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Price Discrimination under Customer Recognition and Mergers*

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Abstract

This paper studies the interaction between horizontal mergers and price discrimination by endogenizing the merger formation process in the context of a repeated purchase model with two periods and three firms wherein firms may engage in Behaviour-Based Price Discrimination (BBPD). From a merger policy perspective, this paper’s main contribution is two-fold. First, it shows that when firms are allowed to price discriminate, the (unique) equilibrium merger gives rise to significant increases in profits for the merging firms (the ones with information to price-discriminate), but has no ex-post effect on the outsider firm’s profitability, thereby eliminating the so called (static) ‘free-riding problem’. Second, this equilibrium merger is shown to increase industry profits at the expense of consumers’ surplus, leaving total welfare unaffected. This then suggests that competition authorities should scrutinize with greater zeal mergers in industries where firms are expected to engage in BBPD.

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Keywords: Behaviour-Based Price Discrimination, Customer Poaching, Horizontal Mergers.

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1 Introduction

The large body of previous literature on the effects of horizontal mergers on firms’ pricing policies has mainly focused on the balance between anticompetitive price (market power) effects and pro-competitive merger-related efficiency improvements.\(^1\) It should be noted, however, that market power and efficiencies are not the only important channels through which horizontal mergers can affect the pricing policies of a merged firm.

Particularly important is the use of the merging partners’ customer information databases for price discrimination policies after the merger. An interesting variant of price discrimination is the so-called behaviour-based price discrimination (henceforth BBPD),\(^2\) which occurs when firms have information about consumers’ past behaviour and use this information to offer different prices to consumers with different purchasing histories.\(^3\)

The main objective of this paper is to study the interaction between horizontal mergers and price discrimination in a context where information about consumers is a key asset of the firms in the industry. In so doing, this paper proposes and explores a new motive for horizontal mergers. In our setting, the pooling of the merging firms’ purchase history databases, through a merger, will improve the profitability of price discrimination and the value of each merger partner’s databases. This will, in turn, promote the profitability of mergers.

The fact that the new Horizontal Merger Guidelines (HMGs), issued by the U.S. Antitrust Agencies on August 19, 2010,\(^4\) include an enlarged and more detailed discussion of price discrimination constitutes an important signal that the Agencies are willing to devote more attention to the alleged effects of price discrimination in their merger investigations.\(^5\) In particular, the new HMGs identify price discrimination as an independent competitive arm, thereby suggesting

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\(^1\)See, for instance, Motta (2004, chapter 5) and Whinston (2006, chapter 3), for general discussions of the effects of horizontal mergers.

\(^2\)For a comprehensive survey on BBPD, see Chen (2005), Fudenberg and Villas-Boas (2007) and Esteves (2009b).

\(^3\)As pointed out by Fudenberg and Villas-Boas (2007), “[t]his sort of ‘behavior-based price discrimination’ (BBPD) and use of ‘customer recognition’ occurs in several markets, such as long-distance telecommunications, mobile telephone service, magazine or newspaper subscriptions, banking services, credit cards, labor markets; it may become increasingly prevalent with improvements in information technologies and the spread of e-commerce and digital rights management.” (p. 2) Along these lines, Gehrig and Stenbacka (2005) highlight that, “[a] typical example of behaviour-based price discrimination is a pricing scheme, which is contingent on the history of internet clicks.” (p. 132)


\(^5\)As pointed out by Langenfeld (2010), “[c]learly the Agencies must believe there needs to be a much better understanding of the way they view the impact of price discrimination on merger analysis. Moreover, since [in the new HMGs] the Agencies devote so much space to the topic, the Agencies presumably believe that there will be many mergers involving price discrimination that should be challenged.”
that the potential for price discrimination should be a key factor in any competitive analysis of mergers.\textsuperscript{6} As McDavid and Stock (2010, p.5) highlight, the expanded discussion of price discrimination issues in this new version of the HMGs (when compared to the previous version) illustrates “a greater willingness on the part of the Agencies to pursue theories of competitive harm based on alleged effects on narrow categories of customers that can be specially targeted for a price increase.” Indeed, the new HMGs provide that “[w]hen price discrimination is feasible, adverse competitive effects on targeted consumers can arise, even if such effects will not arise for other consumers.” (p.6) Along these lines, Shapiro (2010, p.746) highlights that “DOJ investigations often begin by asking whether there are particular types of customers who are most likely to be harmed by the merger. We often find that some types of customers are more vulnerable than others to adverse competitive effects. We look for pre-existing price discrimination and we consider the possibility of post-merger price discrimination. ... The Guidelines are focused on whether the merger is likely to enhance market power. Price discrimination is highly relevant to this question if the merger may enhance market power over some customers but not others.”

The recognition that each firm’s customer-information databases can become more valuable through the process of mergers when price discrimination is likely to occur after the merger raises a number of interesting questions. What is the impact of price discrimination on firms’ merger decisions and on merger analysis? What are the consumer and welfare effects of mergers when price discrimination is feasible? Are there reasons for antitrust agencies to challenge mergers involving price discrimination? Despite the empirical relevance of the interaction between horizontal mergers and price discrimination, the literature has devoted scarce attention to this topic.\textsuperscript{7}

This paper contributes to close this gap in the literature by endogenizing the merger formation process in the context of a repeated purchase model with two periods and three firms wherein firms may engage in BBPD. Consumers are assumed to be heterogeneous: some consumers are captive to a given firm and others are shoppers in the sense that they consider competing firms’ products as perfect substitutes and are, therefore, price-sensitive consumers. In the first period, firms cannot distinguish a captive consumer from a shopper (although the size

\textsuperscript{6}In particular, in the new section 3 of the HMGs on “Targeted Customers and Price Discrimination”, it is stated that “[w]hen examining possible adverse competitive effects from a merger, the Agencies consider whether those effects vary significantly for different customers purchasing the same or similar products. Such differential impacts are possible when sellers can discriminate, e.g., by profitably raising price to certain targeted customers but not to others.”

\textsuperscript{7}Two noteworthy exceptions are Reitzes and Levy (1995) and Cooper et. al (2005, Section III).
of each customer segment is common knowledge to all firms in the industry). Thus, oligopolists necessarily compete in uniform prices. In the second period, however, if price discrimination is permitted, firms can condition prices on observed purchase histories. In particular, they can differentiate between the prices they charge to customers with whom they have established a customer relationship and the prices by which they try to attract new consumers. We also assume that, in the beginning of the second period, i.e. before price competition takes place for the second time, a two-firm merger may occur allowing the merging parties to join their customer-information databases.

Within this theoretical framework, some novel results are obtained. First, if firms are not allowed to merge but price discrimination is permitted, then, in the second-period pricing game, all firms end up earning the same profit regardless of having or not gained access to the required information to engage in price discrimination. This is because, in the second period of the game, discriminating firms will compete very fiercely for shoppers and, as a result, end up not making extra profits in this segment of the market. This result should be compared with Esteves (2009a), who, for the two-firm case, shows that price discrimination boosts both the discriminating and the non-discriminating firm’s second period profit. Hence, by relaxing the standard assumption that there are only two firms in the industry, the model proposed in this paper yields new economic insights which contrast with previous results in the literature.

Second, if instead mergers are possible it follows that: (i) a merger will only occur in equilibrium in case price discrimination is permitted; (ii) the equilibrium merger configuration is unique; and (iii) the merger will involve the two firms with information to price discriminate in the second-period pricing game. In addition, even though, in equilibrium, this merger gives rise to significant increases in profits for the merging firms, the firm which is excluded from participation in the merger (the non-discriminating outsider firm) is not affected (ex-post) by the merger in terms of profits. This result then eliminates the so called ‘free-riding problem’ identified by

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8 The firm charging the lowest price in the first period of the game ends up selling not only to its captive consumers, but also to the entire group of shoppers. Hence, it learns nothing useful for price discrimination and is, thus, forced to sell at a single price in the second period of the game. In contrast, the remaining two firms sell only to their captive consumers in the first period and, therefore, by being able to recognize these captive consumers, will have the required information to engage in price discrimination in the subsequent period of the game.

9 When there are three or more firms in the market, then, in the second period, prices are set at the marginal cost level for shoppers whereas captive consumers are charged the monopoly (reservation) price.

10 As Chen (2005) highlighted in a report on the pros and cons of price discrimination, an important extension of the existing models of BBPD is to allow for more than two firms.
the previous horizontal mergers literature (e.g. Salant, Switzer and Reynolds (1983)). In the present paper, an important role of the merger is to eliminate competition between price discriminators for shoppers. By pooling the merging partners’ customer-information databases, the merger increases the value of this information to each merger partner since the merger entity will then be able to fully separate its (aggregate) segment of locked in customers from the segment of shoppers that bought from the outsider before and to price differently accordingly. Moreover, while the merged firm has information to engage in BBPD, the outsider has not. Thus, the merged entity is able to entice some of the rival’s previous customers to switch, without damaging the profit from its locked in segment, whereas the outsider firm cannot protect its previous customers from price cuts. This then softens the outsider firm’s pricing behaviour and boosts the merged firm’s profits from poached customers.

Lastly, and perhaps most importantly, we show that the equilibrium merger will increase industry profits at the expense of consumers’ surplus, leaving total welfare unaffected. Our results, thus, carry an important merger policy implication: irrespective of the welfare standard adopted by competition authorities to appraise a proposed merger, they should scrutinize the mergers in industries wherein firms are expected to engage in BBPD with greater zeal.

This paper is mainly related to two strands in the literature. It is related to the literature on endogenous horizontal mergers since we explicitly model the merger formation process by making use of the coalition formation game which was first proposed by Hart and Kurz (1983). In particular, at the beginning of the second period, each of the three firms in the market simultaneously announces a list of players (including itself) that it wishes to form a coalition with. Firms that make exactly the same announcement then form a coalition together (i.e., merge).

