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How quorum rules distort referendum outcomes: evidence from a pivotal voter model[1]

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Abstract

In many jurisdictions, whether referendum results are binding depend on certain legally defined quorum requirements. With a pivotal-voter model, we examine how quorum requirements affect voter's behavior. We conclude that quorums can be the cause of lower turnout and that they can deliver outcomes that are an inadequate basis to make inferences about collective preferences. We further conclude that quorums may help minorities to impose their will on majorities and that they may create a bias against the status quo. Finally, they generate situations under which the secrecy of the vote is called into question.

Keywords: Quorum, Referendum, Pivotal-voter model.

JEL Classification: D72

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1. Introduction

Do quorum rules affect the extent to which referendum results correctly represent collective preferences on a particular issue? If so, in what way and under what circumstances? Quorum requirements are part of the 'bewildering variety' of institutional rules that govern direct democracy (Matsusaka 2005). In many jurisdictions, whether results are to be treated as valid is contingent upon the level of turnout reaching a legally defined threshold (participation/turnout quorum) or on the majority representing a particular share of the electorate (approval quorum). What are the consequences of such quorums for referendum outcomes, in comparison with referendums determined by a simple majority rule?

The answer to these questions is of considerable scholarly and practical importance. Direct democracy is becoming increasingly popular in representative democracies, not only in the United States and Europe, but also in Latin American and East Asia. The number of jurisdictions providing for direct democracy mechanisms has increased in the last decades, as has the frequency with which actual votes on referendums and initiatives have taken place.[2] Nowadays, it has become nearly impossible to discuss developments in several policy areas – such as abortion rights, gay rights, European integration, constitutional reforms or local budgetary politics – without reference to role played by direct democracy.[3] Moreover, referendums are receiving increasing support not only from citizens in established democracies,[4] but also from the scholarly community itself, which has begun to increasingly stress the advantages of direct democracy rather than its shortcomings. Traditional concerns with the decision-making costs involved in direct democracy have been somewhat balanced by findings suggesting how it may actually bring policy closer to median-voter preferences, break the control of party cartels over the political agenda, raise levels of information about issues and produce positive effects on subjective well-being.[5]

Several international organizations and think tanks in charge of aiding efforts of democratic consolidation, election monitoring, and constitutional reform have begun to

[2] See Smith (1986), Butler and Ranney (1994), Setälä (1999), Barczak (2001), Le Duc (2003), Qvortrup (2005), Matsusaka (2005).

[3] See, among many, Roh and Haider-Markel (2003), Hug and Sciarini (2000), Christin and Hug (2002), Hug (2003), Clarke and Kornberg (1994), Pattie *et al.* (1999), Megdal (1993), and Feld and Matsusaka (2003)

[4] Dalton *et al*. (2001), Donovan and Karp (2006), Bowler *et al*. (2007).

[5] Gerber (1996), Bowler and Donovan (1998), Frey and Stutzer (1999), Hajnal *et al.* (2002), Benz and Stutzer (2004). For a review of the literature, see Lupia and Matsusaka (2004).

pay substantial attention to features of referendum design, including quorum requirements.[6] Especially in light of the recurrent finding that turnout in referendums tends to be lower than in legislative elections (Blais 2000), quorum rules are said to 'prevent low turnout from producing a distorted result' (Le Duc 2003: 172) and to work as an 'effective safeguard against so called 'false' majorities (a minority's exploitation of voter apathy)' (Qvortrup 2005: 173). However, as May (1952) famously showed, any voting scheme that is not based on a simple majority rule is bound to fail to respond positively to individual preferences. Poorly designed rules may cause voting paradoxes and stimulate the voters to hide or disguise their real preferences. The 1919 Constitution of the German Weimar Republic provides the most famous historical example of some of the potential detrimental consequences of quorum rules. In 1926, a referendum over the confiscation of princely properties, held under a 50 percent turnout quorum rule, resulted in a very large majority supporting that confiscation (well over 90 percent) but a turnout level of 36 percent, rendering the result invalid. This was largely the result of a campaign for abstention organized by the nobility, its allies in the party system and the church, as well as of systematic voter intimidation (West 1985; Suksi 1993; Verhulst and Nijeboer 2007). This failure was seen as one of the first and most visible signs of the latent crisis of the institutions of Weimar Germany, and the notion that the quorum rule had 'impeded the legitimate expression of majority will' was largely responsible for a long-lived legacy of distrust regarding direct democracy in Germany (West 1985).

Quorums and their consequences, however, are much more than a mere historical curiosity. No less than fourteen of the European Union's member-states have today some combination of turnout or approval quorums in at least some types of referendums. In almost all states in the German Federation, state and municipal referendums to be valid include the requirement posed by approval quorums. Some American states, such as Wyoming and Minnesota (approval quorums) or Massachusetts, Mississippi, and Nebraska (turnout quorums), have these rules too. And approval or turnout quorums can also be found in places as diverse as Colombia, Belarus, Venezuela, and Taiwan. And yet, although the burgeoning literature on direct democracy has given considerable attention to the consequences of variations in the

[6] Such as, for example, the Council of Europe's Venice Commission (Venice Commission 2007), the International Institute for Democracy and Electoral Assistance (IDEA 2008), or the Initiative & Referendum Institute (Kaufmann et al. 2008).

design of referendum rules,[7] the effects of quorum rules have seldom been systematically and rigorously examined. Among the few exceptions, Côrte-Real and Pereira (2004) use a decision axiomatic approach to show that different types of quorums imply different assumptions about the interpretation of abstention. Herrera and Mattozzi (2009), in a group turnout model, show how both turnout and approval quorums affect the behavior of political actors, distorting the incentives to mobilize voters and allowing groups that are in favor of preserving the status quo to use a 'quorum-busting strategy'. They also show that, in the context of their model, participation and approval quorums are essentially equivalent. Finally, Aguiar-Conraria and Magalhães (2008), in the first empirical examination of the subject, estimate significant negative impacts of participation quorums on turnout levels.[8]

In this paper, we wish to further contribute to the examination of the consequences of quorum rules, while going beyond the issue of turnout. We are interested in examining on how deviations from voting schemes employing simple majority rule in referendums may have consequences for the outcome as whole (including turnout), as well as in the extent to which that outcome is likely to correctly reveal collective preferences. We examine these issues using the framework provided by the pivotal-voter model originally proposed by Ledyard (1984) and Palfrey and Rosenthal (1983 and 1985).[9]

We demonstrate that quorum requirements may produce several paradoxical and, we argue, detrimental consequences. First, although quorums are avowedly introduced in order to lend further popular legitimacy to outcomes and protect policy from the will of narrow majorities in contexts of low turnout, they can be themselves causes of lower turnout and, in fact, allow minorities to impose their will on majorities. Second, the — conceivably desirable — bias they introduce in favor of the status quo may indeed work in the opposite direction, as a bias against the status quo. Third, they can produce similar outcomes under very different distributions of preferences, rendering outcomes an inadequate basis to make inferences about citizens' actual views on issues. Finally, they create situations under which one of the basic principles of democracy, the secrecy of the vote, becomes seriously imperiled, opening the door to undemocratic forms of social and political pressure.

