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Oil Shocks and the Euro as an Optimum Currency Area*†

Luís Aguiar-Conraria‡ Teresa Maria Rodrigues§ Maria Joana Soares¶

April 11, 2012

Abstract

We use wavelet analysis to study the impact of the Euro adoption on the oil price macroeconomy relation in the Euroland. We uncover evidence that the oil-macroeconomy relation changed in the past decades. We show that after the Euro adoption some countries became more similar with respect to how their macroeconomies react to oil shocks. However, we also conclude that the adoption of the common currency did not contribute to a higher degree of synchronization between Portugal, Ireland and Belgium and the rest of the countries in the Euroland. On the contrary, in these countries the macroeconomic reaction to an oil shock became more asymmetric after adopting the Euro.

JEL classification: Q43; C22; E32; F44;

Keywords: Oil prices; Business cyles, the Euro, Optimum Currency Areas; Wavelet analysis

*To replicate our results, the reader can use a Matlab wavelet toolbox that we wrote. It is freely available at http://sites.google.com/site/aguiarconraria/joanasoares-wavelets. Our data is also available in that website.

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1 Introduction

The literature on business cycle synchronization is related to the literature on optimal currency areas. If several countries delegate on some supranational institution the power to perform a common monetary policy, then they lose this policy stabilization instrument. Obviously, business cycle synchronization is not sufficient to guarantee that a monetary union is desirable; however, it is, arguably, a necessary condition: a country with an asynchronous business cycle will face several difficulties in a monetary union, because of the 'wrong' stabilization policies.

In the economics literature, to test if a group of countries form an Optimum Currency Area (OCA), it is common to check if the different countries face essentially symmetric or asymmetric exogenous shocks (e.g. see Peersman 2011). In the latter case, it is more difficult to argue for a monetary union. However, even if the shock is symmetric, one still has to check if its impact is similar across countries. If this is not the case, the symmetric shock will have asymmetric effects, which deteriorates the case for a monetary union.

There is a caveat to the previous argument. Some authors argue that even if a region is not *ex ante* an OCA it may, *ex post*, become one. The argument for this endogenous OCA is simple and intuitive: by itself the creation of a common currency area will create the conditions for the area to become an OCA. For example, Frankel and Rose (1998) and Rose and Engel (2002) argue that, because currency union members have more trade, business cycles are more synchronized across currency union countries. Imbs (2004) makes a similar argument for financial links. After the creation of a currency area, the finance sector will become more integrated and hence business cycles will become more synchronized. In effect, Inklaar *et al.* (2008) conclude that convergence in monetary and fiscal policies has a significant impact on business cycle synchronization. However, Baxter and Kouparitsas (2005) conclude otherwise and Camacho *et al.* (2008) present evidence that differences between business cycles in Europe have not been disappearing.

We tackle this issue by focusing on one shock that every country faces: oil price changes. We study the relation between oil and the macroeconomy in the 11 countries that first joined the Euro in 1999. We investigate how this relation changed after the adoption of the Euro and test if it became more or less asymmetric after the Euro adoption. The analysis is performed in the time-frequency domain, using wavelet analysis.

We are not the first authors to use wavelets to analyse the oil price-macroeconomy relationship. Naccache (2011) and Aguiar-Conraria and Soares (2011a) have already relied on this technique to assess this relation. Actually, wavelet analysis is particularly well suited for this purpose for several reasons. First, because oil price dynamics is highly nonstationary, it is important to use a technique, such as wavelet analysis, that does not require stationarity. Second, wavelet analysis is particularly useful to study how relations evolve not only across time, but also across frequencies, as it is unlikely that these relations remain invariant. Third, Kyrtsou *et al.* (2009) presented evidence showing that several energy markets display consistent nonlinear dependencies. Based on their analysis, the authors call for nonlinear methods to analysis the impact of oil shocks. Wavelet analysis is one such method. We should also add that wavelets have already proven to be insightful when studying business cycles synchronizations, e.g. see Aguiar-Conraria and Soares (2011b) and Crowley and Mayes (2008).

We use data on the Industrial Production for the first countries joining the Euro and estimate the coherence between this variable and oil prices. The statistical procedure is similar to the one used by Vacha and Barunick (2012) to study co-movements in the time-frequency space between energy commodities. By itself, this analysis will allow us to characterize how the relationship evolved and how the 2000s are different from the 1980/1990s. We will see that in late 1980s and early 1990s, the strongest coherence is for cycles with periods that range between 4 and 8 years, while in more recent times it became a shorter run relation, with coherence being higher for cycles with periods between 2 and 4 years.