The paper is also related to the stream of research on competitive BBPD where firms engage

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11 See also Rothschild et al. (2000) and Reitzes and Levy (1995), for spatial models with price discrimination wherein the gains from merger participants exceed those of the outsider firms.

12 It should be highlighted, however, that even though the static version of the ‘free-riding problem’ identified by the previous horizontal mergers literature disappears in the proposed setting, our analysis also reveals that BBPD creates a dynamic ‘free-riding problem’ via the race to embark in the equilibrium anticipated merger.

13 It should be noted, however, that the adoption of the consumers’ welfare standard appears to be the current practice in the major antitrust jurisdictions. As Lyons (2002, p. 1) highlights, “most major competition authorities operate under legislation and guidelines that reject this [total surplus] standard, and no major competition authority seems to apply it consistently. Instead, they overwhelmingly focus on consumers, including industrial consumers, to the exclusion of the welfare of merging firms.” See also Pittman (2007).

14 This model has, for instance, been applied by Vasconcelos (2006) to derive an upper bound to industry concentration in ‘endogenous sunk cost industries’ (Sutton (1991, 1998)).

15 Some other important contributions in this area are Gowrisankaran (1999), Kamien and Zang (1990), Fauli-Oller (2000) and Horn and Persson (2001a), to name a few.
in price discrimination based on information about the consumers’ past purchases. Like other forms of price discrimination, BBPD can have antitrust and welfare implications. While in the switching cost approach purchase history discloses information about exogenous switching costs (e.g. Chen (1997) and Taylor (2003)), in the brand preference approach purchase history discloses information about a consumer’s exogenous brand preference for a firm (e.g. Villas-Boas (1999), Fudenberg and Tirole (2000)). A common finding in this literature is that BBPD tends to intensify competition and potentially benefit consumers.

Behaviour-based pricing tends to intensify competition and reduce profits in duopoly models where the market exhibits best response asymmetry, firms are symmetric and both have information to engage in BBPD (e.g. Shaffer and Zhang (1995), Bester and Petrakis (1996), Chen (1997), Villas-Boas (1999), Fudenberg and Tirole (2000), Taylor (2003) and Esteves (2010)). There are, however, some models where firms can benefit from BBPD. This happens when firms are asymmetric (e.g. Shaffer and Zhang (2000)), firms’ targetability is imperfect and asymmetric (Chen, et al. (2001)) and when only one of the two firms can recognize customers and price discriminate (Chen and Zhang (2009) and Esteves (2009a)).

The present paper will put forward that a change from two firms to three firms leads to qualitative differences in the economic outcomes derived and raises issues not covered in the literature so far. For antitrust policies, our analysis suggests that behaviour-based pricing can boost industry profit and harm consumers when a merger involving the firms with the necessary information to price discriminate is likely to occur.

The remainder of the paper is organized as follows. Section 2 lays out the formal framework. Section 3 presents two no-merger benchmark cases: (i) the case where price discrimination and mergers are not permitted; and (ii) the case where price discrimination is permitted while mergers are not allowed. Section 4 looks at the endogenous mergers game. The competitive and welfare effects induced by the equilibrium merger are studied in Section 5. Section 6 discusses the merger policy implications that can be derived from our analysis. Section 7 discusses some of the limitations of the proposed model. Finally, Section 8 concludes.

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16 Following Corts (1998), the market exhibits best response asymmetry when one firm’s “strong” market is the other’s “weak” market. In BBPD models there is best-response asymmetry because each firm regards its previous clientele as its strong market and the rival’s previous customers as its weak market.

17 For other recent papers on BBPD see also Caillaud and De Nijs (2011), Chen and Pearcy (2010), Esteves and Regiani (2012) and Gehrig, Shy and Stenbacka (2011), (2012).
2 The model

Consider a market where $N = 3$ firms produce a nondurable good at a constant marginal cost which we normalize to zero without further loss of generality. There are two periods, 1 and 2. On the demand side of the market, there is a unit mass of consumers. All consumers have a common reservation value, $v$, and each consumer wishes to buy at most a single unit of the product in each period. Assume that each firm has a segment of captive (price-insensitive) consumers who have a high preference for its product in the sense that they consider buying only from that firm as long as the price at the firm is below $v$.\footnote{In a repeated purchase model, captive consumers cannot be poached by rival firms.} The proportion of consumers captive to firm $i$ is given by $\gamma_i$. Thus, the total number of consumers who are captive to some firm is $3\gamma$. The remaining consumers are shoppers (price-sensitive customers) who are indifferent between the firms. Consumers in this segment have less intense preference for brands and they buy the product from the cheapest firm, as long as the price is not above $v$. In a repeated interaction, price-sensitive customers might be willing to leave their previous supplier. The size of this customer segment is given by $\beta = 1 - 3\gamma$. In this market firms compete only for the price sensitive consumers. As we are interested in the case where $\beta > 0$, it follows that $\gamma < \frac{1}{3}$. This set up is commonly used in the literature (e.g. Varian (1980), Narasimhan (1988), Iyer et al. (2005), Chen and Zhang (2009) and Esteves (2009a)).

2.1 Timing

In the first period, firms cannot distinguish a captive consumer from a shopper (although the size of each customer segment is common knowledge to all firms in the industry). Thus, in the absence of purchase histories, oligopolists necessarily compete in uniform prices. In the second period, however, firms may have learnt consumers’ types by observing their first period choices. If price discrimination is permitted, firms can then differentiate between the prices they charge to customers with whom they have established a customer relationship and the prices by which they try to attract new consumers (those that bought from a rival before). Like in Chen and Zhang (2009) and in Esteves (2009a) a firm will not be able to distinguish between its captive customers and shoppers if it has sold to them both in the first period. In that case, the firm does not have the required information to price discriminate. In contrast, if a firm sells to only one segment in the first period, then it has the required information to recognize these “old” captive customers in the second period. Consequently, the firm can charge two prices in the
second period: one for the recognized captive segment and the other for the rest of the market which is not recognized (namely for shoppers who bought from a rival before).

Henceforth, we will designate as informed, a firm which, before a merger takes place, is endowed with a purchase history database which allows it to distinguish a captive customer from a shopper in period 2. In contrast, an uniformed firm is one that in period 1 sold its product both to its captive consumers and to (all) price sensitive consumers. Its database then does not allow it to distinguish a captive customer from those other consumers who might be willing to switch.

In both periods, firms set their prices simultaneously. The firms act to maximize their profits using a common discount factor $\delta \in [0, 1]$. Furthermore, consumers are assumed to be naive in the sense that they do not anticipate any poaching attempt by firms in the future neither their incentives to merge.

Finally, we also assume that, in the beginning of the second period, i.e. before price competition takes place for the second time, a two-firm merger may occur, allowing the merging parties to join their customer-information databases.\footnote{It is important to note that, in the proposed model, the merger allows firms to pool existing customer information, but does not make them better at acquiring new information.}

2.2 The merger formation game

In order to determine the merger pattern, we make use of an endogenous merger model based on the coalition formation game which was first proposed by Hart and Kurz (1983). In particular, each firm $i \in \{1, 2, 3\}$ simultaneously announces a list of players (including itself) that it wishes to form a coalition with. Firms that make exactly the same announcement then form a coalition together. For example, if firms 1 and 2 both announced coalition $\{1, 2, 3\}$, while firm 3 announced something different ($\{3\}$ or something else), then players 1 and 2 form a coalition.

In formal terms, firm $i$’s strategy is to choose a set of firms $\hat{S}^i$, which is a subset of the set of firms in the industry $\{1, 2, 3\}$ and includes firm $i$. The set of strategies for firm $i$ is, therefore, $\Sigma^i = \left\{ \hat{S} \subset \{1, 2, 3\} \mid i \in \hat{S} \right\}$. Given firms’ announcements $\alpha \equiv (\hat{S}^1, \hat{S}^2, \hat{S}^3)$, the resulting coalition structure is $C = \{C_1, ..., C_T\}$, where $T$ denotes the number of different lists chosen by the 3 firms. $C_i \cap C_j = \emptyset$ for $i \neq j$ and $\cup_{i=1}^{T} C_i = \{1, 2, 3\}$. Firms $i$ and $j$ belong to the same coalition $C_k$ if and only if $\hat{S}^i = \hat{S}^j$.

Two remarks are in order at this point. First, notice that $\hat{S}^i$ (respectively, $\hat{S}^j$) is the largest set of firms firm $i$ (respectively, firm $j$) would be willing to be associated with in the same
coalition. As a result, the coalition $C_k$ may in general be different from $\hat{S}^i$ (respectively, $\hat{S}^j$). A coalition corresponds to an equivalent class, with respect to equality of strategies. Second, since in the benchmark model we restrict attention to two-firm mergers, the resulting coalition structure $C$ will be composed of at most three coalitions ($T \in \{2, 3\}$) and each coalition will be composed of at most two firms.

3 No-Merger Benchmark Cases

This section investigates the case where mergers cannot occur.

3.1 No price discrimination and no merger

Consider first the case where firms cannot employ price discrimination strategies either because they have no information (e.g. consumers behave anonymously) or because price discrimination is not permitted. Here the model is reduced to two replications of the static model. Following a similar reasoning as in Chen and Zhang (2009), we can prove that a pure strategy equilibrium in prices fails to exist.\(^\text{20}\) Although firm $i$ can always guarantee itself a profit equal to $v\gamma$ merely focusing on its captive customers, the presence of a positive fraction of shoppers creates a tension between its incentives to price low, in order to attract them, and to price high, in order to extract rents from its captive customers. This tension results in an equilibrium displaying price dispersion. More specifically, there is a mixed strategy Nash equilibrium (henceforth, MSNE), the existence of which is proved by construction, as explained in what follows.

