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Costly Investment, Complementarities, International Technological-Knowledge Diffusion and the Skill Premium

Oscar Afonso,[*] Pedro Neves[†] and Maria Thompson[‡]

Abstract

We examine the behavior of the skill premium in a two-country general equilibrium growth model assuming (i) technological-knowledge diffusion; (ii) internal costly investment in both physical capital and R&D; and (iii) complementarities between intermediate goods in production. We find that these three economic features affect the steady-state growth rate in both countries. However, only in the imitator country do they influence the skill premium. We also find that the steady-state skill premium in the innovator country is affected by its relative labor productivity rather than by its relative labor endowments. This result contrasts with most skill-biased technological change models and suggests that the sustained increase in the skill premium observed in several developed countries over the last three decades may have been due to increases in the relative productive advantage of skilled labor.

Keywords: technological-knowledge bias, skill premium, complementarities, costly investment, technological-knowledge diffusion

JEL Classification: F43, J31, O31, O33, O47.

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1. Introduction

With this paper, we analyze theoretically the skill premium behavior in an economic environment characterized by a two-country skill-biased technological-change model with vertical differentiation and three assumptions: (i) technological-knowledge diffusion; (ii) internal costly investment in both physical capital and R&D; and (iii) complementarities between intermediate goods in production.

Building on Acemoglu and Zilibotti (2001), and introducing vertical differentiation as in Aghion and Howitt (1992), our baseline framework is an R&D-based growth model, in which perfectly competitive final goods are produced with labor and quality-adjusted intermediate goods, whose productions under monopolistic competition requires innovation. New designs are obtained through vertical R&D. Each final good is produced by one of two technologies: one that uses skilled labor and skilled-specific intermediate inputs; the other uses unskilled labor and unskilled-specific intermediate goods.

Following Barro and Sala-i-Martin (1997), we develop our baseline framework, into a two-country model with technological-knowledge diffusion. In the innovator (developed) country, firms involved in R&D undertake innovative research. In the imitator (less developed) country, firms involved in R&D undertake imitative research. The two countries differ in (i) productivity levels; (ii) labor endowments; (iii) R&D capacity; and (iv) technological-knowledge levels. In the absence of international trade, technological-knowledge diffusion between countries is assumed to occur as a result of, for example, international mobility of students (Park, 2004; Le, 2010) or communication patterns (Keller, 2003; Wong, 2004).

Following Thompson (2008), we introduce two additional assumptions: (1) total investment in both physical capital and R&D requires internal adjustment costs, specified as in Hayashi (1982); and (2) intermediate goods are complementary to each other in final goods production, in an Evans et al.'s (1998) specification.

We examine the impact of (i) technological-knowledge diffusion; (ii) internal costly investment, and (iii) complementarities, on the technological-knowledge bias, the skill premium, and economic growth in both countries. We find that the three introduced economic features affect the growth rate in both countries. In particular, technological-knowledge diffusion and the complementarities degree influence growth positively, whereas costly investment affects growth negatively. We also find that only in the imitator country do these three assumptions affect the skill premium and the technological-knowledge bias.

Further, we analyze the effects on the skill premium of an increase in the skilled labor relative supply in the innovator country. An important feature of wage patterns in several developed countries, over the last three decades, has been the simultaneous rise in both the skilled labor relative supply and the skill premium As Richardson (1995), He and Liu (2008), among others, review, the skill-biased technological-change theory is the most accepted approach for explaining such pattern. Wishing to contribute theoretically to the academic debate on this important question, we find that the skilled labor relative supply has a positive impact on the growth rate, but does not affect the equilibrium skill premium. In the proposed model, the steady-state skill premium depends solely on the productivity levels of each type of labor. Our model suggests that the sustained increase in the skill premium in several developed countries may not be related to increases in skilled labor relative supply, rather it may be linked to increases in skilled labor's productive advantage.

The remainder of the paper is structured as follows. We set up the model in Section 2 and derive the equilibrium in Section 3. In Section 4, we analyze the impact of the three introduced assumptions on the equilibrium variables. Concluding Remarks in Section 5 bring closure to the paper.

2. The Model

2.1. Consumption Side

Each country is populated by a time-invariant number of heterogeneous households who supply labor, consume final goods and own firms. They are endowed with ability level $a \in$ [0,1] and supply one of two types of labor: unskilled, L_a, if $a \leq \bar{a}$, and skilled, H_a, if $a > \bar{a}$. The amounts of both types of labor, L and H, are fixed. We assume that the innovator country is relatively more abundant in skilled labor; that is, $H_I / L_I > H_P / L_P$, where indexes I and P represent the innovator and the imitator countries, respectively. All households have identical preferences described by a constant relative risk aversion lifetime utility function,

$$U_a(t) = \int_0^\infty e^{-\rho t} \frac{C_a(t)^{1-\theta} - 1}{1-\theta} dt \tag{1}$$

where $C_a(t)$ is household a's consumption at time t, ρ is the subjective discount rate, and θ is the relative risk aversion parameter. Each household's budget constraint equalizes income to consumption plus savings. Savings consist of accumulation of financial assets, E, with return r, in the form of ownership of intermediate goods firms. Each firms' value corresponds to its respective patent's value. Each household's budget constraint is:

$$\dot{E}_a(t) = r(t)E_a(t) + w_M(t)M_a(t) - C_a(t), \text{ where } M = \{L, H\} \tag{2}$$

Household a maximizes function (1) subject to equation (2). The solution, independent of the individual, is the Standard Euler equation:

$$\frac{\dot{C}_a(t)}{C_a(t)} = \frac{\dot{C}(t)}{C(t)} = \frac{r(t) - \rho}{\theta} \tag{3}$$

2.2. Final-Goods Sector

Building on Acemoglu and Zilibotti (2001), in country Z, $Z = \{I, F\}$, the final goods sector is composed by competitive firms indexed by $n_Z \in [0,1]$. Two substitute production technologies are available in each country. The unskilled technology - L-technology - uses unskilled labor combined with a continuum of unskilled-specific intermediate goods indexed by $j \in [0, J_{L,Z}]$. The skilled technology - H-technology - uses skilled labor combined with a continuum of skilled-specific intermediate goods indexed by $j \in [0, J_{H,Z}]$. Intermediate goods enter complementarily in the production function, following Thompson (2008), in an Evans et al. (1998) specification. The output of firm n in country Z at time t, $Y_{n,Z}(t)$, is given by:

$$Y_{n,Z}(t)=A_Z\left\{\left[(1-n_Z)lL_{n,Z}\right]^{1-\alpha}\left[\int_0^{J_{L,Z}}\left(q^{k_{j,L,Z}(t)}x_{n,j,L,Z}(t)\right)^{\gamma}dj\right]^{\phi}+\left[n_Z hH_{n,Z}\right]^{1-\alpha}\left[\int_0^{J_{H,Z}}\left(q^{k_{j,H,Z}(t)}x_{n,j,H,Z}(t)\right)^{\gamma}dj\right]^{\phi}\right\} \quad (4)$$

Variable A_Z is an exogenous constant representing each country's level of productivity, considered dependent on a country's institutions such as property rights, tax laws and government services. Given the general perception that institutions are of better quality in developed countries, which are more innovative, we assume that $A_I > A_F$.

The contribution of intermediate goods to production is represented by expressions within square brackets. Under the Schumpeterian tradition, firm n's output depends on the quantity of the M-type intermediate good j, $x_{n,j,M,Z}(t)$, adjusted by quality. The size of each quality upgrade obtained with each successful R&D is denoted by constant $q>1$. The quality ladder rungs are indexed by k, with a higher value of k denoting a higher quality. As in Thompson (2008), we impose two parameter restrictions: $\gamma\phi=\alpha$, so as to have constant returns to scale; and $\phi>1$, so that intermediate goods are complementary to one another, i.e., so that an increase in the quantity of each j increases the marginal productivity of the others.

Expressions with exponent (1-α) represent the contribution of labor inputs to production. Variables $L_{n,Z}$ and $H_{n,Z}$ are the amounts of, respectively, unskilled and skilled labor, while parameters l and h stand for, respectively, unskilled and skilled labor's productivity. Two productive advantages are here assumed. Firstly, we assume that $h>l\geq 1$, which constitutes an absolute productivity advantage of skilled over unskilled labor. Secondly, terms (1-n) and n imply that unskilled (skilled) labor is relatively more productive with lower (higher) index final goods. The use of these adjustment terms also implies the existence of an endogenous threshold final good, $\bar{n}_Z(t)$, such that final goods $n_Z\in\left[0,\bar{n}_Z(t)\right]$

are produced exclusively under the L-technology, whereas final goods $n_Z \in [\bar{n}_Z(t), 1]$ are produced solely under the H-technology.

Normalizing to one the price of the composite final good, Y_Z, and naming $P_{n,Z}(t)$ the price of final good n, aggregate output in country Z, at time t is:

$$Y_Z(t) = \int_0^1 P_{n,Z}(t) Y_{n,Z}(t) dn = e^{\int_0^1 \ln Y_{n,Z}(t) dn} \tag{5}$$

The demand for the M-type intermediate good j by each final goods firm can be obtained from its profit maximization problem:

$$\max_{x_{n,j,M,Z}(t)} P_{n,Z}(t) Y_{n,Z}(t) - \int_0^{J_{L,Z}} R_{j,L,Z}(t) x_{n,j,L,Z}(t) dj - \int_0^{J_{H,Z}} R_{j,H,Z}(t) x_{n,j,H,Z}(t) dj - w_{L,Z}(t) L_{n,Z} - w_{H,Z}(t) H_{n,Z}$$

where $R_{j,M,Z}$ is the price of the M-type j, and $w_{M,Z}$ is the wage paid for each unit of the M-type labor. The existence of a threshold final good means that each final good producer uses only one type of technology, thus profits are maximized in order to either $x_{n,j,L,Z}(t)$ or $x_{n,j,H,Z}(t)$. The demand for intermediate goods by the n^{th} final good firm is, then:

$$x_{n,j,L,Z}(t) = \left(\frac{\alpha A_Z P_{n,Z}(t)}{R_{j,L,Z}(t)} \right)^{\frac{1}{1-\gamma}} \left[(1-n_Z) l L_{n,Z} \right]^{\frac{1-\alpha}{1-\gamma}} \left[\int_0^{J_{L,Z}} \left(q^{k_{j,L,Z}(t)} x_{n,j,L,Z}(t) \right)^{\gamma} dj \right]^{\frac{\phi-1}{1-\gamma}} q^{k_{j,L,Z}(t) \frac{\gamma}{1-\gamma}}, \text{ if } 0 \le n_Z < \bar{n}_Z \tag{6}$$

$$x_{n,j,H,Z}(t) = \left(\frac{\alpha A_Z P_{n,Z}(t)}{R_{j,H,Z}(t)} \right)^{\frac{1}{1-\gamma}} \left[n_Z h H_{n,Z} \right]^{\frac{1-\alpha}{1-\gamma}} \left[\int_0^{J_{H,Z}} \left(q^{k_{j,H,Z}(t)} x_{n,j,H,Z}(t) \right)^{\gamma} dj \right]^{\frac{\phi-1}{1-\gamma}} q^{k_{j,H,Z}(t) \frac{\gamma}{1-\gamma}}, \text{ if } \bar{n}_Z \le n_Z \le 1 \tag{7}$$