[7] See, for example, Hug and Tsebelis (2002), Lupia and Matsusaka (2004), Hug (2004), Freitag and Vatter (2006).
[8] Other related work is Zwart (2007), who argues that quorum rules depend on the propensity to vote of different interest groups.
[9] See Dhillon and Peralta (2002) for an extensive survey on the several voting theories.

The paper proceeds as follows. In section two, we introduce the pivotal-voter model, as applied to referendums with a binary choice, and modify it to accommodate simple majority voting, turnout and approval quorum rules. In sections three to five, we show, by computational simulations and for different calibrations, capturing the preferences among the electorate and the intensity of such preferences, that several extra equilibriums emerge because of quorum rules. Section six concludes.

2. A pivotal voter model

Most of the 'partial-equilibrium' rational choice models are incapable of explaining observed levels of turnout in a satisfactory way. This happens because the exogenous probability that a single vote is decisive is very close to zero. Therefore, if the costs of voting are positive, these models predict an abstention rate of 100 percent. However, if, instead, we consider a pivotal-voter model, the probability of casting the decisive vote becomes endogenous. Citizens rationally anticipate the probability that their votes will be pivotal and they will vote if the expected benefit outweighs the cost of voting. Assuming away quorum requirements, a strictly positive level of turnout is assured in equilibrium: if no citizen were expected to vote, any deviator would be pivotal with probability one.

The pivotal-voter model is widely used and several implications have been derived with it. For example, Campbell (1999) showed that small minorities, with very strong feelings about the issue to be voted for, can impose their view on an apathetic majority, while Börgers (2004) used a pivotal model to show that voluntary voting Pareto-dominates compulsory voting.

Our support model is the traditional pivotal voter model modified to accommodate different quorum rules. We focus on referenda with a binary choice:

1. the 'Yes' option: change the 'status quo';
2. the 'No' option: conserve the 'status quo'.

Members of the 'Yes' supporters' group are the 'changers'. Opponents are the 'conservatives'.

Each citizen must decide whether to vote or not. It is trivial to show that, if they choose to vote, they will vote according to their preferences: changers will vote 'Yes', conservatives will vote 'No'.

We assume that there are n electors $(i = 1, \dots, n)$ and that each faces a cost of voting given by c_i, where c_i is the realization of a uniformly distributed random variable, $c_i \sim U[0, c]$.

If 'Yes' wins the election, then supporters obtain a benefit b, while opponents suffer a loss x. Each elector knows his/her own type and knows the probability, μ, that each

individual elector favors the proposal. Each voter has perfect knowledge about his/her own costs, but only knows the distribution of the other voters.

A voter derives utility from voting only if he/she is pivotal, i.e. if he/she casts the decisive vote. The probability of being pivotal depends on the strategies of the other voters. This is a game of incomplete information, with preferences and costs exogenously given. The 'Yes' option wins with if it receives more votes than the 'No' option and if the quorum requirements are satisfied. If the quorum is met and there is a tie in the number of votes, the 'Yes' option wins with 50 per cent probability.

A strategy for elector i is a function that specifies if he/she votes or abstains for each possible realization of c_i. We look for symmetric Bayesian-Nash equilibriums: given the strategies of the other citizens and the distribution of supporters and voting costs, each citizen must be happy with his/her own strategy. Symmetry implies that all members of a group (changers or conservatives) follow the same strategy. An elector will vote if the voting cost is below some threshold. Let γ_s and γ_o be those cut-off values for supporters and opponents, respectively.

2.1. The model with no quorum requirements

We start by assuming no frictions, meaning no quorum requirements: whoever gets the majority of the votes wins. To be able to borrow Börgers (2004)'s results we assume that if both alternatives receive exactly the same number of votes, each is selected with probability $1/2$.

Consider the choices an elector i faces. Assuming that the remaining $n-1$ electors follow their equilibrium strategies, each changer will vote if his/her voting cost is less than γ_s, while conservatives will vote if their voting cost is less than γ_o. Let $\rho(v_s, v_o; \gamma_s, \gamma_o, c)$ be the probability that, among the $n-1$ individuals, v_s vote 'Yes' and v_o vote 'No', given their equilibrium strategies, γ_s and γ_o, and the voting cost distribution.

Given our tiebreak rule, a changer will be pivotal in two cases.

- First: when, among the $n-1$ other electors, the number of 'Yes' votes is equal to the number of 'No'. In this case, the changer's vote is decisive to guarantee victory for the 'Yes' option.

- Second, when, among the $n-1$ other electors, the number of 'Yes' votes is equal to the number of 'No' votes minus one. In this case, the changer's vote is decisive to guarantee a chance of 50% of victory for the 'Yes' option.

Therefore, the expected benefit of voting is

$$\sum_{v=0}^{\frac{n}{2}-1}\left[\frac{\rho(v,v;\gamma_s,\gamma_o,c)}{2}+\frac{\rho(v,v+1;\gamma_s,\gamma_o,c)}{2}\right]b.$$

He/she will vote if the expected benefit exceeds the cost of voting. In equilibrium, this means that

$$\sum_{v=0}^{\frac{n}{2}-1}\left[\frac{\rho(v,v;\gamma_s,\gamma_o,c)}{2}+\frac{\rho(v,v+1;\gamma_s,\gamma_o,c)}{2}\right]b=\gamma_s \quad (1)$$

For the opponents, the reasoning is analogous. The expected benefit of voting is

$$\sum_{v=0}^{\frac{n}{2}-1}\left[\frac{\rho(v,v;\gamma_s,\gamma_o,c)}{2}+\frac{\rho(v+1,v;\gamma_s,\gamma_o,c)}{2}\right]x.$$

In equilibrium, we have

$$\sum_{v=0}^{\frac{n}{2}-1}\left[\frac{\rho(v,v;\gamma_s,\gamma_o,c)}{2}+\frac{\rho(v+1,v;\gamma_s,\gamma_o,c)}{2}\right]x=\gamma_o. \quad (2)$$

We have two equations (1 and 2) and two unknowns. To compute the equilibrium we need to derive the function $\rho(v_s,v_o;\gamma_s,\gamma_o,c)$.

As we have defined before, μ is the probability that each individual is a 'Yes' supporter. Therefore, the probability, $P(s)$, that there are s supporters among the remaining $n-1$ electors is given by

$$P(s)=\binom{n-1}{s}\mu^s(1-\mu)^{n-1-s}.$$

Among the s supporters, only the ones whose individual costs are smaller than the expected benefits will vote. Therefore, the probability that v_s of those will vote is

$$V(v_s)=\binom{s}{v_s}\left(\frac{\gamma_s}{c}\right)^{v_s}\left(1-\frac{\gamma_s}{c}\right)^{s-v_s}$$

Similarly the probability that, among the other $n-1-s$ electors, v_o will vote 'No' is

$$V(v_o) = \binom{n-1-s}{v_o}\left(\frac{\gamma_o}{c}\right)^{v_o}\left(1-\frac{\gamma_o}{c}\right)^{n-1-s-v_o}.$$

Putting all these equations together, we have

$$\rho(v_s, v_o; \gamma_s, \gamma_o, c) =$$

$$= \sum_{s=v_s}^{n-1-v_o} \binom{s}{v_s}\left(\frac{\gamma_s}{c}\right)^{v_s}\left(1-\frac{\gamma_s}{c}\right)^{s-v_s}\binom{n-1-s}{v_o}\left(\frac{\gamma_o}{c}\right)^{v_o}\left(1-\frac{\gamma_o}{c}\right)^{n-1-s-v_o} P(s).$$

Introducing this expression in equations 1 and 2, we have a system of two nonlinear equations and two unknowns.