Suppose that a symmetric mixed strategy involves firms charging a price no higher than $p$ with probability $F(p)$ with support $[p_{\text{min}}, v]$. If firm $i$ chooses price $p$ while the other firms use a mixed strategy, its expected profit is:

$$E(\pi_i) = p\gamma \left[1 - (1 - F(p))^2\right] + p(\gamma + \beta)[1 - F(p)]^2 = p\gamma + p\beta[1 - F(p)]^2.$$ (1)

In equilibrium, firm $i$ must be indifferent between quoting any price that belongs to the equilibrium support, where $p_i \in [p_{\text{min}}, v]$. Thus, $p\gamma + p\beta[1 - F(p)]^2 = v\gamma$. This yields:

$$F(p) = 1 - \left(\frac{v - p}{p\beta}\right)^2.$$ (2)

\(^{20}\) A complete proof can be obtained from the authors upon request.
Now, from the conditions which establish that \( F(p_{\text{min}}) = 0 \), it follows that \( p_{\text{min}} = \frac{v\gamma}{1+p}\). In addition, in the two period game, overall expected profit for a representative firm is equal to

\[
E(\pi) = (1 + \delta) \left\{ p\gamma + p\beta [1 - F(p)]^2 \right\} = (1 + \delta) v\gamma.
\]

It is then straightforward to obtain the next proposition.

**Proposition 1.** In the no-merger and no-discrimination benchmark case, there is a symmetric subgame perfect MSNE where:

(i) each firm chooses a price randomly from the distribution function

\[
F(p) = \begin{cases} 
0 & \text{for } p < \frac{v\gamma}{1+p} \\
\left(\frac{v}{p} - 1\right)^{\frac{1}{2}} & \text{for } \frac{v\gamma}{1+p} \leq p \leq v \\
1 & \text{for } p > v 
\end{cases}
\]

(ii) and each firm’s expected profit is equal to \( E(\pi) = (1 + \delta) v\gamma \).

### 3.2 Price discrimination with no merger

Consider now the case where price discrimination is permitted. As usual, we solve the game working backwards from the second period.

In a repeated purchase model, by collecting information about customers’ past behaviour, a firm might be able to learn whether a consumer is a captive or a shopper who bought from a rival before and to price accordingly. The lowest-price firm in period 1 will sell both to the entire group of shoppers and to its captive consumers. Hence, it will learn nothing useful for price discrimination purposes and will set a single price in period 2.²¹ In contrast, the firms selling exclusively to their captive customers in period 1 will be able to recognize these “old” customers in the subsequent period. In particular, when a firm realizes that part of its potential market (of size \( \beta \)) did not buy its good in period 1, it learns that it charged a high first-period price and that all of its first-period customers are captive. In sum, the high price firms in period 1 become informed and so they can charge, in the subsequent period, one price for the recognized captive segment and another one for the rest of the market (consumers who are not recognized as being captive to the firm).

²¹In what follows when we say “the firm learns nothing” we mean “it learns nothing useful” for price discrimination. In fact, the lowest price firm learns who its captives customers are not.
For a given price $p$ chosen by firm $i$ in period 1, firm $i$ is the lowest first-period price firm (or the non-discriminating firm in period 2) with a probability equal to $\prod_{j \neq i} [1 - F_j (p)]$, where $F_j (p)$ denotes the probability that firm $j$’s price is less than or equal to $p$ in period 1. Firm $i$ serves exclusively the segment of its captive customers in period 1 (and can, thus, engage in price discrimination in period 2) with a probability equal to $1 - \prod_{j \neq i} [1 - F_j (p)]$.

With no loss of generality, suppose that in the first-period firms 1 and 2 sell only to their segment of captive customers and become informed firms. If price discrimination is permitted, firms 1 and 2 will price discriminate accordingly in period 2. Let $p^o_i$ and $p^r_i$ denote the price set in the second period by firm $i$ ($i = 1, 2$) to its own captive customers and to the rival’s previous (price sensitive) customers, respectively. Firm 3, on the other hand, being the lowest-price firm in period 1, is uninformed and, therefore, will not be able to price discriminate in period 2.

**Proposition 2.** In a market with 3 firms, there are 2 firms that will be able to distinguish a shopper from a captive consumer and price discriminate accordingly in the second period of the game. Therefore:

(i) all captive consumers pay the monopoly price $v$ whereas shoppers pay the marginal cost price;

(ii) price discrimination has no effect neither on second period profit nor on first-period price decisions.

This proposition highlights that moving from two firms to three firms makes a substantial qualitative difference.\(^{22}\) This is in sharp contrast with the results obtained in a duopoly model where just one firm achieves the discriminating position in the second period, in which price discrimination boosts both firms’ second-period profit (see Chen and Zhang (2009) and Esteves (2009a)).

The reason why three firms is a key number of firms is that with three firms there will be two firms competing *à la* Bertrand (in the second period) for uncommitted customers. These firms will then bid away their profits from these shoppers in an attempt to attract them.\(^{23}\) As a result, each discriminating firm charges shoppers the marginal cost price and ends up making no additional profits in this segment. The ability of the informed firms to fully separate their captive customers from consumers that bought from a rival firm before, together with the incapability

\(^{22}\)Proposition 2 is robust to assuming $N > 3$.

\(^{23}\)By extending Chen’s (1997) model of BBPD in the switching cost approach to a triopoly market, Taylor (2003) shows that the results derived with three firms are different from those obtained in a duopoly market.
of the other firms to poach any of their captive customers, allows these informed firms to charge their captive customers the reservation price \( v \), without fearing any poaching attempt by the uninformed rival. Total second-period profit for a discriminating firm is thus \( \pi^d = v \gamma \).

Consumers remain anonymous to the non-discriminating firm, which has no choice but to charge the same price to all consumers in period 2. Since the rival (informed) firms set a price equal to marginal cost for all uncommitted buyers, the uninformed firm’s best response is to set the highest possible price \( v \) so as to extract the valuation of its captive customers. Total second-period profit for the non-discriminating firm then equals \( \pi^{nd} = v \gamma \). Consequently, all firms earn the same profit in the second period of the game, regardless of whether they have achieved the discriminating position or not.

**Remark 1** Absent the merger possibility, the ability to price discriminate does not lead to higher profits.

Consider next the equilibrium first-period pricing. Firms make their pricing choices simultaneously and rationally anticipating how such decisions will affect their profits in the subsequent period. As second period profits with discrimination are equal to second period profits with no discrimination, price discrimination has no effect on first-period pricing decisions. Therefore, in the price discrimination and no-merger scenario, there is a symmetric subgame perfect MSNE where in period 1 firms behave as in Proposition 1. Each firm chooses a price randomly from the distribution function \( F(p) \) (see equation (3)) and each firm earns an overall profit equal to

\[
E(\Pi) = (1 + \delta) v \gamma. \tag{4}
\]

## 4 Endogenous Mergers

The objective of this section is to investigate the interaction between (endogenous) merger decisions and information-based price discrimination.

When mergers are permitted in the beginning of the second period, i.e. before price competition takes place for the second time, a two-firm merger may occur allowing the merging parties to join their customer-information databases. After first period decisions have been made, each consumer that bought from firm \( i \) will have a record on firm \( i \)’s database. If firms \( i \) and \( j \) decide to merge, they will join their databases. Starting from an initial market with three independent symmetric firms, the model investigates which merger configuration is likely to emerge in
equilibrium. We assume that a merger to monopoly would not be permitted by competition authorities.

4.1 Mergers with no discrimination

Let us start by investigating a scenario in which a merger to duopoly is permitted while behaviour-based pricing practices are, for any reason, not allowed. Consider, for instance, the case where firm 1 and 2 merge and sell their two goods (1 and 2) potentially at different prices. This is not a form of price discrimination. In this case, the merging entity, say firm $M$, second-period pricing strategy is to choose $p^k_M$ where $k = 1, 2$. Firm 3 is the outsider firm, say firm $O$, and its pricing strategy is to choose $p_O$.

In the post-merger game, the firm that results from the merger is endowed with a database of locked in customers equal to $2\gamma$ ($\gamma$ prefer good 1 and $\gamma$ prefer good 2). The outsider firm, on the other hand, has a group of locked in customers equal to $\gamma$. It is then straightforward to show that firm $M$ will price one of its products (for instance, product 1) at the monopoly price $v$ and the other (product 2) will be sold at a price randomly chosen from the distribution function $H(p)$. As firms $M$ and $O$ compete for the segment of shoppers, this creates a tension between the firms’ incentives to price low, in order to attract them, and to price high, in order to extract rents from captive customers. This tension results in an equilibrium displaying price dispersion, as shown by the next proposition.

**Proposition 3.** In the post-merger game without price discrimination:

(i) The merged firm $M$ prices one of its product at price $v$ and the other product’s price is chosen randomly from the distribution

$$H(p) = \begin{cases} 0 & \text{for } p < p_{\text{min}} \\ 1 - \frac{\gamma}{\beta} \left( \frac{v - p}{p} \right) & \text{for } p_{\text{min}} \leq p \leq v \\ 1 & \text{for } p > v \end{cases} \quad (5)$$

with support $[p_{\text{min}}, v]$ where $p_{\text{min}} = \frac{v \gamma}{\beta + \gamma}$.\(^{24}\)

(ii) The outsider firm $O$ chooses a price randomly from the same distribution function $H(p)$.

