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Costly Investment, Complementarities and the Skill Premium

Óscar Afonso\textsuperscript{a} and Maria Thompson\textsuperscript{b}

\textsuperscript{a} CEFUP, OBEGEF and Faculdade de Economia da Universidade do Porto. Email: oafonso@fep.up.pt; Address: Rua Roberto Frias 4200-464 Porto, Portugal; Phone: +351225571100 (ext. 531); Fax: +351225505050

\textsuperscript{b} NIPE, and Escola de Economia e Gestão da Universidade do Minho. Address: Gualtar, 4710-057 Braga, Portugal; Email: mjthompson@eeg.uminho.pt; Phone: +351253604510 (ext 5548).

Abstract: We propose a new framework to analyse the wage premium behaviour. Building on Acemoglu and Zilibotti (2001), we introduce physical capital and two assumptions: (i) internal costly investment in both capital and R&D; (ii) complementarities between capital goods in production. We find that, for economies relatively abundant in high-skilled labour, a rise in the relative endowment of high-skilled labour is accompanied by a rise in the skill premium. We further find that a rise (i) in investment costs or (ii) in the complementarities degree, requires an increase in the relative endowment of high-skilled labour, for the economy to remain in the same growth equilibrium. For economies relatively abundant in high-skilled labour, such rises are also accompanied by an increase in the skill premium.

Keywords: R&D; Technological-Knowledge Bias; Wage Inequality; Complementarities; Costly Investment.
JEL Classifications: J31, O31, O33, O47.

1 Introduction

With the present paper we propose a theoretical contribution to the growing literature on wage inequality.

Our contribution stems from an important debate in the related empirical literature on whether an increase in the relative supply of high-skilled labour contributes to the widening of wage inequality. On one hand, empirical evidence for many developed countries, for instance, Kranz (2006), Acemoglu (2003) and Zhu and Treffer (2005), reveals a simultaneous rise in the relative supply of high-skilled workers and the wage inequality in favor of high-skilled labour, since the 1980s. As surveyed by Richardson (1995), Aghion et al. (2003) and He and Liu (2008), the skill-biased technological change theory\textsuperscript{1} is the most accepted theory for explaining a simultaneous rise in the skill premium and the

\textsuperscript{1}See also Bound and Johnson (1992), Katz and Murphy (1992), Juhn et al. (1993).
relative supply of high-skilled labour\textsuperscript{2}. The main argument is that technological-knowledge progress induces an increase in the relative demand of high-skilled workers which exceeds the increase in its relative supply, thus raising the skill premium. Acemoglu (1998, 2002) and Acemoglu and Zilibotti (2001) further develop this hypothesis by considering that technological-knowledge progress responds to shifts in labour endowments. These authors interpret the rise in the skill premium as a direct consequence of the increase in the relative supply of high-skilled workers. When the supply of one type of labour increases, the market for technologies used by this type of labour broadens, creating additional incentives for R&D activities aimed at those specific technologies. Consequently, technological-knowledge progress steers towards those technologies, which, in turn, increases the demand for their complementary type of labour.

On the other hand, some empirical evidence, such as that documented by, for instance, Acemoglu (2003), Ćrnč (2005) and Robertson (2004), does not always find a simultaneous rise in the skill premium and the relative supply of high-skilled labour, the suggested explanations for these results resting on international trade.

Our closed-economy model suggests that both these seemingly contradictory results are possible. It predicts that an increase in the relative supply of high-skilled labour contributes to the widening of the wage gap in economies relatively abundant in high-skilled labour, whereas in economies relatively scarce in high-skilled labour the opposite result prevails. The model builds on Acemoglu and Zilibotti (2001), adding physical capital and two assumptions new to this literature, namely: (i) an Hayashi’s (1982) internal cost of investment in both physical capital and R&D; and (ii) complementarities between intermediate goods in the production function of final goods, as in Evans et al. (1998).

We bring the costly nature of investment into the model with a twofold motivation. Firstly, we follow the argument of Benavie et al. (1996) and Romer (1996) that growth models should treat investment as a decision variable of firms, and this requires costs to accumulating capital\textsuperscript{3}. Secondly, we also wish to accommodate in the model the idea that R&D expenses should be considered capital investment expenses, as Anagnostopoulos (2008) reviews. In the proposed framework, R&D investment is part of total capital investment. Hence, we can analyse the wage inequality behaviour in an environment with internal capital investment costs which include R&D expenses.

We also incorporate another relevant feature of industrialised economies, that of the existence of complementarities between intermediate goods in final-goods production function. The presence of complementarities means that if the number of its complementary goods increases, the production of a capital good

\textsuperscript{2}Other theories on wage inequality include, for example, openness to international trade, changes in the unionization rate, and change in real minimum wages. A general consensus is that the skill-biased technological change theory provides a more compelling story than these other theories. For a survey on this literature, see, for example, Acemoglu (2002) and Aghion (2002).

\textsuperscript{3}Turnovsky (1995) reviews the implications of the introduction of an adjustment costs function in macroeconomics, and in particular its link with the Tobin-q.
will increase. In turn, by increasing its output, a producer of an intermediate good raises the demand for its complementary intermediate goods. Personal computers, printers and communication networks are familiar examples of such complementarities. As Matsuyama (1995) notes, complementarities should be an essential feature in explaining economic growth, business cycles and under-development⁴. This is our motivation for considering them in our study of the wage inequality behaviour.

We find that for economies relatively abundant in high-skilled labour, there is a positive relationship between the relative supply of high-skilled labour and the wage premium. In this case, the proposed framework is in agreement with the skill-biased technological change theory. It disagrees with the mainstream theory, when it comes to economies relatively scarce in high-skilled labour. Hence the model provides a potential basis for consensus between the two opposite streams of empirical results above mentioned.

