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The Portuguese Business Cycle: Chronology and Duration Dependence

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Abstract

This paper tries to identify, for the first time, a chronology for the Portuguese business cycle and test for the presence of duration dependence in the respective phases of expansion and contraction. A duration dependent Markov-switching vector autoregressive model is employed in that task. This model is estimated over monthly and year-on-year (monthly) growth rates of a set of relevant economic indicators, namely, industrial production, a composite leading indicator and, additionally, civilian employment. The estimated specifications allow us to identify four main periods of contraction during the last three decades and the presence of positive duration dependence in contractions, but not in expansions.

Keywords: *business cycles; duration dependence; Markov-switching.*

JEL Classification: *E32, C41, C24.*

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1 Introduction

This study intends to identify a chronology for the Portuguese business cycle and test for the presence of duration dependence in expansions and contractions. The identification of peaks and troughs in the business cycle and the respective phases of expansion and contraction dates back to the seminal works of Fisher (1925) and Burns and Mitchell (1946). These authors were the first to analyse the mechanisms by which output alternates between states of expansion and contraction and to study the effect of their duration on the transition probabilities between those states.

Based on their studies, the National Bureau of Economic Research (NBER) has been publishing a business cycle chronology for the United States (US) since 1929. In 1978, the NBER introduced some new procedures and the business cycle turning points for the US economy started to be officially determined by a Committee of experts using a large range of macroeconomic indicators and employing a consistent methodology.[1] More recently, the Economic Cycle Research Institute (ECRI) and the Centre for Economic Policy Research (CEPR) started to produce similar chronologies for other countries and for the Euro Area, respectively, based on NBER's methodology and also relying on a Committee of experts.[2]

The main weakness of this methodology of dating the business cycle is the significant lag of time that usually elapses from the occurrence of a turning point until its announcement by the respective Committees. Nevertheless, the careful analysis of relevant economic indicators by a group of experts guarantees a very reliable identification of the respective peaks and troughs in the economy. Unfortunately, no national or international organization have produced, so far, a similar chronology for Portugal. Therefore, we cannot rely in the traditional duration analysis to test for the presence of duration dependence in the phases of the Portuguese business cycle.[3]

An alternative procedure that has been widely used to date the business cycle is the Markov-switching (MS) approach. This procedure, introduced by Hamilton (1989), models

[1] For further details contact the NBER directly at http://www.nber.org/cycles/cyclesmain.html.

[2] For further details contact the ECRI at http://www.businesscycle.com/resources/cycles/ and the CEPR at http://www.cepr.org/data/Dating/.

[3] On duration analysis using duration models see, among others, Sichel (1991), Diebold et al. (1993), Zuehlke (2003), Davig (2007) and Castro (2010).

the business cycle as the outcome of a Markov process that switches between two discrete states: expansions and contractions. This method regards the business cycle as an unobserved stochastic process, so that the reference turning point dates identified by the NBER, CEPR or ECRI are not necessary *a priori*. Moreover, besides dating the business cycle, it allows for a simultaneous estimation of some parameters of interest, like the mean growth rates in each state. Some later developments and improvements make it possible to include duration dependence parameters in the model (Durland and McCurdy, 1994; and Kim and Nelson, 1998) and to estimate it over a vector of relevant time series and not simply over a single time series (Krolzig, 1997).[4]

Pelagatti (2001, 2002) combines these two improvements and employs Bayesian inference in the estimation of the model, which gives rise to the so called duration dependent Markov-switching vector autoregressive model (henceforth, DDMSVAR). This author applies this model to the US economy and show that it reproduces quite accurately the business cycle turning points identified by the NBER. Moreover, the model indicates the presence of positive duration dependence in US contractions but not in expansions. Some evidence of duration dependence is also find by Chen and Shen (2006) and Turk (2009), respectively, in the Taiwanese and Turkish business cycle phases, using this model.

Given the purpose of this paper and the limitations of the other approaches, the DDMSVAR model is the preferable choice to proceed with our work. On one hand, it allows us to date the Portuguese business cycle relying on a set of relevant economic indicators. On the other hand, it permits to control for the presence of duration dependence in the business cycle phases. To our knowledge, this represents the first attempt to analyse such issues for the Portuguese economy. The few existent studies for Portugal look at the business cycle from very different perspectives: Cavalcanti (2007) analyses the effects of economic efficiency on the Portuguese business cycle using a stochastic growth model, while Afonso et al. (2011)

[4] Another basic procedure to date the business cycle is the algorithm proposed by Bry and Boschan (1971) to pin-point the relevant turning points in a dataseries. However, it presents an important drawback: it is only applicable to a single monthly series. Harding and Pagan (2002) solves part of the problem extending the algorithm to quarterly data, but its application remains restricted to a single series. An even simpler procedure is the GDP growth rule, which defines a recession as a period of negative growth of real GDP that lasts two or more consecutive quarters. But, once again, it only relies on a single series which means that not all relevant information can be considered. Hence, these two procedures may not be able to capture the true underlying business cycle.

employ an MS model to study the changes in fiscal policy regimes and its behaviour over the economic cycle in Portugal.

The DDMSVAR model is estimated over a set of relevant (and available) monthly economic indicators, like the industrial production index, a composite leading indicator and, additionally, civilian employment for the period 1978-2010. The estimated specifications generate an interesting chronology for the Portuguese business cycle, permitting us to identify four main periods of contraction during the last 33 years. Moreover, results also show the presence of positive duration dependence in contractions, while the likelihood of expansions ending is not affected by their duration.

The rest of the paper is organized as follows. Section 2 reviews the existing literature on the identification and duration analysis of business cycles. Section 3 presents the econometric model and the estimation procedures. The empirical results are reported and discussed in Section 4. Finally, Section 5 concludes emphasizing the main findings of this paper.

2 Literature

The issue of whether business cycles exhibit duration dependence, i.e. whether expansions or contractions in economic activity are more likely to end as they become older, has been under the scope of the business cycles literature over the last decades. To analyse this issue, researchers have employed either parametric and non-parametric duration models or Markov-switching (MS) models. As parametric duration models and MS models have proved to be more reliable in detecting the presence of positive duration dependence for expansions and contractions than non-parametric duration models, we will focus this brief review of the literature on those two kinds of approaches.[5]

Most of the literature on the duration of expansions and contractions is devoted to the analysis of the US business cycle. This is the case because their turning point dates have been well documented by the NBER for a long period of time. Nevertheless, other industrial

[5] Diebold and Rudebusch (1990) and Ohn et al. (2004) represent important references on non-parametric duration analysis applied to the US business cycle. See Castro (2010), for further references on non-parametric duration approaches to the analysis of the duration of expansions and contractions.

countries have also been under the scope of some studies. Moreover, the CEPR and the ECRI have recently started to establish a business cycle chronology for, respectively, the Euro Area and a group of twenty market oriented economies. Like the NBER, these organizations rely on a Committee to define the business cycle turning points. The methodology is also similar to the one employed by the NBER. However, given their early stage of development and the short number of cycles identified so far by the CEPR for the Euro Area, their use remains quite limited in comparison with the data provided by the NBER.

Using several parametric duration models and the NBER monthly chronology for the US over about a century and a half, Sichel (1991), Diebold et al. (1993), Zuehlke (2003) and Davig (2007) show significant evidence of positive duration dependence for pre-WWII expansions and post-WWII contractions; for the other phases the evidence has proven to be weaker or statistically insignificant. Diebold et al. (1990) reach a similar conclusion in a study for France, Germany and United Kingdom for the pre-WWII period.

Abderrezak (1998) also uses a parametric duration model to analyse the issue of duration dependence in a group of eleven industrial countries. However, instead of considering the classical business cycles, this author uses growth cycles.[6] Results from individual-country and pooled regressions provide evidence of positive duration dependence in both the whole growth cycles and growth phases (upswings and downswings).

Extending the analysis to a panel of thirteen industrial countries, for which the ECRI provides business cycle turning points, and using a (discrete-time) duration model, Castro (2010) finds significant evidence of positive duration dependence for both expansions and contractions in the post-WWII period. Moreover, like Sichel (1991) and Davig (2007), he also notices that the probability of a contraction ending increases more quickly with its age than an expansion and that shorter contractions are preceded by longer expansions.[7]

Other authors have modelled the business cycle as the outcome of a Markov process that switches between the states of expansion and contraction. Contrary to the approaches

[6] Contrary to the classical business cycles, growth cycles are simply identified by increases and decreases in GDP growth rates.

[7] The aim of that paper is not only to find evidence of positive duration dependence, but also to look at other factors that may affect the duration of an expansion or contraction. To do so, the author employs a discrete-time duration model since the additional variables to be included in the analysis are time-varying (leading indicators, investment, price of oil, etc.).

described above, this method regards the business cycle as an unobserved stochastic process, so that the reference cycle turning point dates identified by the NBER, CEPR or ECRI are not necessary. Moreover, it has the advantage of being employed to identify the business cycle chronology in countries for which there is no organization in charge of doing that. Hamilton (1989) was the first to implement this kind of analysis to the US business cycle. A multivariate generalization of the MS model was developed by Krolzig (1997), which resulted in the so called MSVAR model. Krolzig (2001), Artis et al. (2004) and Krolzig and Toro (2005) apply some variants of this model to identify the presence of a common European business cycle. Using both an index of industrial production and GDP, they end up proposing some turning point dates for the European business cycle. More recently, Schirwitz (2009) also employs some MSVARs to identify a chronology for German business cycle.