Rewriting equations (6) and (7) with respect to $\left(q^{k_{j,L,Z}(t)} x_{n,j,L,Z}(t) \right)^{\gamma}$ and $\left(q^{k_{j,H,Z}(t)} x_{n,j,H,Z}(t) \right)^{\gamma}$ and integrating both sides of the resulting expressions, we get:

$$\int_0^{J_{L,Z}} \left(q^{k_{j,L,Z}(t)} x_{n,j,L,Z}(t) \right)^{\gamma} dj = \left(\frac{\alpha A_Z P_{n,Z}(t)}{R_{j,L,Z}(t)} \right)^{\frac{\gamma}{1-\gamma}} \left[(1-n_Z) l L_{n,Z} \right]^{\gamma} \left[\int_0^{J_{L,Z}} \left(q^{k_{j,L,Z}(t) \frac{\gamma}{1-\gamma}} \right) dj \right]^{\frac{1-\gamma}{1-\alpha}}, \text{ if } 0 \le n_Z < \bar{n}_Z$$

$$\int_0^{J_{H,Z}} \left(q^{k_{j,H,Z}(t)} x_{n,j,H,Z}(t) \right)^{\gamma} dj = \left(\frac{\alpha A_Z P_{n,Z}(t)}{R_{j,H,Z}(t)} \right)^{\frac{\gamma}{1-\gamma}} \left[n_Z h H_{n,Z} \right]^{\gamma} \left[\int_0^{J_{H,Z}} \left(q^{k_{j,H,Z}(t) \frac{\gamma}{1-\gamma}} \right) dj \right]^{\frac{1-\gamma}{1-\alpha}}, \text{ if } \bar{n}_Z \le n_Z \le 1$$

Plugging these two equations into equation (4), the supply of the final good n in Z is:

$$Y_{n,Z}(t) = A_Z^{\frac{1}{1-\alpha}} \left(\frac{\alpha P_{n,Z}(t)}{R_{j,M,Z}(t)} \right)^{\frac{\alpha}{1-\alpha}} \left[n_Z h H_{n,Z} Q_{H,Z}(t)^{1+\varepsilon} + (1-n_Z) l L_{n,Z} Q_{L,Z}(t)^{1+\varepsilon} \right] \tag{8}$$

where: $\varepsilon \equiv \dfrac{\phi - 1}{1-\alpha}$ is a positive constant and $Q_{L,Z}(t)$ and $Q_{H,Z}(t)$ are two aggregate quality indexes measuring technological-knowledge in each range of intermediate goods, defined by:

$$Q_{L,Z}(t) \equiv \int_0^{J_{L,Z}} \left(q^{k_{j,L,Z}(t)\frac{\gamma}{1-\gamma}} \right) dj \text{ and } Q_{H,Z}(t) \equiv \int_0^{J_{H,Z}} \left(q^{k_{j,H,Z}(t)\frac{\gamma}{1-\gamma}} \right) dj$$

2.3. Intermediate-Goods Sector

Under monopolistic competition, intermediate firms produce quality-adjusted intermediate goods to supply final-good firms. A wide variety of intermediate goods being produced in the economy, at each t, total production of each variety is provided by one firm alone – the one that uses the top quality. This Schumpeterian leadership is temporary, as the top quality is subject to destruction by new qualities resulting from successful innovation (in Z=I) or imitation (in Z=F) by potential entrants (e.g., Segerstrom et. al., 1990; Grossman and Helpman, 1991; Barro and Sala-i-Martin, 2004, Ch. 7).

Production in the intermediate-goods sector requires both physical capital and R&D capital. We assume that it takes one unit of physical capital to produce one physical unit of each intermediate good j. Thus the physical-capital stock in each t is given by the amount of intermediate goods produced in the economy, $X_Z(t)$. R&D capital is required to invent new designs that lead to better quality intermediate goods. The R&D capital is$\Omega_Z(t)$. Total capital is $\mathrm{K_Z(t) = X_Z(t) + \Omega_Z(t)}$.

Following Thompson (2008), we consider that total investment, $I_Z(t) = \dot{K}_Z(t) = \dot{X}_Z(t) + \dot{\Omega}_Z(t)$, involves an internal cost. With zero capital depreciation, installing $I_Z(t)$ new units of total capital, requires spending an amount given by:

$$C_Z(t) = I_Z(t) + \frac{1}{2}\varphi \frac{I_Z(t)^2}{K_z(t)} \tag{9}$$

where $\frac{1}{2}\varphi\frac{I_Z(t)^2}{K_Z(t)}$ represents the Hayashi's (1982) installation cost, with $\varphi > 0$ standing for the adjustment cost parameter. The Hamiltonian is chosen so as to maximize the present discounted value of cash flows. The current-value Hamiltonian is:

$$H_Z(t) = Y_z(t) - I_z(t) - \frac{1}{2}\varphi\frac{I_z(t)^2}{K_z(t)} + a_z(t)\left[I_z(t) - \dot{K}_z(t)\right]$$

where $a_Z(t)$ is the capital market value.

As we will see later on, in steady state, $X_Z(t)$ and $\Omega_Z(t)$ grow at the same constant rate, which is equal to the output-growth rate, *g*, common to both countries. Hence, $K_Z(t)$ also grows at the rate *g*. This means that aggregate output is a linear function of total capital. Recalling that $I_z(t)/K_Z(t) = \dot{K}_Z(t)/K_Z(t) = g$, the first order condition of the optimal control problem says that, in steady state:

$$a = 1 + \varphi g \tag{10}$$

Facing an aggregate demand given by $X_{j,M,Z}(t) = \int_0^1 x_{n,j,M,Z}(t)dn$, each intermediate good firm's maximization problem is:

$$\max_{R_{j,M,Z}(t)} R_{j,M,Z}(t)X_{j,M,Z}(t) - ar_Z(t)X_{j,M,Z}(t) \tag{11}$$

where $ar_Z(t)$ is the production cost of one unit of *j*. This problem leads to the mark-up price (12), equal across intermediate goods and quality grades:

$$R_{j,Z}(t) = \frac{ar_Z(t)}{\gamma} \tag{12}$$

Since the leader firm is the only one legally allowed to produce the top quality, it will use pricing to wipe out sales of lower quality. The lowest price that the closest follower can charge without negative profits is $ar_Z(t)$. Hence, the leader can capture the entire market by selling at a price slightly below $qar_Z(t)$, as *q* is the quality advantage over the closest follower. Thus, *q* is also an indicator of the incumbent's market power. The limit price is:

$$R_{j,Z}(t) = qar_Z(t) \tag{13}$$

Depending on whether $q\gamma$ is greater or lesser than $ar_Z(t)$, the leader firm will opt for either the monopoly pricing or the limit pricing. Like, e.g., Grossman and Helpman (1991a, Ch. 4), we assume that the limit pricing strategy is binding, used by all firms.

2.4. R&D Sector

R&D activities constitute the search for new designs that lead to a higher quality of the existing intermediate goods. In each intermediate goods industry, only entrants undertake R&D and the innovation/imitation process follows a Poisson process. Patent value depends on the profit-yields accrued by the monopolist at each *t*, and on the monopoly's duration. The monopoly's duration depends on the probability of successful R&D. The instantaneous probability of successful innovation (in *Z*=*I*) or imitation (in *Z*=*F*) at *t* in the next quality intermediate good *j*, which complements the *M*-type labor, $pb_{j,M,Z}(t)$, is:

$$pb_{j,M,Z}(t) = \left[\omega_{j,M,Z}(t)\right]\left[\beta_Z q^{k_{j,M,Z}(t)}\right]\left[\zeta_Z^{-1} q^{k_{j,M,Z}(t)(-1/(1-\gamma))} Q_{M,Z}(t)^{-\varepsilon}\right]\left[e^{PE_P+IT_P} f_P\left(\tilde{Q}_M(t)\right)^{\sigma_P\left(\tilde{Q}_M(t)\right)}\right]^{\Gamma_Z} \tag{14}$$