We further find that, everything else being constant, (i) a rise in investment costs or (ii) a rise in the degree of complementarities between intermediate goods, requires a rise in the relative endowment of high-skilled labour for the economy to remain in the same balanced growth path equilibrium. For economies relatively abundant in high-skilled labour, this (i) rise in investment costs or (ii) rise in the degree of complementarities between intermediate goods is accompanied by a rise in the skill premium.

After this Introduction, in Section 1 the economy is characterised, the model is set up and solved, and main results are discussed. The effects of an increase in the investment cost parameter are analysed in Section 2, while the effects of an increase in the complementarities parameter are studied in Section 3. Concluding Remarks close the present research.

2 Specification and Results of the Model

2.1 Consumption Side

The economy is populated by infinitely-lived households, with zero population growth. Households supply labour, consume final goods and own firms. Households are endowed with ability level $a \in [0,1]$ and supply one of two types of labour. They supply low (high)-skilled labour, $L_a$ ($H_a$), if $a \leq \bar{a}$ ($a > \bar{a}$). The amount of low (high)-skilled labour supplied to the economy is $L = \int_0^\bar{a} L_a da \left( H = \int_\bar{a}^1 H_a da \right)$, and is paid at a wage rate $w_L$ ($w_H$); thus, $L + H = 1$.

All households have identical preferences characterized by a constant relative risk aversion lifetime utility function, $\int_0^\infty e^{-\rho t} C_a(t)^{1-\sigma} \frac{1}{1-\sigma} dt$, where $C_a(t)$ is the consumption of household $a$ at time $t \in \mathbb{R}_0^+$, $\rho$ is the subjective discount rate, and $\sigma$ is the coefficient of relative risk aversion.

⁴This idea has long been conveyed by authors such as Hicks (1950), Kaldor (1985) and Myrdal (1957). In particular, Bryant (1983) stressed the importance of complementarities between capital goods in production.
Households accumulate assets, $E$, which earn returns at the interest rate $r(t)$. Each household’s stock of assets is composed by his/her net savings, given by the difference between his/her income (interest and wages) and his/her consumption. The flow budget constraint of household $a$ is $\dot{E}_a(t) = r(t)E_a(t) + w_F(t)F_a(t) - C_a(t)$, where $\dot{E}_a(t)$ is the change in the assets stock of $a$, $F = L$ if $a \leq \pi$ and $F = H$ if $a > \pi$.

Household $a$ maximizes lifetime utility subject to the budget constraint and the “no Ponzi games” condition $\lim_{t \rightarrow \infty} E_a(t) e^{-pt} = 0$. The solution for the consumption path, which is independent from the household, is the standard Euler equation:

$$\frac{\dot{C}_a(t)}{C_a(t)} = \frac{\dot{C}(t)}{C(t)} = \frac{r(t) - \rho}{\sigma},$$

where $\dot{C}(t)$ is the change in aggregate consumption. It says that in a balanced growth path the interest rate must be constant.

### 2.2 Production Side

#### 2.2.1 Complementarities between Intermediate Goods

Building on Acemoglu and Zilibotti (2001), in the final good $Y$ sector, competitive firms are indexed by $n$ over the range $[0, 1]$. Two substitute production technologies are available. The Low (High)-technology, which we name the $L$ ($H$)-technology, uses a combination of Low (High)-skilled labour and a continuum of Low (High)-specific intermediate goods indexed by $j \in [0, A_L(t)]$ ($j \in [0, A_H(t)]$). Intermediate goods enter complementarily in the production function as in Evans et al. (1998). The production function of firm $n$ is given by:

$$Y_n(t) = ((1 - n)L_n)^{1 - \alpha} \left( \int_0^{A_L(t)} x_{jn} (t)^\gamma\, dj \right)^{\phi} + (nhH_n)^{1 - \alpha} \left( \int_0^{A_H(t)} x_{jn} (t)^\gamma\, dj \right)^{\phi},$$

where $x_{jn}(t)$ represents the quantity of the intermediate good $j$ used to produce the final good $n$. Variables $A_L(t)$ and $A_H(t)$ represent, respectively, the number of Low and High intermediate goods or, in other words, the Low and High technological-knowledge stock, at time $t$. The integral terms in (2) are the contributions of intermediate goods to production. The ratio $A_H/A_L = A$ is our measure of the technological-knowledge bias.

The restriction $\gamma\phi = \alpha$ is imposed so as to have constant returns to scale, since $0 < \alpha < 1$, and $\alpha$ is the aggregate intermediate-goods input share. The restriction $\phi > 1$ is made so that intermediate goods are complementary to one another; that is, so that an increase in the quantity of one intermediate good increases the marginal productivity of the other intermediate goods.

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5 See also Acemoglu (1998, 2002), and Afonso (2006, 2008).
Terms \((1 - n) \ell L_n\) and \(nh H_n\) sum up the contribution of labour to production: (i) Variables \(L_n\) and \(H_n\) are the amounts of, respectively, Low and High-skilled labour used by firm \(n\); (ii) \(l\) and \(h\) are the productivity parameters of, respectively, Low and High-skilled labour, and an absolute productivity advantage of High-skilled over Low-skilled labour is accounted for by assuming that \(1 \leq l < h\); (iii) Terms \((1 - n)\) and \(n\) imply that Low (High)-skilled labour is relatively more productive in producing lower (higher)-index final goods. Both \(\int_0^1 L_n dn = L\) and \(\int_0^1 H_n dn = H\) hold at any moment in time.