These approaches assume that the likelihood of a country switching from an expansion to a contraction (or vice-versa) is not affected by its own duration. Some studies have relaxed this assumption allowing for state transition probabilities to be duration dependent. Durland and McCurdy (1994) apply such a refinement to the US real GNP growth rate series and provide evidence of duration dependence for contractions but not for expansions after WWII.[8] A similar result is obtained by Kim and Nelson (1998) and Pelagatti (2001) for the US business cycle, but applying a Bayesian approach.[9] This approach has some advantages over the standard (asymptotic) maximum likelihood theory for inference used by Durland and McCurdy (1994): first, it does not rely on asymptotics, i.e the Bayesian inference does not depend on the sample size of the real-world data, which in latent variable models, where the number of degrees of freedom is low, *asymptopia* can be difficult to reach; second, inference on the latent variables is not conditional on the estimated parameters, but incorporates also the parameters' variability. Furthermore, Pelagatti (2001, 2002) develops and employs a generalized multivariate duration dependent Markov-switching model in which

[8] Perruchoud (2008) reaches the same conclusion employing a similar Markov-switching approach over the Swiss business cycle: he only finds evidence of positive duration dependence for Swiss contractions. Extending Durland and McCurdy's (1994) model by allowing for duration dependence not only in transition probabilities but also in mean growth rates and heteroscedasticity in the noise component, Lam (2004) shows that the probability of an expansion ending decreases *gradually* as it gets older, while the probability of a contraction ending increases *rapidly* a its age increases.

[9] Also using a Bayesian approach identical to the one employed by Kim and Nelson (1998), Kim (1996) and Iiboshi (2007) provide some evidence of positive duration dependence for, respectively, Korean and Japanese expansions and contractions.

the inference on the state variable is carried out using a multi-move Gibbs sampler, while Kim and Nelson (1998) rely on a univariate specification model and on a single-move Gibbs sampler for inference, which results in a slower convergence to the invariant distribution.

These are some advantages that make the DDMSVAR developed by Pelagatti (2001, 2002) an appealing model to be employed in the study carried out in this paper. There are at least two studies – for other countries, other than the US – that apply this model to the analysis of business cycle duration dependence. Chen and Shen (2006) employ the DDMSVAR model to test for the presence of duration dependence in the Taiwan's business cycle and find evidence of positive duration dependence for (pre-1990) expansions and (post-1990) contractions (but not in the other periods). Ozun and Turk (2009) apply it to the study of the Turkish business cycle and also find the presence of some duration dependence. Our study intends to extend its application to the analysis of the Portuguese business cycle, in order to identify, for the first time, its chronology and to test for the presence of duration dependence. As there is no formal business cycle chronology identified by any organization (like the ECRI or the NBER) for the Portuguese economy or even any study to test for the presence of duration dependence in its business cycle, this paper intends to fill that gap in the literature. To do so, we employ Pelagatti's (2002, 2003) DDMSVAR model, which presents an important practical advantage over the traditional duration models, given that no reference cycle turning point dates are formally identified for the Portuguese business cycle.

Finally, a slightly different approach to the study of business cycle dynamics was implemented by Di Venuto and Layton (2005) and Layton and Smith (2007). They develop a multinomial regime-switching logit model to examine the issue of duration dependence in the Australian and US business cycles, respectively. However, contrary to the Markov-switching approach, this discrete-time approach assumes the ex-post observation of business cycle phases (as in the duration models). As this model allows for the use of time-varying covariates, they also include in the equation some leading indices as explanatory variables. Their findings provide evidence of positive duration dependence for both expansions and contractions and their indicators show some power in predicting the termination of either phase.[10] Although their model is not the most suitable for our study, the relevance of the information contained in a composite leading indicator will also be taken into account in the determination of the Portuguese business cycle chronology and in testing for the presence of duration dependence. However, before presenting the empirical analysis, it is useful to make a brief description of the model to be employed in our study.

3 Econometric Model

The econometric model, which is drawn upon the work of Pelagatti (2002, 2003), is presented in this section. The duration dependent Markov-switching vector autoregressive (DDMSVAR) model is defined by:

$$\mathbf{y}_t = \boldsymbol{\mu}_0 + \boldsymbol{\mu}_1 S_t + \mathbf{A}_1(\mathbf{y}_{t-1} - \boldsymbol{\mu}_0 - \boldsymbol{\mu}_1 S_{t-1}) + ... + \mathbf{A}_p(\mathbf{y}_{t-p} - \boldsymbol{\mu}_0 - \boldsymbol{\mu}_1 S_{t-p}) + \boldsymbol{\varepsilon}_t \quad (1)$$

where $\mathbf{y}_t$ is a vector of observable variables, S_t is a binary unobservable random variable following a Markov chain with varying transition probabilities and that takes value 1 when the economy is in expansion and 0 when it is in contraction, $\mathbf{A}_1, ..., \mathbf{A}_p$ are coefficient matrices of a stable VAR process, and $\boldsymbol{\varepsilon}_t$ is a gaussian white noise vector with covariance matrix $\boldsymbol{\Sigma}$. The parameter vectors $\boldsymbol{\mu}_0$ and $\boldsymbol{\mu}_0 + \boldsymbol{\mu}_1$ represent, respectively, the average growth rates of $\mathbf{y}_t$ in state 0 (contraction) and state 1 (expansion).

Under the assumption of constant transition probability proposed by Hamilton (1989), the state variable, S_t, is assumed to follow a first-order Markov chain with the following transition probabilities:

$$\begin{array}{ll} \Pr(S_t = 0|S_{t-1} = 0) = p_{0|0} & \Pr(S_t = 1|S_{t-1} = 0) = 1 - p_{0|0} \\ \Pr(S_t = 1|S_{t-1} = 1) = p_{1|1} & \Pr(S_t = 0|S_{t-1} = 1) = 1 - p_{1|1} \end{array} \quad (2)$$

In order to achieve duration dependence for S_t, a Markov chain is now built for the pair $(S_t; D_t)$, where D_t is the duration variable. This variable (D_t) counts the number of periods

[10] Castro (2010) confirms the relevance of the information contained in these indicators for the likelihood of a contraction or expansion ending.

in which S_t has been in the current state. The probability of S_t being in a particular state is assumed to be dependent on the previous state S_{t-1} and the duration dependent variable D_{t-1}. Hence, given S_{t-1}, S_t and D_{t-1}, we can determine D_t as follows:

$$D_t = \begin{cases} D_{t-1}+1 & if \quad S_t = S_{t-1} \\ 1 & if \quad S_t \neq S_{t-1} \end{cases} \tag{3}$$

It is assumed that the maximum duration periods are equal to τ, with $0 < \tau < T$, where T is the length of the time series being modelled. This maximum value (τ) for the duration variable D_t must be fixed, so that the Markov chain $(S_t; D_t)$ is defined on the finite state space:

$$\{(0,1),(1,1),(0,2),(1,2),(0,3),(1,3),...,(0,\tau),(1,\tau)\} \tag{4}$$

with finite dimensional transition probabilities matrix:

$$\mathbf{P} = \begin{bmatrix} 0 & p_{0|1}(1) & 0 & p_{0|1}(2) & 0 & p_{0|1}(3) & \cdots & 0 & p_{0|1}(\tau) \\ p_{1|0}(1) & 0 & p_{1|0}(2) & 0 & p_{1|0}(3) & 0 & \cdots & p_{1|0}(\tau) & 0 \\ p_{0|0}(1) & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & p_{1|1}(1) & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & p_{0|0}(2) & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & p_{1|1}(2) & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & \cdots & p_{0|0}(\tau) & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & p_{1|1}(\tau) \end{bmatrix} \tag{5}$$

where $p_{i|j}(d) = \Pr(S_t = i|S_{t-1} = j; D_{t-1} = d)$.

To understand the transition matrix $\mathbf{P}$, consider that $\mathbf{P}[i,j]$ denotes row i and column j of the matrix $\mathbf{P}$. Hence, the probability of $\mathbf{P}[1,1]$ in the matrix $\mathbf{P}$ is the following:

$$\Pr(S_t = 0; D_t = 1|S_{t-1} = 0; D_{t-1} = 1) = 0 \tag{6}$$

which means that, conditional on the fact that in the previous period $(t-1)$ the economy is in contraction (state is 0) with duration equal to 1, the probability of remaining in contraction in period t, with duration equal to 1, is zero. Similarly, the probability of $\mathbf{P}[2,1]$ in the matrix $\mathbf{P}$ will be:

$$\Pr(S_t = 1; D_t = 1 | S_{t-1} = 0; D_{t-1} = 1) = p_{1|0}(1) \tag{7}$$

that is, conditional on the fact that in the previous period $(t-1)$ the economy is in contraction $(S_{t-1} = 0)$ with duration equal to 1, the probability of switching to expansion (state 1) in period t, with duration equal to 1, is $p_{1|0}(1)$. The other transition probabilities in the matrix $\mathbf{P}$ can be analysed in the same way. In general, when $D_t = \tau$, only four events are given non-zero probabilities:

$$\begin{aligned} &(S_t = i, D_t = \tau)|(S_{t-1} = i; D_{t-1} = \tau), \qquad i = 0,1 \\ &(S_t = i, D_t = \tau)|(S_{t-1} = j; D_{t-1} = \tau), \qquad i \neq j, \quad i,j = 0,1 \end{aligned} \tag{8}$$

This means that when the economy has been in state i at least τ times, the additional periods in which the state remains i influence no more the probability of transition.

Assuming that the vector of observable variables $\mathbf{y}_t$ is dependent upon the unobserved states from S_t to S_{t-p}, and the duration dependent variable D_{t-1}, Hamilton (1994) suggests that is always possible to write the likelihood function of $\mathbf{y}_t$, depending only on the state variable at time t, even though in the model a p-order autoregression is present. Pelagatti (2002, 2003) notices that this can be done using the extended state variable $S_t^* = (D_t, S_t, S_{t-1}, ..., S_{t-p})$, which encompasses all the possible combinations of the states of the economy in the last p periods. He also shows that if $\tau \geq p$, the maximum number of non-negligible states is given by $u = \sum_{i=1}^{p}(2^i) + 2(\tau - p)$. Moreover, the transition matrix $\mathbf{P}^*$ of the Markov chain S_t^* is a sparse $(u \times u)$ matrix with a maximum number of 2τ independent non-zero elements to be estimated.

Durland and McCurdy (1994) modelled the transition probabilities by using a logistic functional form. In this study, we employ a probit specification, as suggested by Kim and Nelson (1998) and Pelagatti (2002, 2003), to characterize the duration dependence in the business cycle. The use of a probit model in this framework presents two important advantages: first, it reduces to four the number (2τ) of elements in $\mathbf{P}^*$ to be estimated; second, it makes easier to handle in the Gibbs sampler for the Bayesian inference.