Existence of a solution is not a problem (see Ledyard 1984 and Palfrey and Rosenthal 1985), but there are no general uniqueness results. Multiple solutions are possible. To our knowledge, the only uniqueness result derived so far is for the case of $\mu = 0.5$ and $b = x$ — Börgers (2004).

2.2. The model with an approval quorum requirement

If there is an approval quorum, a change in the 'status quo' requires the support of at least 50% of the voters and the support of a certain percentage of the total electorate. In our calculations, we will consider that certain percentage to be 25%.

For a conservative, the only modification one has to make to equation 2 is to consider the summation from $v = n/4$ instead of $v = 0$:

$$\sum_{v=\frac{n}{4}}^{\frac{n}{2}-1}\left[\frac{\rho_{aq}(v, v; \gamma_s, \gamma_o, c)}{2} + \frac{\rho_{aq}(v+1, v; \gamma_s, \gamma_o, c)}{2}\right] x = \gamma_o \qquad (3)$$

This modification happens because if the number of 'Yes' votes is smaller than $v = n/4$ the 'status quo' wins, independently of who receives the majority of the votes.

For a person who favors the proposal, there are two possibilities of being pivotal. If the quorum is satisfied, a changer is pivotal if, among the other electors, $v-1$ vote 'Yes' and v vote 'No'. On the other hand, the elector can also be pivotal if his/her vote is decisive to guarantee that the quorum is satisfied. In equilibrium, we have:

$$\sum_{v=\frac{n}{4}}^{\frac{n}{2}-1}\left[\frac{\rho_{aq}(v,v;\gamma_s,\gamma_o,c)}{2}+\frac{\rho_{aq}(v,v+1;\gamma_s,\gamma_o,c)}{2}\right]b+\frac{\rho_{aq}\left(\frac{n}{4}-1,\frac{n}{4};\gamma_s,\gamma_o,c\right)}{2}b+\cdots$$

$$+\sum_{v=0}^{n/4-1}\rho_{aq}\left(\frac{n}{4}-1,v;\gamma_s,\gamma_o,c\right)b=\gamma_s. \quad (4)$$

The system of equations to solve is given by equations (3) and (4).

Again, existence of a solution is not a problem, as not voting is always an equilibrium strategy. To realize this, just note that if one believes that nobody else is going to vote, his/her incentives to vote are zero. Whether he/she is a supporter (the vote is not enough to meet the quorum), or an opponent (the 'status quo' will win any way) is irrelevant. Uniqueness is not guaranteed either:

Proposition: *For some parameter values, it is possible to have more than one equilibrium strategy.*

Proof: To prove this proposition it is enough to create one example with two solutions.

Consider the case of $\mu = 0.5$ and $b = x$. For a given $c > 0$, chose *x* large enough so that the solution to system of equations (1) and (2) implies a very high turnout rate, close to 100%. Let $\gamma_s^*$and γ_o^* be that solution. This means that γ_s^*/c and γ_o^*/c are very close to 1. In turn, this implies that $\sum_{v=1}^{n/4-1}\rho_{aq}\left(\frac{n}{4}-1,v;\gamma_s^*,\gamma_0^*,c\right)$ in the system of equations (3) and (4) is very close to zero.

On the other hand, looking again at the system of equations (1) and (2), with very high γ_s^* and γ_0^*, $\rho(v,v;\gamma_s^*,\gamma_0^*,c)$, $\rho(v,v+1;\gamma_s^*,\gamma_0^*,c)$ and $\rho(v+1,v;\gamma_s^*,\gamma_0^*,c)$ will be close to zero for $v \leq n/4$.

Deleting the terms close to zero, the two systems of equations are identical. Using a continuity argument, one concludes that one of the solutions to the system of equations (3) and (4) will be in the neighborhood of (γ_s^*,γ_o^*).

Given that $(0,0)$ is also a solution, the proposition is proved. □

2.3. The model with a participation quorum requirement

With a participation quorum, a change in the status quo requires the support of the majority of 50% of the voters and that a given percentage of registered voters take part in the vote. We will consider that certain percentage to be 50%.

For a person that supports the proposal, the modifications to introduce to equation 1 are straightforward:

$$\sum_{v=\frac{n}{4}}^{\frac{n}{2}-1}\left[\frac{\rho_{pq}(v,v;\gamma_s,\gamma_o,c)}{2}+\frac{\rho_{pq}(v-1,v;\gamma_s,\gamma_o,c)}{2}\right]b+$$

$$+\sum_{v=\frac{n}{4}+1}^{\frac{n}{2}}\rho_{pq}\left(v-1,\frac{n}{2}-v;\gamma_s,\gamma_o,c\right)b=\gamma_s. \quad (5)$$

The first term applies when the quorum is met, meaning that an elector is pivotal if, among the other voters, $v-1$ vote 'Yes' and v vote 'No'. The second term captures the possibility of being pivotal when his/her vote is decisive to guarantee that the quorum is satisfied — note that $(v-1)+(n/2-v)=(n/2-1)$.

The most interesting case is for a person who opposes a proposal. A conservative can be pivotal in two contradicting ways. He/she can be pivotal because his/her vote is decisive to guarantee a 'No' majority. However, he/she can also be decisive to meet the quorum requirement. In such case, even if the person votes 'No', his/her vote is decisive to guarantee that the `Yes' wins (or at least enough to guarantee a 50% chance of victory for the 'Yes' side). Therefore, his/her utility decreases. Accordingly, we have:

$$\sum_{v=\frac{n}{4}}^{\frac{n}{2}-1}\left[\frac{\rho_{pq}(v+1,v;\gamma_s,\gamma_o,c)}{2}+\frac{\rho_{pq}(v,v;\gamma_s,\gamma_o,c)}{2}\right]x-\frac{\rho_{pq}\left(\frac{n}{4},\frac{n}{4}-1;\gamma_s,\gamma_o,c\right)}{2}x-\cdots$$

$$-\sum_{v=\frac{n}{4}+1}^{\frac{n}{2}}\rho_{pq}\left(v-1,\frac{n}{2}-v;\gamma_s,\gamma_o,c\right)x=\gamma_o. \quad (6)$$

Once more, existence of a solution is not a problem, as not voting is always an equilibrium strategy. Uniqueness is not guaranteed either:

Proposition: *For some parameter values, it is possible to have more than one equilibrium strategy.*