To solve the model for a constant growth rate, we impose the following parameter restriction:

\[\xi = \frac{\phi - 1}{1 - \alpha}.\]

Considering \(p_n(t)\) the price of final good \(n\), and normalising the price of the composite final good, \(Y\), at each time \(t\) to one, this economy’s output is obtained by integration over the \(n\) final goods:

\[Y(t) = \int_0^1 P_n Y_n(t) dn.\]

According to production function (2), Low-skilled labour is more productive when it comes to producing low-index final goods, whereas High-skilled labour is more productive at producing high-index final goods. This means that, as shown below, there is an endogenous threshold \(\overline{\pi}(t)\) such that the production of final goods \(n \in [0, \overline{\pi}(t)]\) uses only the \(L\)-technology, whereas the production of final goods \(n \in [\overline{\pi}(t), 1]\) uses only the \(H\)-technology. Hence, we have that:

\[Y(t) = \int_0^{\overline{\pi}(t)} P_n L Y_{nL}(t) dn + \int_{\overline{\pi}(t)}^1 P_n H Y_{nH}(t) dn,
\]

where \(P_n L (P_n H)\) is the price of final good \(n\) produced with \(L\) (\(H\))-technology.

The second productive activity concerns the production of physical machines for each of the already invented types of intermediate goods. Assuming that it takes one unit of physical capital to produce one physical unit of any type of intermediate good, physical capital \(K(t)\) is related to intermediate goods by the rule:

\[K(t) = K_L(t) + K_H(t), \quad K_L(t) = \int_0^{A_L(t)} x_{Lj}(t) dj \text{ and } K_H(t) = \int_0^{A_H(t)} x_{Hj}(t) dj,
\]

where \(x_{Lj}(t) (x_{Hj}(t))\) is the total quantity that each \(L\)-technology (\(H\)-technology) intermediate good firm produces.

Turning to R&D activities, following Rivera-Batiz and Romer (1991), we assume that new designs are invented with the same technology as that of the production of the final good and of intermediate goods. We further assume that the invention of the \(L(H)\)-technology patent \(i\) (\(i = A_L\) or \(i = A_H\)) requires,
like in Evans et al. (1998), \( i^\xi \) units of foregone output, meaning that there is a higher cost for designing goods with a higher index.

Total investment in each period is then given by:

\[
\dot{Z}(t) = \dot{Z}_L(t) + \dot{Z}_H(t);
\]

where:

\[
\dot{Z}_L(t) = \dot{K}_L(t) + \dot{A}_L(t)A_L(t)^\xi \quad \text{and} \quad Z_H(t) = \dot{K}_H(t) + \dot{A}_H(t)A_H(t)^\xi.
\]

Thus, \( K_L \) and \( K_H \) represent investment in physical capital, and \( \dot{A}_L A_L^\xi \) and \( \dot{A}_H A_H^\xi \) represent investment in the invention of new designs in the \( L \)- and \( H \)-technology, respectively. Total capital at time \( t \), \( Z(t) \), is equal to:

\[
Z(t) = K_L(t) + K_H(t) + \frac{1}{1+\xi} \left( A_L(t)^{1+\xi} + A_H(t)^{1+\xi} \right).
\]

(3)

It is shown, in the appendix, that in a balanced growth path, the ratio \( \frac{Y(t)}{Z(t)} \) is constant, meaning that we can write this economy’s production function as:

\[
Y(t) = BZ(t),
\]

(4)

where \( B \), the marginal productivity of total capital, is constant.

### 2.2.2 Internal Costly Investment

Following Thompson (2008), we consider that investment in total capital \( Z(t) \) involves an internal cost. We assume that, with zero capital depreciation, installing \( I(t) = \dot{Z}(t) \) new units of total capital requires spending an amount given by:

\[
J(t) = I(t) + \frac{1}{2} \theta \frac{I(t)^2}{Z(t)}.
\]

where \( \frac{1}{2} \theta \frac{I(t)^2}{Z(t)} \) represents the Hayashi’s (1982) installation cost, with \( \theta > 0 \) standing for the adjustment cost parameter.

The investment rate is chosen so as to maximise the present discounted value of cash flows. Having in mind equation (4), the current-value Hamiltonian is:

\[
H(t) = BZ(t) - I(t) - \frac{1}{2} \theta \frac{I(t)^2}{Z(t)} + q(t) \left[ I(t) - \dot{Z}(t) \right],
\]

where \( q(t) \) is the market value of capital and the transversality condition for this optimization problem is \( \lim_{t \to \infty} e^{-rt} q(t) Z(t) = 0 \). Recalling that the growth rate of output is \( g = \frac{\dot{Z}}{Z} \), the first-order condition is equivalent to:

\[
q = 1 + \theta g,
\]

(5)
and the co-state equation is, in a balanced growth path, equivalent to:

$$ q = \frac{B + \frac{1}{2} q_g^2}{\rho}. \quad (6) $$

Proceeding with the setting up of the model, let us look at the decisions made by each of the \( n \) final-good firms. Functioning in a perfect competition environment, each final-good firm maximises its profits, taking as given prices and wages:

$$ \max_{x_{nL_j}(t)} \pi_n(t) = P_n(t)Y_n(t) - w_L(t)L_n(t) - w_H(t)H_n(t) - $$

$$ - \int_0^{A_L(t)} R_{nL_j}(t)x_{nL_j}(t) dj - \int_0^{A_H(t)} R_{nH_j}(t)x_{nH_j}(t) dj. $$