A latent variable, $S_t^\bullet$, can then be expressed in the following linear model:

$$S_t^\bullet = (\beta_1 + \beta_2 D_{t-1})\, S_{t-1} + (\beta_3 + \beta_4 D_{t-1})\,(1 - S_{t-1}) + \epsilon_t \tag{9}$$

with $\epsilon_t \sim N(0,1)$ and the latent variable, $S_t^\bullet$, defined by:

$$\begin{aligned}
\Pr(S_t^\bullet \geq 0|S_{t-1}; D_{t-1}) &= \Pr(S_t = 1|S_{t-1}; D_{t-1}) \\
\Pr(S_t^\bullet < 0|S_{t-1}; D_{t-1}) &= \Pr(S_t = 0|S_{t-1}; D_{t-1})
\end{aligned} \tag{10}$$

Therefore, the corresponding transition probabilities for the expansion and contraction phases, under the probit specification, are the following:

$$\begin{aligned}
p_{1|1}(d) &= \Pr(S_t = 1|S_{t-1} = 1; D_{t-1} = d) \\
&= \Pr(S_t^\bullet \geq 0|S_{t-1} = 1; D_{t-1} = d) \\
&= \Pr\left[(\beta_1 + \beta_2 D_{t-1})\, S_{t-1} + (\beta_3 + \beta_4 D_{t-1})\,(1 - S_{t-1}) + \epsilon_t \geq 0\right] \\
&= \Pr\left[\epsilon_t \geq (-\beta_1 - \beta_2 D_{t-1})\, S_{t-1} + (-\beta_3 - \beta_4 D_{t-1})\,(1 - S_{t-1})\right] \\
&= \Pr(\epsilon_t \geq -\beta_1 - \beta_2 d) \\
&= 1 - \Phi(-\beta_1 - \beta_2 d)
\end{aligned} \tag{11}$$

$$\begin{aligned}
p_{0|0}(d) &= \Pr(S_t = 0|S_{t-1} = 0; D_{t-1} = d) \\
&= \Pr(S_t^\bullet < 0|S_{t-1} = 0; D_{t-1} = d) \\
&= \Pr\left[(\beta_1 + \beta_2 D_{t-1})\, S_{t-1} + (\beta_3 + \beta_4 D_{t-1})\,(1 - S_{t-1}) + \epsilon_t < 0\right] \\
&= \Pr\left[\epsilon_t < (-\beta_1 - \beta_2 D_{t-1})\, S_{t-1} + (-\beta_3 - \beta_4 D_{t-1})\,(1 - S_{t-1})\right] \\
&= \Pr(\epsilon_t < -\beta_3 - \beta_4 d) \\
&= \Phi(-\beta_3 - \beta_4 d)
\end{aligned} \tag{12}$$

where $d = 1, ..., \tau$, and $\Phi(\cdot)$ is the standard normal cumulative distribution function. Now the transition probability matrix $\mathbf{P}^*$ is completely defined by the four parameters of $\boldsymbol{\beta} = (\beta_1, \beta_2, \beta_3, \beta_4)$. If $\beta_2 = \beta_4 = 0$, then we have fixed transition probabilities or no business cycle duration dependence.

To obtain parameter estimates from the duration dependent Markov-switching model, we may employ either a quasi-maximum likelihood estimator or a Gibbs sampler approach. In this study we will adopt the latter approach.[11] Next, we present a brief description of the steps involved in the implementation of the Gibbs sampling approach to the DDMSVAR model.[12]

The unknown parameters of the DDMSVAR model can be expressed as:

$$\boldsymbol{\theta} = \left(\boldsymbol{\mu}, \mathbf{A}, \boldsymbol{\Sigma}, \boldsymbol{\beta}, \{(S_t, D_t)\}_{t=1}^{T}\right) \tag{13}$$

where $\boldsymbol{\mu} = (\boldsymbol{\mu}_0', \boldsymbol{\mu}_1')'$, $\mathbf{A} = (\mathbf{A}_1, ..., \mathbf{A}_p)$ and $\boldsymbol{\beta} = (\beta_1, \beta_2, \beta_3, \beta_4)$. Bayesian inference on these unknowns is carried out using Markov chain Monte Carlo (MCMC) methods. The used prior distribution is:

$$p\left(\boldsymbol{\mu}, \mathbf{A}, \boldsymbol{\Sigma}, \boldsymbol{\beta}, S_0^{\bullet}\right) = p(\boldsymbol{\mu}).p(\mathbf{A}).p(\boldsymbol{\Sigma}).p(\boldsymbol{\beta}).p(S_0, D_0) \tag{14}$$

where $\boldsymbol{\mu} \sim N(\mathbf{m}_0, \mathbf{M}_0)$, $vec(\mathbf{A}) \sim N(\mathbf{a}_0, \mathbf{A}_0)$, $p(\boldsymbol{\Sigma}) = |\boldsymbol{\Sigma}|^{-\frac{1}{2}[rank(\boldsymbol{\Sigma})+1]}$, $\boldsymbol{\beta} \sim N(\mathbf{b}_0, \mathbf{B}_0)$, and $p(S_0, D_0)$ is a probability function that assigns a prior probability to every element of the state-space of (S_0, D_0). Alternatively, it is possible to let $p(S_0, D_0)$ be the ergodic probability function of the Markov chain $\{S_t, D_t\}$.

Let $\boldsymbol{\theta}_k$, $k = 1, ..., K$, be a partition of the set containing all the unknowns of the model, and $\boldsymbol{\theta}_{-k}$ represent the set without the elements in $\boldsymbol{\theta}_k$. In order to implement a Gibbs sampler, to sample from the joint posterior distribution of all the unknowns of the model, it is sufficient to find the full conditional posterior distribution $p(\boldsymbol{\theta}_k|\boldsymbol{\theta}_{-k}, \mathbf{Y})$, with $\mathbf{Y} = (\mathbf{y}^1, ..., \mathbf{y}^T)$ and $k = 1, ..., K$. A Gibbs sampler iteration is a generation of random numbers from $p(\boldsymbol{\theta}_k|\mathbf{Y}, \boldsymbol{\theta}_{-k})$, $k = 1, ..., K$, where the elements of $\boldsymbol{\theta}_{-k}$ are substituted with the most recent generated values. Considering, like Pelagatti (2002, 2003), that the Markov chain defined for $\boldsymbol{\theta}^{(i)}$ – where $\boldsymbol{\theta}^{(i)}$ is the value generated at the i^{th} iteration of the Gibbs sampler – converges to its stationary distribution, which is assumed to be the true posterior distribution $p(\boldsymbol{\theta}|\mathbf{Y})$,

[11] To estimate the uknown MSVAR parameters, Durland and McCurdy (1994) use a quasi-maximum likehood estimator, while Kim and Nelson (1998) and Pelagatti (2002, 2003) employ the Bayesian Markov Chain Monte Carlo (MCMC) method. An interesting analysis of several specifications and estimations of the MS model can be found in Kim and Nelson (1999).

[12] For a more detailed explanation on how the Gibbs sample is implemented, see Pelagatti (2002, 2003).

it will be enough to define an initial burn-in period of M iterations, such that the Markov chain may "forget" the starting values $\boldsymbol{\theta}^{(0)}$, to sample from the joint posterior distribution. Therefore, the samples obtained for each element of $\boldsymbol{\theta}$ will be samples from the marginal posterior distribution of each parameter.

To make clear how the Gibbs sample is implemented, we may split $\boldsymbol{\theta}$ into four groups ($K = 4$):

$$\boldsymbol{\theta} = (\boldsymbol{\theta}_1', \boldsymbol{\theta}_2', \boldsymbol{\theta}_3', \boldsymbol{\theta}_4')' \tag{15}$$

where $\boldsymbol{\theta}_1' = (\mathbf{A}', \boldsymbol{\Sigma}')'$, $\boldsymbol{\theta}_2' = (\boldsymbol{\mu}_0', \boldsymbol{\mu}_1')'$, $\boldsymbol{\theta}_3' = (\beta_1, \beta_2, \beta_3, \beta_4)'$, and $\boldsymbol{\theta}_4' = (\{(S_t, D_t)\}_{t=1}^{T})'$. Given all the observed data $\mathbf{Y}$, the conditional distribution of $\boldsymbol{\theta}_k$ can be denoted as follows:

$$p\left(\boldsymbol{\theta}_k | \mathbf{Y}, \boldsymbol{\theta}_{-k}\right), \qquad k = 1, ..., 4. \tag{16}$$

From the arbitrary initial values $\boldsymbol{\theta}^{(0)} = \left(\boldsymbol{\theta}_1^{(0)\prime}, \boldsymbol{\theta}_2^{(0)\prime}, \boldsymbol{\theta}_3^{(0)\prime}, \boldsymbol{\theta}_4^{(0)\prime}\right)'$, we obtain the i^{th} realization of $\boldsymbol{\theta}$ by considering the following procedures:

$$\begin{array}{lllll} (1) & Draw & \boldsymbol{\theta}_1^{(i)} & from & p\left(\boldsymbol{\theta}_1 | \mathbf{Y}, \boldsymbol{\theta}_2^{(i-1)}, \boldsymbol{\theta}_3^{(i-1)}, \boldsymbol{\theta}_4^{(i-1)}\right) \\ (2) & Draw & \boldsymbol{\theta}_2^{(i)} & from & p\left(\boldsymbol{\theta}_2 | \mathbf{Y}, \boldsymbol{\theta}_1^{(i)}, \boldsymbol{\theta}_3^{(i-1)}, \boldsymbol{\theta}_4^{(i-1)}\right) \\ (3) & Draw & \boldsymbol{\theta}_3^{(i)} & from & p\left(\boldsymbol{\theta}_3 | \mathbf{Y}, \boldsymbol{\theta}_1^{(i)}, \boldsymbol{\theta}_2^{(i)}, \boldsymbol{\theta}_4^{(i-1)}\right) \\ (4) & Draw & \boldsymbol{\theta}_4^{(i)} & from & p\left(\boldsymbol{\theta}_4 | \mathbf{Y}, \boldsymbol{\theta}_1^{(i)}, \boldsymbol{\theta}_2^{(i)}, \boldsymbol{\theta}_3^{(i)}\right) \end{array} \tag{17}$$