Where: (i) $\omega_{j,M,Z}(t)$ is R&D capital in *j*, which defines the framework as a lab-equipment model; (ii) $\beta_Z q^{k_{j,M,Z}(t)}$ represents learning-by-R&D in *j*, (e.g., Grossman and Helpman, 1991, Ch. 12). β_Z is the positive coefficient on past successful R&D experience and we assume that $0 < \beta_F < \beta_I$, i.e. there are greater learning effects in country *F*. Moreover, for *j*, $q^{k_{j,M,Z}(t)}$ is the highest quality level attained by innovation or imitation. Producers in *F* are only required to imitate technologies on one quality rung above the current level, since they only sell the imitated intermediate goods domestically, hence $k_I \geq k_F$; (iii) $\zeta_Z^{-1} q^{k_{j,M,Z}(t)(-1/(1-\gamma))} Q_{M,Z}(t)^{-\varepsilon}$ is the adverse effect caused by the increasing complexity of quality improvements (e.g. Kortum, 1997). For a given *M*-type *j*, the complexity cost increases not only with the quality rung, $q^{k_{j,M,Z}(t)}$, but also, given complementarities between intermediate goods, with the average quality of all M-type intermediate goods, $Q_{M,Z}(t)$. The positive learning effect $\beta_Z q^{k_{j,M,Z}(t)}$ is modeled in such a way that, together with the complexity cost $\zeta_Z^{-1} q^{k_{j,M,Z}(t)(-1/(1-\gamma))} Q_{M,Z}(t)^{-\varepsilon}$, totally offset the positive influence of the quality rung on the profits of each j firm, as can be seen below. This is the technical reason for the presence of the parameters γ and ε in term $\zeta_Z^{-1} q^{k_{j,M,Z}(t)(-1/(1-\gamma))} Q_{M,Z}(t)^{-\varepsilon}$. This term further includes a country-firm specific fixed research cost, ζ_Z, which, in line with several authors (e.g. Mansfield et. al., 1981), is higher for innovation than for imitation, $\zeta_I > \zeta_F > 0$; (iv) $e^{PE_F + IT_F} f_F\left(\tilde{Q}_M(t)\right)^{\sigma_F\left(\tilde{Q}_M(t)\right)}$ is the catching-up term, specific to the imitator country – hence $\Gamma_I = 0, \Gamma_F = 1$ – that sums up two important determinants of the imitation probability: the imitation capacity (i.e., the capacity to learn, assimilate and implement advanced technologies), and the backwardness effect (according to which the successful imitation rate is an increasing function of the gap between F and I). PE_F and IT_F are two positive exogenous variables that capture important determinants of imitation capacity, namely domestic policies promoting R&D (e.g., Aghion et al., 2001) and openness to international trade (e.g., Coe et al., 1997). The benefits of relative backwardness, in turn, are captured by function $f_F\left(\tilde{Q}_M(t)\right)$, which, in line with Papageorgiu (2002), is equal to:

$$f_F\left(\tilde{Q}_M(t)\right) = \begin{cases} 0 & , \ if\ 0 < \tilde{Q}_M(t) \leq d \\ -\tilde{Q}_M(t)^2 + (1+d)\tilde{Q}_M(t) - d, & if\ d < \tilde{Q}_M(t) < 1 \end{cases}$$

where $\tilde{Q}_M(t) \equiv Q_{M,F}(t) / Q_{M,I}(t) < 1$ is the relative technological-knowledge of the imitator's M-specific intermediate goods, measuring the technological-knowledge gap between countries. If the gap is not too large, i.e. $\tilde{Q}_M(t)$ is above threshold d, country F can benefit from a backwardness advantage, as in Barro and Sala-i-Martin (1997). When $\tilde{Q}_M(t)$ is below d, backwardness is no longer an advantage. Technological-knowledge is diffused only up to a certain point: Country F can grow rapidly only if an adequate minimum of development base is initially present. Function $f_F\left(\tilde{Q}_M(t)\right)$ is quadratic over the range of main interest and, once affected by the exponent, $\sigma_F\left(\tilde{Q}_M(t)\right)=-\bar{\sigma}+\tilde{Q}_M(t)$, yields an increasing backwardness advantage. Above threshold d, the higher the gap, the higher the imitation probability, consequently the faster the technological-knowledge progress and growth.

The experience-adjusted probability of successful R&D being:

$$\Phi_{M,Z}(t)\equiv\beta_Z\zeta_Z^{-1}\left[e^{PE_F+IT_F}f_F\left(\tilde{Q}_M(t)\right)^{\sigma_F\left(\tilde{Q}_M(t)\right)}\right]^{\Gamma_Z} \tag{15}$$

equation (14) can be rewritten as

$$pb_{j,M,Z}(t)=\omega_{j,M,Z}(t).\Phi_{M,Z}(t).q^{k_{j,M,Z}(t)(-\gamma/(1-\gamma))}Q_{M,Z}(t)^{-\varepsilon} \tag{16}$$

3. Equilibrium

Let us derive the equilibrium. Firstly, for a given technological-knowledge bias, i.e., for given aggregate quality indexes $Q_{L,Z}(t)$ and $Q_{H,Z}(t)$, we obtain equilibrium values for the threshold final good, final-good prices, aggregate output and physical capital, and wages. Secondly, we derive the equilibrium values for successful R&D probability, aggregate R&D capital, and the technological-knowledge path. The steady state in both countries in then characterized.

3.1. Equilibrium for a given Technological-Knowledge Bias

Threshold Final Good

As mentioned above, there is and endogenous threshold final good, $\bar{n}_Z(t)$, such that the production of final goods $n_Z\in\left[0,\bar{n}_Z(t)\right]$ uses only the L-technology, whereas the production of final goods $n_Z\in\left[\bar{n}_Z(t),1\right]$ uses only the H-technology. Then production function (8) is:

$$Y_{n,Z}(t)=\begin{cases}(1-n_Z)^{1-\alpha}A_Z^{\frac{1}{1-\alpha}}\left(\dfrac{\alpha P_{n,Z}(t)}{R_{L,Z}(t)}\right)^{\frac{\alpha}{1-\alpha}}lL_{n,Z}Q_{L,Z}(t)^{1+\varepsilon}, & if\ 0\le n_Z<\bar{n}_Z\\[2ex] n_Z^{1-\alpha}A_Z^{\frac{1}{1-\alpha}}\left(\dfrac{\alpha P_{n,Z}(t)}{R_{H,Z}(t)}\right)^{\frac{\alpha}{1-\alpha}}hH_{n,Z}Q_{H,Z}(t)^{1+\varepsilon}, & if\ \bar{n}_Z\le n_Z\le 1\end{cases} \tag{1}$$

Taking into account that, in each period, we have

$$L_{n,Z} = \frac{L_z}{\bar{n}_Z} \text{ and } H_{n,Z} = \frac{H_z}{1-\bar{n}_Z} \tag{2}$$

$$P_{L,Z}(t)^{\frac{1}{1-\alpha}} = \frac{P_{n,L,Z}(t)^{\frac{1}{1-\alpha}}}{\bar{n}_Z} \text{ and } P_{H,Z}(t)^{\frac{1}{1-\alpha}} = \frac{P_{n,H,Z}(t)^{\frac{1}{1-\alpha}}}{(1-\bar{n}_Z)} \tag{3}$$

after replacing $R_{j,M,Z}(t)$ by (13), equation (17) becomes:

$$Y_{n,Z}(t) = \begin{cases} (1-n_Z)^{1-\alpha} A_Z^{\frac{1}{1-\alpha}} \left(\frac{\alpha P_{L,Z}(t)}{qar_Z(t)} \right)^{\frac{\alpha}{1-\alpha}} lL_Z Q_{L,Z}(t)^{1+\varepsilon}, & if\ 0 \le n_Z < \bar{n}_Z \\ n_Z^{1-\alpha} A_Z^{\frac{1}{1-\alpha}} \left(\frac{\alpha P_{H,Z}(t)}{qar_Z(t)} \right)^{\frac{\alpha}{1-\alpha}} hH_Z Q_{H,Z}(t)^{1+\varepsilon}, & if\ \bar{n}_Z \le n_Z \le 1 \end{cases} \tag{4}$$

Both expressions in (19) must hold for $n_Z = \bar{n}_Z$, for which a firm using the *L*-technology and a firm using the *H*-technology breakeven. It follows that:

$$\frac{P_{H,Z}(t)}{P_{L,Z}(t)} = \left(\frac{\bar{n}_Z(t)}{1-\bar{n}_Z(t)} \right)^{1-\alpha} \tag{5}$$

Since $P_{n,Z}(t).Y_{n,Z}(t)$ is constant for all n_Z, equations (19), (20), and (21) together give:

$$\bar{n}_Z(t) = \left\{ 1 + \left[\frac{hH_z}{lL_z} \hat{Q}_z(t)^{1+\varepsilon} \right]^{\frac{1}{2}} \right\}^{-1} \tag{6}$$

where $\hat{Q}_z(t) \equiv Q_{H,Z}(t) / Q_{L,Z}(t)$ is the technological-knowledge bias in *Z*. In line with previous considerations, we assume that $\hat{Q}_I(t) > \hat{Q}_F(t)$.

Final-Good Prices

The normalized price of Y implies that $P_Z = 1 = e^{\int_0^1 \ln P_{n,Z}(t)dn}$, yielding, after some algebra:

$$\bar{n}_Z(t)\ln\left(P_{L,Z}(t)\right) + \left(1-\bar{n}_Z(t)\right)\ln\left(P_{L,Z}(t)\right) + \left(1-\alpha\right)\left[1 + \left(1-\bar{n}_Z(t)\right)\ln\left(\frac{1-\bar{n}_Z(t)}{\bar{n}_Z(t)}\right) + \ln\left(\bar{n}_Z(t)\right)\right] = 0$$

Using equations (21) and (22), the equilibrium final goods prices are:

$$P_{L,Z}(t) = e^{-(1-\alpha)} \left\{ 1 + \left[\frac{hH_z}{lL_z} \hat{Q}_z(t)^{1+\varepsilon} \right]^{\frac{1}{2}} \right\}^{1-\alpha} \tag{7}$$

$$P_{H,Z}(t) = e^{-(1-\alpha)} \left\{ 1 + \left[\frac{hH_z}{lL_z} \hat{Q}_z(t)^{1+\varepsilon} \right]^{-\frac{1}{2}} \right\}^{1-\alpha} \tag{8}$$

$$\frac{P_{H,Z}(t)}{P_{L,Z}(t)} \equiv \hat{P}_Z(t) = \left[\frac{lL_z}{hH_z} \hat{Q}_z(t)^{-(1+\varepsilon)} \right]^{\frac{1-\alpha}{2}} \tag{9}$$

Macroeconomic Aggregates

Using equations (19), (20), (23) and (24) in (5), we rewrite Y as a function of technological-knowledge levels:

$$Y_Z(t) = e^{-1} A_Z^{\frac{1}{1-\alpha}} \left(\frac{\alpha}{qar_Z(t)} \right)^{\frac{\alpha}{1-\alpha}} \left[\left(lL_Z Q_{L,Z}(t)^{1+\varepsilon} \right)^{\frac{1}{2}} + \left(hH_Z Q_{H,Z}(t)^{1+\varepsilon} \right)^{\frac{1}{2}} \right]^2 \tag{10}$$

Next, we rewrite equation (4) with respect to $\left(q^{k_{j,L,Z}(t)} x_{n,j,L,Z}(t)\right)^{\gamma}$ and $\left(q^{k_{j,H,Z}(t)} x_{n,j,H,Z}(t)\right)^{\gamma}$:

$$\int_0^{J_{L,Z}} \left(q^{k_{j,L,Z}(t)} x_{n,j,L,Z}(t) \right)^{\gamma} dj = Y_{L,Z}^{\frac{1}{\phi}}(t) A_Z^{-\frac{1}{\phi}} \left\{ (1-n_Z) lL_{n,Z} \right\}^{-\frac{1-\alpha}{\phi}}$$

$$\int_0^{J_{H,Z}} \left(q^{k_{j,H,Z}(t)} x_{n,j,H,Z}(t) \right)^{\gamma} dj = Y_{H,Z}^{\frac{1}{\phi}}(t) A_Z^{-\frac{1}{\phi}} \left\{ n_Z hH_{n,Z} \right\}^{-\frac{1-\alpha}{\phi}}$$