After estimating, for each country, the coherencies between industrial production and oil prices, we propose a metric to compare these coherencies and measure and test the degree of synchronization among countries. Interestingly, we show that the relation between oil and the macroeconomy in the different countries was more similar before than after the euro adoption. This is particularly true for Portugal, Ireland and Belgium. It seems that, at least for these three countries, the endogenous OCA theory is refuted.

This paper follows a very simple structure. We describe the data and present our results in section two and section three concludes. In the appendix, we provide a brief introduction to the mathematics of wavelets and explain how to derive the metric that we use to compare the oil-macroeconomy relation in the different countries.

2 The Oil-Macroeconomy Relationship and the Euro

We analyze the oil price-macroeconomy relation in the Euro area by looking both at the coherency content and phasing of cycles. We look at the first 11 countries joining the Euro: Austria, Belgium, Finland, France, Germany, Ireland, Italy, Luxembourg, Netherlands, Portugal and Spain. For this type of purpose, to measure real economic activity, most studies use either real GDP or an Industrial Production Index. We use the Industrial Production Index because wavelet analysis is quite data demanding, and having monthly data is a plus. We use seasonally adjusted data from the OECD Main Economic Indicators database, from January of 1986 to December of 2011. We have, therefore, 26 years of data. Exactly 13 years before and 13 years after the Euro adoption. The oil price data is the West Texas Intermediate Spot Oil Price taken from the Federal Reserve Economic Data – FRED – St. Louis Fed.

For each country, we estimate the wavelet coherence between the yearly rate of growth of Industrial Production and the oil price. It is known that oil price increases are more important than oil price decreases. Because of that, Hamilton (1996 and 2003) proposed a nonlinear transformation of the oil price series. In our computations, we use the Hamilton's Net Oil Price. Because we focus our analysis on business cycle frequencies, we estimate the coherence for frequencies corresponding to periods between 1.5 and 8 years.

In Figure 1, we have our first set of results. For each country, on the left (a) we have the wavelet coherency between Industrial Production and Oil Prices.[1] On the right, we have the phase-differences: on the top (b), we have the phase-difference in the $2 \sim 4$ years frequency band (chosen to capture the region of high coherency that appears in most of the countries after 2000); in the bottom (c), we have the phase-difference in the $4 \sim 8$ years frequency band, which captures the regions of high coherency in the late 1980s and in the first half of

[1] The black contour designates the 5% significance level, obtained by 1000 Monte Carlo simulations based on two independent ARMA(1,1) processes as the null. Coherency ranges from blue (low coherency) to red (high coherency). The cone of influence is shown with a thick line, which is the region subject to border distortions.

the 1990s — recall that it only makes sense to interpret the phase-differences in the regions of high coherency.