The i^{th} realization of $\boldsymbol{\theta}$ is then $\boldsymbol{\theta}^{(i)} = \left(\boldsymbol{\theta}_1^{(i)\prime}, \boldsymbol{\theta}_2^{(i)\prime}, \boldsymbol{\theta}_3^{(i)\prime}, \boldsymbol{\theta}_4^{(i)\prime}\right)'$. Repeating steps (1) to (4) I times, we obtain the Gibbs sequence $\left(\boldsymbol{\theta}^{(1)}, \boldsymbol{\theta}^{(2)}, \boldsymbol{\theta}^{(3)}, ..., \boldsymbol{\theta}^{(I)}\right)$.[13] Hence, as indicated above, the joint and marginal distributions of the generated $\left(\boldsymbol{\theta}^{(1)}, \boldsymbol{\theta}^{(2)}, \boldsymbol{\theta}^{(3)}, ..., \boldsymbol{\theta}^{(I)}\right)$ will converge to the joint and marginal distribution of $\boldsymbol{\theta} = (\boldsymbol{\theta}_1', \boldsymbol{\theta}_2', \boldsymbol{\theta}_3', \boldsymbol{\theta}_4')'$ as $I \to \infty$, i.e. $\boldsymbol{\theta}^{(I)} \xrightarrow{d} p(\boldsymbol{\theta}|\mathbf{Y})$. To perform all these econometric procedures we use the DDMSVAR code for Ox developed by Pelagatti (2003).[14]

[13] The order of the Gibbs sampler's steps is not relevant, because any ordering of the steps tends to provide the same ergodic distribution.

[14] We would like to thank Mateo Pelagatti for making his DDMSVAR code for Ox publicly available in his website: http://www.statistica.unimib.it/utenti/p_matteo/

4 Empirical Results

To test for the presence of duration dependence in the US business cycle, Pelagatti (2002, 2003) apply the DDMSVAR model to the monthly growth rate of industrial production, nonfarm-employment, manufacturing and trade sales, and personal income over the period January 1960 to August 2001.[15] These are considered the most important time series on which the NBER relies to date the US business cycle. The ECRI employs a set of similar monthly economic indicators – industrial production, employment, real personal income, sales, and monthly estimates of real GDP – to identify the business cycle turning points in some economies.

Given the importance of these variables, we decided to consider them to be used in our estimations to identify the Portuguese business cycle. However, monthly data are not available for most of those series. Only the industrial production has been recorded on a monthly basis for a reasonable period of time (since 1955).[16] As a way of overcoming this problem, we decided to combine the information contained in the industrial production index (IP) with a composite leading indicator (CLI) computed by the OECD, for which data is available since 1977. On one hand, the CLI can be considered as a proxy for the information contained in the other series since it comprises data on related variables, which are detailed in Table 1. On the other hand, leading indicators are designed to signal fluctuations in economic activity and, therefore, they are considered important in explaining the transition probabilities between expansions and contractions and able to improve the quality and predictive power of the underlying model.[17] For these reasons, the CLI will be used, together with the IP, in the estimation of the DDMSVAR model for the Portuguese economy.[18] In particular, the empirical model will be applied to 100 times the difference of the logarithm (or monthly growth rate) of these two variables for the period January 1978 to

[15] See also the extension to the Turkish economy provided by Ozun and Turk (2009).

[16] Monthly data are also available for sales, but this series starts only in the 1990s.

[17] See, for example, Filardo (1994), Filardo and Gordon (1998), Kim and Nelson (1998), Di Venuto and Layton (2005), Layton and Smith (2007) and Castro (2010).

[18] An alternative would be to rely on the fluctuation of GNP or GDP series – like Pelagatti (2001) and Chen and Shen (2006) – but the available quarterly data for these series for Portugal start only in 1996. Despite the small sample period some attempts were made, but the model did not work well: expansions and contractions were not clearly identifiable. The same happened when IP was the only series used in the model.

October 2010. These variables are defined in detail in Table 1 and some descriptive statistics are provided in Table 2.[19]

<Insert Table 1 around here>

<Insert Table 2 around here>

The estimates of the DDMSVAR model for the monthly growth rates of IP ($dlIP_1$) and CLI ($dlCLI_1$) are presented in Table 3. The scalar for the maximum duration (τ) is assumed to be 60 months (or 5 years), which shall be enough to identify the presence of duration dependence in the business cycle phases. The number of lags (p) was set as equal to 0. Some lags were tried, but the model did not work well, making the identification of expansions and contractions unclear. The priors to the vectors of parameters $\boldsymbol{\mu}$ and $\boldsymbol{\beta}$ were chosen to focus the sampling in an economically reasonable set of values.[20] The Gibbs sampler was run for 11000 iterations, of which the first 1000 were discarded, and the remaining 10000 sample points were used to estimate the densities and the posteriors presented in Table 3.[21]

<Insert Table 3 around here>

Besides the mean and standard deviation of the posteriors, the median (50%) and the 95%-credibility intervals of the posterior distributions – based on the 2.5^{th} and the 97.5^{th} percentiles of the 10000 simulated draws – are also presented. Considering the mean of the posterior distributions for the estimates of IP and CLI growth rates ($\boldsymbol{\mu}$), we obtain mean monthly growth rates of about -0.2% and -0.4% (i.e. about -2.5% and -4.9%, on a yearly basis), respectively, during a contraction (μ_0), and expansion mean growth rates ($\mu_0 + \mu_1$) of about 0.3% and 0.4% (i.e. about 3.6% and 4.9%, on a yearly basis), respectively. According to the 95%-credibility intervals, only $\mu_0 dlIP_1$ may not be statistically different from zero: its interval comprehends the value 0.

[19] There, we also find information for the annual growth rate of these variables (compared to the same month of the previous year). These are to be employed in some later estimations, as well as the civilian employment variable. Monthly data for employment was obtained by linear interpolation of the available quarterly data for the period 1983-2010.

[20] Other priors were tried, including 0 for all variables, but results were quite similar. As in Pelagatti (2001, 2002), no specific prior was defined for matrix $\boldsymbol{\Sigma}$.

[21] The Gibbs sampler always reached convergence to its stationary distribution. To save space their graphs are not presented here, but they are available upon request. The same applies to the kernel density estimates/distributions of $\boldsymbol{\mu}$ and $\boldsymbol{\beta}$.

The estimates of $\boldsymbol{\beta}$ are displayed next. The constants (β_1 and β_3) present the expected signs and are clearly different from zero. However, the coefficients of most interest are the duration dependence coefficients β_2 and β_4. The concentration of the posterior of the parameter β_2 around zero seems to indicate that the probability of falling into a contraction is independent of how long the economy has been in expansion. Hence, expansions are not duration dependent. The posterior of β_4 is away from zero, but statistically it cannot be considered different from zero since its 95%-confidence interval includes the value 0. However, it remains some evidence of positive duration dependence of the transition probability of moving into an expansion after a period of contraction ($Pr(S_t = 1|S_{t-1} = 0, D_{t-1} = d)$), which can be confirmed in Figure 1. The probability of a contraction ending indeed increases over time, but at a slow pace. Hence, there is some (weak) evidence of positive duration dependence for contractions, but no duration dependence is found for expansions.[22]

<Insert Figure 1 around here>

Next we intend to identify the periods of expansion and contraction estimated by the model. The estimated probabilities of the Portuguese economy being in expansion over the period 1978-2010 are presented in Figure 2. The model proved to have a reasonable capability of discerning expansions and contractions, as the probabilities of expansions, in general, tend to assume high and low values. These probabilities of expansion can be used to identify the turning points (peaks and troughs) in the Portuguese economy over the period 1978-2010. Making use of Hamilton's (1989) 0.5-rule to determine the state of the economy, we end up with the business cycle chronology presented in Table 4.

<Insert Figure 2 around here>

<Insert Table 4 around here>

We identify seven cycles of contraction and expansion, based on NBER and ECRI's procedure of not considering a phase of expansion or contraction with less than five months.[23] Four of them coincide with world crises. The contraction that started in June 1983 and ended

[22]Note that, in Figure 1, the mean of the transition probability of moving into a contraction after a period of expansion, i.e. $Pr(S_t = 0|S_{t-1} = 1, D_{t-1} = d)$, is flat.

[23]Note also that, on average, Portuguese expansions tend to last twice as much as contractions.

in January 1984, follows the international crisis caused by a monetary policy tightening in the US to control inflation, which boosted a debt crisis in some less developed countries. In this period, Portugal was also economically affected by the introduction of some austerity measures imposed by the IMF in exchange for financial help to avoid bankruptcy. The contraction in the beginning of the 1990s is the repercussion of the stock crash in the late 1980s in the US that lead to a recession in much of the Industrial world.[24] The early 2000s contraction is in line with the crisis caused by the burst of the Dot Com bubble and the September 2001 attacks. Finally, the contraction January 2007-April 2009 matches the financial crises that has affected the world in the late 2000s. The other three contractions seem to be the repercussion of the previous ones and motivated by internal issues. The political instability felt in the first half of the 2000s in Portugal – three different governments were in office during this period – may have contributed to the two additional (short) contractions identified in this period.[25]

Nevertheless, as there is no organization in charge of identifying the Portuguese business cycle, we do not have a formal point of reference to compare our results with. Our sensitivity tells us that a reasonable comparison could be made using the annual growth of real GDP as an indicative reference. The series of real GDP growth over the period 1971-2010 is presented in Figure 3.

<Insert Figure 3 around here>

In general, the contraction periods identified by the DDMSVAR for the Portuguese economy match reasonably well the years of low or negative growth of real GDP: 1983-1984; 1992-1993; 2002-2003; 2005; and 2008-2009. However, the years of 1994 and, in particular, 1995 seem to be years of recovery and not contraction (as pointed out by the estimated model). Looking at Figure 2, we see that – however below 0.5 – the probability of expansion is not as close as zero as the others and it is not very well defined. The presence of this additional contraction may be due to a higher volatility of monthly than annual growth

[24] The Golf War, the German reunification and the problems with the European Exchange Rate Mechanisms also contributed, in some degree, to the international recession.