Using the above two equations in (6) and (7), and replacing $Y_{L,Z}^{\frac{1}{\phi}}$ and $Y_{H,Z}^{\frac{1}{\phi}}$ with equation (20), we rewrite X as a function of the technological-knowledge level:

$$x_{n,j,L,Z}(t) = \left(\frac{\alpha A_Z P_{L,Z}(t) \left(1-n_Z\right)^{-(1-\alpha)}}{qar_Z(t)} \right)^{\frac{1}{1-\alpha}} \left(\frac{lL_Z}{n_Z} \right) Q_{L,Z}(t)^{1+\varepsilon} q^{k_{j,L,Z}(t)\frac{\gamma}{1-\gamma}}$$

$$x_{n,j,H,Z}(t) = \left(\frac{\alpha A_Z P_{H,Z}(t) n_Z^{-(1-\alpha)}}{qar_Z(t)} \right)^{\frac{1}{1-\alpha}} \left(\frac{hH_Z}{1-n_Z} \right) Q_{H,Z}(t)^{1+\varepsilon} q^{k_{j,H,Z}(t)\frac{\gamma}{1-\gamma}}$$

Integrating these two equations in *n,* each intermediate firm's output is:

$$X_{j,L,Z}(t) = \int_0^{\bar{n}_Z} x_{n,j,L,Z}(t) dn = \left(\frac{\alpha A_Z P_{L,Z}(t)}{qar_Z(t)} \right)^{\frac{1}{1-\alpha}} \left(lL_Z \right) Q_{L,Z}(t)^{1+\varepsilon} q^{k_{j,L,Z}(t)\frac{\gamma}{1-\gamma}} \tag{11}$$

$$X_{j,H,Z}(t) = \int_{\bar{n}_Z}^{1} x_{n,j,H,Z}(t) dn = \left(\frac{\alpha A_Z P_{H,Z}(t)}{qar_Z(t)} \right)^{\frac{1}{1-\alpha}} \left(hH_Z \right) Q_{H,Z}(t)^{1+\varepsilon} q^{k_{j,H,Z}(t)\frac{\gamma}{1-\gamma}} \tag{12}$$

Integrating these two equations in j, we obtain X as a function of Q_L and Q_H :

$$
\begin{aligned}
X_Z(t) &= \int_0^{J_{L,Z}} X_{j,L,Z}(t)dj + \int_0^{J_{H,Z}} X_{j,H,Z}(t)dj \\
&= e^{-1} A^{\frac{1}{1-\alpha}} \left(\frac{\alpha}{qar_Z(t)} \right)^{\frac{1}{1-\alpha}} \left[\left(lL_Z Q_{L,Z}(t)^{1+\varepsilon} \right)^{\frac{1}{2}} + \left(hH_Z Q_{H,Z}(t)^{1+\varepsilon} \right)^{\frac{1}{2}} \right]^2
\end{aligned} \tag{13}
$$

Wages

Deriving production function (26) with respect to L and H, wages are:

$$
W_{L,Z}(t) = e^{-1} A^{\frac{1}{1-\alpha}} \left(\frac{\alpha}{qar_Z(t)} \right)^{\frac{\alpha}{1-\alpha}} \left[\left(lL_Z Q_{L,Z}(t)^{1+\varepsilon} \right)^{\frac{1}{2}} + \left(hH_Z Q_{H,Z}(t)^{1+\varepsilon} \right)^{\frac{1}{2}} \right] \left[lL_Z Q_{L,Z}(t)^{1+\varepsilon} \right]^{-\frac{1}{2}} lQ_{L,Z}(t)^{1+\varepsilon} \tag{14}
$$

$$
W_{H,Z}(t) = e^{-1} A^{\frac{1}{1-\alpha}} \left(\frac{\alpha}{qar_Z(t)} \right)^{\frac{\alpha}{1-\alpha}} \left[\left(lL_Z Q_{L,Z}(t)^{1+\varepsilon} \right)^{\frac{1}{2}} + \left(hH_Z Q_{H,Z}(t)^{1+\varepsilon} \right)^{\frac{1}{2}} \right] \left[hH_Z Q_{H,Z}(t)^{1+\varepsilon} \right]^{-\frac{1}{2}} hQ_{H,Z}(t)^{1+\varepsilon} \tag{15}
$$

$$
\frac{W_{H,Z}(t)}{W_{L,Z}(t)} \equiv \hat{W}_Z(t) = \left[\frac{hL_z}{lH_z} \hat{Q}_z(t)^{1+\varepsilon} \right]^{\frac{1}{2}} \tag{16}
$$

Equation (32) says that the skill premium, $\hat{W}_Z(t)$ is greater when: (i) h is greater; (ii) L_Z / H_Z is lower; and (iii) $\hat{Q}_z$ is higher.

3.2. Equilibrium R&D

Monopoly Profit Explicit Flow and Duration

Each intermediate firm's expected profits current value, $V_{k,j,M,Z}(t)$, depends on profits, $\Pi_{k,j,M,Z}(t)$, and on their expected duration, which depends on successful R&D's probability:

$$
V_{k,j,M,Z}(t) = \frac{\Pi_{k,j,M,Z}(t)}{r_Z(t) + pb_{k,j,M,Z}(t)} \tag{17}
$$

Working equations (13), (27), and (28) into (11), monopolists' profits at each t are:

$$
\Pi_{k,j,L,Z}(t) = (q-1)ar_Z(t) \left(\frac{\alpha A_Z P_{L,Z}(t)}{qar_Z(t)} \right)^{\frac{1}{1-\alpha}} lL_Z Q_{L,Z}(t)^{\varepsilon} q^{k_{j,L,Z}(t)\frac{\gamma}{1-\gamma}} \tag{18}
$$

$$
\Pi_{k,j,H,Z}(t) = (q-1)ar_Z(t) \left(\frac{\alpha A_Z P_{H,Z}(t)}{qar_Z(t)} \right)^{\frac{1}{1-\alpha}} hH_Z Q_{H,Z}(t)^{\varepsilon} q^{k_{j,H,Z}(t)\frac{\gamma}{1-\gamma}} \tag{19}
$$

Equilibrium Probability of Successful R&D and R&D Capital

Under free-entry R&D equilibrium, expected returns match spent resources:

$$pb_{k,j,M,Z}(t)\,V_{k+1,j,M,Z}(t)=\omega_{k,j,M,Z}(t) \tag{20}$$

Working equations (16), (33), (34) and (35) into (36), the equilibrium probabilities of innovation (Z=I) and imitation (Z=F) are:

$$pb_{L,Z}(t)=\left(q-1\right)q^{\varepsilon+\frac{\gamma}{1-\gamma}-\frac{1}{1-\alpha}}lL_Z\Phi_{L,Z}(t)\left(ar_Z(t)\right)^{\frac{-\alpha}{1-\alpha}}\left(\alpha A_Z P_{L,Z}(t)\right)^{\frac{1}{1-\alpha}}-r_Z(t) \tag{21}$$

$$pb_{H,Z}(t)=\left(q-1\right)q^{\varepsilon+\frac{\gamma}{1-\gamma}-\frac{1}{1-\alpha}}hH_Z\Phi_{H,Z}(t)\left(ar_Z(t)\right)^{\frac{-\alpha}{1-\alpha}}\left(\alpha A_Z P_{H,Z}(t)\right)^{\frac{1}{1-\alpha}}-r_Z(t) \tag{22}$$

Plugging these equations in (14), each firm's equilibrium R&D capital is:

$$\omega_{j,L,Z}(t)=\left(q-1\right)q^{\varepsilon+\frac{\gamma}{1-\gamma}-\frac{1}{1-\alpha}}lL_Z Q_{L,Z}^{\varepsilon}(t)q^{k_{j,L,Z}(t)}\left(ar_Z(t)\right)^{\frac{-\alpha}{1-\alpha}}\left(\alpha A_Z P_{L,Z}(t)\right)^{\frac{1}{1-\alpha}}-r_Z(t)/\Phi_{L,Z}(t) \tag{23}$$

$$\omega_{j,H,Z}(t)=\left(q-1\right)q^{\varepsilon+\frac{\gamma}{1-\gamma}-\frac{1}{1-\alpha}}hH_Z Q_{H,Z}^{\varepsilon}(t)q^{k_{j,H,Z}(t)}\left(ar_Z(t)\right)^{\frac{-\alpha}{1-\alpha}}\left(\alpha A_Z P_{H,Z}(t)\right)^{\frac{1}{1-\alpha}}-r_Z(t)/\Phi_{H,Z}(t) \tag{24}$$

Total R&D capital in each country is:

$$\begin{aligned}\Omega_Z(t)&=\int_0^{J_{L,Z}}\omega_{j,L,Z}(t)dj+\int_0^{J_{H,Z}}\omega_{j,H,Z}(t)dj\\&=L_Z Q_{L,Z}^{1+\varepsilon}(t)\Phi_{L,Z}^{-1}(t)pb_{L,Z}(t)+H_Z Q_{H,Z}^{1+\varepsilon}(t)\Phi_{H,Z}^{-1}(t)pb_{H,Z}(t)\end{aligned} \tag{25}$$

Aggregate Quality Indexes Behavior

Following Barro and Sala-i-Martin (2004, Ch. 7), we derive the laws of motion of $Q_{L,Z}(t)$ and $Q_{H,Z}(t)$ These variables definition says that in case of a quality improvement in *j*, the proportional change in the quality grade is ($q^{\frac{\gamma}{1-\gamma}}-1$). Equations (37) and (38) imply that the equilibrium R&D probability per period is equal across intermediate goods. The expected proportional change in $Q_{M,Z}(t)$ is then $pb_{M,Z}(q^{\frac{\gamma}{1-\gamma}}-1)$. The number of intermediate goods being large enough, according to the Law of Large Numbers, the average growth rate of $Q_{M,Z}$ is close to $pb_{M,Z}(q^{\frac{\gamma}{1-\gamma}}-1)$. The laws of motion of $Q_{L,Z}(t)$ and $Q_{H,Z}(t)$ are:

$$\frac{\dot{Q}_{L,Z}(t)}{Q_{L,Z}(t)}=pb_{L,Z}(t)\left(q^{\frac{\gamma}{1-\gamma}}-1\right) \tag{26}$$

$$\frac{\dot{Q}_{H,Z}(t)}{Q_{H,Z}(t)} = pb_{H,Z}(t)\left(q^{\frac{\gamma}{1-\gamma}} - 1\right) \tag{27}$$