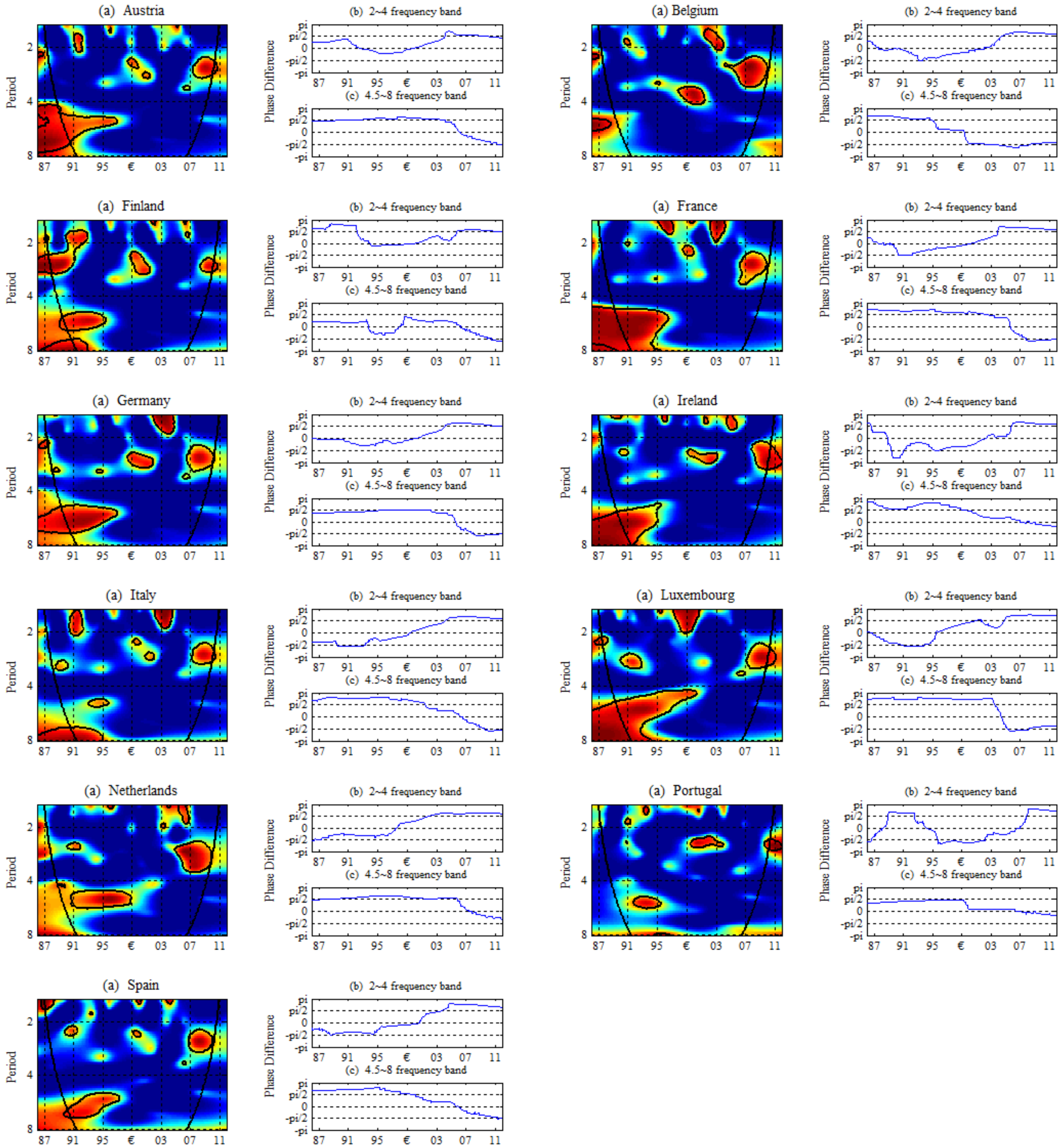

Figure 1: On the left — Wavelet Coherency between each country's Industrial Production and Oil Prices. On the right — Phase-difference between Industrial Production and Oil Prices at $2 \sim 4$ years (top) and $4 \sim 8$ years (bottom) frequency bands.

For most countries the region with the strongest coherency is located between the mid-1980s and mid-1990s at the $4 \sim 8$ years frequency band. And for most countries, the phase-difference is consistently between $\pi/2$ and π, suggesting that oil price increases anticipate downturns in the Industrial Production. After the Euro adoption, in 1999, for most of the countries, the strongest region of high coherency is in the $2 \sim 4$ years frequency band after 2005. Again, the phase differences are located between $\pi/2$ and π, consistent with the idea that negative oil shocks anticipate downturns in the Industrial Production. The most interesting aspect is this change in the predominant frequencies.

These results are consistent with the results of other authors, who conclude that, in the more recent times, the negative impact of oil shocks is shorter-lived than before. This may happen because the oil exporting countries follow different pricing strategies — see, for example, Aguiar-Conraria and Wen (2012) —, because the nature of oil shocks was different — see, for example, Hamilton (2009) or Kilian (2008 and 2009) — or because the western macroeconomies became more flexible — see, for example, Blanchard and Gali (2010) who argue that less rigid wages as well as a smaller share of oil in the production are candidate explanations for the shorter-lived impact of oil shocks.

To assess if the oil price-macroeconomy relation is similar between two countries, we compute the distance between the complex wavelet coherencies matrix associated with both countries, using formula (12) in the appendix. This measure takes into account both the real and the imaginary parts of the cross-wavelet transform. A value very close to zero means that two countries have a very similar complex wavelet coherency. This means that (1) the contribution of cycles at each frequency to the total correlation between oil prices and the industrial production is similar in both countries, (2) this contribution happens at the same time in both countries and, finally, (3) the leads and lags between the oil price cycles and the industrial production cycles are similar in both countries.

To test if the similitude is statistically significant, we rely on Monte Carlo simulations. We compute the complex wavelet coherencies matrix between a surrogate for the oil price and a surrogate for the industrial production of each country. Then we compute the distances between the two cross spectra, using formula (12). For each pair of countries, we do this 1000 times and compute the distance for each trial. From the computed distances, we extract the critical values at 1, 5 and 10%.