[25] In a recent paper, Aisen and Veiga (2011) study this relation using a panel of countries and show that higher degrees of political instability are indeed strongly associated with lower economic growth.

rates. Therefore, monthly growth rates may reveal some (weaker) contractions that might be diluted on a year-on-year basis. To check whether that might be the case, we decided to re-estimate the model using the annual growth rates of IP ($dlIP_{12}$) and CLI ($dlCLI_{12}$)[26] – compared to the same month of the previous year – instead of monthly rates. Results are shown in Table 5.

<Insert Table 5 around here>

The priors to the vectors of parameters $\boldsymbol{\mu}$ and $\boldsymbol{\beta}$ were, once again, chosen to centre the sampling in an economically reasonable set of values for year-on-year monthly growth rates.[27] The results show annual growth rates of about -2.5% (4.2%) for IP and -2.9% (4.4%) for CLI during contractions (expansions). On a yearly basis, these average rates are not very different from the ones identified above, but now all the coefficients of the vector $\boldsymbol{\mu}$ are statistically significant according to the 95%-credibility intervals. Additionally, the duration dependence coefficients (β_2 and β_4) confirm the lack of evidence of duration dependence for expansions and the presence of some (but statistically weak) positive duration dependence for contractions. Figure 4 confirms that weaker evidence in comparison with the one identified above when monthly growth rates are used.[28]

<Insert Figure 4 around here>

The periods of expansion and contraction estimated by the model with annual growth rates for the period 1978-2010 are depicted in Figure 5. Also using Hamilton's (1989) 0.5-rule to determine the state of the economy, we are able to establish the corresponding business cycle chronology, which is reported in Table 6.

[26] The OECD call it the year-on-year growth rate of the CLI and considers it as the preferred pointer to identify turning points because it is less volatile and provides earlier and clearer signals for their identification than the CLI itself.

[27] Other priors were tried, including 0 for all variables, but results were quite similar. No specific prior was set for matrix $\boldsymbol{\Sigma}$. This specification also considers $\tau = 60$ and $p = 0$. As before, the Gibbs sampler was run for 11000 iterations, of which the first 1000 were discarded, and the remaining 10000 sample points were used to estimate the densities and the posteriors.

[28] The use of year-on-year growth rates could be seen as an immediate justification for that weaker evidence since they are smoother and less volatility than monthly growth rates. However, we will see below that this may not indeed be the reason, since the additional information contained in the (annual growth rate of the) employment variable will be helpful in unveiling the presence of positive duration dependence in contractions.

<Insert Figure 5 around here>

<Insert Table 6 around here>

Now, we only identify four cycles of contraction and expansion. These cycles are even clearer and better defined than the ones identified using monthly rates. As mentioned above, this can be the result of using smoother and less volatile year-on-year monthly growth rates instead of simple monthly growth rates. Hence, some weaker states end up being absorbed by the "nearest" and "strongest" phase. This generates a chronology that matches quite well the years of low or negative growth of real GDP depicted in Figure 3. Moreover, the model is now considering 1994 and 1995 as years of expansion, which is in line with the graph in Figure 3. It also suggests that the period May 2001-February 2006 can be considered as a single contraction. This is indeed a period of very low growth of real GDP, which means that the two small contractions identified above – in the model with monthly growth rates – can be aggregated in a single contraction. Adding to this the contraction period from May 2007 until November 2009, we may argue that the 2000s can be seen as a "lost decade" for Portugal in terms of economic expansion.

Although annual growth rates are able to produce a clearer chronology of the Portuguese business cycle, the presence of positive duration dependence in contractions – however weak – seems to fade away. Given this evidence, we could be tempted to conclude that either the likelihood of a contraction ending is not indeed affected by its duration or annual growth rates are simply hiding out some useful information to detect its presence. To explore a little more this issue, we decided to keep the annual growth rates of IP and CLI in the model and tried to add additional information/variables to the model. The problem is – as already noticed – that monthly data for other useful variables (employment, sales, income,...) are not available for a reasonable period of time or they are not available at all. However, we found that quarterly data for an index of civilian employment (Emp) is available since 1983. As the changes in this variable are more or less smooth over time, we consider that a linear interpolation of quarterly data, to generate monthly data, would produce a series that might be very close to the actual monthly time series. Following this procedure, we generated monthly data for Emp and then computed the respective annual growth rates compared to the same month of the previous year ($dlEmp_{12}$).[29] This variable was then added to the model. The results of the DDMSVAR model with $dlIP_{12}$, $dlCLI_{12}$ and $dlEmp_{12}$ for the shorter period of June 1984 to October 2010 are presented in Table 7.

<Insert Table 7 around here>

Once again, the choice of the priors to the vectors of parameters $\boldsymbol{\mu}$ and $\boldsymbol{\beta}$ was based on a set of sensible values.[30] The results present annual growth rates of about -2.1% (3.8%) for IP, -3.9% (3.8%) for CLI, and -1.1% (1.5%) for Emp during contractions (expansions), and all the respective estimated coefficients are statistically significant according to the 95%-credibility intervals.

The estimates of the duration dependence coefficients (β_2 and β_4) present quite interesting results. First, the concentration of the posterior of the parameter β_2 around zero confirms that the probability of falling into a contraction is independent of how long the economy has been in expansion, strengthening the previous conclusion that Portuguese expansions are not duration dependent. Second, the posterior of β_4 now lays significantly away from zero; statistically, it can be considered different from zero since its 95%-confidence interval does not include the value 0. Therefore, positive duration dependence is present in Portuguese contractions. This evidence can be confirmed in Figure 6, where it is very clear the increase, over time, in the transition probability of the economy moving into an expansion after a period of contraction ($Pr(S_t = 1|S_{t-1} = 0, D_{t-1} = d)$).

<Insert Figure 6 around here>

Given these results, we cannot simply blame the use of year-on-year monthly growth rates for the weaker evidence of positive duration dependence in contractions. The argument that some information is missing seems to make more sense here. In particular, the additional information contained in the employment variable has proved to be relevant to detect the

[29] See Table 1 for definitions and Table 2 for descriptive statistics.

[30] Other priors were tried but results were quite similar. This specification also considers $\tau = 60$ and $p = 0$, and the Gibbs sampler was run for the same number of iteractions as the other estimations presented above.

presence of positive duration dependence in contractions.[31] We should notice that these results must be analysed with some careful since the gains in terms of additional economic information were obtained at the cost of some missing years of observations and linear interpolation from quarterly data for civilian employment.

Finally, it would be interesting to identify the business cycle chronology that results from this new specification of the DDMSVAR model. The periods of expansion and contraction for the period 1984-2010 are presented in Figure 7 and the corresponding chronology is reported in Table 8.

<Insert Figure 7 around here>

<Insert Table 8 around here>

The general pattern of the periods of contraction and expansion in the Portuguese economic activity is very close to the one identified above in Figure 5 and Table 6. There are, however, some differences that should be considered. First, as the time period is now shorter (it starts only in June 1984), it is impossible to identify the contraction that took place in 1983. Nevertheless, Figure 7 is suggestive of its presence since it indicates probabilities lower than 1 in 1984.

Second, the model now indicates that the contraction in the early 1990s seems to have started in December 1991 and not in March 1991 – as estimated by the model without the annual growth rate of Emp – but the ending date is the same (November 1993). With little differences, both models are successful in identifying this contraction.

Third, the model considers the period of July 1995 to July 1996 as a period of contraction, but the probability of expansion is not close to 0 as in the other cases of contraction; in fact, it is very close to the 0.5-threshold, which makes this a less relevant and "weak" contraction. Moreover, it does not find support neither in Table 3 nor in Figure 5.

Fourth, both models that use annual growth rates identify May 2001 as the starting month for the first contraction in the 2000s, but the model with Emp seems to indicate

[31] Several combinations of annual and monthly growth rates of IP, CLI and Emp were also tried but results and conclusions regarding the presence of duration dependence and the respective business cycle chronology remained practically the same. In particular, when only monthly growth rates of those three variables are used, results are quite similar to the ones presented, in first place, in this section. The problem is that the time period is shorter, which means that the contraction in 1983 is missed. All those estimations and results are not reported here due to space limitations, but they are available upon request.

that the recovery has started sooner. Nevertheless, this recovery is not very strong, since the probability of expansion only jumps to values close to 1 in the beginning of 2006. Not surprisingly, this matches closely the date reported by the model without Emp for the end of this contraction (February 2006). The differences identified in the estimates provided by both models might be due to the fact that 2004 and 2005 are characterised by a low economic growth in comparison with the average standards, as can be confirmed in Figure 3: the annual growth rate of GDP is low in 2004 and even falls in 2005. Hence, the model without Emp identifies it as a state of contraction (given that the average growth rate is in the low state), while the model with Emp considers it a period of expansion, however, not very strong given that the probability of expansion is far from 1 until the beginning of 2006. Given all this evidence and the picture with monthly growth rates, we prefer consider it as a period of contraction (or not full recovery).

Finally, the model with Emp is quite successful in identifying the recent contraction in the Portuguese economic activity caused by the recent financial crisis (July 2007-October 2009). In particular, we should stress that the dates are very close to the ones reported by the model without Emp (see Table 6).

Thus, the DDMSVAR model with the annual growth rates of IP, CLI and Emp (estimated over the period June 1983-October 2010) together with the model without Emp (estimated over the period January 1978-October 2010) seem to provide a reasonable picture of the Portuguese business cycle chronology over the last 33 years and also some support for the presence of positive duration dependence in contractions.

5 Conclusions

The identification of the business cycle chronology for the US economy has been undertaken by the NBER for a long period of time. More recently, the CEPR and the ECRI have extended such task to the Euro Area and to other market oriented economies, respectively. However, the identification of the Portuguese business cycle chronology is out of their scope. Moreover, to our knowledge, no national or international organization or scientific study has dedicated, so far, to that identification. Given this lack of attention over the Portuguese case, we decided to implement some econometric procedures to identify the periods of expansion and contraction in the Portuguese economy over the last (almost) four decades of democracy.

