-a_o}$, increases, the model may go through a point of discontinuity at which $1+\gamma-\beta(1-\delta)\frac{a_n}{1-a_o} = 0$ and $\det(B)$ and $\mathrm{tr}(B)$ both change sign from $+\infty$ to $-\infty$, if the condition $1 - a_o - a_k > 0$ still holds. Clearly, when these terms are negative infinity, the conditions for $\det(B) < 1$ and $\mathrm{tr}(B) < 1 + \det(B)$ are trivially satisfied. But to reach the discontinuity point

such that the second term in (1.6) and the third term in (1.7) are negative, we need

$$\beta(1-\delta)\frac{a_n}{1-a_o} > 1+\gamma. \tag{1.8}$$

(1.8) is an important necessary condition for indeterminacy. Clearly, the larger a_o is, the easier this condition can be satisfied. To facilitate interpreting and comparing this condition with the literature, we map the monopolistic competition model into a one-sector competitive model with production externalities (see Benhabib and Farmer, 1994), in which the aggregate production function is replaced by

$$y_t = k_t^{\alpha_k(1+\eta)} n_t^{\alpha_n(1+\eta)} o_t^{\alpha_o(1+\eta)},$$

and the reduced-form production function is replaced by

$$y_t = A k_t^{\frac{\alpha_k(1+\eta)}{1-\alpha_o(1+\eta)}} n_t^{\frac{\alpha_n(1+\eta)}{1-\alpha_o(1+\eta)}}$$

where $(\alpha_k + \alpha_n + \alpha_o) = 1$ and the parameter η measure the degree of production externalities. This model is identical to the monopolistic competition model if $\lambda a_k = \alpha_k$, $\lambda a_n = \alpha_n$, $\lambda a_o = \alpha_o$, and $(a_k + a_n + a_o) = 1 + \eta$. This gives $\lambda(a_k + a_n + a_o) = \lambda(1+\eta) = 1$, implying that in the corresponding monopolistic competition model the intermediate goods producing firms earn zero profits. In the externality version of the model, aggregate returns to scale are measured by $1 + \eta$. With this change in framework, equations (1.6) and (1.7) become

$$\det(B) = \frac{1}{\beta}\left[1 + \frac{(1+\gamma)(1-\beta(1-\delta))\frac{\eta}{1-\alpha_o(1+\eta)}}{1+\gamma-\beta(1-\delta)\frac{\alpha_n(1+\eta)}{1-\alpha_o(1+\eta)}}\right] \tag{1.9}$$

$$\mathrm{tr}(B) = 1 + \det(B) + \frac{(1-\beta(1-\delta))(1+\gamma)\left(\frac{1-(\alpha_o+\alpha_k)(1+\eta)}{1-\alpha_o(1+\eta)}\right)\delta\frac{1-s}{s}}{1+\gamma-\beta(1-\delta)\frac{\alpha_n(1+\eta)}{1-\alpha_o(1+\eta)}} \tag{1.10}$$

Clearly, indeterminacy is not possible if $\eta = 0$, which implies $\det(B) = 1/\beta > 1$. This shows that monopoly power in the previous version of the model pertains to externality in the current version of the model. Condition 1.8 thus becomes

$$\beta(1-\delta)\frac{\alpha_n(1+\eta)}{1-\alpha_o(1+\eta)} - 1 > \gamma,$$

which can also be expressed as

$$\eta > \frac{(1+\gamma)(1-\alpha_o) - \beta(1-\delta)\alpha_n}{\beta(1-\delta)\alpha_n + (1+\gamma)\alpha_o}. \tag{1.11}$$

Condition (1.11) is analogous to that derived by Benhabib and Farmer (1994) in a continuous

time model when $\delta \to 0$ and $\beta \to 1$. In a continuous time version of the model, this condition simplifies to

$$\eta > \frac{1-(\alpha_o+\alpha_n)+\gamma(1-\alpha_o)}{(\alpha_o+\alpha_n)+\gamma\alpha_o}.$$

If $\alpha_o = 0$ (i.e., production does not require the imported factor), then this condition for indeterminacy is identical to that in Benhabib and Farmer (1994). Since the right hand side is a decreasing function of α_o, this necessary condition for indeterminacy is much easier to satisfy than that in the Benhabib-Farmer model.