	Au	Be	Fi	Fr	Ge	Ir	It	Lx	Ne	Pt	Sp
Austria		0.091	0.055	0.042	0.050	0.097	0.048	0.066	0.056	0.095	0.049
Belgium	0.056		0.084	0.085	0.082	0.126	0.077	0.087	0.074	0.070	0.093
Finland	0.077	0.067		0.061	0.078	0.097	0.053	0.073	0.059	0.093	0.065
France	0.049	0.076	0.075		0.054	0.100	0.049	0.069	0.049	0.086	0.047
Germany	0.041	0.067	0.074	0.053		0.089	0.043	0.061	0.062	0.083	0.054
Ireland	0.056	0.063	0.072	0.054	0.064		0.075	0.106	0.098	0.104	0.078
Italy	0.060	0.066	0.078	0.048	0.058	0.060		0.059	0.051	0.084	0.040
Luxembourg	0.056	0.066	0.079	0.065	0.057	0.059	0.050		0.072	0.083	0.067
Netherlands	0.059	0.075	0.067	0.073	0.059	0.060	0.067	0.060		0.080	0.053
Portugal	0.075	0.077	0.074	0.068	0.071	0.070	0.056	0.069	0.062		0.092
Spain	0.063	0.076	0.078	0.052	0.065	0.045	0.060	0.066	0.055	0.083	
Gray scale code:			p < 0.01			p < 0.05			p < 0.10		

Table 1: Lower triangle – Complex Wavelet Dissimilarities before the Euro. Upper triangle – Complex Wavelet Dissimilarities after the Euro.

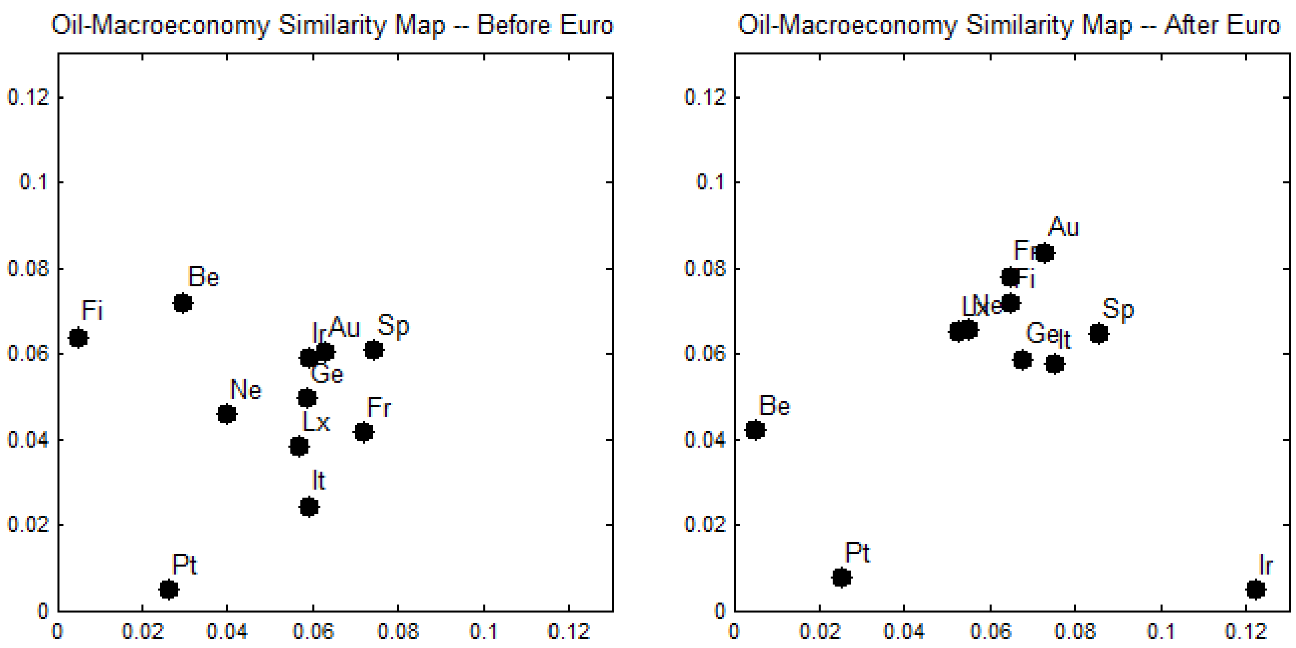

Figure 2: Multidimensional scaling maps

For each pair of countries we estimate two distances: one before the Euro adoption and the other after the adoption. It is as if we divide each of the pictures in Figure 1 in two halves: left and right. To measure the distances between two countries before the Euro, we compare the left halves. And we compare the right halves to measure the distance after 1999. Given that, by definition, a distance matrix is symmetric, to save space, we use the lower triangle for the distances before the Euro adoption and in the upper-triangle we have the distances after the euro adoption. These results are described in Table 1.