Another task of this study is to analyse the presence of duration dependence in the phases of the Portuguese business cycle. The issue of whether the likelihood of an expansion or contraction ending is dependent on its age has been studied in several papers for a reasonable group of countries, with special attention given to the US. Duration analysis and Markov-switching models have been the mainly used approaches in those studies. Most of them have been successful in finding evidence of positive duration dependence for expansions and/or contractions. Unfortunately, no study has analysed yet this issue for the Portuguese economy. As far as we are concerned, our study represents the first attempt to analyse the presence of duration dependence in the phases of the Portuguese business cycle.

With the aims of identifying the business cycle chronology for the Portuguese economy and the presence of duration dependence in its phases, we decided to employ a model that is able to deal with both tasks at the same time: the DDMSVAR model developed by Pelagatti (2001, 2002). In its specification we combined monthly data of the industrial production index with monthly information contained in the OECD composite leading indicator, which aggregates some variables that are expected to influence the business cycle. This model proved to have a good capability of discerning periods of contraction and expansion and in finding the presence of duration dependence. In particular, it was able to identify a reasonable chronology for the Portuguese business cycle, especially when year-on-year monthly growth rates of the variables are used. Four important periods of contraction were identified by this model for the period 1978-2010: October 1983-June 1984; March 1991-November 1993; May 2001-February 2006; and May 2007-November 2009. However, we should notice that the ending date for the first contraction in the 2000s reveals to be different when some additional information from civilian employment is added to the model. In that case, the ending date turns out to be September 2003. This result should be analysed with a grain of salt because that variable presents some drawbacks: first, monthly series for that variable are not available, so they had to be generated by linear interpolation from quarterly data; second, its growth rate is only available from June 1984 onwards, which reduces the time-span of the analysis.

Despite these limitations, the two contractions registered in the 2000s and the low economic growth that has caracterised the short periods of expansion during this decade show that Portugal has lost some momentum in achieving higher levels of economic convergence to the European Union average during that decade. This represents a big concern for the next years since strong economic expansion is needed to serve private and public debt that was accumulated during the last decade.

Finally, the model was also able to detect the presence of positive duration dependence for contractions, while the likelihood of an expansion ending is not affected by its duration. Therefore, we can conclude that the likelihood of a contraction ending increases over time, but for expansions it remains constant. In sum, these results for the Portuguese business cycles are quite similar to the ones obtained in several papers for the US: contractions are duration dependent, while expansions are not.[32]

The conclusions reached in this study are quite promising since this analysis can be extended to the study of the business cycle in other countries for which no organization is dating their business cycle turning points. The model employed here may also be useful to detect the presence of any duration dependence in the phases of their cycles. That might not always be the case, but this model can be a good starting point for that analysis. When monthly data for the most important variables that characterize de business cycle are not available, quarterly series of GDP or GNP can be an alternative. We also tried to use quarterly series of GDP in our study for the Portuguese economy, but the model did not work well with the short time-span available for quarterly data for the growth rate of that series: 1996-2010. This prevented us from making an identification of the business cycle phases and of finding the presence of duration dependence with quarterly data of GDP. We hope to be more successful in the future when a longer time-span is available for that series.

Additionally, we could employ the model to detect the presence of duration dependence

[32] See, for example, Diebold and Rudebusch (1990), Diebold et al. (1993), Sichel (1991), Durland and McCurdy (1994), Kim and Nelson (1998), and Pelagatti (2002).

in other areas where cycles might be present, like in the stock or housing markets. We believe that an adequate study of their cycles and respective phases can provide more useful information to better understand the economic business cycles and the presence of duration dependence detected, mainly, in contractions.

References

[1] Abderrezak, A., 1998. On the duration of growth cycles: An international study. International Review of Economics and Finance 7, 343-355.

[2] Afonso, A., Claeys, P., Sousa, R., 2011. Fiscal regime shifts in Portugal. Portuguese Economic Journal, Forthcoming.

[3] Aisen, A., Veiga, F., 2011. How does political instability affect economic growth? IMF Working Paper, WP/11/12.

[4] Artis, M., Krolzig, H.-M., Toro, J., 2004. The European business cycle. Oxford Economic Papers 56, 1-44.

[5] Bry, G., Boschan, C., 1971. Cyclical Analysis of Time Series: Selected Procedures and Computer Program. National Bureau of Economic Research, New York.

[6] Burns, A.F., Mitchell, W.C., 1946. Measuring Business Cycles. NBER Studies in Business Cycles, No. 2. National Bureau of Economic Research, New York.

[7] Castro, V., 2010. The duration of economic expansions and recessions: More than duration dependence. Journal of Macroeconomics 32, 347-365.

[8] Cavalcanti, T., 2007. Business cycle and level accounting: the case of Portugal. Portuguese Economic Journal 6, 47-64.

[9] Chen, S., Shen, C., 2006. Is there a duration dependence in Taiwan's business cycles. International Economic Journal 20(1), 109-127.

[10] Davig, T., 2007. Change-points in US business cycle durations. Studies in Nonlinear Dynamics and Econometrics 11 (2), Article 6.

[11] Di Venuto, N., Layton, A., 2005. Do the phases of the business cycle die of old age? Australian Economic Papers 44, 290-305.

[12] Diebold, F., Rudebusch, G., 1990. A nonparametric investigation of duration dependence in the American business cycle. Journal of Political Economy 98, 596-616.

[13] Diebold, F., Rudebusch, G., Sichel, D., 1990. International evidence on business cycle duration dependence. Institute for Empirical Macroeconomics, DP 31.

[14] Diebold, F., Rudebusch, G., Sichel, D., 1993. Further evidence on business cycle duration dependence. In: J. H. Stock and M. W. Watson Eds. Business Cycles, Indicators, and Forecasting, pp. 87-116. Chicago: University of Chicago Press.

[15] Durland, M., McCurdy, T., 1994. Duration-dependent transitions in a Markov model of US GNP growth. Journal of Business and Economic Statistics 12, 279-288.

[16] Fisher, I., 1925. Our unstable dollar and the so-called business cycle. Journal of American Statistical Association 20, 179-202.

[17] Hamilton, J., 1989. A new approach to the economic analysis of nonstationary time series and the business cycle. Econometrica 57, 357-384.

[18] Hamilton, J., 1994. Time Series Analysis. Princeton University Press.

[19] Harding, D., Pagan, A., 2002. Dissecting the cycle: A methodological investigation. Journal of Monetary Economics 49, 365-381.

[20] Iiboshi, H., 2007. Duration dependence of the business cycle in Japan: A Bayesian analysis of extended Markov switching model. Japan and the World Economy 19, 86-111.

[21] Kim, M., 1996. Duration dependence in the Korean business cycle: evidence and its implication based on Gibbs sampling approach to regime-switching model. Seoul Journal of Economics, 9(2), 123-144.

[22] Kim, C., Nelson, C., 1998. Business cycle turning points, a new coincident index, and tests of duration dependence based on a dynamic factor model with regime switching. Review of Economics and Statistics 80, 188-201.

[23] Kim, C., Nelson, C., 1999. State Space Models with Regime Switching: Classical and GIBBS-Sampling Approaches with Applications. The MIT Press, Cambridge.

[24] Krolzig, H.-M., 1997. Markov-Switching Vector Autoregressions. Modelling, Statistical Inference and Application to Business Cycle Analysis. Springer, Berlin.

[25] Krolzig, H.-M., 2001. Markov-switching procedures for dating the Euro-Zone business cycle. Quarterly Journal of Economic Research 70, 339-351.

[26] Krolzig, H.-M., Toro, J., 2005. Classical and modern business cycle measurement: The European case. Spanish Economic Review 7, 1-21.

[27] Lam, P. S., 2004. A Markov-switching model of GNP growth with duration dependence. International Economic Review 45, 175-204.

[28] Layton, A., Smith, D., 2007. Business cycle dynamics with duration dependence and leading indicators. Journal of Macroeconomics 29, 855-875.

[29] Ohn, J., Taylor, L., Pagan, A., 2004. Testing for duration dependence in economic cycles. Econometrics Journal 7, 528-549.

[30] Ozun, A., Turk, M., 2009. A duration-dependent regime switching model for an open emerging economy. Romanian Journal of Economic Forecasting 4, 66-81.

[31] Pelagatti, M., 2001. Gibbs sampling for a duration dependent Markov-switching model with application to the US business cycle. Working Paper, Quaderno di Dipartimento QD2001/2 (March), Dipartimento di Statistica, Universita degli Studi di Milano Bicocca.

[32] Pelagatti, M., 2002. Duration-dependent Markov-switching VAR models with application to the US business cycle analysis. Proceedings of the XLI Scientific Meeting of the Italian Statistics Society.

[33] Pelagatti, M., 2003. DDMSVAR for Ox: A software for time-series modeling with duration-dependent Markov-switching autoregressions. 1st OxMetrics User Conference, London, September 1-2, 2003.

[34] Perruchoud, A., 2008. Analyzing the Swiss business cycle. Applied Economics Quarterly 54, 255-91.

[35] Sichel, D., 1991. Business cycle duration dependence: A parametric approach. Review of Economics and Statistics 73, 254-260.

[36] Schirwitz, B., 2009. A comprehensive German business cycle chronology. Empirical Economics 37, 287-301.

[37] Zuehlke, T., 2003. Business cycle duration dependence reconsidered. Journal of Business and Economic Statistics 21, 564-569.

List of Tables

Table 1 - Description of the variables

Variable	Description
IP	Index of industrial production which refers to the volume of output generated by production in the following industrial sectors: mining, manufacturing, electricity, gas and water; Seasonally adjusted with base year 2005 = 100.
$dlIP_1$	Monthly growth rate of the industrial production index (IP).
$dlIP_{12}$	Annual growth rate of IP compared to the same month of the previous year.
CLI	Composite Leading Indicator (trend restored) computed by the OECD, which aggregates a variety of indicators or variables that are expected to influence the business cycle; For Portugal, it comprises: industrial production index for electricity, gas and water; production (future tendency in manufacturing); order books/demand (in manufacturing); export order books/demand (tendency); share prices index; and unfilled job vacancies.
$dlCLI_1$	Monthly growth rate of the composite leading indicator (CLI).
$dlCLI_{12}$	Annual growth rate of CLI compared to the same month of the previous year.
Emp	Civilian employment; Index, non seasonally adjusted, base year 2005 = 100.
$dlEmp_1$	Monthly growth rate of the civilian employment (Emp).
$dlEmp_{12}$	Annual growth rate of Emp compared to the same month of the previous year.