To further pin down the full set of conditions for indeterminacy, note that as long as $(\alpha_o + \alpha_k)(1+\eta) < 1$, the second term in the determinate of B and the third term in the trace of B must pass through $-\infty$ for large enough η and moves to a finite negative number as η keeps increasing. Since we are interested only in the smallest value of η that gives rise to indeterminacy, we can therefore limit our attention to the following simpler one-sided conditions as necessary and sufficient conditions for indeterminacy:

$$\det(B) > -1 \text{ and } \mathrm{tr}(B) > -(1+\det(B)),$$

assuming the necessary condition (1.11) is satisfied.

The condition $\det(B) > -1$ implies

$$\eta > \frac{(1+\gamma)(1-\alpha_o)-\beta(1-\delta)\alpha_n}{\beta(1-\delta)\alpha_n+(1+\gamma)\alpha_o-\frac{1+\gamma}{1+\beta}(1-\beta(1-\delta))}.$$

Note that if this condition is satisfied, then condition (1.11) is also satisfied since they differ only by a positive term, $\frac{1+\gamma}{1+\beta}(1-\beta(1-\delta))$. In a continuous time version of this model ($\delta \to 0, \beta \to 1$), this condition simplifies to

$$\eta > \frac{1-(\alpha_o+\alpha_n)+\gamma(1-\alpha_o)}{(\alpha_o+\alpha_n)+\gamma\alpha_o},$$

which is identical to (1.11). Hence, an increase in α_o, either holding α_n constant or holding $(\alpha_o + \alpha_n)$ constant, will decrease the right hand side, making indeterminacy easier to arise.

The condition $\mathrm{tr}(B) > -(1+\det(B))$ implies

$$\frac{1+\eta}{1-\alpha_o(1+\eta)} > \frac{2(1+\gamma)(2-\delta)+(1+\gamma)\delta\frac{1-s}{s}(1-\beta(1-\delta))}{2(1+\beta)(1-\delta)\alpha_n-(1+\gamma)\frac{\delta\alpha_k}{s}(2-(1-s)(1-\beta(1-\delta)))}.$$

Clearly, the presence of α_o on the left-hand side makes the inequality easier to satisfy the larger the value of α_o is. Alternatively, we can consider a continuous time version of the model ($\delta \to 0, \beta \to 1$), then the above condition simplifies to

$$\frac{1+\eta}{1-\alpha_o(1+\eta)} > \frac{1+\gamma}{\alpha_n},$$

which implies

$$\eta > \frac{1-(\alpha_o+\alpha_n)+\gamma(1-\alpha_o)}{(\alpha_o+\alpha_n)+\gamma\alpha_o}.$$

This is identical to the condition implied by $\det(B) > -1$. Hence, the necessary and sufficient conditions for indeterminacy are all easier to be satisfied if $\alpha_o > 0$.

4 A Calibrated Exercise with Capacity Utilization

The previous analysis, based on a simple benchmark model, provides the essential understanding on the mechanism as to how the dependence of production on imported energy can increase the likelihood of indeterminacy under increasing returns to scale. Now we derive a more realistic neoclassical growth model with variable capital utilization. The reason for introducing capacity utilization is three-fold: (i) As argued by Schmitt-Grohé (1997) and many others, the Benhabib-Farmer model is empirically implausible since it requires extremely large increasing returns to scale to generate indeterminacy. Hence it is not a good reference point for calibration. (ii) Both empirical and theoretical studies show that allowing for capacity utilization can dramatically reduce the estimated returns to scale in the data (see, e.g., Burnside et al., 1995) and the required returns to scale in the model (see Wen, 1998). For the U.S. and most European countries, the estimated returns to scale after controlling for capacity utilization are around 1.1,[4] which is slightly below the lower bond for generating indeterminacy in models with capacity utilization (see Wen, 1998). Given this low level of increasing returns to scale, it is interesting to study whether a cost share of foreign energy around 5% is significant enough for triggering indeterminacy. (iii) Our analysis with capacity utilization also serves as a robustness check for the results obtained in the Benhabib-Farmer model.

We show that when capacity utilization is allowed to vary, dependence on foreign energy can also significantly reduce the required level of increasing returns to scale for indeterminacy. Especially, given that most European countries have estimated returns to scale around 1.1, which is right at the threshold of indeterminacy in a model with capacity utilization, even a 5% cost share of foreign energy in domestic production can turn an otherwise stable economy into an indeterminate economy, making it vulnerable to sunspots shocks.