Proof: As before, to prove this proposition it is enough to construct one example with two solutions. It is a simple exercise to create an example analogous to the one use in the proof of the previous proposition. □

2.4. Expected outcomes

Let γ_o^* and γ_s^* be the equilibrium strategies. Note that γ_s^*/c and γ_o^*/c provide the expected value of the percentage of supporters and opponents, respectively, that will cast their vote. Therefore, the expected turnout rate, the expected percentages of 'Change' and 'Status Quo' votes and the margin of victory are easy to compute:

$$E(Turnout) = \mu\frac{\gamma_s^*}{c} - (1-\mu)\frac{\gamma_o^*}{c};$$

$$E(\%\ Change) = \frac{\mu\gamma_s^*}{\mu\gamma_s^* + (1-\mu)\gamma_o^*};$$

$$E(\%\ Status\ Quo) = \frac{\mu\gamma_o}{\mu\gamma_s^* + (1-\mu)\gamma_o^*};$$

$$E(\%Margin) = \left|\frac{\mu\gamma_s^* - (1-\mu)\gamma_o^*}{\mu\gamma_s^* + (1-\mu)\gamma_o^*}\right|.$$

2.5. Probability of Change

Assuming that we know μ— the probability that each individual is a changer — b, x and c — the benefits and the cost of voting — and the quorum rules, we can compute the equilibrium strategies γ_o^* and γ_s^*. With all this information, one can compute the probability of each election result, from which the probability that 'Yes' wins can easily be computed. With this, we have a direct estimation of the effects of quorum rules and of its usefulness. For example, if a quorum rule is meant to bias the results for the status quo, then the quorum requirement is efficient only if the probability of change decreases. If the objective of the quorum is to obstruct active minorities from imposing their agenda, then the quorum does not meet its objectives unless there is a drop in the probability that the minority agenda wins.

Let $\rho_n(v_s, v_o; \gamma_s^*, \gamma_o^*, c)$ be the probability that, among the n individuals, v_s vote 'Yes' and v_o vote 'No', given their equilibrium strategies, γ_o^* and γ_s^*, and the voting cost distribution.[10]

If there is no quorum, the probability that 'Change' wins is given by the sum of probabilities of all possible outcomes in which 'Yes' receives more votes than 'No', plus one half of the sum of all possible ties:

$$P(Change|\ No\ quorum) =$$

$$= \sum_{v_s=v_o+1}^{n} \left(\sum_{v_o=0}^{\frac{n}{2}-1} \rho_n(v_s, v_o; \gamma_s^*, \gamma_o^*, c) \right) + \frac{1}{2} \sum_{v=0}^{\frac{n}{2}} \rho_n(v, v; \gamma_s^*, \gamma_o^*, c).$$

If there is a quorum requirement, the reasoning is similar. The only difference is that the summation is restricted to outcomes that meet the quorum. In the case of an approval quorum of 25%, we have

$$P(Change|\ Approval\ quorum) =$$

$$= \sum_{v_s=\frac{n}{4}}^{n} \left(\sum_{v_o=0}^{\frac{n}{4}-1} \rho_{n,aq}(v_s, v_o; \gamma_s^*, \gamma_o^*, c) \right) +$$

$$+ \sum_{v_s=v_o+1}^{n} \left(\sum_{v_o=\frac{n}{4}}^{\frac{n}{2}-1} \rho_{n,aq}(v_s, v_o; \gamma_s^*, \gamma_o^*, c) \right) + \frac{1}{2} \sum_{v=\frac{n}{4}}^{\frac{n}{2}} \rho_{n,aq}(v, v; \gamma_s^*, \gamma_o^*, c).$$

With a participation quorum, we have

$$P(Change|\ Participation\ quorum) =$$

$$= \sum_{v_s=\frac{n}{2}-v_o}^{n} \left(\sum_{v_o=0}^{\frac{n}{4}-1} \rho_{n,pq}(v_s, v_o; \gamma_s^*, \gamma_o^*, c) \right) +$$

$$+ \sum_{v_s=v_o+1}^{n} \left(\sum_{v_o=\frac{n}{4}}^{\frac{n}{2}-1} \rho_{n,pq}(v_s, v_o; \gamma_s^*, \gamma_o^*, c) \right) + \frac{1}{2} \sum_{v=\frac{n}{4}}^{\frac{n}{2}} \rho_{n,pq}(v, v; \gamma_s^*, \gamma_o^*, c).$$

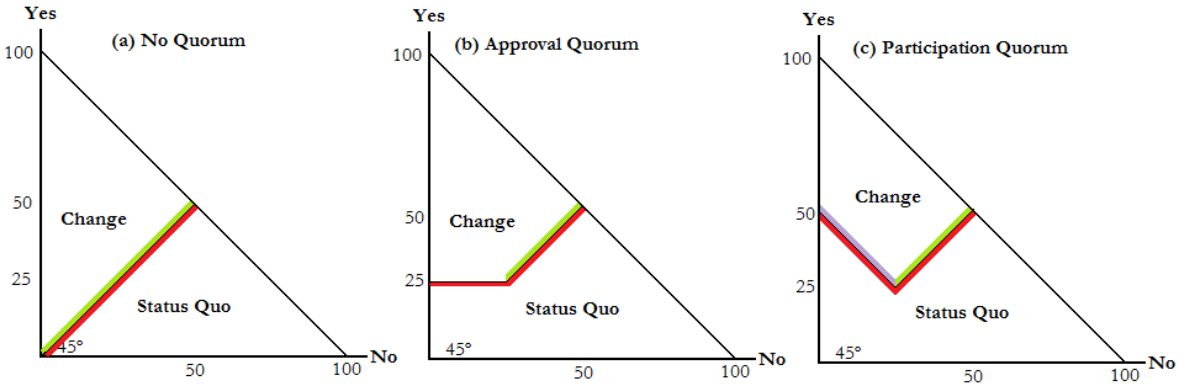
2.6. Graphical illustration

The ideas formalized so far can be illustrated with the aid of a simple picture. In Figure 1, let the vertical axis represent the percentage of the population that favors the proposal submitted to referendum. In the horizontal axis, we have the percentage of people that

[10] We do not derive the formulas here, because they are analogous to the derivations of $\rho(v_s, v_o; \gamma_s^*, \gamma_o^*, c)$, $\rho_{aq}(v_s, v_o; \gamma_s^*, \gamma_o^*, c)$ and $\rho_{pq}(v_s, v_o; \gamma_s^*, \gamma_o^*, c)$. The only difference is that our population is now formed by n individuals instead of $n-1$.

oppose the proposal. If there are no quorum requirements (left picture), there is a change in the status quo, with at least 50% probability, if the outcome of the referendum places the results on or above the 45-degree line (meaning that at least 50% of the voters vote 'Yes'). Therefore, a supporter who believes that the referendum outcome will be placed on the red line believes that he/she will be pivotal. On the other hand, a conservative believes that he/she will be pivotal if he/she believes that the outcome will be on the green line.