The profit maximising conditions,

$$ \frac{d\pi_n}{dx_{nL_j}} = \frac{R_{nL_j}}{P_{nL}} \quad \text{and} \quad \frac{d\pi_n}{dx_{nH_j}} = \frac{R_{nH_j}}{P_{nH}}, $$

lead to the following demand functions faced by the \( L \)- and \( H \)-technology intermediate good firms:

$$ R_{nL_j} = \alpha P_{nL} \left[(1-n)l_n\right]^{1-\alpha} \left[ \int_0^{A_L} x_{nL_j}^\gamma dj \right]^{\phi-1} x_{nL_j}^{-\gamma-1}, \quad (7a) $$

$$ R_{nH_j} = \alpha P_{nH} \left[nh_n\right]^{1-\alpha} \left[ \int_0^{A_H} x_{nH_j}^\gamma dj \right]^{\phi-1} x_{nH_j}^{-\gamma-1}. \quad (7b) $$

Turning now to the intermediate good firms production decisions: Once invented, the physical production of each unit of the specialised intermediate good requires one unit of capital. Thus, in each period, the monopolistic \( L \) (\( H \))-technology intermediate good producer maximises its profits, taking as given the demand curve of each final-good firm \( n \) for its good:

$$ \max_{x_{nL_j}(t)} \pi_{nL_j}(t) = R_{nL_j}(t)x_{nL_j}(t) - rqx_{nL_j}(t), $$

$$ \max_{x_{nH_j}(t)} \pi_{nH_j}(t) = R_{nH_j}(t)x_{nH_j}(t) - rqx_{nH_j}(t), $$

which leads to the markup rule:

$$ R_{nL_j} = R_{nH_j} = R_L = R_H = R = \frac{rq}{\gamma}, $$

meaning that intermediate-good producers charge the same price for their differentiated goods, \( R \), to all final good producers. \( R \) is constant in a balanced growth path.
The symmetry of the model also implies that all \( L (H) \)-technology intermediate good producers sell the same quantities of their goods to each final-good firm \( n \). We can then rewrite equations (7a) and (7b) as, respectively:

\[
\begin{align*}
R_L &= \alpha P_{nL} [(1 - n) \ell L_n]^{1-\alpha} A_{L}^{\alpha - 1} \pi_n^{\alpha - 1} \\
R_H &= \alpha P_{nH} [nhH_n]^{1-\alpha} A_{H}^{\alpha - 1} \pi_n^{\alpha - 1}.
\end{align*}
\]

Taking into consideration that, in each period, we have

\[
L_n = \frac{L}{\pi} \quad \text{and} \quad H_n = \frac{H}{1 - \pi},
\]

and normalising prices so that:

\[
P_{\pi L} = \frac{P_{nL}}{\pi} \quad \text{and} \quad P_{\pi H} = \frac{P_{nH}}{(1 - \pi)}.
\]

equations (8) are equivalent to:

\[
x_{nL} = (1 - n) A_{\pi L} \ell L \left[ \frac{\alpha P_L}{R_L} \right]^{\frac{1}{\pi}} \quad \text{and} \quad x_{nH} = n A_{\pi H} hH \left[ \frac{\alpha P_H}{R_H} \right]^{\frac{1}{1-\pi}}.
\]

It follows that the total quantity that each \( L (H) \)-technology intermediate good firm produces and sells is, respectively:

\[
x_L = \int_0^{\pi} x_{nL} dn = \left( \frac{\pi (2 - \pi)}{2} \right) A_{\pi L} \ell L \left[ \frac{\alpha P_L}{R_L} \right]^{\frac{1}{\pi}},
\]

\[
x_H = \int_{\pi}^1 x_{nH} dn = \left( \frac{1 - \pi^2}{2} \right) A_{\pi H} hH \left[ \frac{\alpha P_H}{R_H} \right]^{\frac{1}{1-\pi}}.
\]

Each \( L (H) \)-technology intermediate good producer’s profits are, respectively:

\[
\pi_L = (1 - \gamma) x_L R_L = \left( \frac{\pi (2 - \pi)}{2} \right) (1 - \gamma) A_{\pi L} \ell L \left( \frac{\alpha P_L}{R_L} \right) R_L^{\frac{1}{\pi}},
\]

\[
\pi_H = (1 - \gamma) x_H R_H = \left( \frac{1 - \pi^2}{2} \right) (1 - \gamma) A_{\pi H} hH \left( \frac{\alpha P_H}{R_H} \right) R_H^{\frac{1}{1-\pi}}.
\]

At time \( t \), in order to enter the market and produce the \( A_L (A_H) \)th intermediate good, a firm must spend up-front an amount given by \( A_L(t) \) \((A_H(t)\)\). Hence, the dynamic zero-profit conditions are:

\[
A_L(t)^\xi = \int_t^\infty e^{-r(\tau-t)} \pi_{LJ}(\tau) d\tau \quad \text{and} \quad A_H(t)^\xi = \int_t^\infty e^{-r(\tau-t)} \pi_{HJ}(\tau) d\tau,
\]

which, assuming no bubbles, are, for each technology, equivalent to:

\[
g_{A_L} = \frac{1}{\xi} \left[ r - \frac{\pi_L}{A_L} \right] \quad \text{and} \quad g_{A_H} = \frac{1}{\xi} \left[ r - \frac{\pi_H}{A_H} \right].
\]
In a balanced growth path solution $g_{A_L} = g_{A_H}$, which means that the ratios $\frac{\bar{n}_L}{A_L}$ and $\frac{\bar{n}_H}{A_H}$ must be equal and constant. This implies that the threshold final good, $\bar{n}$, and prices, $P_H$, $P_L$, as well as the price ratio $\frac{P_L}{P_L} = \mathcal{P}$ are constant. It also implies that:

$$\mathcal{P}^{\frac{1}{\bar{n}}} = \frac{\bar{n} (2 - \bar{n})}{(1 - \bar{n})} \tilde{H}^{-1}, \quad \tilde{H} = \frac{h}{I} \mathcal{H} \quad \text{and} \quad \mathcal{H} = \frac{H}{L}. \quad (15)$$