It is interesting to note that the endogeneity of the OCAs does not survive our analysis, at least when one considers the case of Portugal, Belgium and, even more strongly, Ireland. Before the Euro adoption, Portugal was synchronized with Italy (1% significance), France, Netherlands, Luxembourg (5% significance), Austria, Finland, Germany and Ireland (10% significance). In the second half of the sample, Portugal is only synchronized with Belgium. Similar results hold for Belgium. The case of Ireland is even stronger. Before the birth of the Euro, Ireland was synchronized with every country except Finland. With 1% significance in the majority of the cases. After the Euro adoption, Ireland is simply synchronized with Italy and Spain at 10% significance. The only country that clearly became more synchronized after the Euro adoption was Finland.

The same information is displayed in Figure 2, where we use the distances of Table 1 to plot a map with the countries into a two-axis system — see Camacho *et al.* (2006).[2] This cannot be performed with perfect accuracy because distances are not Euclidean. In these maps it is clear that while most of the countries became slightly tighter, particularly in the case of Finland who moved to the core after 1999, this was not the case for Belgium, Portugal and Ireland, who now look like three isolated islands with no strong connections to mainland.

3 Conclusions

Unlike most previous studies on OCAs and on business cycle synchronization, which rely on time domain methods — such as VAR, gravity and panel data models —, we relied on time-frequency domain methods. To be more precise, we used wavelet analysis to study the impact of the Euro adoption on the member countries' macroeconomic reaction to one of the most common shocks: oil shocks. Given that energy is such an important production input, and that due to several reasons (including ecological, political and economic reasons) it is such

[2] Basically, we reduce each of the distance matrices to a two-column matrix, called the configuration matrix, which contains the position of each country in two orthogonal axes.

a volatile sector, the transmission mechanism of oil shocks to the macroeconomy is bounded to have important effects. If a group of countries have asymmetric responses to the same oil shock, it is highly unlikely that those countries form an OCA.

We estimated the wavelet coherency between the Industrial production of the 11 countries that first joined the Euro and the oil price. We uncovered evidence that shows that the oil-macroeconomy relation changed in the past decades. In the second half of 1980s and in the first half of 1990s, oil price increases preceded macroeconomic downturns. This effect occurred at frequencies with periods around six years. However, in the last decade, the regions of high coherencies were located at frequencies that corresponded to shorter-run cycles (cycles with periods around three years).

We also showed that after the Euro adoption some countries became more similar with respect to how their macroeconomies react to oil shocks. This is true for Austria, France, Germany, Italy, Luxembourg, Netherlands, and Spain and even more true for Finland, who had a rather asymmetric reaction to oil shocks before the Euro adoption. However, we also showed that at least three countries do not share a common response to oil shocks: Portugal, Ireland and Belgium. Particularly interesting is the conclusion that the adoption of the common currency did not contribute to a higher degree of synchronization between these countries and the rest of the countries in the Euroland. This effect is particular surprising in the case of Ireland, who was highly synchronized before 1999.

4 Appendix — Wavelets: Frequency Analysis Across Time

Wavelet analysis performs the estimation of the spectral characteristics of a time-series as a function of time, revealing how the different periodic components of a particular time-series evolve over time. This technical presentation is, necessarily brief. For a detailed technical overview, the reader can check Aguiar-Conraria and Soares (2011c). Alternatively, for a thorough intuitive discussion on these concepts, the reader is referred to Cazelles *et al.* (2007) and Aguiar-Conraria *et al.* (2012).