Notes: For further details on the components of the CLI and on the methodology to compute it, contact the OECD directly at http://www.oecd.org/std/cli.
Source: OECD, Main Economic Indicators, December 2010.

Table 2 - Descriptive Statistics

Variable	period	obs.	mean	st.dev.	min.	max.
$dlIP_1$	$1978M1 - 2010M10$	394	0.152	2.871	-9.13	12.29
$dlIP_{12}$	$1978M1 - 2010M10$	394	2.054	5.120	-16.88	14.61
$dlCLI_1$	$1978M1 - 2010M10$	394	0.166	0.512	-2.27	1.27
$dlCLI_{12}$	$1978M1 - 2010M10$	394	2.104	4.783	-15.56	11.43
$dlEmp_1$	$1984M6 - 2010M10$	317	0.061	0.496	-2.53	1.72
$dlEmp_{12}$	$1984M6 - 2010M10$	317	0.727	2.616	-7.82	7.32

Notes: See Table 1.

Table 3 - Estimates of the DDMSVAR model: monthly growth rates ($dlIP_1, dlCLI_1$)

	Prior		Posterior				
Parameter	mean	var.	mean	st.dev.	2.5%	50.0%	97.5%
$\mu_0 dlIP_1$	−0.200	1.000	−0.2119	0.2416	−0.7093	−0.2081	0.2325
$\mu_0 dlCLI_1$	−0.200	1.000	−0.4161	0.0552	−0.5274	−0.4152	−0.3092
$\mu_1 dlIP_1$	0.500	1.000	0.5108	0.2753	0.0437	0.4972	1.0775
$\mu_1 dlCLI_1$	0.500	1.000	0.8178	0.0470	0.7274	0.8178	0.9120
β_1	1.000	2.000	1.7474	0.2611	1.2698	1.7293	2.3031
β_2	0.000	2.000	0.0033	0.0077	−0.0127	0.0036	0.0175
β_3	−1.000	2.000	−1.6922	0.3278	−2.3763	−1.6755	−1.0916
β_4	0.000	2.000	0.0262	0.0247	−0.0203	0.0255	0.0769

Notes: See Table 1. Time-period: January 1978 - October 2010.

Table 4 - Business cycle chronologies based on the monthly growth rates ($dlIP_1, dlCLI_1$)

Business cycle reference dates		Duration (in months)			
Peak	Trough	Contraction	Expansion	Cycle	
		Peak-Trough	Trough-Peak	Trough-Trough	Peak-Peak
–	December 1977$^+$	–	–	–	–
June 1983	January 1984	7	66^+	73^+	–
November 1991	May 1993	18	94	112	101
October 1994	February 1996	16	17	33	35
November 2000	February 2003	27	57	84	73
October 2003	May 2004	7	8	15	35
March 2005	October 2005	6	10	16	17
January 2007	April 2009	27	16	43	22
March 2010	October 2010$^+$	7^+	11	18^+	38
	Average (7 cycles)	15	30	51	46

Notes: $^+$ indicates that the duration can be higher because the date of the respective troughs has been censored since they are out of the sample (January 1978-October 2010) and they are not known. December 1977 and October 2010 are assumed to be the reference (censored) dates, but the real troughs might be further away in the past or in the future, respectively. The censored durations are not considered in the computation of the averages.

Table 5 - Estimates of the DDMSVAR model: annual growth rates ($dlIP_{12}, dlCLI_{12}$)

	Prior		Posterior				
Parameter	mean	var.	mean	st.dev.	2.5%	50.0%	97.5%
$\mu_0 dlIP_{12}$	−2.000	4.000	−2.4900	0.3740	−3.2263	−2.4887	−1.7522
$\mu_0 dlCLI_{12}$	−2.000	4.000	−2.8757	0.3189	−3.5060	−2.8720	−2.2633
$\mu_1 dlIP_{12}$	5.000	4.000	6.6779	0.4368	5.8138	6.6793	7.5343
$\mu_1 dlCLI_{12}$	5.000	4.000	7.3153	0.3587	6.6018	7.3186	8.0285
β_1	1.000	5.000	2.5681	0.5167	1.6409	2.5429	3.6787
β_2	0.000	5.000	−0.0085	0.0104	−0.0296	−0.0082	0.0117
β_3	−1.000	5.000	−2.4945	0.5209	−3.6485	−2.4576	−1.5876
β_4	0.000	5.000	0.0260	0.0225	−0.0076	0.0225	0.0886

Notes: See Table 1. Time-period: January 1978 - October 2010.

Table 6 - Business cycle chronologies based on annual growth rates ($dlIP_{12}, dlCLI_{12}$)

Business cycle reference dates		Duration (in months)			
Peak	Trough	Contraction	Expansion	Cycle	
		Peak-Trough	Trough-Peak	Trough-Trough	Peak-Peak
–	December 1977^{+}	–	–	–	–
October 1983	June 1984	8	70^{+}	78^{+}	–
March 1991	November 1993	32	81	113	89
May 2001	February 2006	57	90	147	122
May 2007	November 2009	30	15	45	72
October 2010^{+}	–	–	11^{+}	–	41^{+}
	Average (4 cycles)	32	62	102	94

Notes: $^{+}$ indicates that the duration can be higher because the date of the respective trough or peak has been censored since they are out of the sample (January 1978-October 2010) and they are not known. December 1977 and October 2010 are assumed to be the reference (censored) dates, but the real trough/peak might be further away in the past or in the future, respectively. The censored durations are not considered in the computation of the averages.

Table 7 - Estimates of the DDMSVAR model: annual growth rates (with $dlEmp_{12}$)

	Prior		Posterior				
Parameter	mean	var.	mean	st.dev.	2.5%	50.0%	97.5%
$\mu_0 dlIP_{12}$	−2.000	4.000	−2.0846	0.9541	−3.8030	−1.9082	−0.5439
$\mu_0 dlCLI_{12}$	−2.000	4.000	−3.9111	0.6851	−5.3355	−3.8666	−2.7388
$\mu_0 dlEmp_{12}$	−2.000	4.000	−1.0954	0.2873	−1.6921	−1.0818	−0.5664
$\mu_1 dlIP_{12}$	5.000	4.000	4.8975	1.2662	2.9557	4.5708	7.1275
$\mu_1 dlCLI_{12}$	5.000	4.000	7.7015	0.5425	6.7127	7.6876	8.7994
$\mu_1 dlEmp_{12}$	5.000	4.000	2.5978	0.3047	2.0107	2.5946	3.1882
β_1	1.000	5.000	2.5837	0.6553	1.5418	2.4987	4.2629
β_2	0.000	5.000	−0.0112	0.0139	−0.0415	−0.0102	0.0130
β_3	−1.000	5.000	−3.3109	1.0260	−5.8325	−3.1299	−1.8330
β_4	0.000	5.000	0.0878	0.0563	0.0084	0.0787	0.2242

Notes: See Table 1. Time-period: June 1984 - October 2010.

Table 8 - Business cycle chronologies based on annual growth rates (with $dlEmp_{12}$)

Business cycle reference dates		Duration (in months)			
Peak	Trough	Contraction	Expansion	Cycle	
		Peak-Trough	Trough-Peak	Trough-Trough	Peak-Peak
–	May 1984^{+}	–	–	–	–
December 1991	November 1993	23	91^{+}	114^{+}	–
July 1995	July 1996	12	20	32	43
May 2001	September 2003	28	58	86	70
July 2007	October 2009	46	27	73	55
October 2010^{+}	–	–	12^{+}	–	58^{+}
	Average (4 cycles)	27	50	76	56

Notes: $^{+}$ indicates that the duration can be higher because the date of the respective trough or peak has been censored since they are out of the sample (June 1984-October 2010) and they are not known. May 1984 and October 2010 are assumed to be the reference (censored) dates, but the real trough/peak might be further away in the past or in the future, respectively. In this case, looking at Figure 5 and Table 7, May 1984 might be considered the actual trough, hence the computation of the averages will take it into account, but not the censored duration for the conjectured peak in October 2010.

List of Figures

Figure 1 - Transition Probabilities (monthly growth rates)

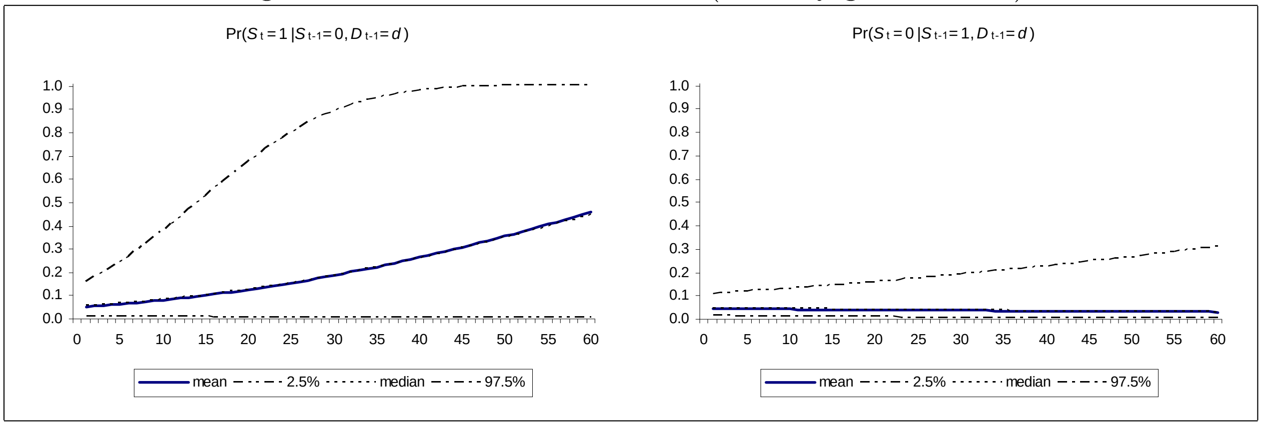


Figure 2 - Probability of expansion (monthly growth rates)