Carrying on, the symmetry of the model also allows us to write the production function for firm $n$ as:

$$Y_n = \begin{cases} 
[(1 - n) \xi L_n] A_{L_n}^{1+\xi} \left( \frac{a}{\bar{n}} \right)^{1/n} P_{nL}^{\frac{\alpha}{\bar{n}}} & \text{if } n \leq \bar{n} \\
[nhH_n] A_{H_n}^{1+\xi} \left( \frac{a}{\bar{n}} \right)^{1/n} P_{nH}^{\frac{\alpha}{\bar{n}}} & \text{if } n > \bar{n}
\end{cases} \quad (16)$$

Being in a perfect competition environment, it must be that $P_{nu}Y_{nu} = P_{nu}Y_{nv}, \forall u, v = 0, ..., \bar{n}$, and that $P_{nu}Y_{nu} = P_{nu}Y_{nv}, \forall u, v = \bar{n}, ..., 1$. Hence, for the threshold final good, $\bar{n}$, we have that:

$$\mathcal{P}^{\frac{1}{\bar{n}}} = \frac{1 - \bar{n}}{\bar{n}} A^{1+\xi} \tilde{H}^{-1}, \quad (17)$$

where, recall, $\mathcal{P} = \frac{P_L}{P_L}$, and $A = \frac{A_{L_n}}{A_{H_n}}$. For $\bar{n}$ it is also true that $P_{L\bar{n}} = P_{H\bar{n}}$, from which, using equation (10), it follows that:

$$\mathcal{P}^{\frac{1}{\bar{n}}} = \frac{\bar{n}}{1 - \bar{n}} \quad (18)$$

Expressions (15) and (18) together give us the value for $\bar{n}$:

$$\bar{n} = \frac{2 - \tilde{H}}{1 + \tilde{H}} \quad (19)$$

which implies restrictions:

$$2\tilde{H}^{-1} \geq 1 \quad \text{and} \quad 2\tilde{H} \geq 1 \quad (20)$$

Expressions (17) and (18) together give us the endogenously determined technological-knowledge bias, $A$:

$$A^{1+\xi} = \left( \frac{1 - \bar{n}}{\bar{n}} \right)^{2} \left( \frac{1 + \bar{n}}{2 - \bar{n}} \right)^{2} = \left( \frac{2\tilde{H} - 1}{2 - \tilde{H}} \right)^{2} \tilde{H}^{-1} \quad (21)$$

To sum up, profit maximization (by perfectly competitive final goods producers and by monopolist producers of intermediate goods) and the full employment equilibrium in factor markets, given the labour supply, $L$ and $H$, determine: (i) the endogenous threshold final good, $\bar{n}$; (ii) the technological-knowledge bias, $A$; and (iii) the price ratio, $\mathcal{P}$. That is, together with the values assigned to $h$
and \( l \), labour endowments determine the balanced growth path values of \( \pi \), \( A \) and \( \mathcal{P} \).

Equations (13a), (13b), (18), (19) and (21) are useful for understanding the action of both the market-size channel and the price (of final goods) channel from the \( \mathcal{H} \) ratio into the \( A \) ratio. For example, an increase in \( \mathcal{H} \) decreases \( \pi \) – see (19) – implying that more final goods are produced with the \( H \)-technology, which generates positive market-size incentives on \( A_H \) – see (13a) and (13b). That is, profit opportunities in the production of intermediate-goods used by high-skilled labour bias the technological-knowledge in favour of \( A_H \).

Regarding the price channel, small \( \pi \) implies that the relative price of \( H \)-technology final goods is also low and, conversely, the relative price of \( L \)-technology final goods is high – see (18). Thus, profit opportunities in the production of intermediate-goods used by the relatively high-priced \( L \)-technology final goods bias the technological-knowledge in favour of \( A_L \) – see (13a) and (13b); i.e., there are more incentives to develop the technologies used to produce the final goods that command higher prices.

The positive market-size channel (through higher \( \mathcal{H} \) and lower \( \pi \)) on \( A \) dominates the negative price channel (through lower \( \mathcal{P} \) ratio) on \( A \) – see (21) and our detailed analysis on the skill premium later on.

Continuing with the setting up of the model, recalling equations (12a) and (12b), this economy’s production function is rewritten as:

\[
Y = \left[ \frac{\alpha}{\beta} \right]^{\frac{\alpha}{\beta}} \left\{ A_L^{1+\xi} \left( \frac{\pi(2 - \pi)}{2} \right) l L P_L^{\frac{1}{\alpha+\beta}} + A_H^{1+\xi} \left( \frac{1 - \pi^2}{2} \right) h H P_H^{\frac{1}{\alpha+\beta}} \right\}. \tag{22}
\]

Log-differentiation of equation (22) shows that the growth rate of aggregate output is \( g_y = (1 + \xi)g_A \). Hence, one of the equations in (14) can be rewritten in order to define the Technology curve:

\[
g_y = \frac{1 + \xi}{\xi} \left[ r - \frac{\pi L}{A_L^{1+\xi}} \right] = \frac{1 + \xi}{\xi} \left[ r - \frac{D_H}{(r q)^{1+\xi}} \right] = \frac{1 + \xi}{\xi} \left[ r - \frac{D_L}{(r q)^{1+\xi}} \right], \tag{23}
\]

since \( D_H \) and \( D_L \) are equal to each other in a balanced growth path and such that:

\[
2D_H = (1 - \pi^2)^{1+\xi} H \Phi E_H \quad \text{and} \quad 2D_L = \pi(2 - \pi)^{1+\xi} L \Phi E_L,
\]

\[
\Phi = (1 - \gamma) \gamma^{1+\xi}, \quad E_H = (\alpha P_H)^{\frac{1}{1+\xi}} \quad \text{and} \quad E_L = (\alpha P_L)^{\frac{1}{1+\xi}}.
\]