3.3. Steady-State Equilibrium

Let us now derive the steady-state growth rate, interest rate, technological-knowledge bias, technological-knowledge gap, probability of successful R&D, final-good prices, and the skill premium for the innovator and the imitator countries separately.[1]

Steady-State Common Features

In equilibrium, all macroeconomic aggregates grow at the same constant rate, g_Z:

$$\frac{\dot{Y}_Z}{Y_Z} = \frac{\dot{X}_Z}{X_Z} = \frac{\dot{\Omega}_Z}{\Omega_Z} = \frac{\dot{C}_Z}{C_Z} = g_Z \tag{28}$$

Equations (3) and (44) tell us that the steady-state growth rate is equal to:

$$\frac{r_Z - \rho}{\theta} = g_Z \tag{29}$$

In a balanced growth path, aggregate quality indexes grow at the same constant rate:

$$\frac{\dot{Q}_{L,Z}}{Q_{L,Z}} = \frac{\dot{Q}_{H,Z}}{Q_{H,Z}} \tag{30}$$

which implies: (i) a constant equilibrium technological-knowledge bias, $\hat{Q}_z$; (ii) a constant probability of successful R&D for both technologies – given (42) and (43):

$$pb_{L,Z} = pb_{H,Z} \equiv pb_Z \tag{31}$$

Considering (37) and (38), a constant steady-state probability of successful R&D implies constant equilibrium values for the interest rate, the final-good prices, and $\Phi_{M,Z}$. From equations (26), (29), (41) and (46), it follows that *Y*, *X* and *RS* grow at the rate:

$$(1+\varepsilon)\frac{\dot{Q}_{L,Z}}{Q_{L,Z}} = (1+\varepsilon)\frac{\dot{Q}_{H,Z}}{Q_{H,Z}} = g_Z \tag{32}$$

Equations (45) and (48) give us each country's economic growth rate.

[1] Since all these variables are constant in steady-state, the time index *t* will be suppressed.

Steady-State in the Innovator Country

Equalizing (37) and (38) and using (47), the equilibrium technological-knowledge bias in I is:

$$\hat{Q}_I^{1+\varepsilon} = \frac{hH_I}{lL_I} \tag{33}$$

Plugging (49) into (23), (24), (32), (37) and (38), we obtain country I' equilibrium values for the final-good prices, the skill premium, and the probability of successful R&D:

$$P_{L,I} = e^{-(1-\alpha)}\left(1+\frac{hH_I}{lL_I}\right)^{1-\alpha} \tag{34}$$

$$P_{H,I} = e^{-(1-\alpha)}\left(1+\frac{lL_I}{hH_I}\right)^{1-\alpha} \tag{35}$$

$$\hat{W}_I = \frac{h}{l} \tag{36}$$

$$pb_I = e^{-1}\left(q-1\right)q^{\varepsilon+\frac{\gamma}{1-\gamma}-\frac{1}{1-\alpha}}\beta_I\zeta_I^{-1}\left(\alpha A_I\right)^{\frac{1}{1-\alpha}}\left(hH_I+lL_I\right)\left(ar_I\right)^{\frac{-\alpha}{1-\alpha}}-r_I \tag{37}$$

Equation (52) shows an important result of our model: The equilibrium skill premium in country I does not depend on labor supply. It depends solely and positively on the productive advantage of skilled over unskilled labor. This result has two important implications. Firstly, it means that, considering (32), the immediate effect on wages resulting from changes in the labor supply is exactly offset by changes in the demand resulting from technological-knowledge progress. Secondly it suggests that the sustained increase in the skill premium in several developed countries may be due to increases in the relative productive advantage of skilled labor, rather than to increases in its relative supply.[2]

Recalling that $g_I = (1+\varepsilon)(q^{\frac{\gamma}{1-\gamma}}-1)pb_I$, replacing pb_I by (53) and considering (45), we derive country I's equilibrium growth rate and interest rate, by solving the system:[3]

[2] This result does not imply that the skill premium does not react to changes in the labor supply. Equation (32) shows that it does, during the transitional dynamics. An increase in skilled labor, for example, generates an immediate fall in the skill premium – the supply effect – and a subsequent increase during the transition dynamics – the demand effect –, before converging to the initial steady-state level.

[3] See proof of the existence of steady-state in the appendix.

$$\begin{cases} g_I = \dfrac{r_I - \rho}{\theta} \\ g_I = (1+\varepsilon)(q^{\frac{\gamma}{1-\gamma}} - 1)\left[e^{-1}\left(q-1\right) q^{\varepsilon + \frac{\gamma}{1-\gamma} - \frac{1}{1-\alpha}} \beta_I \zeta_I^{-1} \left(\alpha A_I\right)^{\frac{1}{1-\alpha}} \left(h_I H_I + l_I L_I\right)\left((1+\varphi g) r_I\right)^{\frac{-\alpha}{1-\alpha}} - r_I \right] \end{cases}$$

This system yields the following implicit expression for the steady-state growth rate:

$$g_I = (1+\varepsilon)(q^{\frac{\gamma}{1-\gamma}} - 1)\left[\Psi_I \left(hH_I + lL_I\right)\left(g_I\theta + g_I^2\theta\varphi + \rho + \rho\varphi g_I\right)^{\frac{-\alpha}{1-\alpha}} - \theta g_I - \rho \right] \quad (38)$$

where $\Psi_I = e^{-1}\left(q-1\right) q^{\varepsilon + \frac{\gamma}{1-\gamma} - \frac{1}{1-\alpha}} \beta_I \zeta_I^{-1} \left(\alpha A_I\right)^{\frac{1}{1-\alpha}}$ is a positive constant.

Steady-State in the Imitator Country

The equilibrium in the imitator country is determined considering four conditions:

(i) Both technologies have equal imitation probability– see (47). Hence the equilibrium technological-knowledge bias; price levels; and skill premium are:

$$\hat{Q}_F^{1+\varepsilon} = \left(\frac{hH_F}{lL_F}\right)\left(\frac{f(\tilde{Q}_H)^{2\sigma(\tilde{Q}_H)}}{f(\tilde{Q}_L)^{2\sigma(\tilde{Q}_L)}}\right) \quad (39)$$

$$P_{L,F} = e^{-(1-\alpha)}\left[1 + \left(\frac{hH_F}{lL_F}\right)\left(\frac{f_F(\tilde{Q}_H)^{2\sigma_F(\tilde{Q}_H)}}{f_F(\tilde{Q}_L)^{2\sigma_F(\tilde{Q}_L)}}\right)\right]^{1-\alpha} \quad (40)$$

$$P_{H,F} = e^{-(1-\alpha)}\left[1 + \left(\frac{lL_F}{hH_F}\right)\left(\frac{f_F(\tilde{Q}_L)^{2\sigma_F(\tilde{Q}_L)}}{f_F(\tilde{Q}_H)^{2\sigma_F(\tilde{Q}_H)}}\right)\right]^{1-\alpha} \quad (41)$$

$$\hat{W}_F = \frac{h}{l}\left(\frac{f(\tilde{Q}_H)^{\sigma(\tilde{Q}_H)}}{f(\tilde{Q}_L)^{\sigma(\tilde{Q}_L)}}\right) \quad (42)$$

Equations (55) to (58) show that the technological-knowledge bias, final-good prices and wage inequality in country *F* depend on the same variables as in country *I.* Due to technological-knowledge diffusion, they also depend on $\tilde{Q}_H$ and $\tilde{Q}_L$. Thus, calculation of the values of $\hat{Q}_F^{1+\varepsilon}$ in (55) involves determining the values of $\tilde{Q}_H$ and $\tilde{Q}_L$, constant in steady-state.

(ii) Due to technological-knowledge diffusion, the equilibrium growth and interest rates are the same in both countries, and determined by country I. Since condition $g_I = (1+\varepsilon)(q^{\frac{\gamma}{1-\gamma}} - 1) p b_F$ must hold, given equations (37) and (38), we have:

$$g_I = (1+\varepsilon)(q^{\frac{\gamma}{1-\gamma}} - 1)\left[\Psi_F l L_F P_{L,F}^{\frac{1}{1-\alpha}} f_F\left(\tilde{Q}_L(t)\right)^{\sigma_F\left(\tilde{Q}_L(t)\right)}\left(g_I\theta + g_I^2\theta\varphi + \rho + \rho\varphi g_I\right)^{\frac{-\alpha}{1-\alpha}} - \theta g_I - \rho\right] \tag{43}$$

$$g_I = (1+\varepsilon)(q^{\frac{\gamma}{1-\gamma}} - 1)\left[\Psi_F h H_F P_{H,F}^{\frac{1}{1-\alpha}} f_F\left(\tilde{Q}_H(t)\right)^{\sigma_F\left(\tilde{Q}_H(t)\right)}\left(g_I\theta + g_I^2\theta\varphi + \rho + \rho\varphi g_I\right)^{\frac{-\alpha}{1-\alpha}} - \theta g_I - \rho\right] \tag{44}$$

where $\Psi_F = (q-1)q^{\varepsilon + \frac{\gamma}{1-\gamma} - \frac{1}{1-\alpha}} \beta_F \zeta_F^{-1} (\alpha A_F)^{\frac{1}{1-\alpha}} e^{PE_F + IT_F}$ is a positive constant and $g_I = g_F \equiv g$ is determined by equation (54).

(iii) Definitions of technological-knowledge bias and technological-knowledge gap, imply:

$$\hat{Q}_I \tilde{Q}_H = \hat{Q}_F \tilde{Q}_L \tag{45}$$

(iv) As the technological-knowledge bias is higher in country *I*, equation (61) implies:

$$\hat{Q}_I > \hat{Q}_F \Leftrightarrow \tilde{Q}_L > \tilde{Q}_H \tag{46}$$

4. Steady-State Effects

We now examine the response of the relevant variables[4] in both countries to changes in the: (i) skilled labor supply in *I*; (ii) investment cost parameter, φ; (iii) degree of complementarities between intermediate goods, ϕ; (iv) nature of the two countries interaction. With Case (i) we compare the proposed model with other skill-biased technological-change models regarding the effects on the skill premium of an increase in the skilled labor supply. With Cases (ii), (iii) and (iv), we examine the effects on the skill premium and growth of the three introduced assumptions.