Figure 1: Pivotal lines



The picture in the middle describes a situation where there is an approval quorum of 25%. Therefore, a change the status quo requires the results to be above the 45-degree line and above the 25 percent-Yes line. In this case, the 'Status Quo' region increases. There is also a qualitative change on the probability of being pivotal. A supporter may be pivotal if his vote is decisive to reach 50% of the votes (given that the quorum is satisfied) or to reach the quorum (given that at least 50% of the voters choose 'Yes'). The red line represents these possibilities. For the opponent, the situation is different. A 'No' vote that guarantees a 'No' majority is decisive only if the quorum is met. Therefore, the pivotal green line is reduced, when compared to the first case.

Finally, on the right, we describe a situation where there is a participation quorum of 50%. The 'Change' region is reduced to the area above the 50 percent participation rate and above the 45-degree line. A supporter is pivotal if his/her vote is necessary either to reach 50% of the votes (given that the quorum is met) or to meet the quorum (given that the tie is guaranteed). The red line represents this. A conservative may have the decisive vote in two different ways. Given that the quorum is satisfied, then a conservative is pivotal, in a good way, if his/her vote guarantees the majority (green line). On the other hand, given that 'Yes' receives majority, an opponent is pivotal, in a bad way, if his/her

vote is decisive to meet the quorum requirement (violet line). It is pivotal in a bad way, because the vote is decisive to guarantee an undesired outcome.

From the pictures, it is clear what the main objective of imposing a quorum requirement is. The idea is to create a bias for the status quo, by enlarging the 'Status Quo' region. This way, it would be more difficult for very active minorities to change the status quo. We will see in the following sections whether this objective is actually achieved.

3. Benchmark scenarios

For now, we assume away overwhelming ex ante majorities, leaving the analysis of that case to section 4. It is known that a pivotal voter model is a bad to explain big margins of victory (Coate *et al.* 2008). This is so because electors only have an incentive to vote if there is a reasonable chance that their vote is pivotal. Equilibrium with a large margin of victory is, therefore, rather difficult to obtain, precisely because the high margin implies that the probability of casting the decisive vote is close to zero. As we will see, this reasoning is no longer valid if there are quorum requirements, which do change the structure of the decision.

Because there is no closed form solution to this problem, we have to rely on numerical methods to find solutions. To simulate our model, we fix the number of eligible voters. For computational reasons we consider only $n = 200$ voters and, each time we have to compute a binomial probability, with parameters p and N, we use a Normal approximation if $pN > 5$ and$(1-p)N > 5$. If not, we use the exact binomial probabilities. Again, for computational reasons, we consider a simplified version of the model. We assume that in case of a tie, the status quo wins. Other than a small asymmetry, this assumption has negligible effects on the final results and simplifies computations by a great deal. To look for equilibriums, we start with a thin grid search. After that, using a standard algorithm, we individually iterate from the most promising candidates.

We have to calibrate the parameter values for $b, x, \mu,$ and c. For our benchmark, we consider that supporters and opponents have equally strong feelings about the issue, $b = x = 22.5$. The choice of c is irrelevant, as we can see in the system of equations (1) and (2), because only the ratios γ_o/c and γ_s/c matter. We consider $c = 1$.

3.1. Very competitive referendum

For our competitive benchmark, it is not clear who is in majority, $\mu = 0.5$. We chose the values for b and x in such way that, with no quorum requirements, the participation rate is close to 75%. Therefore, without quorum requirements, we have a competitive election with a relatively high turnout rate.

The scenarios in which 'Change' has more than 55% chance of winning are highlighted in green. Scenarios in which the 'Status Quo' has more than 55% chance to prevail are shaded in red and, when the chances are even (neither side has more than 55% chances

of winning) the background is in white. The probability that 'Change' wins is given in the last column.

Table 1 tells us the possible equilibriums in our benchmark scenario. In the case of no quorum, 73.8% of conservatives cast their vote. The percentage among the changers is similar.[11]

Under a participation quorum, several equilibriums emerge. Two were to be expected, given the proofs of proposition 1 and 2: one equilibrium, the high participation one (eq. 1), is similar to the equilibrium with no quorum; another possible equilibrium is simply the no-show equilibrium (eq. 5). Nevertheless, there are also some intermediate cases.

Table 1: Equilibrium outcomes for $n = 200, b = 22.5, x = 22.5, \mu = 0.50$

		Expected Conservatives turnout rate	Expected Changers turnout rate	Expected General turnout rate	Expected Percentage of Votes for Change (total voters)	Expected Percentage of Votes for Change (total electors)	Probability that Change Wins
No quorum	eq1	73.8%	74.0%	73.9%	50.1%	37.0%	48.9%
Participation Quorum	eq1	73.8%	74.0%	73.9%	50.1%	37.0%	48.9%
	eq2	53.0%	62.6%	57.8%	54.2%	31.3%	79.5%
	eq3	0.0%	100.0%	50.0%	100.0%	50.0%	52.8%
	eq4	0.0%	94.1%	47.0%	100.0%	47.0%	22.1%
	eq5	0.0%	0.0%	0.0%	NA	0.0%	0.0%
Approval Quorum	eq1	73.8%	74.0%	73.9%	50.1%	37.0%	48.9%
	eq2	48.1%	59.1%	53.6%	55.1%	29.6%	83.3%
	eq3	0.0%	57.9%	29.0%	100.0%	29.0%	90.7%
	eq4	0.0%	39.8%	19.9%	100.0%	19.9%	4.6%
	eq5	0.0%	0.0%	0.0%	NA	0.0%	0.0%

In equilibrium 2, conservatives vote less than changers. The turnout rate is smaller than in the case of no quorum, but it is still enough to reach the participation requirement. Participation quorums can and are sometimes described as introducing abias for the 'status quo', in order to protect an equilibrium that the society achieved. As we saw in Figure 1.c, this is thought to be accomplished by increasing the 'status quo' region. However, what this exercise shows is that, in some cases, quorums may be a blessing in disguise to changers. If equilibrium 2 prevails, change will occur with a probability of 80%.

[11] Actually, it is slightly more because we assumed that for supporters a tie was not enough to win the election.