### 2.3 General Equilibrium

Log-differentiation of equation (3) tells us that, in a balanced growth path equilibrium, the growth rate of total capital is \( g_K = (1 + \xi)g_A \), equal to the growth rate of total output. Hence, in the general equilibrium solution, as labour is constant, the per-capita economic growth rate is given by:

\[
g_c = g_y = g_K = g_L = g = (1 + \xi)g_A;
\]

\[
g_{\pi} = g_{\pi_L} = g_{\pi_H} = g_R = g_{\pi_L} = g_{\pi_H} = g_{\pi_L} = 0.
\]
The general equilibrium solution is obtained by solving the system of two equations, (1) and (23), in two unknowns, \( r \) and \( g \):

\[
\begin{align*}
  g &= \frac{1}{\xi}(r - \rho) \\
  g &= \frac{1+\xi}{\xi} \left[ r - \frac{D_L}{(qr)^{1+\phi}} \right], \text{ and } r > g > 0,
\end{align*}
\]

where the restriction \( r > g > 0 \) is imposed so that present values will be finite, and also so that our solution(s) have positive value(s) for the interest rate and the growth rate.

The Euler equation is linear and positively sloped in the space \((r, g)\). Application of the implicit function theorem shows that the nonlinear Technology curve is also positively sloped in the first quadrant of the space \((r, g)\):

\[
\frac{dr}{dg} = -\frac{\frac{\partial F(r, g)}{\partial g}}{\frac{\partial F(r, g)}{\partial r}} = -\frac{\xi}{(1 + \xi) + q \left( -\frac{\alpha}{\xi} \right) \left( \frac{1+\xi}{\xi} \right) D_L (qr)^{\frac{\xi}{1-\phi}}} > 0
\]

In order to determine numerically the general equilibrium solution(s) and visualise it graphically, the chosen parameter values are:

\( \alpha = 0.4; \gamma = 0.1; \sigma = 2; \rho = 0.02; \theta = 30; l = 1; h = 1.2; H = 0.35; L = 0.65, \)

where the values for \( \alpha, \gamma \) and consequently \( \phi = \frac{\alpha}{\gamma} \) and \( \xi = \frac{\alpha - 1}{\gamma} \) are the same as those used by Evans et al. (1998) in their numerical example. The values for the preference parameters \( \sigma \) and \( \rho \) are in agreement with those found in empirical studies (e.g., Barro and Sala-i-Martin, 2004, Chap. 2). The values for \( \theta, l, h \) and for the initial conditions, \( L \) and \( H \), are in line with our theoretical assumptions, allowing us at the same time to round the U.S. average growth rate in the post-war period (see, e.g., Jones, 1995). The unique real equilibrium can be imaged in Figure 1, and is characterised by:

\[
g = 0.0267 \text{ and } r = 0.0733
\]

### 2.4 Skill Premium

Let us now determine the skill premium, \( \frac{W_{\text{H}}}{W_{\text{L}}} = W \), in the balanced growth path solution. Working with the production function (16) and equations (9) and (10) it follows that wages are equal to:

\[
\begin{align*}
  W_L &= \frac{\partial (P_{nL} Y_{nL})}{\partial L} = \left( (1 - \pi) l - \frac{\partial \pi}{\partial L} l L \right) A_L^{1+\xi} P_L^{\frac{\gamma}{1-\phi}} \left[ \frac{\alpha}{R} \right]^{\frac{\mu}{1+\phi}}, \\
  W_H &= \frac{\partial (P_{nH} Y_{nH})}{\partial H} = \left( \frac{\partial \pi}{\partial H} h H + \pi h \right) A_H^{1+\xi} P_H^{\frac{\gamma}{1-\phi}} \left[ \frac{\alpha}{R} \right]^{\frac{\mu}{1+\phi}}.
\end{align*}
\]

Hence, using equation (17), we find the skill premium to be equal to:
Figure 1:

\[
\frac{W_H}{W_L} = \mathcal{W} = \frac{-2l^3h^{-2} + 6l^2hH^{-1} - 2h^3H - 3lh^2}{-2l^3H^{-1} + 6lh^2H - 2h^3H^2 - 3l^2h^2}. \hspace{1cm} (24)
\]

It follows that the influence of the relative endowment of high-skilled labour on the skill premium is felt according to:

\[
\frac{\partial \mathcal{W}}{\partial H} = \frac{15l^2h^4 - 20l^2h^3H^{-1} + 15l^4h^2H^{-2} - 3l^5hH^{-3} - l^6H^{-4} - 3lh^5H - h^6H^2}{(-2l^3H^{-1} + 6lh^2H - 2h^3H^2 - 3l^2h^2)^2}. \hspace{1cm} (25)
\]

Equation (19) tells us that, for our parameter values \( l = 1 \) and \( h = 1.2 \), the relative endowment of high-skilled labour, \( H \), can only assume values in the interval \([0.417, 1.667]\). According to equation (25), the derivative \( \frac{\partial \mathcal{W}}{\partial H} \) is positive for values of \( H \) in the interval \([0.512, 1.357]\). Hence, we have the following result:

\[
\text{If } 0.512 \leq H \leq 1.357 \text{ then } \frac{\partial \mathcal{W}}{\partial H} > 0. \hspace{1cm} (26)
\]

Our proposed model is hence in agreement with the skill-biased technological change theory, if we consider values for \( H \) between 0.512 and 1.357, that is, for economies relatively rich in high-skilled labour. It disagrees with the skill-biased technological change theory otherwise.