The Model: The model is based on Wen (1998). A representative agent chooses sequences of consumption (c), hours (n), capacity utilization (e), and capital accumulation (k) to solve

$$\max\ E_0\sum_{t=0}^{\infty}\beta^t\left(\log c_t - b\frac{n_t^{1+\gamma}}{1+\gamma}\right)$$

subject to

$$c_t + [k_{t+1} - (1-\delta_t)k_t] + p_t o_t = y(e_t k_t, n_t, o_t),$$

[4] See, e.g., Hansen and Knowles 1998, Laitner and Stolyarov 2004 and Inklaar 2006, among others.

where the home country pays the amount $p_t^o o_t$ in terms of output to foreigners to receive the amount o_t as energy input,[5] and where the production technology is given by

$$y(e_t k_t, n_t, o_t) = \Phi_t \left(e_t k_t\right)^{\alpha_k} n_t^{\alpha_n} o_t^{\alpha_o},$$

in which $e_t \in [0,1]$ denotes capital utilization rate, and Φ_t is a measure of production externalities and is defined as a function of average aggregate output which individual firms take as parametric:

$$\Phi_t = \left[(e_t k_t)^{\alpha_k} n_t^{\alpha_n} o_t^{\alpha_o}\right]^{\eta}, \quad \eta \geq 0.$$

The rate of capital depreciation, δ_t, is time variable and is endogenously determined in the model. In particular, it is assumed capital depreciates faster if it is used more intensively:

$$\delta_t = \frac{1}{\theta} e_t^{\theta}, \quad \theta > 1;$$

which imposes a convex cost structure on capital utilization.[6]

Proposition 1 *The necessary and sufficient conditions for indeterminacy under variable capacity utilization are given by*

$$\eta > \frac{\theta\left[(1+\gamma)(1-\alpha_o) - \beta\alpha_n\right] - (1+\gamma)\alpha_k}{\theta\beta\alpha_n + (1+\gamma)(\alpha_k + \alpha_o\theta) - \theta(1+\gamma)\frac{1-\beta}{1+\beta}}, \tag{1.13}$$

$$1 + \eta > \frac{\left[2\left(1+\gamma\right) + \left(1-\beta\right)\phi\right]\theta}{\alpha_n\left(1+\beta\right)\theta - \alpha_k\left(1+\beta\right)\phi\left(\theta-1\right) + \left[2\left(1+\gamma\right) + \left(1-\beta\right)\phi\right]\left(\alpha_k + \theta\alpha_o\right)}, \tag{1.14}$$

where $\phi = \left[(1-\alpha_o)\frac{\theta}{\alpha_k} - 1\right](1+\gamma)\frac{\delta}{2}$

Proof. See Appendix 2.■

Calibration: We calibrate the model's structural parameters following Benhabib and Wen (2004) and Wen (1998). Namely, we set the time period in the model to a quarter, the time discounting factor $\beta = 0.99$, the steady-state rate of capital depreciation $\delta^* = 0.025$ (which implies $\theta = 1.404$), the inverse labor supply elasticity $\gamma = 0$, and the labor elasticity of output $\alpha_n = 0.7$.

Given these parameter values, the following table shows the relationship between the share of foreign energy in GDP and the threshold value of the production externality for inducing indeterminacy (η^*).

[5] Note that trade is balanced in every period since the cost of intermediate goods – energy imports – are paid for with exports of output. Hence national income is given by $y - po$, which equals domestic consumption and capital investment.

[6] The equivalence between a representative-agent model with aggregate externalities and an imperfect competition model with increasing returns to scale at the firm level can be easily established. We choose to work with the representative agent model because it is easier to calibrate.

Table 2. The Effect of Factor Shares on Indeterminacy

Energy Imports Share (α_o)	Required Externality	Reduction of η^*
0.00	0.1037	0
0.04	0.0898	13%
0.08	0.0763	26%
0.12	0.0631	39%
0.16	0.0502	52%

We observe in Table 2 that as the share of foreign factor in domestic production increases, the threshold value of the production externality for inducing indeterminacy (η^*) decreases significantly. For example, when we increase the share parameter of imported energy α_o from zero percent to 10 percent, the reduction in the externality is 33%. And if we increase the share parameter to 16 percent, then the reduction in the externality is 52%.[7]

If we compare the values of table 1 with table 2, we see that the required returns to scale for indeterminacy may vary between 1.05 and 1.10 in the presence of foreign energy imports. These values imply that many countries are in the dangerous zone of indeterminacy. For example, Laitner and Stolyarov (2004) found the estimated returns to scale around $1.09 - 1.11$ for the U.S. economy. Inklaar (2006) found the estimated returns to scale around 1.16 for Germany and 1.12 for France. Hansen and Knowles (1998) found the average estimated returns to scale around 1.105 for high income OECD countries (including Australia, Belgium, Canada, Finland, France, West Germany, Japan, Norway, Sweden, the United Kingdom and the United States). Miyagawa et al. (2006) found estimated returns to scale in Japan about 1.075, and Kwack and Sun (2005) found it to be around 1.1 for South Korea. With these numbers in mind, it is clear the dependence on imported energy can significantly increase a country's risk of indeterminacy, thereby making the country more susceptible to sunspots-driven fluctuations.