4.1 The Continuous Wavelet Transform

A wavelet is simply a rapid decaying oscillatory function. Mathematically, for $\psi(t)$ to be called a wavelet, it must satisfy $\int_{-\infty}^{\infty} |\psi(t)|^2 dt < \infty$ and a certain technical condition which, for functions with sufficient decay, is equivalent to requiring that it has zero mean, i.e. $\int_{-\infty}^{\infty} \psi(t)\, dt = 0$. The continuous wavelet transform (CWT) of a given time-series x_t is given by

$$W_x(\tau, s) = \int_{-\infty}^{\infty} x(t) \frac{1}{\sqrt{|s|}} \overline{\psi}\left(\frac{t-\tau}{s}\right) dt, \tag{1}$$

where s is a scaling or dilation factor that controls the width of the wavelet and τ is a translation parameter controlling the location of the wavelet. Here, and throughout, the bar denotes complex conjugate.

When the wavelet $\psi(t)$ is chosen as a complex-valued function, as we do, the wavelet transform $W_x(\tau, s)$ is also complex-valued. In this case, the transform can be separated into its real part, $\Re(W_x)$, and imaginary part, $\Im(W_x)$, or in its amplitude, $|W_x(\tau, s)|$, and phase, $\phi_x(\tau, s) : W_x(\tau, s) = |W_x(\tau, s)|\, e^{i\phi_x(\tau,s)}$.[3] For real-valued wavelet functions, the imaginary part is constantly zero and the phase is, therefore, undefined. Hence, in order to separate the phase and amplitude information of a time-series, it is important to make use of complex wavelets.

[3]The phase-angle $\phi_x(\tau, s)$ of the complex number $W_x(\tau, s)$ can be obtained from the formula: $\tan(\phi_x(\tau, s)) = \frac{\Im(W_x(\tau,s))}{\Re(W_x(\tau,s))}$, using the information on the signs of $\Re(W_x)$ and $\Im(W_x)$ to determine to which quadrant the angle belongs to.

In order to describe the time-frequency localization properties of the CWT, we have to assume that both the wavelet ψ and its Fourier transform $\hat{\psi}$ are well localized functions. More precisely, these functions must have sufficient decay to guarantee that the quantities defined below are all finite.[4] In what follows, for simplicity, assume that the wavelet has been normalized so that $\int_{-\infty}^{\infty} |\psi(t)|^2 \, dt = 1$. With this normalization, $|\psi(t)|^2$ defines a probability density function. The mean and standard deviation of this distribution are called, respectively, the center, μ_ψ, and radius, σ_ψ, of the wavelet. They are, naturally, measures of of localization and spread of the wavelet. The center $\mu_{\hat{\psi}}$ and radius $\sigma_{\hat{\psi}}$ of $\hat{\psi}$, the Fourier transform of the wavelet ψ, are defined in a similar manner. The interval $\left[\mu_\psi - \sigma_\psi, \mu_\psi + \sigma_\psi\right]$ is the set where $\psi(t)$ attains its "most significant" values whilst the interval $\left[\mu_{\hat{\psi}} - \sigma_{\hat{\psi}}, \mu_{\hat{\psi}} + \sigma_{\hat{\psi}}\right]$ plays the same role for $\hat{\psi}(f)$. The rectangle $H_\psi := \left[\mu_\psi - \sigma_\psi, \mu_\psi + \sigma_\psi\right] \times \left[\mu_{\hat{\psi}} - \sigma_{\hat{\psi}}, \mu_{\hat{\psi}} + \sigma_{\hat{\psi}}\right]$ in the (t, f) −plane is called the Heisenberg box or window for the function ψ. We then say that ψ is localized around the point $\left(\mu_\psi, \mu_{\hat{\psi}}\right)$ of the time-frequency plane, with uncertainty given by $\sigma_\psi \sigma_{\hat{\psi}}$. The Heisenberg uncertainty principle establishes that the uncertainty is bounded from below by the quantity $1/2$.

The Morlet wavelet became the most popular of the complex valued wavelets for several reasons. Among then we highlight two: (1) the Heisenberg box area reaches its lower bound with this wavelet, i.e. the uncertainty attains the minimum possible value; (2) the time radius and the frequency radius are equal, $\sigma_\psi = \sigma_{\hat{\psi}} = \frac{1}{\sqrt{2}}$,and, therefore, this wavelet represents the best compromise between time and frequency concentration. The Morlet wavelet is given by

$$\psi_{\omega_0}(t) = \pi^{-\frac{1}{4}} e^{i\omega_0 t} e^{-\frac{t^2}{2}}, \tag{2}$$

where ω_0 is a localization parameter. Strictly speaking $\psi_{\omega_0}(t)$ is not a true wavelet, however, for sufficiently large ω_0 (e.g. $\omega_0 > 5$), for all practical purposes can be considered as such. For the most common choice of ω_0, $\omega_0 = 6$, we have that $f \simeq \frac{1}{s}$ facilitating the conversion from scales to frequencies. To our knowledge, in economics, every paper uses $\omega_0 = 6$.