More specifically, due to the combination between types of labour and types of intermediate goods in final goods production, changes in wage inequality are closely related to labour endowments – as equation (24) clearly shows. A rise in the relative supply of \( H \) increases the high-skilled labour premium when
0.512 \leq \mathcal{H} \leq 1.357.  In fact, the market-size channel (higher \mathcal{H} implies higher \mathcal{A}, which implies higher \mathcal{W}) outweighs the price channel (higher \mathcal{H} implies lower \mathcal{P}, which implies lower \mathcal{W}).

Indeed, the technological-knowledge bias, \mathcal{A}, is also fully determined by the labour endowments and their respective productivity parameters. According to (21), we have that:

\[
\frac{\partial [A^{1+\xi}]}{\partial \pi} = \frac{-4\pi + 5\pi^2 - 2\pi^3 + \pi^4}{(2\pi^2 - \pi^3)^2} < 0,
\]

and we can also find that:

\[
\frac{\partial \pi}{\partial \mathcal{H}} = \frac{-3h}{(1 + \mathcal{H})} < 0,
\]

hence, we know that ratios \mathcal{A} and \mathcal{H} move in the same direction:

\[
\frac{\partial [A^{1+\xi}]}{\partial \mathcal{H}} = \frac{\partial [A^{1+\xi}]}{\partial \pi} \times \frac{\partial \pi}{\partial \mathcal{H}} > 0.
\]

3 Effects of Costly Investment

How does an increase in the adjustment cost parameter, \theta, affect the skill premium? Let us find out what must happen to the relative high-skilled labour endowment, \mathcal{H}, \textit{ceteris paribus}, so that this economy remains in the same general equilibrium balanced growth path as before, despite having to accommodate a higher internal investment cost \theta.

**Proposition 1** Facing a higher investment cost parameter, \theta, this economy must have a higher \mathcal{H} ratio, \textit{ceteris paribus}, if it is to remain in the same balanced growth path solution as before.

**Proof.** Firstly, recall our Technology curve (23):

\[
g = \frac{1 + \xi}{\xi} \left[ r - \frac{D_H}{(rq)^{1-\nu}} \right] = \frac{1 + \xi}{\xi} \left[ r - \frac{6lL \mathcal{H} - 3h \mathcal{H} \Phi E_H}{(1+\mathcal{H})^2 (rq)^{1-\nu}} \right],
\]

with \(E_H\) constant. Working with restrictions (20), we find that:

\[
\frac{\partial D_H}{\partial \mathcal{H}} = \frac{3h [5(hH)^2 + 4LhH - (Ll)^2]}{(Ll)^2 (1 + \mathcal{H})^4} > 0 \quad \text{and} \quad \frac{\partial D_H}{\partial L} = \frac{6l [(hH)^2 - LhH - 2(Ll)^2]}{(hH)^2 (\mathcal{H} - 1 + 1)^4} < 0.
\]

\[\text{If we choose instead to work with the Technology curve in terms of } L, \text{ i.e. with } D_L, \text{ we find that } \frac{\partial D_L}{\partial \mathcal{H}} > 0, \text{ and } \frac{\partial D_L}{\partial \mathcal{H}} < 0.\]
Secondly recalling equation (6), we have that:

\[
\frac{\partial \left[ (rq)^{\frac{\alpha}{\alpha-1}} \right]}{\partial \theta} = \frac{1}{2} \frac{\alpha}{1-\alpha} q^2 \left( B + \frac{1}{2} \theta q^2 \right)^{-\frac{\alpha-1}{\alpha}} > 0,
\]  

(28)

In order to accommodate a higher value for \( \theta \) while remaining in the same general equilibrium solution as before, we must have that:

\[
dD_H = d \left[ (rq)^{\frac{\alpha}{\alpha-1}} \right],
\]

equivalent to:

\[
\frac{\partial D_H}{\partial H} dH + \frac{\partial D_H}{\partial L} dL = \frac{\partial \left[ (rq)^{\frac{\alpha}{\alpha-1}} \right]}{\partial \theta} d\theta.
\]  

(29)

Since \( H + L \) is constant, it must be that \( dH = -dL \). Hence, let us first suppose that: \( dL > 0 \wedge dH < 0 \). Then, given (27) and (28), equation (29) assumes the following relations, where symbol \( \oplus \) stands for a positive term and symbol \( \ominus \) represents a negative term:

\[
\oplus \times \ominus + \ominus \times \ominus = \ominus \times \ominus,
\]

which is impossible.

It follows that it can only be that: \( dH > 0 \wedge dL < 0 \). In this case, equation (29) displays a possible relation:

\[
\ominus \times \oplus + \ominus \times \ominus = \ominus \times \ominus
\]

Concluding, a higher \( \theta \) implies a higher \( \mathcal{H} \) ratio in order to remain in the same equilibrium as before.

Bearing in mind result (26), the rise in \( \mathcal{H} \in [0.512, 1.357] \) motivated by an increase in \( \theta \) requires a rise in the skill premium. Indeed, in this case, the increase in the relative supply of high-skilled labour induces technological-knowledge bias that strongly stimulates the demand for \( H \) – see the previous subsection.

Putting it in another perspective, looking at two high-skilled labour abundant economies – \( \mathcal{H} \in [0.512, 1.357] \) – similar in everything but their investment costs, if they are growing at the same rate, the economy with higher investment costs must have a higher relative endowment of skilled labour, and consequently display a larger skill premium.