Equilibriums 3 and 4 show more radical results with very important consequences. In these cases, the abstention rate among opponents is 100%. This way, abstention is an almost functional equivalent of a 'No' vote and, as a result, although the 'change' option receives 100% of the votes, the 'status quo' wins with almost 50% chance in equilibrium 3 and with 80% chance in equilibrium 4. This sort of outcomes is far from being a mere theoretical curiosity. The case of the Italian 'abrogative referendums' — called to wholly or partially repeal an existing law — the requirement that over half of the electorate needs to participate in order for results to be valid has given those who support the law two alternative courses of action: to support the 'No' option or, instead, to simply abstain from voting in order to render the result null and void (Uleri 2002). As a result, of the 49 issues decided by referendum in Italy from 1987 to 2007, 24 failed to meet the required turnout quorum (Kaufmann et al. 2008: 218). Many other contemporary and historical examples of massive demobilization by status quo supporters under turnout quorum requirements can be found, including the Weimar referendums, several referendums held in Eastern Europe since the early 1990s, and others.[12]

The approval quorum has similar, although not identical, effects. Approval quorum equilibriums 1, 2 and 5 are similar to the participation quorum equilibriums 1, 2 and 5. The main difference is that there are two different consequences when opponents decide not to participate. In one of them, turnout among supporters is almost 58%, while in the other it is less than 40%. In one case, the quorum is met with a probability of 90%. In the other case, the 'status quo' prevails in 95% of the times, generating the same awkward situation described before. Again, real-world examples of massive demobilization on the part of conservatives under approval quorums are available, including the two constitutional referendums held under the Danish 1920 Constitution (Suksi 1993: 211; Svensson 1996: 38-40) and several referendums held at the state and

[12] The two referendums held under the German Weimar Republic, under a 50 percent turnout quorum rule, resulted in overwhelming majorities voting 'Yes' but very low levels of turnout and the invalidation of the results (Suksi 1993: 95). In Slovakia, for example, where a 50 percent turnout requirement for validity remains in force, only one of the six referendums held since 1994 has surpassed that threshold and, in five of them, the share of the vote for 'Yes' was equal or above 87 per cent. And in Colombia, an October 2003 referendum pushed by President Alvaro Uribe on no-less than 15 different issues was the object of a campaign for abstention organized by trade unions and opposition parties, and turnout levels ultimately failed to meet the 25 percent requirement for validity (IDEA 2008: 182).

municipal level in Germany (Verhulst and Nijeboer 2007: 19-21) and at the national level in Eastern Europe.[13]

3.2. What if majority is for the status quo?

In this example, the only difference is that we now assume that $\mu = 0.45$. This means that the percentage of conservatives among potential voters will be close to 55%. Therefore, a referendum outcome that mirrors the majority must be one in which the 'No' is expected to win.

Table 2: Equilibrium outcomes for $n = 200, b = 22.5, x = 22.5, \mu = 0.45$

		Expected Conservatives turnout rate	Expected Changers turnout rate	Expected General turnout rate	Expected Percentage of Votes for Change (total voters)	Expected Percentage of Votes for Change (total electors)	Probability that Change Wins
No quorum	eq1	58.8%	63.1%	60.8%	46.7%	28.4%	22.1%
Participation Quorum	eq1	59.2%	63.6%	61.2%	46.8%	28.6%	22.6%
	eq2	47.4%	68.5%	56.9%	54.2%	30.8%	78.5%
	eq3	0.0%	0.0%	0.0%	NA	0.0%	0.0%
Approval Quorum	eq1	58.9%	63.2%	60.8%	46.7%	28.4%	22.2%
	eq2	0.0%	63.8%	28.7%	100.0%	28.7%	89.4%
	eq3	0.0%	44.7%	20.1%	100.0%	20.1%	5.4%
	eq4	0.0%	0.0%	0.0%	NA	0.0%	0.0%

Table 2 gives us quite striking results that can only be labeled as ironic. If there is no quorum, the 'No' is expected to win with a comfortable margin of almost 7 percentage points. In our simulations, this implies that the status quo prevails almost 80% of the times. If there is a quorum requirement, no matter which one, the first equilibrium is in the neighborhood of the 'no quorum' equilibrium. However, the expected outcome is reversed in the equilibrium 2. With a participation quorum, the second possible equilibrium implies a smaller percentage of conservatives voting. This happens because a conservative vote has two contradicting consequences: on the one hand, it contributes to the status quo majority, but on the other hand, it helps 'Change' to reach the quorum. The implication is that if a conservative is afraid that a majority of votes supports the proposal, his/her best option may be to abstain, rather than voting. Under this

[13] In three of four referendums held in Hungary since the mid-1990s, under a 25 percent approval quorum requirement, percentages of voters for 'Yes' have been above 80 percent. In Latvia, under a 50 per cent approval quorum rule, the August 2008 referendum on the introduction of the possibility of dissolving parliament by popular vote resulted in a 97 per cent majority for 'Yes', but only 42 percent turnout.

equilibrium, 68% of the supporters will vote, which is enough to give them a solid majority (8.4 percentage points ahead).

Under an approval quorum requirement, the second and third equilibriums involve the total abstention of conservatives, an outcome we had already observed under competitive conditions. If supporters are able to coordinate to show up in the polling stations in big numbers (equilibrium 2), they will win the referendum, imposing their will upon a majority of conservatives. Note that the approval quorum requirement is that 25% of the electorate votes 'Yes' and equilibrium 2 implies that almost 29% of the electorate votes `Yes'.

Therefore, once more, one of the arguments for the quorum requirements loses further its strength. The quorum requirement, instead of promoting the 'status quo' may actually be working the other way around, creating a bias for 'Change'. To be harmless, the equilibrium under the quorum requirement should be in the neighborhood of the 'no quorum' equilibrium, but, of course, in this case it would just be simpler not to have quorum.

3.3. What if majority is for change?

We consider now the reverse scenario: $\mu = 0.55$. An expected majority of people is for change. Which of the three systems is the best to reflect these choices? Looking at Table 3, once again, we confirm that the design whose outcome is closer to the preferences of the electors is the no quorum requirement.

Table 3: Equilibrium outcomes for $n = 200, b = 22.5, x = 22.5, \mu = 0.55$

		Expected Conservatives turnout rate	Expected Changers turnout rate	Expected General turnout rate	Expected Percentage of Votes for Change (total voters)	Expected Percentage of Votes for Change (total electors)	Probability that Change Wins
No quorum	eq1	64.5%	60.5%	62.3%	53.4%	33.3%	76.3%
Participation Quorum	eq1	0.0%	95.3%	52.4%	100.0%	52.4%	77.8%
	eq2	0.0%	84.7%	46.6%	100.0%	46.6%	18.6%
	eq3	0.0%	0.0%	0.0%	NA	0.0%	0.0%
Approval Quorum	eq1	64.5%	60.5%	62.3%	53.4%	33.3%	76.3%
	eq2	56.7%	55.7%	56.1%	54.5%	30.6%	81.9%
	eq3	0.0%	53.0%	29.2%	100.0%	29.2%	91.7%
	eq4	0.0%	35.9%	19.7%	100.0%	19.7%	4.0%
	eq5	0.0%	0.0%	0.0%	NA	0.0%	0.0%

Both the participation and approval quorum equilibriums show two possibilities where, under a majority of supporters, the status quo wins, and they involve the massive demobilization of conservatives. Particularly striking are the equilibriums associated with the participation rule: all the equilibriums that we found have one common characteristic: the desertion of people who favor the status quo. This makes sense: as it is very unlikely that conservatives will have the majority of votes, by showing up they would help changers to meet the quorum requirement. Thus, they desert the polls.