4.1. An Increase in the Skilled Labor Supply in Country I

Proposition 1.1: If, *ceteris paribus*, the skilled labor supply in country *I*, H_I, increases, the technological-knowledge bias in this country, $\hat{Q}_I$, will increase, and the skill premium, $\hat{W}_I$, remains unchanged. The growth rate in both countries, *g*, will increase.

Proof: See appendix.

[4] That is, technological-knowledge bias, technological-knowledge gap, growth rate and skill premium.

Proposition 1.2: If, *ceteris paribus*, the skilled labor supply in country *I*, H_I, increases, the technological-knowledge bias in *F*, $\hat{Q}_F$, will increase and the *H*-technological-knowledge gap, $\tilde{Q}_H$, will decrease. The *L*-technological-knowledge gap, $\tilde{Q}_L$, and the skill premium in *F*, $\hat{W}_F$, may either increase or decrease.

Proof: See appendix.

Due to the market-size channel, an increase in the skilled labor supply in *I* raises its steady-state probability of innovation, thus accelerating the technological-knowledge progress and growth. Moreover, it affects the technological-knowledge bias, both positively (by making R&D investment in the *H*-technology more attractive) and negatively (by raising the prices of the final goods using this type of technology). Since the former mechanism (market-size channel) is stronger than the latter (price channel), the net effect on the technological-knowledge bias will be positive. This positive effect, in turn, translates into an increased demand for skilled labor. According to our model, such an increase exactly offsets the initial reduction in the supply, hence the stead-state skill premium does not change.

An increase in the skilled labor supply in *I* also affects the steady-state in *F*, as, due to technological-knowledge diffusion, the growth rate of the latter is determined by that of the former. Besides, technological-knowledge progress in the new steady-state is also more biased towards the *H*-technology. This increase in the technological-knowledge is less pronounced in *F*, as can be seen by the reduction in $\tilde{Q}_H$. Due to the indefinite effect on $\tilde{Q}_L$, it is not possible to predict the final effect on the skill premium in *F*.

4.2. An Increase in the Internal Investment Cost in Country *I*

Proposition 2.1: If, *ceteris paribus*, the investment cost parameter, φ, increases in country *I*, both the technological-knowledge bias, $\hat{Q}_I$, and the skill premium, $\hat{W}_I$, in this country remain unchanged. The growth rate in both countries, *g*, will fall.[5]

Proof: Similar to the proof of Proposition 1.1.; the difference being that we must now calculate $dg / d\varphi$ and show it is lower than zero.

Proposition 2.2: If, *ceteris paribus*, the investment cost parameter, φ, increases in country *I*, the technological-knowledge bias in *F*, $\hat{Q}_F$, will fall and both technological-knowledge gaps, $\tilde{Q}_L$ and $\tilde{Q}_H$, will increase. Thus, the skill premium in *F*, $\hat{W}_F$, may either increase or decrease.[6]

Proof: See appendix.

An increase in φ in *I* raises the marginal cost of producing intermediate goods, thereby reducing intermediate firms profits, deeming successful innovation less likely. The common growth rate will hence decrease. Despite not affecting the technological-knowledge bias nor the skill premium in *I* – see (49) and (52) –, this change affects such variables in *F* through international diffusion. In particular, given (59) and (60) and due to the backwardness hypothesis, a fall in the growth rate and consequently in the imitation probability, implies a smaller technological-knowledge gap between the two countries, hence the rise in $\tilde{Q}_H$ and $\tilde{Q}_L$ The effect on the skill premium in *F* will be indeterminate. Besides, since $\tilde{Q}_L$ increases more than $\tilde{Q}_H$, and $\hat{Q}_I$ remains constant, the technological-knowledge bias in *F*, $\hat{Q}_F$, will decrease.

4.3. An Increase in the Degree of Complementarities in Country *I*

[5] If we considered instead an increase in the internal cost of investment in *F*, the effects would be a maintenance of not only $\hat{Q}_I$ and $\hat{W}_I$, but also *g*.

[6] If we considered instead an increase in the internal cost of investment in *F*, the effects on $\hat{Q}_F$, $\tilde{Q}_L$ and $\tilde{Q}_H$ would be exactly the opposite.

Proposition 3.1: If, *ceteris paribus*, the degree of complementarities between intermediate goods, ϕ, increases in *I*, both the technological-knowledge bias, $\hat{Q}_I$, and the skill premium, $\hat{W}_I$, remain unchanged. The growth rate in both countries, *g*, increases.[7]

Proof: Similar to the proof of Propositions 1.1. and 2.1.; only now we must now calculate $dg / d\varepsilon$ and show it is higher than zero (a higher value of ϕ implies a higher value of ε).

Proposition 3.2: If, *ceteris paribus*, the degree of complementarities between intermediate goods, ϕ, increases in *I*, the technological-knowledge bias in *F*, $\hat{Q}_F$, will increase and technological-knowledge gaps, $\tilde{Q}_L$ and $\tilde{Q}_H$, will decrease. Thus, the skill premium in *F*, $\hat{W}_F$, may either increase or decrease.[8]

Proof: Similar to the proof of Proposition 2.2., with all variables varying in the opposite way.

An increase in the degree of complementarities (an increase in ε) between intermediate goods in *I* affects growth positively, via two channels. Firstly, a higher degree of complementarities raises the total demand of intermediate goods, thereby raising global output – this effect is captured by (48). Secondly, as (37) and (38) show, it also raises the steady-state innovation probability as well as the aggregate quality indexes' growth rate, thereby fostering technological progress.

Given (59) and (60) and due to the backwardness hypothesis, a rise in the global growth rate and, thus, in the imitation probability, implies a wider technological-knowledge gap between *I* and *F*, hence the fall in $\tilde{Q}_H$ and $\tilde{Q}_L$. Since $\tilde{Q}_L$ decreases more than $\tilde{Q}_H$, and $\hat{Q}_I$ remains constant, the technological-knowledge bias in *F*, $\hat{Q}_F$, will increase.

[7] If we considered instead an increase in the degree of complementarities between intermediate goods in *P*, the effects would be a maintenance of not only $\hat{Q}_I$ and $\hat{W}_I$, but also *g*.

[8] If we considered instead an increase in the degree of complementarities between intermediate goods in *F*, the effects on $\hat{Q}_F$, $\tilde{Q}_L$ and $\tilde{Q}_H$ would be exactly the opposite.

4.4. Effects of Technological-Knowledge Diffusion

Proposition 4.1: The introduction of technological-knowledge diffusion does not alter the steady-state values of the technological-knowledge bias, $\hat{Q}_I$, the skill premium, $\hat{W}_I$, and growth rate, g_I, in country *I*.

Proof: Since the world growth rate is determined by *I* and there is not any feedback effect from *F*, the steady-state values of all variables in the former are the same with and without technological-knowledge diffusion.

Proposition 4.2: Technological-knowledge diffusion raises the steady-state values of the technological-knowledge bias, $\hat{Q}_F$, skill premium, $\hat{W}_F$, and growth rate, g_F, in country *F*.

Proof: Without technological-knowledge diffusion, firms in *F* cannot imitate the innovations of *I*, thus *F* behaves as an innovator. Therefore, all the relevant steady-state expressions for *F* are given by equations (49)-(54), with index *I* replaced by *F*. Given that: $\tilde{Q}_L > \tilde{Q}_H$; $A_I > A_F$; $h > l$; and $H_I / L_I > H_F / L_F$, technological-knowledge diffusion increases: (a) *F*'s growth rate – in (54), *g* is higher when *Z=I* than when *Z=F* –; (b) the technological-knowledge bias, which is evident by the comparison of (49), for *Z=F*, with (55); and (c) the skill premium, which is evident by comparison of (52), for *Z=F*, with (58).

With technological-knowledge diffusion, technological-knowledge progress in *F* will be more *H*-biased. This has two consequences: Firstly, it raises the skill premium. Secondly, it raises the country's growth rate. Thus, only *F* benefits with technological-knowledge diffusion, as the growth rate in *I* remains unaffected. Moreover, since it brings about steady state growth rates equalization between countries, diffusion causes convergence. But this convergence occurs only in growth rates, not in levels, because technological-knowledge in *F* will remain lower than that of *I*. Our model then predicts that international technological-knowledge diffusion brings about conditional convergence.

5. Concluding Remarks

We have developed a dynamic general-equilibrium model with growth driven by vertical R&D that includes three elements of contemporary economies: internal costly investment in both physical capital and R&D, complementarities between intermediate goods in production, and technological-knowledge diffusion between innovator and imitator countries.

Our first finding is that an increase in the skilled labor supply, an increase in the complementarities degree, and a decrease in the internal investment cost have a positive impact on the innovator country's long-term growth. However, they do not affect the equilibrium skill premium which depends solely on the productivity of each type of labor.

In fact, our model carries a proposed contribution to the technological-knowledge bias literature, predicting that changes in the relative supply of skilled workers do not affect the skill premium in the innovator country in steady-state, since the immediate effect resulting from a change in the supply is exactly offset by the subsequent effects resulting from changes in demand. This result suggests that the sustained increase in the skill premium observed in several developed countries may be due to increases in the relative productive advantage of skilled labor, rather than to increases in its relative supply.

Our findings for the imitator country are that, due to technological-knowledge diffusion, the three introduced economic features influence the steady-state growth rate, skill premium, technological-knowledge bias, and the technological-knowledge gaps. Moreover, technological-knowledge diffusion is beneficial for the imitator country, as it brings about world growth rates equalization. As the imitation cost is lower than the innovation cost, the imitator country's technological-knowledge level tends, however, to remain lower than the innovator's. Diffusion enables conditional convergence between countries.

