Fiscal Policy and Macroeconomic Stabilization: What are the Gains from Coordination?

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Abstract

This paper extends the model by Obstfeld and Rogof (2002) to include fiscal policy. We assume that government spending is home biased and welfare enhancing in a non-separable way. The last assumption ensures that increases in government spending affect private expenditure and allows to maintain the log-linearity of the welfare function. We find that from a global perspective, it is optimal to use fiscal policy in response to asymmetric shocks, but not in response to global shocks. However individual countries will have an incentive to unilaterally use fiscal policy in response to global shocks. In a flexible exchange rate regime countries will respond countercyclically to global shocks in the non-cooperative solution, while under a monetary union the incentives are for procyclical responses. Calibrating the model suggests that gains from fiscal policy coordination are likely to be small under a flexible exchange rates, but can instead be high under a monetary union.

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1 Introduction

In the 1980's and 1990's, fears of competitive devaluations and protectionism motivated a first generation of research on the international transmission of shocks and the need for policy coordination. More recently, the formation of the euro area, the swings in the euro-dollar exchange, the escalating deterioration in the US fiscal and current account deficits and the reluctance of Asian countries to let their exchange rates appreciate, have stirred again the debate on the need for international coordination of macroeconomic policies.

The first generation of research in this area was based on the old Keynesian models that did provide a theoretical rationale for policy coordination, but could not generate quantitatively large coordination gains (see Canzoneri et al., 2002a, and references therein). The second generation uses instead "new open economy macroeconomics" (NOEM) models. The NOEM literature, which gained ground with the publication of the seminal Redux model of Obstfeld and Rogoff (1995), is the first influential attempt to substitute the Mundell-Fleming-Dornbush (MFD) model as the workhorse framework for analyzing the international transmission by a micro-founded framework. It builds on the MFD lineage, by considering nominal rigidities, but it provides a rationale for such rigidities through the monopolistic behavior of economic agents. It also substitutes the ad-hoc evaluation of alternative policy regimes by rigorous welfare comparisons, sometimes leading to conclusions which differ substantially from those reached by the old literature.

The initial contributions to the still-emerging second generation of policy coordination models, namely Obstfeld and Rogoff (2002) and Corsetti and Pesenti (2001), which focus on monetary policy, could not produce substantial coordination gains. Canzoneri et al. (2002a) point out the fact that this is due to some simplifying assumptions which ensure analytical tractability but considerably reduce interdependence, making these models at the end as unlikely to produce large gains from coordination as the first-generation old-Keynesian models were. The simplifying assumption of a Cobb-Douglas aggregation for the consumption bundle, for instance, implies that expected employment is either insulated from shocks or is proportional to expected consumption. Therefore, there is no trade-off between the stabilization of consumption and the stabilization of employment, which are the two objectives built into the social welfare function.[1]

When no, or only insignificant, trade-offs are implied by the Nash solution, central banks can mimic or close to mimic the flexible price equilibrium, and since in that case the flexible price solution is equal or is close to the (constrained) optimum, the Nash and the Cooperative solutions coincide or are close to each other. This was one of the main reasons why the old models could only generate gains from achieving efficiency through cooperation that were of a second order when compared to the gains of responding to the shocks themselves (see Canzoneri and Minford, 1988). Apart from the price inertia, there were no other plausible distortions that could drive the cooperative and Nash solutions sufficiently apart.

In the NOEM literature, monopolistic competition is an additional source of inefficiency, but Obstfeld and Rogoff (2002) show that it must still be coupled with other distortions such as financial market frictions or distortionary taxes to generate first-order gains from cooperation. Otherwise, the cooperative solution

[1] Direct utility from money balances tends to be ignored to avoid dealing with the accompanying incentives for central banks to generate surprise inflation or deflation. Direct utility from government spending would create the additional objective of stabilization of government expenditure.

will always target the flexible price equilibrium, which will also be a Nash solution. In their model, which is a stochastic version of the Redux where the intertemporal elasticity of substitution ρ is allowed to be different from 1, such a situation occurs precisely when $\rho = 1$ (the case of the Redux) or when all shocks are symmetric. Whenever these conditions are not met, the sharing of tradable consumption risks is not efficient and there is another distortion in addition to the one caused by monopoly. Hence the optimal cooperative policy will strike a balance between improving the risk sharing and mitigating the price rigidities. However, making ρ differ from one in their model is not sufficient to generate large gains.

These contributions have focused on the coordination of monetary stabilization policies. However, fiscal policies can also play an important stabilization role. In 2002, the US government used tax cuts in order to mitigate the effects of the economic recession, and in the European Monetary Union, there has been a strong debate about whether governments should be allowed more freedom to respond to asymmetric shocks using fiscal policy.[2] In order to investigate whether the same conclusions apply to fiscal policy coordination, this paper extends the model by Obstfeld and Rogoff (2002) to include fiscal policy. We assume home bias in government spending. This guarantees that fiscal expansions may be beneficial, whereas in models where there is no home bias, such as Obstfeld and Rogoff (1995), fiscal expansions tend to be beggar-thy-self. We also assume that government spending affects utility in a non-separable way. Canzoneri, Cumby and Diba (2002b) show that when this is not the case, an increase in government spending does not affect private spending, limiting the scope for fiscal stabilization policies.