[4]The precise requirements are that $|\psi(t)| < C(1 + |t|)^{-(1+\epsilon)}$ and $|\hat{\psi}(f)| < C(1 + |f|)^{-(1+\epsilon)}$, for $C < \infty$, $\epsilon > 0$.

4.1.1 Wavelet and Cross Wavelet Power

In analogy with the terminology used in the Fourier case, the (local) wavelet power spectrum (sometimes called scalogram or wavelet periodogram) is defined as $WPS_x(\tau, s) = |W_x(\tau, s)|^2$. This gives us a measure of the variance distribution of the time-series in the time-scale (frequency) plane.

In our applications, we are interested in detecting and quantifying relationships between two time series. The concepts of cross-wavelet power, cross-wavelet coherency and wavelet phase-difference are natural generalizations of the basic wavelet analysis tools that enable us to deal with the time-frequency dependencies between two time-series.

The cross-wavelet transform of two time-series, $x(t)$ and $y(t)$, is defined as

$$W_{xy}(\tau, s) = W_x(\tau, s)\,\overline{W_y}(\tau, s), \tag{3}$$

where W_x and W_y are the wavelet transforms of x and y, respectively. The cross-wavelet power is simply given by $|W_{xy}(\tau, s)|$. While we can interpret the wavelet power spectrum as depicting the local variance of a time-series, the cross-wavelet power of two time-series depicts the local covariance between these time-series at each time and frequency.

In analogy with the concept of coherency used in Fourier analysis, given two time series $x(t)$ and $y(t)$ one can define their *complex wavelet coherency* ϱ_{xy} by:

$$\varrho_{xy} = \frac{S(W_{xy})}{\left[S\left(|W_x|^2\right) S\left(|W_y|^2\right)\right]^{1/2}}, \tag{4}$$

where S denotes a smoothing operator in both time and scale; smoothing is necessary, because, otherwise, coherency would be identically one at all scales and times.[5] Time and scale smoothing can be achieved by convolution with appropriate windows; see Aguiar-Conraria and Soares (2011c), for details.

The absolute value of the complex wavelet coherency is called the *wavelet coherency* and is denoted by R_{xy}, i.e.

$$R_{xy} = \frac{|S(W_{xy})|}{\left[S\left(|W_x|^2\right) S\left(|W_y|^2\right)\right]^{1/2}}, \tag{5}$$

[5] The same happens with the Fourier coherency.

with $0 \leq R_{xy}(\tau, s) \leq 1$.

The complex wavelet coherency can be written in polar form, as $\varrho_{xy} = \left|\varrho_{xy}\right| e^{i\phi_{xy}}$. The angle ϕ_{xy} is called the *phase-difference* (phase lead of x over y), i.e.

$$\phi_{xy} = \text{Arctan}\left(\frac{\Im\left(S\left(W_{xy}\right)\right)}{\Re\left(S\left(W_{xy}\right)\right)}\right) \tag{6}$$

A phase-difference[6] of zero indicates that the time series move together at the specified time-frequency; if $\phi_{xy} \in (0, \frac{\pi}{2})$, then the series move in phase, but the time series x leads y; if $\phi_{xy} \in (-\frac{\pi}{2}, 0)$, then it is y that is leading; a phase-difference of π (or $-\pi$) indicates an anti-phase relation; if $\phi_{xy} \in (\frac{\pi}{2}, \pi)$, then y is leading; time series x is leading if $\phi_{xy} \in (-\pi, -\frac{\pi}{2})$.

4.1.2 Significance tests

To test for statistical significance of the wavelet coherency we rely on Monte Carlo simulations. However, there are no such tests for the phase-differences, because there is no consensus on how to define the null hypothesis. The advice is that we should only interpret the phase-difference on the regions where coherency is statistically significant.