Ex ante measures of preferences in concrete cases of referendums are highly fallible, resulting invariably from a (often scarce) number of opinion polls whose results may already incorporate the effects of quorum themselves (particularly in terms of the likelihood to vote). However, there is one potentially strong example of the phenomenon in the Italian June 2005 referendum on the abolition of restrictions to in vitro fertilization and embryo research in Italy.[14] Throughout 2004 and early 2005, all opinion polls on the subject indicated that a majority of Italians was generically supportive of in vitro fertilization and therapeutic cloning and that, in spite of a large number of undecided voters, a majority of voting age respondents were in favor of abrogating the existing legislation.[15] These polls, by bringing the clear perception that the "Yes" vote might win, seemed to have played no small role in the reaction from the Catholic Church: the Conference of Bishops, led by Cardinal Ruini, engaged in a vigorous campaign to prevent the abrogation of the existing law, by appealing to abstention. Pope Benedict XVI publicly backed the boycott campaign, which was joined by the Northern League and the Popular Alliance parties, as well as some figures of the governing *Forza Italia*.[16] In the end, in spite of the 'Yes' votes on the different issues

[14] If successful, the referendum would have lifted a ban on the freezing of embryos, permitted their screening to identify genetic defects and allowed more than three to be implanted, eliminated a passage in the existing law that gives embryos the same rights as parents, and repealed the prohibition of homosexual couples and single people from having children via assisted fertility.

[15] In September 2004, a Eurispes poll showed that 65 percent of Italian were in favour of in vitro fertilization and 64 percent in favour of therapeutic cloning. In the same month, an Swg poll showed a majority of 54 per cent supporting the abrogation of the existing legislation and 34 percent against it, while in January 2005, a TNS poll showed that only 11% of respondents leaned towards or had already decided against the abrogation of the law, with 35% were totally or partially in favour of repealing it, with the remaining unsure. See also the Ispo April 27-28 poll and the Istituto Piepoli June 6th poll. All results collected from Angus-Reid Global Monitor, at: http://www.angus-reid.com/polls.

[16] "When opinion polls showed that a win for the 'yes' vote seemed likely bishops throughout the land launched a vigorous campaign urging Italians to boycott the referendum. Pope Benedict XVI, who in addition to being pontiff is also bishop of Rome, publicly backed the boycott campaign." – Roland Flamini, "Church blocks Italian referendum", *United Press International*, June 15th, 2005, at: http://www.religiousconsultation.org/News_Tracker/Church_blocks_Italian_referendum.htm.

reaching shares of the valid vote that ranged from 77 to 88 percent, turnout was only 26 percent, and the law stayed in force.

3.4. Implications

Under the different scenarios examined so far, quorums may produce a series of disturbing consequences. First, the contrast between the outcomes without quorum rules and several equilibriums under such rules reveals what in the literature became known as the 'No-Show paradox': while both the quorum requirements would be reached in the absence of quorums, there are equilibriums where, precisely because of the presence of such quorums, they end up not being met. As Aguiar-Conraria and Magalhães (2008) show, this possibility is empirically very plausible. Second, there are equilibriums where, instead of introducing an alleged bias for 'status quo', quorums actually favor supporters of change. Third, several equilibriums yield the result where, in the presence of quorums, minorities end up imposing their will on majorities. In fact, this possibility becomes even more real when we consider the additional consequence of a very frequent sort of equilibrium in our analysis: the mass demobilization of status quo supporters. To the extent that turnout and abstention are observable, this creates of conditions under which voting choices on the part of the electorate may, in practice, cease to be protected by a veil of secrecy. Weimar is again the textbook case. In the already discussed 1926 referendum, active intimidation of voters by landowners and organizations linked to the Bavarian People's Party or the National Socialists – what the supporters of the measure called *Wahlterror* – took place throughout the country, especially in the rural areas, leading many to stay at home (West 1985: 247; Verhulst and Nijeboer 2007: 82). Similar problems have occurred in the Danish 1920 and 1939 referendums, as well as the Weimar 1932 referendum (Suksi 1993: 94 and 211). And there are good reasons to believe that such concerns about the lack of voter anonymity are not a mere historical curiosity. A few days before the Italian 2005 referendum, a woman living in a small town in the Central Italian region of Lazio reported how the lack of anonymity of the vote in the forthcoming referendum was resulting in less than subtle forms of social pressure.[17]

[17] "In Capranica, a town of 6000 people, voting will be difficult. The local church organized groups of volunteers to intimidate the town's people to prevent them from voting. My town is not unique. There are several municipalities where anonymity does not exist. People are controlled, their votes, their actions and whereabouts. The degree of distress, especially among women, is enormous." At: http://ecumenici.altervista.org/html/pivot/entry.php?id=233

4. Can a small noisy minority be decisive?

Campbell (1999), using a standard pivotal model, showed that it is possible that small minorities, with very strong feelings about the issue to be voted for, impose their view on an apathetic majority. Campbell considers two different normative criteria to check if this outcome is socially desirable. According to a 'democratic criterion' (1), an outcome is desirable only if the majority of the population prefers that same outcome; according to an 'economic criterion' (2), one should sum all the individuals' utility and choose the one that provides the greater total utility.

Campbell shows that both criterions will be frequently violated in the presence of a small very active minority. As pointed out by Qvortrup, the possibility of this type of outcome is one of the most common arguments for the existence of quorum requirements: they attempt to avoid the problem in which, in situations of high voter apathy, 'a special group of committed citizens may well take advantage of the situation by trooping to the polls in great numbers while the majority of the electors stay at home' (2005: 173).

We will provide two counter-examples to this argument. In one of them, a very active small minority group of changers will receive precious help from quorum rules to impose their will. The second scenario is similar, but the active minority is for the 'status quo' instead.

4.1. Active minority for 'Change'

We consider $\mu = 0.30$, meaning that the expected percentage of the population for change is 30%, while an expected overwhelming majority of 70% is against. Following Campbell (1999), we introduce a twist. We assume that the minority is strongly in favor of the proposal while the majority does not have such strong feelings for the subject. This is captured by assuming that $b = 30$, while $x = 15$, so that the utility of winning the referendum is twice as much for the supporters than for the opponents. Note that, according to both the democratic and the economic criterions, it is socially desirable that the 'status quo' prevails.

Table 4: Equilibrium outcomes for $n = 200, b = 30, x = 15, \mu = 0.30$

		Expected Conservatives turnout rate	Expected Changers turnout rate	Expected General turnout rate	Expected Percentage of Votes for Change (total voters)	Expected Percentage of Votes for Change (total electors)	Probability that Change Wins
No quorum	eq1	45.0%	94.9%	60.0%	47.5%	28.5%	27.6%
Participation Quorum	eq1	45.4%	96.3%	60.7%	47.6%	28.9%	28.5%
	eq2	38.0%	100.0%	56.6%	53.0%	30.0%	70.5%
	eq3	0.0%	0.0%	0.0%	NA	0.0%	0.0%
Approval Quorum	eq1	45.1%	95.2%	60.1%	47.5%	28.6%	27.6%
	eq2	34.3%	95.7%	52.7%	54.4%	28.7%	78.6%
	eq3	0.0%	94.6%	28.4%	100.0%	28.4%	87.4%
	eq4	0.0%	67.9%	20.4%	100.0%	20.4%	6.5%
	eq5	0.0%	0.0%	0.0%	NA	0.0%	0.0%

In this scenario, under the no quorum equilibrium, the 'status quo' wins with a probability of more than 70 percent. The same happens in equilibrium 1, under the participation or under the approval quorum. However, in the presence of a quorum requirement, other equilibria emerge under which 'Change' is expected to win. In the case of a participation quorum, one equilibrium involves less conservatives voting and a mass run to the polls by changers. Change wins with a probability of 70 per cent. With an approval quorum, 'Change' wins under two equilibriums. One of them involves partial desertion from conservatives the other total desertion. In one case, 'change' wins with a probability of almost 80 percent. In the other case, there is a probability of more than 87 percent that 'change' wins.