4 Effects of Complementarities between Intermediate Goods

How does an increase in the complementarities parameter \( \phi \), affect the skill premium? Let us find out how the relative supply of high-skilled labour, \( \mathcal{H} \), must change so that our economy, facing a higher degree of complementarities between intermediate goods in the production function, manages to remain in the same balanced growth path solution.
Proposition 2  Facing a higher complementarities parameter, $\phi$, this economy must have a higher $H$ ratio, ceteris paribus, if it is to remain in the same balanced growth path solution as before.

Proof. Firstly, we must remember that, according to our production function (2), a higher value of $\phi$, implies a lower value of $\gamma$, if we are to keep the same value for the capital share in output, $\alpha$.

Our relevant ratio in the chosen Technology curve (23) is once more:

$$\frac{D_H}{(rq)^{\frac{1}{1-\alpha}}} = \frac{(1 - \gamma)\alpha^{\frac{\beta-1}{\beta}}(1 - \pi^2)hHE_h}{2(q)^{\frac{1}{1-\alpha}}}$$

from which it follows that:

$$\frac{\partial D_H}{\partial \gamma} = \left[\frac{\alpha - \gamma}{\alpha - 1}\right]^{\frac{\beta-1}{\beta}} \frac{\partial H}{\partial \alpha} (1 - \pi^2)hHE_h > 0,$$

which is positive because $\alpha > \gamma$, according to our production function assumptions.

In order to accommodate a lower value of $\gamma$ while remaining in the same general equilibrium solution as before, we must have that

$$dD_H = \frac{\partial D_H}{\partial \gamma} d\gamma + \frac{\partial D_H}{\partial H} dH + \frac{\partial D_H}{\partial L} dL = 0,$$

equivalent to:

$$\frac{\partial D_H}{\partial H} dH + \frac{\partial D_H}{\partial L} dL = -\frac{\partial D_H}{\partial \gamma} d\gamma$$

(31)

Suppose first that: $dL > 0 \land dH < 0$. Then, given (27) and (30), equation (31) assumes the following relation, where symbol $\oplus$ stands for a positive term whereas symbol $\ominus$ stands for a negative term:

$$\oplus \times \ominus + \oplus \times \oplus = \ominus \times \ominus$$

which is impossible.

It follows that it can only be that: $dH > 0 \land dL < 0$. In this case, equation (31) displays a possible relation:

$$\oplus \times \ominus + \ominus \times \oplus = \ominus \times \ominus$$

Concluding, a higher $\phi$ implies a higher $H$ ratio so that the economy remains in the same equilibrium as before. ■

According to result (26), the required increase in $H \in [0.512, 1.357]$ resulting from an increase in $\phi$ requires a rise in the skill premium.

Looking at two high-skilled labour abundant economies – $H \in [0.512, 1.357]$ – similar in everything but their degree of complementarities between intermediate goods in the production function, if they are to grow at the same rate, the economy with a higher degree of complementarities must have a higher relative endowment of skilled labour, and consequently display a larger skill premium.
5 Concluding Remarks

We have developed a dynamic general-equilibrium model with growth driven by R&D applied to intermediate goods that: (i) complement either high or low-skilled labour in final-goods production; and (ii) are complementary to each other in their respective high-technology or low-technology side of the production function. Moreover, in the introduced model, there are costs to investment in both R&D and physical capital.

Both assumptions of internal costs of investment and of complementarities between intermediate goods in the final goods production function have not yet been considered in previous studies on wage inequality and, as mentioned in the Introduction, they constitute two important features of current economies.

In the proposed model, the complementarity between intermediate goods and either high or low-skilled labour implies that the wage inequality is directly linked to the relative endowment of high-skilled labour and the technological-knowledge bias, as is suggested by the skill-biased technological change literature. However, due to the introduction of physical capital, and the assumptions of internal costly investment (in both physical capital and R&D) and of complementarities between intermediate goods in final-goods production, the direct relationship between the relative high-skilled labour endowments and the wage inequality exists only for economies relatively abundant in high-skilled labour. For economies relatively scarce in high-skilled labour, the model predicts an inverse relationship between the relative supply of high-skilled labour and the skill premium. Hence the proposed framework constitutes a potential basis for consensus between the opposite results in terms of the relation between the relative endowment of high-skilled labour and the widening of the wage gap gathered by the empirical literature.

Furthermore, we conclude that, with everything else remaining constant, a rise in investment costs or an increase in the degree of complementarities between intermediate goods in final-goods production is associated with a rise in the relative endowment of high-skilled labour. As a result, in economies relatively abundant in high-skilled labour, the skill premium increases with an increase in investment costs or an increase in the degree of complementarities.
Appendix

Variable B is the marginal productivity of total capital:

\[ B = \frac{Y}{Z} \]

\[ = \frac{\left( \frac{\alpha}{\beta} \right)^{\frac{1}{1-\xi}} \left\{ A_{L}^{1+\xi} \left( \frac{m-2m}{2} \right) \ell L P_{L}^{1-\xi} + A_{H}^{1+\xi} \left( \frac{1-m}{2} \right) h H P_{H}^{1-\xi} \right\}}{\left( \frac{\alpha}{\beta} \right)^{\frac{1}{1-\xi}} \left\{ A_{L}^{1+\xi} \left( \frac{m-2m}{2} \right) \ell L P_{L}^{1-\xi} + A_{H}^{1+\xi} \left( \frac{1-m}{2} \right) h H P_{H}^{1-\xi} \right\} + \frac{1}{1-\xi} \left( A_{L}^{1+\xi} + A_{H}^{1+\xi} \right)} \]

constant in a balanced growth path.

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