5 Conclusion

The impact of oil price shocks on economic fluctuations have been widely recognized. But the relationship between economic stability and the reliance on foreign energy has not been fully investigated in the literature. This paper shows dependence of domestic production on imported energy, such as oil or natural gas, can significantly increase the economy's instability in the presence of externalities or increasing returns to scale, because it reduces the required

[7] Table 2. is computed under the assumption that the foreign imported factor is mainly a substitute for capital, hence when α_o increases, α_n remains constant but α_k decreases such that $\alpha_k + \alpha_o$ remains constant (assuming constant returns to scale at the firm level). If we assume imported energy is mainly a substitute for labor instead (i.e., $\alpha_n + \alpha_o$ is fixed), then a larger α_o has the same qualitative consequences, although less dramatic.

degree of returns to scale for indeterminacy. As a result, the economy is more susceptible to endogenous fluctuations driven by self-fulfilling expectations.

6 Appendix 1

Given $B = M1^{-1}M2$, we have $det(B) = \frac{det(M2)}{det(M1)}$. Straightforward re-arrangement shows that

$$\det(B) = \frac{1}{\beta}\left[1 + \frac{(1+\gamma)(1-\beta(1-\delta))\frac{(1-\lambda)}{\lambda(1-a_o)}}{1+\gamma-\beta(1-\delta)\frac{a_n}{1-a_o}}\right].$$

Also, given

$$B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$$

we have $\mathrm{tr}(B) = b_{11} + b_{22}$, with

$$\begin{cases} b_{11} = \frac{\delta}{s}\frac{(1+\gamma)a_k}{(1+\gamma)(1-a_o)-a_n} + 1 - \delta \\ b_{22} = \frac{(1+\gamma)(1-a_o)-a_n}{(1+\gamma)(1-a_o)-\beta(1-\delta)a_n} - \left[\frac{\delta}{s}\frac{(1+\gamma)a_k}{(1+\gamma)(1-a_o)-a_n} - \delta\right]\left[\frac{(1-\beta(1-\delta))(a_n+(1+\gamma)(a_k+a_o-1))}{(1+\gamma)(1-a_o)-\beta(1-\delta)a_n}\right] \end{cases}.$$

Re-arrangement gives

$$\mathrm{tr}(B) = 1 + \det(B) + \frac{(1-\beta(1-\delta))(1+\gamma)\left(\frac{1-a_o-a_k}{1-a_o}\right)\delta\frac{1-s}{s}}{1+\gamma-\beta(1-\delta)\frac{a_n}{1-a_o}}.$$

■

7 Appendix 2

Denote λ_t as the Lagrangian multiplier for the budget constraint, the first order conditions with respect to $\{c, n, e, o, k\}$ and the budget constraint are given respectively by

$$\frac{1}{c_t} = \lambda_t \tag{A}$$

$$bn_t^{1+\gamma} = \lambda_t \alpha_n \left(e_t k_t\right)^{\alpha_k(1+\eta)} n_t^{\alpha_n(1+\eta)} o_t^{\alpha_o(1+\eta)} \tag{B}$$

$$\alpha_k \frac{y_t}{k_t} = e_t^{\theta} \tag{C}$$

$$\alpha_o y_t = p_t o_t \tag{D}$$

$$\lambda_t = \beta\lambda_{t+1}\left[\alpha_k\frac{y_{t+1}}{k_{t+1}} + 1 - \frac{1}{\theta}e_{t+1}^{\theta}\right] \tag{E}$$

$$c_t + k_{t+1} - (1 - \frac{1}{\theta}e_t^{\theta})k_t = (1-\alpha_o)y_t. \tag{F}$$