\(^{24}\)For the merging entity there are in fact an infinite number of such type of equilibrium in which with a given strictly positive probability firm $M$ prices one of its goods at price $v$ and with the complement probability it prices the other good according to $H(p)$.
(iii) The equilibrium second period profit for the merged firm is $\pi_M = 2v\gamma$, and that of the outsider firm equals $\pi_O = v\gamma$.

**Proof.** See the Appendix. ■

Proposition 3 highlights that if price discrimination is not permitted, a merger has no effect on each of the insiders’ second period profit which is equal to $v\gamma$.

**Remark 2** The merger itself without price discrimination does not lead to higher profits.

Although a merger would allow firms to join their customer databases, the merged entity cannot use these databases to boost its (second period) profits. Thus, no firm will embark on a merger in equilibrium if price discrimination is not permitted. Hence, each firm will announce a singleton coalition, and so the resulting equilibrium coalition structure will be $C = \{1\}, \{2\}, \{3\}$. As mergers do not occur in the beginning of the second period, the first period game is similar to the benchmark case with no mergers and no discrimination. Consequently, in period 1, firms behave again as in Proposition 1.

### 4.2 Mergers with price discrimination

Let us now analyze mergers when price discrimination based on purchase history is feasible. We are therefore assuming that price discrimination is legal, firms have the required information to price discriminate and there is no arbitrage among consumers. Two scenarios are relevant. In the first scenario, two informed firms are involved. This will be the case when a merger occurs between those two firms that in period 1 gained the patronage of only their captive customers. The merger of customer purchase histories will then allow the merged entity to distinguish an old (captive) customer from a shopper (who previously bought from a competitor). In the second scenario, even though, in the period 2, one of the merging firms would be able to distinguish a captive consumer from an uncommitted one if it didn’t embark on a merger, when it merges with an uninformed firm, the merger of customer purchase databases will ‘obfuscate’ the merged entity in the sense that it will not be able to completely distinguish consumer types. An interesting point here is that the merger of customer databases will not always give the merged entity the required information to distinguish between all its captive customers and those consumers that might be induced to switch.
4.2.1 Second-period

Both merging firms are informed  Suppose first that the merger is between two firms with information to distinguish whether a customer is a captive one or not. The outsider firm (the lowest-price firm in period 1) cannot distinguish between its captive customers and those willing to switch in the second period. When firms 1 and 2 merge, the merger not only increases the merged firm base of captive customers but also gives this merged firm an information advantage over the outsider to the merger (the uninformed firm 3). Following the same reasoning as in the previous section we will assume that the merging entity, firm $M$, offers two products. Clearly, the merged entity has more flexibility in its pricing strategy because the two products will have a price tailored at its old locked in customers ($p^c_M$) and a potentially different one tailored at the shoppers who previously bought from a rival firm ($p^r_M$). As all the captive consumers have the same reservation price, then the two goods will be priced at the monopoly price ($p^o_M = v$) when the consumer is recognized as a price insensitive one. Thus, firm $M$’s profit in this segment is equal to $\pi^+_M = 2v\gamma$. The outsider firm 3, on the other hand, cannot engage in price discrimination in period 2.

Look next at the second–period price competition for the segment of customers who might be willing to switch from the outsider, and let $b^p_N$ denote the non-discriminating firm’s second-period price.$^{25}$

**Proposition 4.** There is no pure strategy equilibrium in prices for the group of consumers (shoppers) that bought from the outsider (non-discriminating) firm in period 1.

**Proof.** See the Appendix. ■

There is, however, an asymmetric MSNE. Let $G^r_M(\hat{p}_N)$ denote the probability that the merged firm’s price to the rival’s previous customers is no higher than $\hat{p}_N$ and $G^r_N(p^r_M)$ denote the probability that the non-discriminating firm’s price is less than or equal to $p^r_M$.

**Proposition 5.** When the merged firm can engage in price discrimination, whilst the outsider firm cannot, price competition over the group of shoppers gives rise to an asymmetric MSNE in which:

---

$^{25}$Since consumers remain anonymous to the outsider firm, this firm is forced to quote the same price to all consumers.
(i) The non-discriminating outsider firm chooses a price randomly from the distribution

\[
\hat{G}_N(p^*_M) = \begin{cases} 
0 & \text{for } \hat{p}_M^* < \hat{p}_{N \text{ min}} \\
1 - \frac{v^\gamma}{p^\gamma} & \text{for } \hat{p}_{N \text{ min}} \leq p^*_M \leq v \\
1 & \text{for } p^*_M > v
\end{cases}
\]

with support \([\hat{p}_{N \text{ min}}, v]\) and has a mass point at \(v\) with a density equal to \(m = \frac{\gamma}{\gamma + \beta}\), where

\[
\hat{p}_{N \text{ min}} = \frac{v^\gamma}{\gamma + \beta}.
\]

(ii) The discriminating-merged firm chooses a price randomly from the distribution

\[
G^r(\hat{p}_N) = \begin{cases} 
0 & \text{for } \hat{p}_N < \hat{p}_{N \text{ min}} \\
1 - \frac{(v - \hat{p}_N)^\gamma}{p^\gamma} & \text{for } \hat{p}_{N \text{ min}} \leq \hat{p}_N \leq v \\
1 & \text{for } \hat{p}_N \geq v
\end{cases}
\]

with support \([\hat{p}_{N \text{ min}}, v]\).

(iii) The profit for the discriminating firm from poached consumers equals

\[
\pi^r_M = \frac{v^\gamma \beta}{\gamma + \beta},
\]

and the profit for the non-discriminating firm equals \(\hat{\pi}_N = v^\gamma\).

Proof. See the Appendix. ■

Corollary 1. From the equilibrium distribution functions defined by (6) and (8) it follows that:

(i) \(\hat{G}(p^*_M) < G^r(\hat{p}_N)\), that is, \(\hat{G}(p^*_M)\) first-order stochastically dominates \(G^r(\hat{p}_N)\);

(ii) \(E(\hat{p}_N) > E(p^*_M)\); and

(iii) \(m\) is decreasing in \(\beta\) and increasing in \(\gamma\).

Proof. See the Appendix. ■

In the equilibrium derived above, the outsider non-discriminating firm uses a “Hi-Lo” pricing strategy. To squeeze more surplus from its captive customers, it charges the monopoly price \(v\), with probability \(m\); and to avoid being poached and losing the group of customers willing to switch it quotes occasionally a low price. As expected, the shoppers face on average a higher price at the outsider non-discriminating firm than at the merged discriminating firm. As a result
of the merger, the discriminating merged firm has an advantage over its rival because it is able
to entice the shoppers who bought from the rival before, without damaging the profit from its
locked in segment. Conversely, the outsider firm cannot protect all its first-period customers
from price cuts. When it charges a low price, as a way to avoid poaching, it damages the profit
from its captive segment. As the merged discriminating company has less to lose, it can be more
aggressive. Therefore, regarding the price tailored for shoppers, it charges, on average, lower
prices. Further, part (iii) states that the greater is the size of the outsider non-discriminating
firm’s captive group \( \gamma \), the higher is the probability of this firm charging the monopoly price \( v \)
and so the probability of poaching. The reverse happens with respect to the size of the switchable
segment.

So, total second-period expected profit for each of the insider discriminating firms, denoted
\( \pi^2_{i,M} \), equals
\[
\pi^2_{i,M} = \frac{1}{2} (\pi^0_M + \pi^r_M) = v\gamma + \frac{1}{2} \left( \frac{v\gamma}{\gamma + \beta} \right).
\]
while the second-period profit for the outsider non-discriminating firm is \( \hat{\pi}_N = v\gamma \).

As \( \pi^2_{i,M} - \hat{\pi}_N = \frac{1}{2} \left( \frac{v\gamma}{\gamma + \beta} \right) > 0 \) we have a measure of the (second period) benefit of embarking
on a merger when price discrimination is permitted. A merger between the two informed firms
has the strategic effect of eliminating the competition between price discriminators for shoppers.
Thus, the pooling of the informed firms’ purchase history databases, through a merger, will
improve the profitability of price discrimination and the value of each merger partner’s database.
This will, in turn, promote the profitability of mergers.

Now, contrasting the results in Proposition 1 and 5, one immediately sees that a merger
between firms with the required information to engage in behaviour-based pricing boosts the
insiders’ second period profit and has no impact on the outsider’s second-period profit.

**One merging firm has information and the other does not** In this scenario the
merger of customer purchase databases will ‘obfuscate’ the merged entity which after the merger
will not be able to distinguish all customer types. This will be the case when we have a merger
between a firm that in period 1 sold only to its captive customers and a firm that in period 1 sold
to both its captive customers and to shoppers. As before, with no loss of generality, suppose that
in the first-period firms 1 and 2 sell only to their captive customers. Firm 3 is the lowest-price
firm in period 1 and so serves its captive market as well as the shoppers. Suppose that we have a
merger between firms 1 and 3. In this case, after first period decisions have been disclosed, all of
firm 1’s captive customers will have a record on firm 1’s purchase histories database. Similarly, all of firm 3’s captive customers as well as all the shoppers will have a record on firm 3’s customer database. Now, in the second period, by joining firm 1’s and firm 3’s databases, the merged firm will only be able to recognize as being captive customers those that bought from firm 1 before. This means that the merged firm can (in period 2) charge two prices, one price tailored to those customers recognized old captive to firm 1 ($p_{1M}^o$) and another one to all those customers which have a record on firm 3’s database ($\tilde{p}_M$). In contrast, the outsider firm (in this case, firm 2) will be able to distinguish a captive customer from a customer who bought from a rival before. The outsider firm will then charge, in period 2, two prices: one price targeted to its old captive customers ($p_{2O}^o$) and another one targeted to the shoppers previously buying from a rival firm ($p_{2O}^r$).