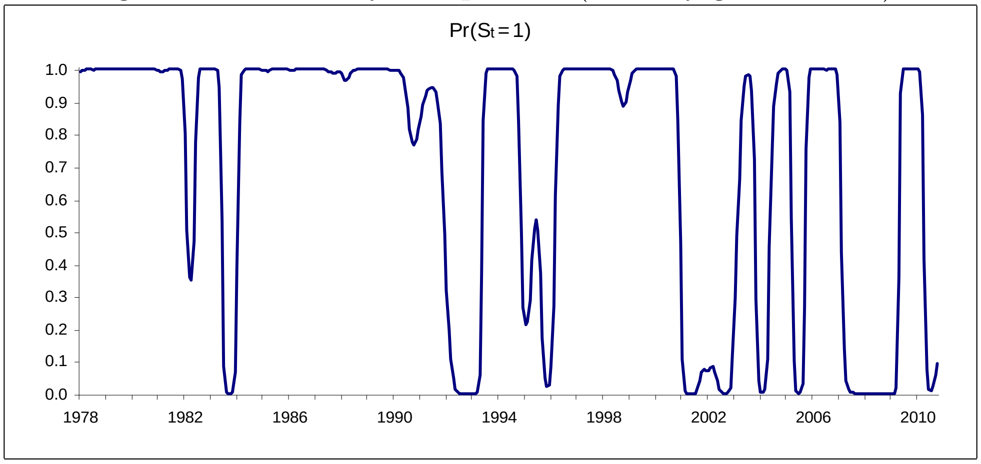
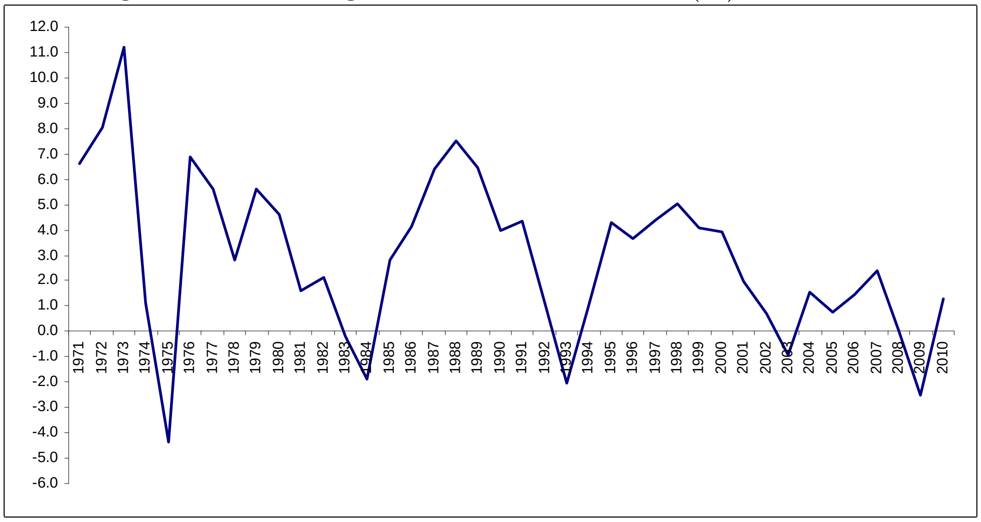


Figure 3 - Annual growth rate of real GDP (%), 1971-2010



Source: OECD, Main Economic Indicators, February 2011.

Figure 4 - Transition Probabilities (annual growth rates)

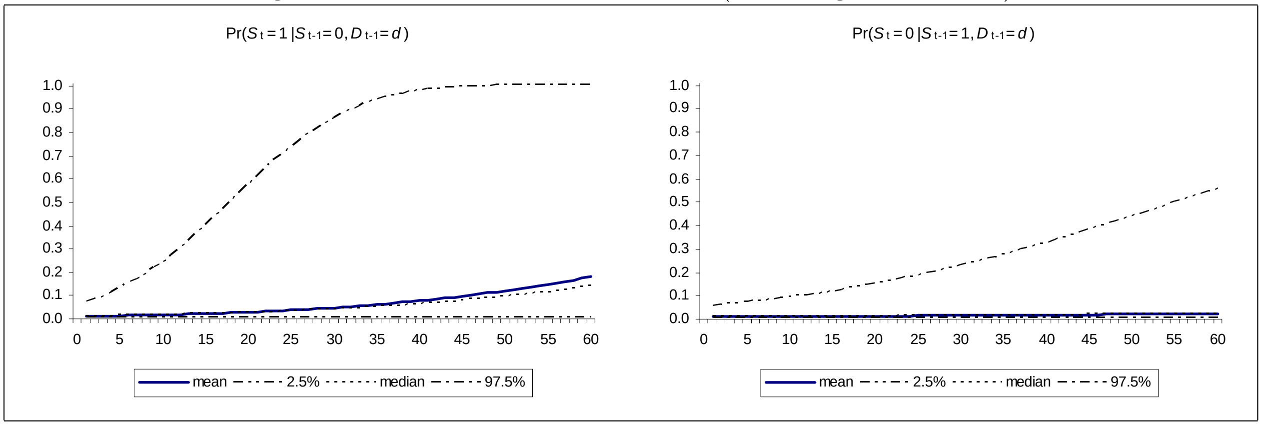


Figure 5 - Probability of expansion (annual growth rates)

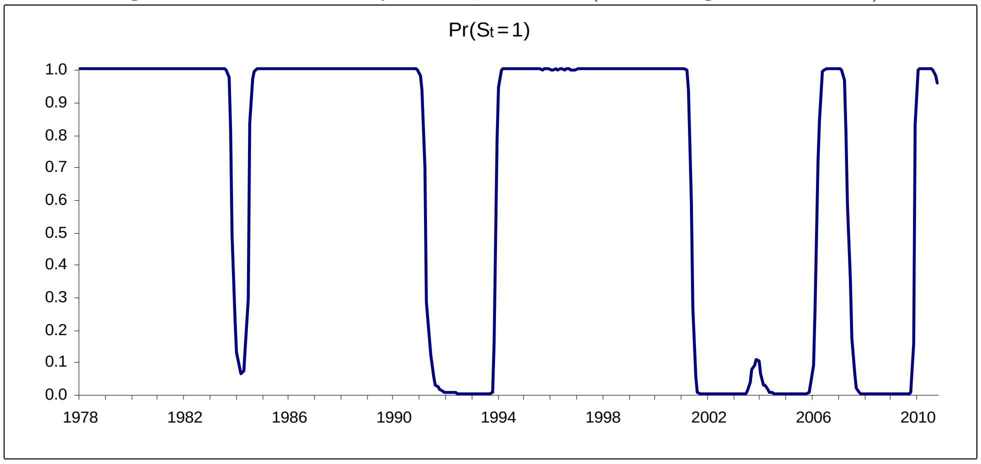


Figure 6 - Transition Probabilities (with $dlEmp_{12}$)

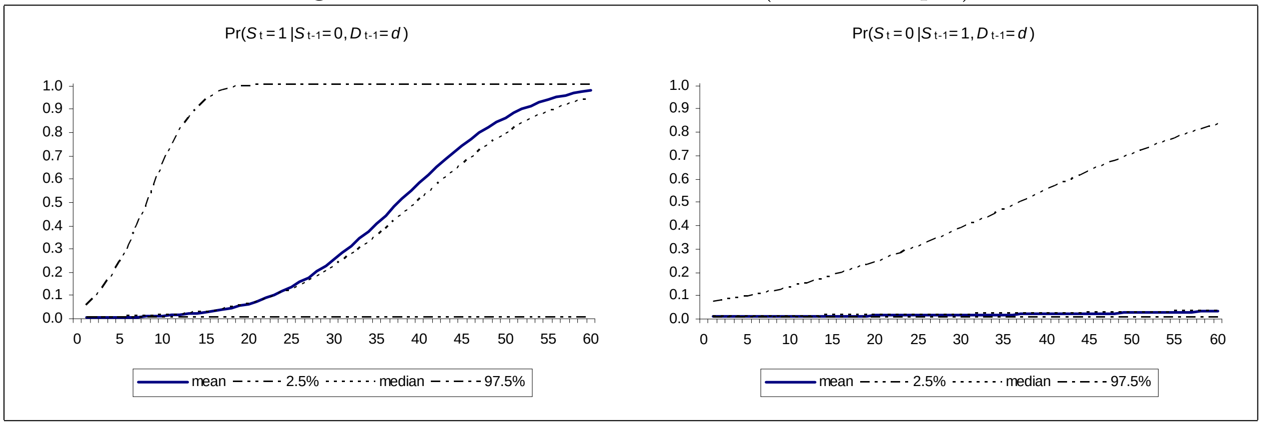
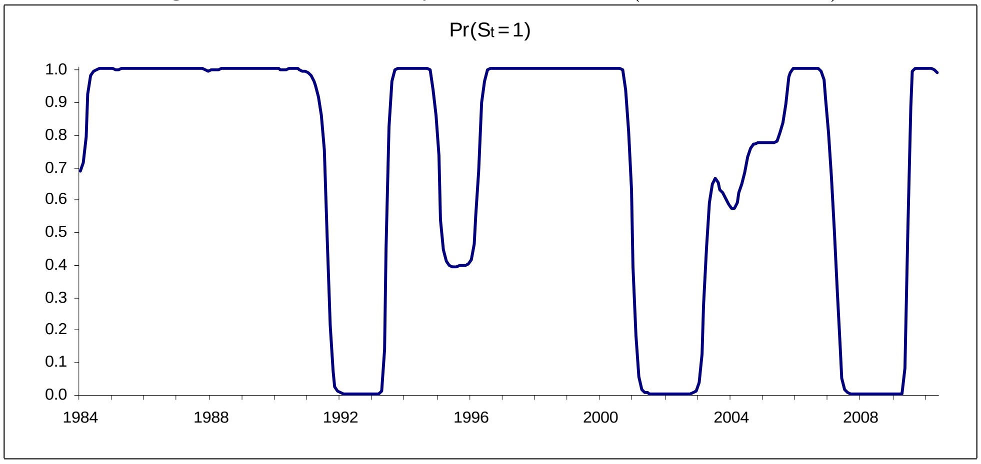


Figure 7 - Probability of expansion (with $dlEmp_{12}$)



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