APPENDIX

Proof of the existence of steady-state equilibrium

Consider the system that determines the equilibrium values of r and g and investigate, for each equation, the signal of dg/dr:

$$\begin{cases} g = \dfrac{r-\rho}{\theta} \\ g = (1+\varepsilon)(q^{\frac{\gamma}{1-\gamma}}-1)\left[e^{-1}\left(q-1\right)q^{\varepsilon+\frac{\gamma}{1-\gamma}-\frac{1}{1-\alpha}}\beta_I\zeta_I^{-1}\left(\alpha A_I\right)^{\frac{1}{1-\alpha}}\left(hH_I+lL_I\right)\left((1+\varphi g)r\right)^{\frac{-\alpha}{1-\alpha}}-r \right] \end{cases}$$

For the first equation, $\dfrac{dg}{dr}=\dfrac{1}{\theta}>0, \forall g,r$. Regarding the second equation, let's define $G1$ as:

$$G1(.) = g-(1+\varepsilon)(q^{\frac{\gamma}{1-\gamma}}-1)\left[e^{-1}\left(q-1\right)q^{\varepsilon+\frac{\gamma}{1-\gamma}-\frac{1}{1-\alpha}}\beta_I\zeta_I^{-1}\left(\alpha A_I\right)^{\frac{1}{1-\alpha}}\left(hH_I+lL_I\right)\left((1+\varphi g)r\right)^{\frac{-\alpha}{1-\alpha}}-r \right]$$

Using the Implicit Function Theorem:

$$\frac{dg}{dr}=-\frac{\frac{\partial G1(.)}{\partial r}}{\frac{\partial G1(.)}{\partial g}}=-\frac{\left(1+\varepsilon\right)\left(q^{\frac{\gamma}{1-\gamma}}-1\right)+\frac{\alpha}{1-\alpha}\left(1+\varepsilon\right)\left(q^{\frac{\gamma}{1-\gamma}}-1\right)\Psi_I\left(hH_I+lL_I\right)\left(1+\varphi g\right)^{\frac{-\alpha}{1-\alpha}}r^{\frac{-1}{1-\alpha}}}{1+\frac{\alpha}{1-\alpha}\left(1+\varepsilon\right)\left(q^{\frac{\gamma}{1-\gamma}}-1\right)\Psi_I\left(hH_I+lL_I\right)\varphi\left(1+\varphi g\right)^{\frac{-1}{1-\alpha}}r^{\frac{-\alpha}{1-\alpha}}}<0, \forall g,r>0$$

The second equation is negatively sloped in the first quadrant of the space (g,r), while the first is positively sloped, hence there is a unique combination of positive values of g and r such that both equations are simultaneously satisfied.

Proof of proposition 1.1.

Regarding an increase in $\hat{Q}_I$ and the maintenance of $\hat{W}_I$, proof is immediate by inspection of (49) and (52). Concerning the increase in g, proof relies on the determination of the signal of dg/dH. From (54), let's define $G2$ as:

$$G2(.) = g-(1+\varepsilon)(q^{\frac{\gamma}{1-\gamma}}-1)\left[\Psi_I\left(h_IH_I+l_IL_I\right)\left(g\theta+g^2\theta\varphi+\rho+\rho\varphi g\right)^{\frac{-\alpha}{1-\alpha}}-\theta g-\rho\right]$$

Using the Implicit Function Theorem:

$$\frac{dg}{dH}=-\frac{\frac{\partial G2(.)}{\partial H}}{\frac{\partial G2(.)}{\partial g}}=-\frac{-\left(1+\varepsilon\right)\left(q^{\frac{\gamma}{1-\gamma}}-1\right)\left(g\theta+g^2\theta\varphi+\rho+\rho\varphi g\right)^{\frac{-\alpha}{1-\alpha}}\Psi_I h_I}{1-\left[-\theta\left(1+\varepsilon\right)\left(q^{\frac{\gamma}{1-\gamma}}-1\right)+\left(1+\varepsilon\right)\left(q^{\frac{\gamma}{1-\gamma}}-1\right)\Psi_I\left(hH_I+lL_I\right)\left(\frac{-\alpha}{1-\alpha}\right)\left(g\theta+g^2\theta\varphi+\rho+\rho\varphi g\right)^{\frac{-\alpha}{1-\alpha}-1}\left(\theta+2g\theta\varphi+\rho\varphi\right)\right]}>0$$

Proof of proposition 1.2.

Consider (59) and (60). To examine the effects on $\hat{Q}_F$, $\tilde{Q}_L$ and $\tilde{Q}_H$, we must check how $P_{L,F}{}^{\frac{1}{1-\alpha}} f_F\left(\tilde{Q}_L(t)\right)^{\sigma_F\left(\tilde{Q}_L(t)\right)}$ and $P_{H,F}{}^{\frac{1}{1-\alpha}} f_F\left(\tilde{Q}_H(t)\right)^{\sigma_F\left(\tilde{Q}_H(t)\right)}$ – the only endogenous components in (59) and (60), apart from g – change in the face of an increase in g. Starting with (59), define $G3$ as:

$$G3(.) = g - (1+\varepsilon)(q^{\frac{\gamma}{1-\gamma}} - 1)\left[\Psi_F l L_F P_{L,F}{}^{\frac{1}{1-\alpha}} f_F\left(\tilde{Q}_L(t)\right)^{\sigma_F\left(\tilde{Q}_L(t)\right)} \left(g_I\theta + g_I^2\theta\varphi + \rho + \rho\varphi g_I\right)^{\frac{-\alpha}{1-\alpha}} - \theta g_I - \rho\right]$$

Using the Implicit Function Theorem:

$$\frac{\partial\left[P_{L,F}{}^{\frac{1}{1-\alpha}} f_F\left(\tilde{Q}_L(t)\right)^{\sigma_F\left(\tilde{Q}_L(t)\right)}\right]}{\partial g} = -\frac{\frac{\partial G3(.)}{\partial g}}{\frac{\partial G3(.)}{\partial\left[P_{L,F}{}^{\frac{1}{1-\alpha}} f_F\left(\tilde{Q}_L(t)\right)^{\sigma_F\left(\tilde{Q}_L(t)\right)}\right]}} =$$

$$= -\frac{-(1+\varepsilon)\left(q^{\frac{\gamma}{1-\gamma}} - 1\right)\left(g\theta + g^2\theta\varphi + \rho + \rho\varphi g\right)^{\frac{-\alpha}{1-\alpha}} \Psi_F l L_F}{1 - \left[-\theta(1+\varepsilon)\left(q^{\frac{\gamma}{1-\gamma}} - 1\right) + (1+\varepsilon)\left(q^{\frac{\gamma}{1-\gamma}} - 1\right)\Psi_F l L_F \left(\frac{-\alpha}{1-\alpha}\right)\left(g\theta + g^2\theta\varphi + \rho + \rho\varphi g\right)^{\frac{-\alpha}{1-\alpha}-1}(\theta + 2g\theta\varphi + \rho\varphi) P_{L,F}{}^{\frac{1}{1-\alpha}} f_F\left(\tilde{Q}_L(t)\right)^{\sigma_F\left(\tilde{Q}_L(t)\right)}\right]} > 0$$

Similarly, using (60): $\dfrac{\partial\left[P_{H,F}{}^{\frac{1}{1-\alpha}} f_F\left(\tilde{Q}_H(t)\right)^{\sigma_F\left(\tilde{Q}_H(t)\right)}\right]}{\partial g} > 0$.

An increase in g implies an increase in $P_{L,F}{}^{\frac{1}{1-\alpha}} f_F\left(\tilde{Q}_L(t)\right)^{\sigma_F\left(\tilde{Q}_L(t)\right)}$ and $P_{H,F}{}^{\frac{1}{1-\alpha}} f_F\left(\tilde{Q}_H(t)\right)^{\sigma_F\left(\tilde{Q}_H(t)\right)}$. As these increases can result from different combinations of prices, $P_{L,F}$ and $P_{H,F}$ paths and functions $f_F\left(\tilde{Q}_L(t)\right)^{\sigma_F\left(\tilde{Q}_L(t)\right)}$ and $f_F\left(\tilde{Q}_H(t)\right)^{\sigma_F\left(\tilde{Q}_H(t)\right)}$, we must investigate which combinations are possible, considering that the four conditions for the determination of the equilibrium in F are satisfied.

- *Result* 1.2.*A*: $f_F\left(\tilde{Q}_H(t)\right)^{\sigma_F\left(\tilde{Q}_H(t)\right)}$ *must rise*

  If $f_F\left(\tilde{Q}_H(t)\right)^{\sigma_F\left(\tilde{Q}_H(t)\right)}$ falls, the following will occur:

  - $\downarrow f_F\left(\tilde{Q}_H(t)\right)^{\sigma_F\left(\tilde{Q}_H(t)\right)} \Rightarrow \uparrow \boldsymbol{\tilde{Q}_H}$ (given that $f_F(.)$ is a decreasing function of $\tilde{Q}$)
  - $\downarrow f_F\left(\tilde{Q}_H(t)\right)^{\sigma_F\left(\tilde{Q}_H(t)\right)} \Rightarrow \uparrow P_{H,F}$ (so that $P_{H,F}{}^{\frac{1}{1-\alpha}} f_F\left(\tilde{Q}_H(t)\right)^{\sigma_F\left(\tilde{Q}_H(t)\right)}$ rises) $\Rightarrow \downarrow \boldsymbol{\hat{Q}_F}$ (eq. 24)
  - $\downarrow \hat{Q}_F \Rightarrow \downarrow P_{L,F}$ (eq. 23) $\Rightarrow \uparrow f_F\left(\tilde{Q}_L(t)\right)^{\sigma_F\left(\tilde{Q}_L(t)\right)}$ (so that $P_{L,F}{}^{\frac{1}{1-\alpha}} f_F\left(\tilde{Q}_L(t)\right)^{\sigma_F\left(\tilde{Q}_L(t)\right)}$ rises) $\Rightarrow \downarrow \boldsymbol{\tilde{Q}_L}$

  Under this scenario, $\hat{Q}_F$ and $\tilde{Q}_L$ decrease and $\hat{Q}_I$ and $\tilde{Q}_H$ increase. However, such a scenario is not possible, as (61) is not satisfied.