As the merged firm does not compete with the outsider firm with regards to part of its captive customers (those $\gamma$ consumers who bought from firm 1 in period 1) it has no incentive to charge them anything other than the monopoly price. Therefore, the merged firm will charge the customers recognized as captive customers the monopoly price, i.e., $p_{1M}^o = v$. However, the merged firm has no way to distinguish a captive customer from a shopper who previously bought from firm 3. As a result, there is no pure strategy equilibrium in prices for the group of consumers that bought from the lowest-price firm in period 1. There is, however, an asymmetric MSNE. Let $G_O^e(\tilde{p}_M)$ denote the probability that the outsider firm’s price to the rival’s previous customers is no higher than $\tilde{p}_M$ and $G_M(p_O^r)$ denote the probability that the merged firm’s price is less than or equal to $p_O^r$.

**Proposition 6.** Price competition over the group of consumers that bought from the lowest-price firm in period 1 gives rise to an asymmetric MSNE in which:

1. The merged firm chooses a price randomly from the distribution

$$G_M(p_O^r) = \begin{cases} 0 & \text{for } p_O^r < \tilde{p}_M^{\text{min}} \\ 1 - \frac{v^\gamma}{p_O^r(\gamma + \beta)} & \text{for } \tilde{p}_M^{\text{min}} \leq p_O^r \leq v \\ 1 & \text{for } p_O^r > v \end{cases}$$

with support $[\tilde{p}_M^{\text{min}}, v]$. It has a mass point at $v$ with density equal to $\tilde{m} = \frac{\gamma}{\gamma + \beta}$, where

$$\tilde{p}_M^{\text{min}} = \frac{v^\gamma}{\gamma + \beta}. \quad (12)$$
(ii) The outsider firm chooses a price randomly from the distribution

$$G_O(\tilde{p}_M) = \begin{cases} 
0 & \text{for } \tilde{p}_M < \tilde{p}_M \text{min} \\
1 - \frac{(v - \tilde{p}_M)\gamma}{\tilde{p}_M^\beta} & \text{for } \tilde{p}_M \text{min} \leq \tilde{p}_M \leq v \\
1 & \text{for } \tilde{p}_M \geq v 
\end{cases}$$

(13)

with support \([\tilde{p}_M \text{min}, v)\).

(iii) The outsider firm’s profit from poached consumers equals

$$\pi_O^r = \frac{v\gamma\beta}{\gamma + \beta},$$

(14)

and the profit earned by the merged firm with its non-recognized customers \(\tilde{\pi}_M = v\gamma\).

**Proof.** See the Appendix. ■

Note that, as for the second-period, the profit earned by the merged firm is \(\pi_M^2 = 2v\gamma\) whereas the outsider firm’s profits equals \(\pi_O^2 = v\gamma + \frac{v\gamma\beta}{\gamma + \beta}\). This therefore suggests that when a merger of customer databases discloses partial information about a customer type, the merger will have no effect on the insiders’ second period profit (which would be equal to \(v\gamma\)) but enhances the outsider firm’s profit.

**Remark 3** Only the merger (involving informed firms) and the possibility of engaging in BBPD lead to higher (insiders’) profits.

Now, combining remarks 1, 2 and 3, one can conclude that a merger will only occur in equilibrium when: (i) price discrimination is permitted and (ii) the merger involves the two firms with information to price discriminate in the post-merger game.\(^{26}\) In other words, in a context wherein price discrimination is permitted, the equilibrium merger will involve the firms with small market shares in the pre-merger game, i.e. those firms selling exclusively to their captive customers in period 1.

As before and with no loss of generality, suppose that firm 1 and 2 sold only to the segment of captive customers in period 1, meaning that they have the required information to recognize all types of customers in period 2. Formally, when price discrimination is permitted, firms 1 and 2 will both announce \(\{1, 2\}\), whereas the lowest first-period price firm, firm 3, will be indifferent between announcing a singleton coalition \(\{3\}\) and announcing \(\{3, 1\}\) or \(\{3, 2\}\). Therefore, the

\(^{26}\)In all other cases, the merger does not enhance insiders’ (aggregate) profits and, hence, will not occur in equilibrium.
resulting equilibrium coalition structure will be \( C = \{1, 2\}, 3 \). Intuitively, as one firm (firm 1) is handicapped in competition after the first period (it cannot price discriminate), this firm is less valuable as a merger partner – only a merger between the two high priced firms forecloses competition in the market structure induced by the merger.

### 4.2.2 First-Period

Consider next the equilibrium first-period pricing. Firms make their pricing choices simultaneously and rationally anticipating how such decisions will affect both the merger game outcome and their profits in the subsequent period. Again it is straightforward to show that there is no subgame perfect Nash equilibrium in pure strategies. There is, however, a MSNE, the existence of which is proved by construction. We have already seen that the firm charging the lowest price in period 1 does not embark on a merger in equilibrium. Therefore, for a given price \( p_i \) chosen by firm \( i \) in period 1, firm \( i \) is the lowest-price firm in period 1 (or the non-discriminating outsider firm in period 2) with a probability equal to \( \prod_{j \neq i} [1 - F_j^1(p_i)] \), where \( F_j^1(p_i) \) denotes the probability that firm \( j \)'s price in period 1 is less than or equal to \( p_i \). On the other hand, firm \( i \) is one of the discriminating insider firms in period 2 with a probability equal to \( 1 - \prod_{j \neq i} [1 - F_j^1(p_i)] \).

Since we are looking for a symmetric MSNE, let \( F_j^1(p) = F^1(p) \) for all firms. Overall expected profit for firm \( i \) when it charges first-period price \( p \), uses a discount factor equal to \( \delta \), and its competitors price according to \( F^1(p) \), is equal to:

\[
E\Pi_i = p\gamma \left\{ 1 - [1 - F^1(p)]^2 \right\} + p(\gamma + \beta) \left[ 1 - F^1(p) \right]^2 \\
+ \delta \left\{ [1 - F^1(p)]^2 \sigma_N^2 + \left[ 1 - [1 - F^1(p)]^2 \right] \sigma_{i,M}^2 \right\}.
\]

Equivalently,

\[
E\Pi_i = p\gamma + \delta \left( v\gamma + \frac{v\gamma\beta}{2(\gamma + \beta)} \right) + [1 - F^1(p)]^2 \left( p\beta - \frac{\delta v\gamma\beta}{2(\gamma + \beta)} \right) 
\tag{15}
\]

**Proposition 7.** When price discrimination is permitted and a two-firm merger occurs, there is a symmetric subgame perfect MSNE in which:
(i) Each firm’s first-period price is randomly chosen from the distribution

\[
F^1(p) = \begin{cases} 
0 & \text{for } p \leq p_{\text{min}} \\
1 - \left( \frac{(v-p)^\gamma}{p^{\beta - \delta} (\frac{v\gamma\beta}{(\beta + \gamma)^2})} \right)^{\frac{1}{2}} & \text{for } p_{\text{min}} \leq p \leq v \\
1 & \text{for } p \geq v
\end{cases}
\]  

with minimum equilibrium price equal to

\[
p_{\text{min}} = \frac{v\gamma}{\gamma + \beta} + \delta \frac{v\gamma\beta}{2(\gamma + \beta)^2};
\]

(ii) Each firm earns expected overall equilibrium profits equal to

\[
E(\Pi^*) = v\gamma + \delta \left( v\gamma + \frac{v\gamma\beta}{2(\gamma + \beta)} \right)
\]  

Proof. See the Appendix. ■

Making use of equations (4) and (17) it follows that the merger possibility when price discrimination is feasible gives rise to a positive effect on individual firm’s expected overall profits:

\[
E(\Pi^*) - E(\Pi) = v\gamma + \delta \left( v\gamma + \frac{v\gamma\beta}{2(\gamma + \beta)} \right) - (1 + \delta)v\gamma = \frac{1}{2(\beta + \gamma)}.
\]

Hence, our paper shows that a merger is profitable only when price discrimination is possible. This result is due to the unilateral effects of the merger, i.e., the strategic elimination of competition for shoppers between the two discriminating firms.\(^{27}\)

5 Competitive and welfare effects

As the new HMGs suggest, when price discrimination is reasonably likely, the Agencies should evaluate the possible adverse competitive effects from a merger. These new guidelines also suggest that the competitive effects should be evaluated separately by type of targeted customer. With this motivation in mind, in what follows we investigate the price effects of mergers in our theoretical framework.

Look first at prices after the merger. The merged discriminating firm raises (or at least

\(^{27}\)If price discrimination were permitted but a merger blocked, the two informed firms would be head-to-head competitors for shoppers and would end up making no additional profit in this segment.
does not reduce) the price to its captive consumers who will pay the monopoly price. In other words, consumers with a strong preference for the product of the merged firms are expected to pay a higher price in the post-merger period. Regarding the price targeted to the segment of shoppers who previously bought from a rival, a comparison between $F^1$ and $G^r$, reveals that the first-period price is stochastically larger than $p^r$. Hence, if poaching occurs, the group of shoppers will pay, on average, a lower second-period price. Finally, in what concerns the price charged to the group of the outsider’s captive consumers the conclusion is less clear-cut. This firm uses a “Hi-Lo” pricing strategy in period 2. With probability equal to $m$, its locked-in customers will pay the monopoly price. Otherwise, because it is not possible to establish a general stochastic order between $F^1$ and $G$, this set of consumers may end up paying a higher or lower second-period price.

Look next at first-period prices when firms anticipate the possibility of a merger when price discrimination is possible. From the equilibrium distribution functions it immediately follows that the effect of a merger on first-period prices depends on whether price discrimination is permitted or not. Similarly, the effect of price discrimination on first-period prices depends on whether a merger to duopoly can or cannot occur.