To simplify the analysis, we use equation (C) to substitute out e in the production function to get

$$y_t = Ak_t^{\alpha_k(1+\eta)\tau_k} n_t^{\alpha_n(1+\eta)\tau_n} o_t^{\alpha_o(1+\eta)\tau_n} \tag{G}$$

where $\tau_k \equiv \frac{\theta-1}{\theta-\alpha_k(1+\eta)}, \tau_n \equiv \frac{\theta}{\theta-\alpha_k(1+\eta)}$. Next, we use equation (D) to substitute out o in the production function (G) to get

$$y_t = \tilde{A} k_t^{\frac{\alpha_k(1+\eta)\tau_k}{1-\alpha_o(1+\eta)\tau_n}} n_t^{\frac{\alpha_n(1+\eta)\tau_n}{1-\alpha_o(1+\eta)\tau_n}}. \tag{H}$$

After similar substitutions in all equations, the above equation system is reduced to

$$c_t = \frac{\alpha_n}{b}\frac{y_t}{n_t^{1+\gamma}} \tag{A'}$$

$$c_{t+1} = \beta c_t \left[(1-\frac{1}{\theta})\alpha_k \frac{y_{t+1}}{k_{t+1}} + 1\right] \tag{B'}$$

$$c_t + k_{t+1} - k_t = (1-\alpha_o - \frac{\alpha_k}{\theta})y_t \tag{C'}$$

where the production function is given by (H). Denote $a^* \equiv \frac{\alpha_k(1+\eta)\tau_k}{1-\alpha_o(1+\eta)\tau_n}, b^* \equiv \frac{\alpha_n(1+\eta)\tau_n}{1-\alpha_o(1+\eta)\tau_n}$, log-linearize the above equations (A'-C') around the steady state and substitute out c_t using (A'), we have the following simplified 2-variable system:

$$(1+\beta(a^*-1))\hat{k}_{t+1} + (\beta b^* - (1+\gamma))\hat{n}_{t+1} = a^*\hat{k}_t + (b^* - (1+\gamma))\hat{n}_t$$

$$\hat{k}_{t+1} = \hat{k}_t + \left[(1-\alpha_o)\frac{\theta}{\alpha_k} - 1\right]\delta(1+\gamma)\hat{n}_t$$

or

$$M_1 \begin{bmatrix} \hat{k}_{t+1} \\ \hat{n}_{t+1} \end{bmatrix} = M2 \begin{bmatrix} \hat{k}_t \\ \hat{n}_t \end{bmatrix}$$

where

$$M_1 = \begin{bmatrix} 1+\beta(a^*-1) & \beta b^* - (1+\gamma) \\ 1 & 0 \end{bmatrix}$$

$$M_2 = \begin{bmatrix} a^* & b^* - (1+\gamma) \\ 1 & \left[(1-\alpha_o)\frac{\theta}{\alpha_k} - 1\right]\delta(1+\gamma) \end{bmatrix}.$$

Hence, the Jacobian is given by

$$B = M_1^{-1}M_2 = \begin{bmatrix} 1 & \left[(1-\alpha_o)\frac{\theta}{\alpha_k} - 1\right]\delta(1+\gamma) \\ \frac{(1-\beta)(1-a^*)}{1+\gamma-\beta b^*} & \frac{1+\gamma-b^*+(1+\beta(a^*-1))\left[(1-\alpha_o)\frac{\theta}{\alpha_k}-1\right]\delta(1+\gamma)}{1+\gamma-\beta b^*} \end{bmatrix},$$

which implies that the determinate and the trace of B are given by (after simplification and re-arrangement):

$$\det(B) = \frac{1}{\beta}\left[1 + \frac{(1+\gamma)(1-\beta) + \delta\beta(1+\gamma)(\alpha_k - (1-\alpha_o)\theta)\frac{a^*}{\alpha_k}}{(b^*\beta - (1+\gamma))}\right]$$

$$\mathrm{tr}(B) = 1 + \det(B) + \frac{(1-\beta)(1-a^*)\left[(1-\alpha_o)\frac{\theta}{\alpha_k} - 1\right]\delta(1+\gamma)}{1+\gamma-\beta b^*}.$$

Following the same discussions in section 3, it can be shown that the value of η that satisfies the condition, $\det(B) > -1$, also satisfies the condition, $\beta b^* > 1+\gamma$, hence the necessary and sufficient conditions for indeterminacy can be limited to the value of η that satisfy:

$$\det(B) > -1 \text{ and } \mathrm{tr}(B) > -(1+\det(B)).$$

These two conditions imply the conditions in proposition 4.1■

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