4.2 Complex Wavelet Coherency Distance Matrix

In this section, we adapt a formula derived by Aguiar-Conraria and Soares (2011b) to find a metric for measuring the distance between a pair of matrices of complex coherencies. Given two $F \times T$ matrices C_x and C_y of complex coherencies, let $C_{xy} = C_x C_y^H$, where C_y^H is the conjugate transpose of C_y, be their covariance matrix. Performing a Singular Value Decomposition (SVD) of this matrix yields

$$C_{xy} = U \Sigma V^H, \tag{7}$$

where the matrices U and V are unitary matrices (i.e. $U^H U = V^H V = I$), and $\Sigma = \text{diag}(\sigma_i)$ is a diagonal matrix with non-negative diagonal elements ordered from highest to lowest, $\sigma_1 \geq \sigma_2 \geq \ldots \geq \sigma_F \geq 0$. The columns, $\mathbf{u}_k$ of the matrix U and the columns $\mathbf{v}_k$ of V

[6] Some authors prefer a slightly different definition, Arctan $\left(\frac{\Im(W_{xy})}{\Re(W_{xy})}\right)$. In this case, one would have $\phi_{xy} = \phi_x - \phi_y$, hence the name phase-difference.

are known, respectively, as the singular vectors for C_x and C_y, and the σ_i are known as the singular values. Let $\mathbf{l}_x^k$ and $\mathbf{l}_y^k$ be the so-called leading patterns, i.e. the $1 \times T$ vectors obtained by projecting each of the matrices C_x and C_y onto the respective k^{th} singular vector (axis):

$$\mathbf{l}_x^k := \mathbf{u}_k^{\mathrm{H}} C_x \quad \text{and} \quad \mathbf{l}_y^k := \mathbf{v}_k^H C_y. \tag{8}$$

It can be shown that each of the matrices C_x and C_y can be written as

$$C_x = \sum_{k=1}^{F} \mathbf{u}_k \mathbf{l}_x^k, \quad C_y = \sum_{k=1}^{F} \mathbf{v}_k \mathbf{l}_y^k, \tag{9}$$

and also that very good approximations can be obtained by using only a small number $K < F$ of terms in the above expressions.

After reducing the information contained in the complex coherency matrices C_x and C_y to a few components, say the K most relevant leading patterns and singular vectors, the idea is to define a distance between the two matrices by appropriately measuring the distances from these components. We compute the distance between two vectors (leading patterns or leading vectors) by measuring the angles between each pair of corresponding segments, defined by the consecutive points of the two vectors, and take the mean of these values. This would be easy to perform if all the values were real. In our case, because we use a complex wavelet, we need to define an angle in a complex vector space. Aguiar-Conraria and Soares (2011b) discuss several alternatives. In this paper, we make use of the Hermitian inner product $\langle \mathbf{a}, \mathbf{b} \rangle_{\mathbb{C}} = \mathbf{a}^H \mathbf{b}$ and corresponding norm $\|\mathbf{a}\| = \sqrt{\langle \mathbf{a}, \mathbf{a} \rangle_{\mathbb{C}}}$ and compute the so-called Hermitian angle between the complex vectors $\mathbf{a}$ and $\mathbf{b}$, $\Theta_H(\mathbf{a}, \mathbf{b})$, by the formula

$$\cos(\Theta_H) = \frac{|\langle \mathbf{a}, \mathbf{b} \rangle_{\mathbb{C}}|}{\|\mathbf{a}\| \|\mathbf{b}\|}, \quad \Theta_H \in [0, \frac{\pi}{2}]. \tag{10}$$

The distance between two complex vectors $\mathbf{p} = (p_1, \ldots, p_M)$ and $\mathbf{q} = (q_1, \ldots, q_M)$ (applicable to the leading patterns and leading vectors) is simply defined by

$$d(\mathbf{p}, \mathbf{q}) = \frac{1}{M-1} \sum_{i=1}^{M-1} \Theta_H \left(\mathbf{s}_i^{\mathbf{p}}, \mathbf{s}_i^{\mathbf{q}} \right) \tag{11}$$

where the i^{th} segment $\mathbf{s}_i^{\mathbf{P}}$ is the two-vector $\mathbf{s}_i^{\mathbf{P}} := (i+1, p_{i+1}) - (i, p_i) = (1, p_{i+1} - p_i)$. To compare the matrix C_x with the complex wavelet coherencies of country x with the corresponding matrix for country y, C_y, we then compute the following distance:

$$\operatorname{dist}(C_x, C_y) = \frac{\sum_{k=1}^{K} \sigma_k^2 \left[d\left(\mathbf{l}_x^k, \mathbf{l}_y^k\right) + d\left(\mathbf{u}_k, \mathbf{v}_k\right) \right]}{\sum_{k=1}^{K} \sigma_k^2}, \tag{12}$$

where σ_k are the k^{th} largest singular values that correspond to the first K leading patterns and K leading vectors.

The above distance is computed for each pair of countries and, with this information, we can then fill a matrix of distances.

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