**Corollary 2.** From the comparison between $F$ and $F^1$, it follows that $F^1$ first-order stochastically dominates $F$ as long as $\delta > 0$. Therefore, $E(p^1) > E(p)$.

**Proof.** See the Appendix. ■

When price discrimination and mergers are permitted, a merger is profitable for the insider firms when all of them have information to engage in price discrimination. Consequently, the benefit of embarking on a merger will give rise to strategic interactions in the pre-merger period. Specifically, firms will have a strategic incentive to raise first-period prices as a way to secure being one of the insider informed-discriminating firms in the subsequent period. This acts to soften first-period price competition and to boost first-period prices.\(^{28}\)

Next we look at the welfare effects of mergers with price discrimination. Without loss of generality, suppose that $\delta = 1$. Although prices play no welfare role here—due to the unit demand assumption, no dropping out of consumers and no switching (or transport) costs—, price discrimination being permitted affects the firms’ merger decisions and so their profits and

\(^{28}\)Note also that regardless of price discrimination being permitted or not, the support of equilibrium prices is $\left[\frac{\alpha^2}{\gamma^2 + \beta}, \mu\right]$ when mergers are blocked. In contrast, the support of equilibrium prices is $\left[\frac{\alpha^2}{\gamma^2 + \beta} + \delta_{\frac{\alpha^2}{2(\gamma + \beta)}}, \mu\right]$ when both mergers and price discrimination are allowed. Since $\delta > 0$, the minimum price is always higher when mergers and price discrimination are permitted.
consumer welfare. Since production costs are assumed to be zero, total welfare \( W \) is equal to the value of the good for all buyers that enter the market in both periods, that is \( W = 2v \). Due to the previous assumptions, a merger will have no effect on overall welfare. Nevertheless, it is important to investigate separately the effects of mergers on industry profit and consumer welfare.

To evaluate the profit and consumer surplus effects of mergers, we first analyze the case where price discrimination is permitted and we move from the no-merger to the merger scenario. As welfare is constant, the effect of a merger is to give rise to a transfer of income and wealth from individual consumers to the firms. When firms are not allowed to merge, it follows that industry profit is equal to \( \pi_{ind}^N = 6v\gamma \). This being the case, consumer surplus equals

\[
CS^N = 2v - 6v\gamma = 2v (1 - 3\gamma) = 2v\beta. \tag{19}
\]

When mergers are instead permitted, then, from equation (18), and recalling that \( \delta = 1 \), we obtain that there is a positive net effect on industry profit which equals

\[
\pi_{ind}^M - \pi_{ind}^N = \frac{3v\gamma\beta}{2(\gamma + \beta)}. \tag{20}
\]

This gain is exactly compensated by a loss in terms of consumer surplus:

\[
CS^M - CS^N = -\frac{3v\gamma\beta}{2(\gamma + \beta)}. \tag{21}
\]

We can then state the following proposition.

**Proposition 8.** In a context where BBPD is possible, when a merger between informed players occurs, industry profit increases at the expense of consumer surplus.

This result is, therefore, in stark contrast with the general presumption of Chen (2005), according to whom “price discrimination by purchase history ... is by and large unlikely to raise significant antitrust concerns. In fact, as the economics literature suggests, such pricing practices in oligopoly markets often intensify competition and potentially benefit consumers.” (p. 123).
6 Merger policy implications

In this section, we discuss what are the main policy implications for mergers that can be derived from our theoretical model.

First, our model shows that when firms are allowed to price discriminate, then the (unique) equilibrium merger reduces competition in such a way as to transfer wealth from customers to the merged firm (and its competitors). This then suggests that if total welfare is the criterion adopted by the competition authorities to appraise a proposed merger, the merger is welfare-neutral. Nonetheless, if consumer surplus is the competition authority welfare standard, as it is the case in most antitrust jurisdictions, then competition authorities should scrutinize with greater zeal mergers in industries wherein firms are expected to engage in price discrimination practices.

Second, as far as prices after the merger are concerned, our analysis reveals that competitive effects should be evaluated separately by type of targeted customer. In particular, we find that, in the post merger period, consumers with a strong preference for the merged entity products will be charged their reservation (monopoly) price. However, due to poaching activities, consumers in the segment of shoppers who bought from the merged firm’s competitor before will pay, on average, a price which is lower than the one they paid before the merger. Our model, therefore, gives a one possible theoretical rationale for the fact that, as the new HMGs emphasize, when price discrimination is feasible, a merger can give rise to adverse competitive effects on targeted consumers. In so doing, it gives support to the new HMGs claiming that price discrimination should be a key factor in any competitive analysis of mergers.

Finally, the outsider’s profit is shown not to be affected by a merger when price discrimination (in the second period) is not feasible. In addition, also in a scenario in which price discrimination is permitted, the firm which is excluded from participation in the equilibrium merger (the non-discriminating outsider firm) is not affected by the merger in terms of its second period profits. This result then eliminates the so called (static) ‘free-riding problem’ identified by the previous horizontal mergers literature regarding outsiders’ profitability in the ex-post industry structure induced by the merger. However, it is important to highlight that, in our setting, BBPD creates instead a dynamic ‘free-riding problem’ via the race to embark on the (two-firm) merger. More specifically, our analysis reveals that in industries where BBPD and mergers are possible, the anticipation of the fact that the equilibrium merger will involve the firms with the necessary information to price discriminate ex-post (i.e. those firms that only served their
captive customers before the merger) will lead all firms in the industry to charge higher prices in the pre-merger market interaction. Moreover, this competition softening effect in turn enhances the overall equilibrium expected profit of all firms in the status quo industry structure. This being the case, and contrary to the results in the extant literature, our analysis discloses that in a context wherein BBPD is possible, the free-riding problem does not stem from strategic interactions among firms post-merger, but from strategic interactions pre-merger.

This last result is of utmost importance for merger policy since it suggests that there might be scope for improving the rules currently used by competition agencies to investigate the potential anticompetitive effects of horizontal mergers. In particular, our analysis reveals that the anticipated merger gives rise to first-period anti-competitive effects and that part of the harm on consumers can be produced in the pre-merger period. Put it another way, at the time the merger is notified to the relevant agency, a substantial part of the harm on consumers has already occurred. As a result, in the context of the model, when considering unilateral effects associated with the merger, a standard rule based on market shares that would only allow mergers among firms with small market shares is particularly bad-designed and can actually turn out to be counterproductive. So, our model suggests that when notifying parties are firms with small market shares, it is very important to ensure that the strategic adoption of soft pricing strategies by the merging parties at an earlier stage is not the reason why their corresponding small market shares have emerged. Competition agencies should, therefore, complement the usual market share test with: (i) a detailed analysis of firms pricing strategies before the merger; and (ii) an in depth evaluation of the potential risk that the likely adoption of post-merger price discrimination strategies might harm in a disproportional way certain consumers or groups of consumers in specific segments of the market (or submarkets).

7 Limitations of the model

In this section we discuss some important limitations of the proposed model.

\footnote{In our setting, “being successful” at the market in the first period (by charging the lowest price) actually hurts the firm, when it comes to being desirable as a merger partner.}

\footnote{We thank an anonymous referee for pointing out this issue.}

\footnote{Given the timing of the proposed game, it seems natural to assume that the merger would be notified to the relevant agency between the two periods.}
7.1 Allowing for more than three firms

The previous analysis focused attention on the effects of a two-firm merger in a setting where there are initially only three firms in the industry. So, it is natural to wonder what would be the equilibrium outcome of the proposed game if one allows for more than three firms in the status quo industry structure. Suppose now that there are \(N > 3\) firms in the industry. Each of these firms is assumed to have a proportion \(\gamma\) of captive consumers, implying that now the number of shoppers is \(\beta = 1 - N\gamma > 0\). If mergers cannot occur, it is straightforward to prove that each firm’s overall expected equilibrium profit equals \(E(\pi) = (1 + \delta) \nu \gamma\), irrespective of whether price discrimination is permitted or not.

In an online Appendix of this article,\(^{32}\) we investigate which merger would occur in equilibrium when price discrimination is possible and \(N > 3\).\(^{33}\) More specifically, we carry out a technical analysis composed of two main steps that we briefly describe in what follows.

First, we focus attention on the induced effects of two-firm mergers and find out that any such merger leaves at least two separate informed firms and, therefore, has no effect on insiders’ profits. As in the baseline model, a two-firm merger can involve either two informed firms or one informed and one uninformed firm. If any pair of informed firms merge, then there will always exist at least one informed outsider. Hence, after the merger, Bertrand competition for shoppers between the merged entity and the informed outsider(s) will result in price equal to marginal cost for the shoppers. The same reasoning applies if instead an informed firm merges with an uninformed one. So, any single merger involving only two of the \(N\) firms in the status quo industry structure, where \(N > 3\), will have no impact on the insider’s equilibrium second-period profits. Therefore, if only a single two-firm merger is allowed, then no merger will occur in equilibrium. Note, however, that even though it seems reasonable to assume that a merger to monopoly would normally be prevented by antitrust authorities, as it is standard in the previous endogenous mergers literature (see, for instance, Fauli-Oller (2000), Horn and Persson (2001b) and Lommerud, Straume and SØrgard (2005)), there is no reason to assume that in a setting where there are more than three firms in the initial industry structure only a two-firm merger might occur. In practice, in situations where there are several firms in the industry, it is often the case that mergers involve more than two firms. This then motivates the second step of our

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\(^{32}\)The online appendix is available at http://esteves.rosabranca.googlepages.com, and contains supplementary material (e.g. the formal proofs) regarding the main theoretical results summarized in the present Section.

\(^{33}\)If price discrimination practices are not allowed, then it is straightforward to conclude that, similarly to what happened in the baseline model, no merger would occur in equilibrium.
formal analysis, that we discuss in turn.