Therefore, in this scenario of a very active minority for change, a quorum requirement, instead of protecting the majority, may cause the opposite: to make the active minority more powerful.

4.2. Active minority for the 'Status Quo'

We now consider the reverse scenario. There is a noisy minority for the status quo and an apathetic majority for change: $\mu = 0.30, b = 15, x = 30$. In this scenario, the vast majority of citizens supports 'Change', which, without quorum requirements, wins with a probability of 70 percent. According to both criterions, the socially desirable outcome is 'change'.

Table 5: Equilibrium outcomes for $n = 200, b = 15, x = 30, \mu = 0.70$

		Expected Conservatives turnout rate	Expected Changers turnout rate	Expected General turnout rate	Expected Percentage of Votes for Change (total voters)	Expected Percentage of Votes for Change (total electors)	Probability that Change Wins
No quorum	eq1	96.6%	46.1%	61.2%	52.7%	32.2%	70.5%
Participation Quorum	eq1	0.0%	73.9%	51.7%	100.0%	51.7%	70.8%
	eq2	0.0%	67.8%	47.4%	100.0%	47.4%	25.6%
	eq3	0.0%	0.0%	0.0%	NA	0.0%	0.0%
Approval Quorum	eq1	96.5%	46.0%	61.2%	52.7%	32.2%	70.6%
	eq2	86.4%	43.1%	56.1%	53.8%	30.1%	77.1%
	eq3	0.0%	41.1%	28.8%	100.0%	28.8%	89.7%
	eq4	0.0%	28.6%	20.0%	100.0%	20.0%	4.9%
	eq5	0.0%	0.0%	0.0%	NA	0.0%	0.0%

Again, both types of quorums generate equilibriums in which the preferences of the status quo minority are prevail.

5. Is it possible that quorum rules enhance turnout?

In table 4, we can see that the first equilibrium with quorum implies a slightly higher turnout than with no quorum. Nevertheless, this effect is very marginal and does not change the general impression that quorum requirements promote abstention. Herrera and Mattozi (2009) also concluded that quorum requirements can promote abstention. However, according to the empirical results of Aguiar-Conraria and Magalhães (2008), analysis of aggregate from all national referendums held in all European Union countries from 1970 to 2007 shows that while participation quorums have detrimental effects on turnout, approval quorums, on average, have either negligible or positive effects of turnout rates. Is our pivotal voter model compatible with these empirical results?

We provide another example, in which the approval quorum may significantly increase turnout.[18] We consider a scenario in which the public opinion is divided ($\mu = 0.50$), changers have weaker feelings about the issue. This implies that, under no quorum distortions, the 'Status Quo' is expected to win, which is the socially desirable outcome according to Campbell's economic criterion.

Table 6: Equilibrium outcomes for $n = 200, b = 15, x = 22.5, \mu = 0.50$

		Expected Conservatives turnout rate	Expected Changers turnout rate	Expected General turnout rate	Expected Percentage of Votes for Change (total voters)	Expected Percentage of Votes for Change (total electors)	Probability that Change Wins
No quorum	eq1	44.5%	34.1%	39.3%	43.4%	17.1%	10.9%
Approval Quorum	eq1	56.7%	49.9%	53.3%	46.8%	25.0%	22.8%
	eq2	0.0%	55.9%	28.0%	100.0%	28.0%	84.5%
	eq3	0.0%	41.5%	20.8%	100.0%	20.8%	8.4%
	eq5	0.0%	0.0%	0.0%	NA	0.0%	0.0%

In this situation, the existence of an approval quorum may fight abstention. The strategic reasoning is clear. Under the no quorum benchmark, conservatives will win the referendum. This happens because conservatives care more deeply about the issue, not because they were in majority. If an approval quorum is introduced, the expected benefit of voting for a changer increases, because it increases the probability of being pivotal (the elector has two chances of being pivotal: his vote may be decisive to reach

[18] We were not able to find a similar example for the participation quorum.

the quorum or to reach majority). Therefore, there is an increase in the participation of the changers. This, in turn, implies a reaction from the conservatives. If they want to win the election, they have to increase their participation too. Overall, participation of members of both groups increases significantly. In this example, as in can see in table 6, in the approval quorum equilibrium 1, the 'status quo' still wins and turnout increases by 14 percentage points.[19] As in previous examples, conservatives desert the ballots in other equilibriums.

[19] It would be easy to construct a similar example in which change wins.

6. Conclusions

Direct democracy is becoming increasingly common in Western democracies and public support for referendums is high and rising. Frey (1995) argued that economic analysis of referenda should focus on two major aspects. First, the importance of the pre-referendum stage (issues like the number and frequency of propositions to be voted for). Frey identifies one second crucial facet: direct participation is a way to keep the agenda-setting power with the voters. Frey argues that fear of 'irresponsible voters' or of the 'excesses of majority' do not have a real basis as long as the institutional framework is adequate. In this paper, we focused precisely on the latter aspect.

The institutional design of referendums is crucial to guarantee correct representation of the collective preferences. The introduction of quorum requirements in referendums has typically been justified as a way to prevent 'distortions' in outcomes caused by low turnout: by allegedly preventing adoption of policy changes supported only by a minority of the population, quorums were supposed to help lending greater legitimacy to outcomes. However, we have shown that quorums can themselves produce several detrimental consequences, including some of the very distortions they set out to prevent. And we have provided several examples which persuade us that, in real-world democracies, such consequences have indeed arisen in several occasions.

To study the effects of quorum requirements on referendum outcomes, we used a standard pivotal-voter model and computed equilibria for different scenarios. Our analysis showed that quorum requirements fail to meet their purposes. First, we have seen that quorum requirements may promote abstention. This means that the introduction of a quorum requirement with the aim of lending legitimacy to the results may be the cause for the lack of legitimacy of the referendum results. Second, the existence of a quorum, instead of biasing the results for the status quo may actually bias the result against the status quo. Third, it is not clear that quorum rules act against small decisive minorities; actually, we showed that it may even help them to achieve their objectives. Finally, quorum requirements may create an additional problem. We described several quorum equilibrium outcomes, in which one whole group of voters, typically conservative, does not vote. In turn, this implies that vote is no longer secret. Whoever votes must belong to the other group. This opens the door to less democratic forms of social pressure.

In sum, our results show that distortions caused by quorum requirements are such that it is rather difficult, based on the results of a particular referendum, to make inferences about the true preferences of the electorate.

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