- *Result* 1.2.*B: $P_{L,F}$ must rise*

  If $P_{L,F}$ falls, the following will occur:

  - $\downarrow P_{L,F} \Rightarrow \downarrow \hat{\boldsymbol{Q}}_F$ (eq. 23)
  - $\downarrow P_{L,F} \Rightarrow \uparrow f_F\left(\tilde{Q}_L(t)\right)^{\sigma_F\left(\tilde{Q}_L(t)\right)}$ (so that $P_{L,F}^{\frac{1}{1-\alpha}} f_F\left(\tilde{Q}_L(t)\right)^{\sigma_F\left(\tilde{Q}_L(t)\right)}$ rises) $\Rightarrow \downarrow \tilde{\boldsymbol{Q}}_{\boldsymbol{L}}$
  - $\uparrow f_F\left(\tilde{Q}_H(t)\right)^{\sigma_F\left(\tilde{Q}_H(t)\right)}$ (from Result 1.2.*A*) $\Rightarrow \downarrow \tilde{\boldsymbol{Q}}_{\boldsymbol{H}}$

  Under this scenario, $\hat{Q}_F$, $\tilde{Q}_L$ and $\tilde{Q}_H$,decrease and $\hat{Q}_I$ increases. Therefore, (61) is satisfied only if $\tilde{Q}_H$ falls more than $\tilde{Q}_L$, and (55) is satisfied only if $f_F\left(\tilde{Q}_L(t)\right)^{\sigma_F\left(\tilde{Q}_L(t)\right)}$ increases more than $f_F\left(\tilde{Q}_H(t)\right)^{\sigma_F\left(\tilde{Q}_H(t)\right)}$. Given the configuration of $f_F\left(\tilde{Q}_L(t)\right)^{\sigma_F\left(\tilde{Q}_L(t)\right)}$ and $f_F\left(\tilde{Q}_H(t)\right)^{\sigma_F\left(\tilde{Q}_H(t)\right)}$, these conditions are simultaneously satisfied only if the initial level of $\tilde{Q}_H$ is higher than the initial level of $\tilde{Q}_L$. Since it violates (62), this scenario is also not possible.

- *The only possible final effects are: $\uparrow \hat{Q}_F$, $\downarrow \tilde{Q}_H$, $\uparrow\downarrow \tilde{Q}_L$.*

  From Results 1.2.*A* and 1.2.*B*, the following must occur:

  - $\uparrow f_F\left(\tilde{Q}_H(t)\right)^{\sigma_F\left(\tilde{Q}_H(t)\right)} \Rightarrow \downarrow \tilde{\boldsymbol{Q}}_{\boldsymbol{H}}$
  - $\uparrow P_{L,F} \Rightarrow \uparrow \hat{\boldsymbol{Q}}_F$ (eq. 23) $\Rightarrow \downarrow P_{H,F}$ (eq. 24)

  Since both $P_{L,F}^{\frac{1}{1-\alpha}} f_F\left(\tilde{Q}_L(t)\right)^{\sigma_F\left(\tilde{Q}_L(t)\right)}$ and $P_{H,F}^{\frac{1}{1-\alpha}} f_F\left(\tilde{Q}_H(t)\right)^{\sigma_F\left(\tilde{Q}_H(t)\right)}$ must increase, $f_F\left(\tilde{Q}_L(t)\right)^{\sigma_F\left(\tilde{Q}_L(t)\right)}$ can either increase or decrease. In the first case, $\tilde{Q}_L$ and $\tilde{Q}_H$ decrease and $\hat{Q}_I$ and $\hat{Q}_F$ increase. In the second case, $\tilde{Q}_H$ decrease and $\tilde{Q}_L$, $\hat{Q}_I$ and $\hat{Q}_F$ increase. Both scenarios are possible, as (55), (61), (62) are satisfied. Thus, given (58), the skill premium may either increase or decrease.

Proof of proposition 2.2.

From the proof of proposition 1.2., (59) and (60) imply that if g drops, $P_{L,F}^{\frac{1}{1-\alpha}} f_F\left(\tilde{Q}_L(t)\right)^{\sigma_F\left(\tilde{Q}_L(t)\right)}$ and $P_{H,F}^{\frac{1}{1-\alpha}} f_F\left(\tilde{Q}_H(t)\right)^{\sigma_F\left(\tilde{Q}_H(t)\right)}$ decrease. Thus, we can derive the following results:

- *Result* 2.2.*A: $f_F\left(\tilde{Q}_H(t)\right)^{\sigma_F\left(\tilde{Q}_H(t)\right)}$ must fall*

  If $f_F\left(\tilde{Q}_H(t)\right)^{\sigma_F\left(\tilde{Q}_H(t)\right)}$ rises, the following will occur:

  - $\uparrow f_F\left(\tilde{Q}_H(t)\right)^{\sigma_F\left(\tilde{Q}_H(t)\right)} \Rightarrow \downarrow \tilde{\boldsymbol{Q}}_{\boldsymbol{H}}$ (given that $f_F(.)$ is a decreasing function of $\tilde{Q}$)

- $\uparrow f_F\left(\tilde{Q}_H(t)\right)^{\sigma_F\left(\tilde{Q}_H(t)\right)} \Rightarrow \downarrow P_{H,F}$ (so that $P_{H,F}^{\frac{1}{1-\alpha}} f_F\left(\tilde{Q}_H(t)\right)^{\sigma_F\left(\tilde{Q}_H(t)\right)}$ rises) $\Rightarrow \uparrow \boldsymbol{\hat{Q}_F}$ (eq. 24)
- $\uparrow \hat{Q}_F \Rightarrow \uparrow P_{L,F}$ (eq. 23) $\Rightarrow \downarrow f_F\left(\tilde{Q}_L(t)\right)^{\sigma_F\left(\tilde{Q}_L(t)\right)}$ (so that $P_{L,F}^{\frac{1}{1-\alpha}} f_F\left(\tilde{Q}_L(t)\right)^{\sigma_F\left(\tilde{Q}_L(t)\right)}$ rises) $\Rightarrow \uparrow \boldsymbol{\tilde{Q}_L}$

Under this scenario, $\tilde{Q}_H$ decreases, $\hat{Q}_F$ and $\tilde{Q}_L$ increase and $\hat{Q}_I$ remains unchanged. Such a scenario is not possible, however, as (55) is not satisfied.

- *Result* 2.2.*B: $P_{L,F}$ must fall*

  If $P_{L,F}$ rises, the following will occur:
  - $\uparrow P_{L,F} \Rightarrow \uparrow \boldsymbol{\hat{Q}_F}$ (eq. 23)
  - $\uparrow P_{L,F} \Rightarrow \downarrow f_F\left(\tilde{Q}_L(t)\right)^{\sigma_F\left(\tilde{Q}_L(t)\right)}$ (so that $P_{L,F}^{\frac{1}{1-\alpha}} f_F\left(\tilde{Q}_L(t)\right)^{\sigma_F\left(\tilde{Q}_L(t)\right)}$ rises) $\Rightarrow \uparrow \boldsymbol{\tilde{Q}_L}$
  - $\downarrow f_F\left(\tilde{Q}_H(t)\right)^{\sigma_F\left(\tilde{Q}_H(t)\right)}$ (from Result 2.2.*A*) $\Rightarrow \uparrow \boldsymbol{\tilde{Q}_H}$

  Thus, under this scenario, $\tilde{Q}_H$, $\tilde{Q}_L$ and $\hat{Q}_F$ increase and $\hat{Q}_I$ remains unchanged. Therefore, (61) is satisfied only if $\tilde{Q}_H$ increases more than $\tilde{Q}_L$, and (55) is satisfied only if $f_F\left(\tilde{Q}_L(t)\right)^{\sigma_F\left(\tilde{Q}_L(t)\right)}$ falls more than $f_F\left(\tilde{Q}_H(t)\right)^{\sigma_F\left(\tilde{Q}_H(t)\right)}$. Given the configuration of $f_F\left(\tilde{Q}_L(t)\right)^{\sigma_F\left(\tilde{Q}_L(t)\right)}$ and $f_F\left(\tilde{Q}_H(t)\right)^{\sigma_F\left(\tilde{Q}_H(t)\right)}$, these conditions are simultaneously satisfied only if the initial level of $\tilde{Q}_H$ is higher than the initial level of $\tilde{Q}_L$. As it violate (62), this scenario is not possible either.

- *The only possible final effects are:* $\downarrow \hat{Q}_F$, $\uparrow \tilde{Q}_H$, $\uparrow \tilde{Q}_L$.

  From Results 2.2.*A* and 2.2.*B*, the following must occur:
  - $\downarrow f_F\left(\tilde{Q}_H(t)\right)^{\sigma_F\left(\tilde{Q}_H(t)\right)} \Rightarrow \uparrow \boldsymbol{\tilde{Q}_H}$
  - $\downarrow P_{L,F} \Rightarrow \downarrow \boldsymbol{\hat{Q}_F}$ (eq. 23) $\Rightarrow \uparrow P_{H,F}$ (eq. 24)

  Since both $P_{L,F}^{\frac{1}{1-\alpha}} f_F\left(\tilde{Q}_L(t)\right)^{\sigma_F\left(\tilde{Q}_L(t)\right)}$ and $P_{H,F}^{\frac{1}{1-\alpha}} f_F\left(\tilde{Q}_H(t)\right)^{\sigma_F\left(\tilde{Q}_H(t)\right)}$ must decrease, $f_F\left(\tilde{Q}_L(t)\right)^{\sigma_F\left(\tilde{Q}_L(t)\right)}$ can either increase or decrease. In the first case, $\tilde{Q}_L$ and $\hat{Q}_F$ decrease, $\tilde{Q}_H$ increases, and $\hat{Q}_I$ remains still, which is impossible as it does not satisfy (61). In the second case, $\hat{Q}_F$ decreases, $\tilde{Q}_L$ and $\tilde{Q}_H$ increase and $\hat{Q}_I$ remains constant. Under this scenario, (61) is satisfied only if $\tilde{Q}_L$ increases more than $\tilde{Q}_H$, and (55) is satisfied only if $f_F\left(\tilde{Q}_L(t)\right)^{\sigma_F\left(\tilde{Q}_L(t)\right)}$ falls more than $f_F\left(\tilde{Q}_H(t)\right)^{\sigma_F\left(\tilde{Q}_H(t)\right)}$. Given the configuration of $f_F\left(\tilde{Q}_L(t)\right)^{\sigma_F\left(\tilde{Q}_L(t)\right)}$ and $f_F\left(\tilde{Q}_H(t)\right)^{\sigma_F\left(\tilde{Q}_H(t)\right)}$, these two conditions are simultaneously satisfied only if the initial level of $\tilde{Q}_H$ is lower than the initial level of $\tilde{Q}_L$, which is compatible with (62). This is the only scenario for which conditions (55) to (62) are all satisfied. Consequently, given (58), the skill premium may either increase or decrease.

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