Second, we study an extended version of our baseline model wherein even though the monopoly grand coalition cannot be formed, each firm can announce any other subset of the firms in the industry (including itself) that it wishes to form a coalition with. By so doing, we show that a situation wherein the subset of informed firms embark on a merger between themselves, leaving outside the merger the uninformed firm, constitutes the unique strong Nash equilibrium of the proposed coalition formation game. Put it another way, a merger between all informed firms is profitable and (if feasible) has the same qualitative effect on first period pricing as the equilibrium two-firm merger studied in our baseline model with three firms.\footnote{A similar qualitative result is obtained using an alternative coalition formation game according to which the merger process is (instead) sequential, i.e. a first merger might trigger other merger(s) and firms anticipate this.}

7.2 Information sharing

In section 7.1, we have shown that a merger (or a sequence of mergers) involving all firms with the necessary information to price discriminate in the second period of the game is a profitable strategy. As explained, the main role of the merger is to eliminate all competition for shoppers between the merging discriminating parties. By pooling the customer information databases of the two informed firms, the merger increases the value of this information to each merger partner since the merger entity will then be able to fully separate different types of consumers and price discriminate accordingly.\footnote{Put it another way, each informed firm’s customer information database becomes more valuable when pooled through a merger with the database belonging to the other informed firm.} This being the case, the following question can be raised: Couldn’t the benefit of customer recognition and price discrimination be realized by the less restrictive means of information exchange agreements between the firms?

Notice that in our proposed theoretical framework (with three firms) there is an important difference between a merger and an information sharing agreement. In particular, if one of the informed firms decides to enter an information exchange agreement with the other informed firm (rather than embark on a merger involving the very same firms), both of these firms become fully informed about the identity of each customer but keep on being independent competitors in the product market. As a result, the pool of information through an information exchange agreement would not eliminate all competition for shoppers between the involved firms: in equilibrium, each firm would charge the marginal cost price to the shoppers and the reservation price $v$ to its group of captive consumers.

In sum, information sharing between rival informed firms only intensifies competition in
the price-sensitive consumers’ segment of the market, where informed firms will compete à la Bertrand for shoppers, thereby being unable to earn any positive profit in this particular market segment. This in turn implies that informed firms have no incentive to participate in a customer information sharing agreement in our setting, whereas by merging they might be able to reduce or even eliminate competition in specific segments of the market. Hence, our analysis suggests that competition authorities should not access mergers and information exchange agreements in the same way, and should be particularly concerned whenever they investigate mergers in industries wherein firms are expected to engage in price discrimination.

7.3 Strategic consumers

So far, we have assumed that consumers are myopic (or naive). Relaxing this naivety assumption in our framework would imply assuming that consumers are highly sophisticated. In particular, apart from anticipating that firms would engage in BBPD practices, consumers would also have to predict the outcome of the endogenous merger formation game that firms play before competing in the marketplace for the second time. In what follows we briefly discuss some of the implications of assuming instead that consumers are sophisticated.36

A shopper has no incentive to behave strategically. In period 1, he must buy from the cheapest firm. If he decided to buy from a high first-period price firm, he would be recognized as a captive consumer in period 2 and, as a result, he would end up paying higher prices in both periods. On the other hand, if he decided to forgo a purchase in the first-period he would be recognized as a shopper. Therefore, he would pay the same price in period 2, but would forgo a positive surplus in period 1.

It only remains to see whether captive consumers have an incentive to forgo a purchase in order to avoid being recognized as captives in the subsequent period. Chen and Zhang (2009) account for this possibility by assuming that \( \psi \) is the proportion of these captive consumers who have forgone a purchase in period 1. They show that in equilibrium none of these consumers has an incentive to forgo a purchase in the first period (i.e., \( \psi = 0 \)). Therefore, given the similarities between our model and that of Chen and Zhang (2009), we believe that the main qualitative results associated with an extended version of our model allowing for the existence of strategic consumers should be identical to those obtained in our baseline model.

36 A full discussion of the implications of strategic consumers in a similar duopoly model of BBPD with no mergers can be found in Chen and Zhang (2009).
8 Conclusion

The economics literature on oligopoly price discrimination by purchase history is relatively new and has focused mostly on markets with two symmetric firms, where the possibility of horizontal mergers is not considered. In these situations, dynamic price discrimination by competing firms often results in intensified competition; and such pricing practices are typically believed not to raise antitrust concerns.

This article has taken a first step in investigating the impact of Behaviour-Based Price Discrimination (BBPD) in markets with more than two firms where horizontal mergers may occur. By so doing, we show that: (i) absent the merger possibility, the ability to price discriminate does not lead to higher profits; (ii) the merger itself, without price discrimination, does not lead to higher profits; and (iii) only the merger with price discrimination lead to an increase in profits, but at the expense of consumers’ welfare. This then suggests that competition authorities should be particularly vigilant with regards to mergers in industries wherein firms are expected to engage in BBPD.

Further, it is shown that in a scenario wherein price discrimination is feasible, the firm which is excluded from the equilibrium merger is not affected by the merger in terms of its ex-post merger profits. This result, therefore, eliminates the so called (static) ‘free-riding problem’ identified by the previous literature on horizontal mergers. However, our analysis also discloses that BBPD creates a dynamic ‘free-riding problem’. More specifically, in industries where BBPD and mergers are possible, the anticipation that the equilibrium merger will involve the firms with the necessary information to price discriminate ex-post (i.e. those firms that in the pre-merger market structure only sell to their captive customers) will induce all firms in the industry to charge high prices in the pre-merger product market competition phase. Put it another way, because of the race between firms in the industry to embark on the anticipated merger, there is a pre-merger competition softening effect that will enhance the overall equilibrium expected profits of all firms in the status quo industry structure. This being the case, and contrary to the results in the extant literature, our analysis reveals that in a context wherein BBPD is possible, the free-riding problem does not stem from strategic interactions among firms post-merger, but from strategic interactions pre-merger. This suggests that the anticipated merger gives rise to first-period anti-competitive effects and that part of the harm on consumers can be produced in the pre-merger period.

An important implication for antitrust policy is then that merger enforcement should be
sensitive to the role of price discrimination in particular industries and how consumer information used to price discriminate is shared and acquired by merging firms. In particular, our analysis suggests that there might be scope for improving the rules currently used by competition agencies to investigate the potential anticompetitive effects of horizontal mergers. The model suggests that when notifying parties are firms with small market shares, it is very important to ensure that the strategic adoption of soft pricing strategies by the merging parties at an earlier stage is not the reason why their corresponding small market shares have emerged. Competition agencies should, therefore, complement the usual market share test with: (i) a detailed analysis of firms pricing strategies before the merger; and (ii) an in depth evaluation of the potential risk that the likely adoption of post-merger price discrimination strategies might harm in a disproportional way certain consumers or groups of consumers in specific segments of the market (or submarkets).

A Proofs

**Proof of Proposition 3:** Suppose that the merging firm prices say product 1 at \( p_M^1 = v \) and prices product 2 at \( p_M^2 \) drawn in an interval according to the distribution function \( H_M^2(.) \). If the outsider firm chooses price \( p_O \) according to \( H_O(.) \) then the merging entity expected profit for product 2 is equal to \( p_M^2 (\gamma + \beta) [1 - H_O(p_M^2)] \). Since in a MSNE any price from a firm’s price support should generate the same expected profit for the firm, we must have:

\[
p_M^2 [\gamma + \beta (1 - H_O(p_M^2))] = v\gamma.
\]

Similarly the expected profit of the outsider firm is equal to \( p_O (\gamma + \beta) [1 - H_M(p_O)] \). Thus in a MSNE we must observe:

\[
p_O [\gamma + \beta (1 - H_M(p_O))] = v\gamma.
\]

At equilibrium one has \( H_O(.) = H_M(.) = H(p) \). Therefore:

\[
H(p) = 1 - \frac{\gamma}{\beta} \left( \frac{v - p}{p} \right).
\]

From \( H(p_{\text{min}}) = 0 \) one obtains \( p_{\text{min}} = \frac{v}{1 + \gamma} \). Q.E.D.

**Proof of Proposition 4:** Suppose \( (p^*, \hat{p}_N^*) \) is an equilibrium in pure strategies. Then, by definition, there is no such \( p^r \), such that \( \pi_M^r (p_M^r, \hat{p}_N^r) > \pi_M^r (p_M^*, \hat{p}_N^*) \). The proof proceeds by contradiction.
(i) If \( p^*_r = \hat{p}_N \), then
\[
\pi^*_i = \frac{1}{2} \beta p^*_r .
\] (22)

If firm \( M \) deviates and quotes \( p'_M = p^*_r - \varepsilon \), with \( \varepsilon > 0 \), its profit from deviation is \( \pi^*_M = \beta (p^*_r - \varepsilon) \). It is then trivial to see that there exists such an \( \varepsilon \) that makes the deviation profitable. A contradiction. Q.E.D.

(ii) Let \( p^*_M < \hat{p}_N \) then
\[
\pi^*_i = \beta p^*_r .
\] (23)

Let \( p^*_M = p^*_r + \varepsilon < \hat{p}_N \), then, firm \( i \)'s profit from deviation is \( \pi^*_i = \beta (p^*_r + \varepsilon) \), from which it is straightforward to see that the deviation is profitable. A contradiction. Q.E.D.

**Proof of Proposition 5:** The existence of such an equilibrium is proved by construction. It is a dominated strategy for each firm to set a price above \( v \). Additionally, the non-discriminating firm can guarantee itself a profit of \( v \gamma \), charging \( v \) to its captive customers. It thus follows that at price \( \hat{p}_N \) the best it can do is to attract all shoppers as well as its captive customers. This means that a necessary condition for it to be willing to charge \( \hat{p}_N \) is \( \hat{p}_N (\gamma + \beta) \geq v \gamma \). In other words, any \( \hat{p}_N < \hat{p}_{N\min} = \frac{v \gamma}{\gamma + \beta} \) is a dominated strategy for the non-discriminating firm. As this firm would never want to price below \( \hat{p}_{N\min} \), by quoting a price \( p'_M \) arbitrarily close to \( \hat{p}_{N\min} \), the discriminating firm poaches all the selective customers that bought previously from the rival, guaranteeing itself a profit of \( \hat{p}_{N\min} \beta = \frac{v \gamma \beta}{\gamma + \beta} \). Thus, any price \( p^*_M < \hat{p}_{N\min} \) is a dominated strategy for the discriminating firm.

Next, we prove that neither firm has a mass point \( p^* \), such that \( \hat{p}_{N\min} < p^* < v \). By way of contradiction, assume that \( p^* \) is chosen with positive probability by firm the discriminating firm. Then by choosing \( \hat{p}_N = p^* - \varepsilon \), where \( \varepsilon \) is arbitrarily small, the non-discriminating firm becomes the low priced firm and can increase its profits. There is a profitable deviation. A contradiction. Assume now that that \( p^* \) is chosen with positive probability by the non-discriminating firm. Then by choosing \( p'_M = p^* - \varepsilon \), where \( \varepsilon \) is arbitrarily small, the discriminating firm has a profitable deviation. A contradiction. By similar arguments it is also straightforward to show that neither firm has a mass point at \( \hat{p}_{N\min} \). It remains to prove that only the non-discriminating firm has a mass point at the highest price \( v \). If the non-discriminating firm has a mass point at \( v \), the discriminating firm is always better off not charging that price but coming arbitrarily close to it. Following Narasimhan (1988) it is also straightforward to prove that both distribution functions are strictly increasing and continuous over the interval with lower bound \( \hat{p}_{N\min} \) and
upper bound $v$. In equilibrium, for the non-discriminating firm, the following condition must be satisfied:

$$\hat{p}_N \gamma + \hat{p}_N \beta [1 - G^r(\hat{p}_N)] = v \gamma$$

It follows that

$$G^r(\hat{p}_N) = 1 - \frac{(v - \hat{p}_N) \gamma}{\hat{p}_N \beta},$$

with $G^r(\hat{p}_{N_{\text{min}}}) = 0$ and $G^r(v) = 1$. This proves part (ii).

Similarly, in equilibrium, the discriminating firm must be indifferent between prices that belong to the half open interval $[\hat{p}_{N_{\text{min}}}, v)$, i.e.:

$$p^r_{M} \beta [1 - \hat{G}_N(p^r)] = \hat{p}_{N_{\text{min}}},$$

from which it follows that:

$$\hat{G}_N(p^r_{M}) = 1 - \frac{\hat{p}_{N_{\text{min}}}}{p^r_{M}} = 1 - \frac{v \gamma}{p^r_{M} \gamma},$$

with $\hat{G}_N(p^r_{M} = \hat{p}_{N_{\text{min}}}) = 0$ and $\hat{G}_N(v) = 1 - \frac{7}{\gamma + \beta}$ which is smaller than 1 as long as $\beta > 0$ which by assumption is always true. This implies that the non-discriminating firm has a mass point at $v$. This completes the proof. Q.E.D.

**Proof of Corollary 1**: To prove part (i) note that $G^r(\hat{p}_N) - \hat{G}_N(p^r_{M})$ can be written as $\left[\frac{v - p}{p} - \frac{v}{\gamma + \beta}\right] \frac{\gamma}{\hat{p}}$. Since $\frac{\gamma}{\hat{p}} > 0$ and $\beta (\gamma + \beta) > 0$, then $G^r(\hat{p}_N) - \hat{G}_N(p^r_{M}) > 0$ as long as $(v - p) \gamma > 0$, which is always true. When (i) holds, result (ii) follows. Q.E.D.

**Proof of Proposition 6**: The existence of such an equilibrium is proved by construction. It is a dominated strategy for each firm to set a price above $v$. Additionally, the merged firm can guarantee itself a profit of $v \gamma$, charging $v$ to its unrecognised captive customers. It thus follows that at price $\hat{p}_M$ the best it can do is to attract all shoppers as well as its unrecognised captive customers. This means that a necessary condition for it to be willing to charge $\hat{p}_M$ is $\hat{p}_M (\gamma + \beta) \geq v \gamma$. In other words, any $\hat{p}_M < \hat{p}_{M_{\text{min}}} = \frac{v \gamma}{\gamma + \beta}$ is a dominated strategy for the merged firm. As this firm would never want to price below $\hat{p}_{M_{\text{min}}}$, by quoting a price $p^r_{O}$ arbitrarily close to $\hat{p}_{M_{\text{min}}}$, the outsider firm poaches all the switchable customers that bought previously from the rival, guaranteeing itself a profit of $\hat{p}_{M_{\text{min}}} \beta = \frac{v \gamma \beta}{\gamma + \beta}$. Thus, any price $p^r_{O} < \hat{p}_{M_{\text{min}}}$ is a dominated strategy for the outsider firm. Following a similar proof as in the proof of proposition 6, it is
straightforward to prove that only the merged firm can have a mass point at \( v \). In equilibrium, for the merged firm, the following condition must be satisfied:

\[
\tilde{p}_M \gamma + \tilde{p}_M \beta \left[ 1 - G^r_O(\tilde{p}_M) \right] = v \gamma
\]

from which we obtain:

\[
G^r_O(\tilde{p}_M) = 1 - \frac{(v - \tilde{p}_M) \gamma}{\tilde{p}_M \beta}.
\]

It thus follows that \( G^r_O(v) = 1 \) and from \( G^r_O(\tilde{p}_{M\text{ min}}) = 0 \) we obtain \( \tilde{p}_{M\text{ min}} = \frac{v \gamma}{\gamma + \beta} \).

Similarly, in equilibrium, the outsider firm must be indifferent between prices that belong to the interval \([\tilde{p}_{M\text{ min}}, v)\), i.e.:

\[
p_O \beta \left[ 1 - \tilde{G}_M(p_O) \right] = \tilde{p}_{M\text{ min}} \beta
\]

It follows that:

\[
\tilde{G}_M(p_O) = 1 - \frac{\tilde{p}_{M\text{ min}}}{p_O} = 1 - \frac{v \gamma}{p_O (\gamma + \beta)}
\]

with \( \tilde{G}_M(p_O = \tilde{p}_{M\text{ min}}) = 0 \) and \( \tilde{G}_M(v) = 1 - \frac{\gamma}{(\gamma + \beta)} \), smaller than 1 as long as \( \beta > 0 \) which by assumption is always true. This implies that the merged firm has a mass point at \( v \) equal to \( \tilde{m} = \frac{\gamma}{\gamma + \beta} \). This completes the proof. \( Q.E.D. \)

**Proof of Proposition 7:** The overall expected profit for firm \( i \), when it charges first-period price \( p_i \), uses a discount factor equal to \( \delta \), and their competitors charge a first-period price equal to \( p_j \) according to \( F^1_j(p_i) \), is equal to:

\[
E\Pi_i = p \gamma \left\{ 1 - [1 - F^1(p)]^2 \right\} + p (\gamma + \beta) \left[ 1 - F^1(p) \right]^2
+
\delta \left[ [1 - F^1(p)]^2 \frac{\pi^2}{\pi_N^2} + \left[ 1 - [1 - F^1(p)]^2 \right] \frac{\pi^2_{i,M}}{\pi_{i,M}} \right].
\]

\[
E\Pi_i = p \gamma + \delta \left( v \gamma + \frac{v \gamma \beta}{2 (\gamma + \beta)} \right) + \left[ 1 - F^1(p) \right]^2 \left( p \beta - \frac{\delta v \gamma \beta}{2 (\gamma + \beta)} \right)
\]

In MSNE the firm must be indifferent between quoting the monopoly price \( v \) or any price in the equilibrium support.

\[
E\Pi_i = p \gamma + \delta \left( v \gamma + \frac{v \gamma \beta}{2 (\gamma + \beta)} \right) + \left[ 1 - F^1(p) \right]^2 \left( p \beta - \frac{\delta v \gamma \beta}{2 (\gamma + \beta)} \right) = v \gamma + \delta \left( v \gamma + \frac{v \gamma \beta}{2 (\gamma + \beta)} \right)
\]
Solving for $F^1(p)$

$$F^1(p) = 1 - \left( \frac{(v - p)\gamma}{p^\beta - \delta \left( \frac{v^\gamma\beta}{2(\gamma + \beta)} \right)} \right)^{\frac{1}{2}}$$

Given that $F^1(p_{\text{min}}) = 0$ we find that $p_{\text{min}} = \frac{v^\gamma}{\gamma + \beta} + \delta \frac{v^\gamma\beta}{2(\gamma + \beta)^2}$. Q.E.D.

**Proof of Corollary 2:** $F^1$ first-order stochastically dominates $F$ as long as $F^1 < F$, i.e.,

$$1 - \left( \frac{(v - p)\gamma}{p^\beta - \delta \left( \frac{v^\gamma\beta}{2(\gamma + \beta)} \right)} \right)^{\frac{1}{2}} < 1 - \left( \frac{(v - p)\gamma}{p^\beta} \right)^{\frac{1}{2}}.$$ This condition is satisfied if $\frac{\delta v^\gamma}{2p^\beta(\beta + \gamma)} > 0$, which is true as long as $\delta > 0$. When this holds it follows that $E(p^1) > E(p)$. Q.E.D.

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