Beetsa and Jensen (2002) also include fiscal policy in a NOEM model to analyse the gains from stabilisation using balanced-budget changes in government spending, but their more general model is not log-linear. For this reason they limit themselves to analysing fiscal stabilisation in a monetary union where the fiscal authorities commit to cooperate and therefore maximize the aggregate welfare of the union. They solve the model using a first order approximation and estimate the overall gains from fiscal stabilisation, but not distinguish between the Nash outcomes and the cooperative solution, because this would require a second-order accurate solution to the model. Kim and Kim (2003) estimate gains from international tax policy cooperation using a second-order accurate solution method but in a rather different model. They use a cashless model where consumers derive utility from consumption and leisure in a non-separable way. They also introduce capital in the model and consider costs to capital accumulation. Their results show that in this model the optimal capital and labour tax policies respond procyclically to productivity shocks (positive productivity shocks prompt a reduction in taxes). They find gains from fiscal policy stabilisation of about 0.007% and 0.001% of output, depending on the type of policy analysed, and additional welfare gains from tax policy coordination relative to the Nash outcome of approximately 33%.

In this paper we try to maintain the log-linearity to follow the same solution strategy of Obstfeld and Rogoff (2002), in order to be able to calculate the Nash solution and therefore identify the additional welfare gains achievable through cooperation. Assuming that monetary policy is neutral, we estimate that the gains from fiscal policy coordination under a flexible exchange rate regime are small. Under a monetary union, however, the gains from coordination gain importance and can even be much larger than those estimated

[2] Currently, countries belonging to EMU are limited to a fiscal deficit of 3% of GDP by the Stability and Growth Pact, and have additionally agreed to bring their fiscal positions close to balance.

by Kim and Kim (2003) for reasonable parameter values. When the parameter of risk aversion lies between 2 and 5, the coordination gains are estimated to reach between 48 to 140% of the stabilization gains (the stabilization gains are found to be of a similar order of magnitude to those found by Kim and Kim). In addition we find that a central planner would not find it optimal to use fiscal policy in response to symmetric shocks, while individual countries have an incentive to do so. According to the model, from the point of view of a central planner maximizing world utility fiscal policy should only be used to stabilize asymmetric shocks.

2 A Two Country Model with Fiscal Spending

There are two countries of equal size, home and foreign. Product and labour markets are modelled as in Obstfeld and Rogoff (2002). Home produces differentiated tradable goods in the interval [0,1] while foreign produces differentiated tradable goods in the interval [1,2]. Each country also produces non-tradable goods in the interval [0,1]. Goods are produced out of labour. In the home traded goods sector, for instance, the output of a differentiated good i is given by:

$$Y_H(i) = \left[\int_0^1 \left[L_H(i,j)^{\frac{\phi-1}{\phi}} dj \right]^{\frac{\phi}{\phi-1}} dj \right]$$

where $L_H(i,j)$ is the demand for labour input j by producer i. Production in the other sectors, including the foreign traded and non-traded goods sectors is similar, with the subscripts F, and N, replacing H. The analysis focuses on a single contracting period, hence time subscripts are omitted. Cost minimization implies that firm i's demand function for labor of type j is:

$$L_H(i,j) = \left[\frac{W(j)}{W} \right]^{-\phi} Y(i)$$

where $W(j)$ is the nominal wage of worker j and W is the aggregate wage index, defined as the minimum cost of producing a unit of output:

$$W = \int_0^1 \left[W(j)^{1-\phi} dj \right]^{\frac{1}{1-\phi}}$$

Consumer preferences are modelled in a similar way as in Obstfeld and Rogoff (2002) with the difference that consumers derive direct utility from government spending. Consumers derive utility from a consumption basket C, including tradable goods (home and foreign) and non-tradable goods:

$$C = \frac{(C_T)^{\gamma} (C_N)^{1-\gamma}}{\gamma^{\gamma}(1-\gamma)^{1-\gamma}}$$

whereC_T is the basket of tradable goods, composed by goods produced at home (subscript H) and abroad (subscript F), defined as:

$$C_T = 2C_H^{1/2} C_F^{1/2}$$

Hence it is assumed that there is no home bias in private consumption. The price of these indexes, defined as the minimum expenditure required to purchase one unit is given by:

$$\begin{aligned} P &= (P_T)^{\gamma} (P_N)^{1-\gamma} \\ P_T &= (P_H)^{1/2} (P_N)^{1/2} \end{aligned}$$

The foreign country consumption and price indexes, denoted by $C^*$and P^*, are parallel, with X_j^* substituting X_j, for $X = C, P$ and $j = T, H, F, N$. The baskets of government spending, on the other hand, are assumed to include only national goods (tradable and non-tradable):

$$G = \frac{(G_H)^{\gamma} (G_N)^{1-\gamma}}{\gamma^{\gamma}(1-\gamma)^{1-\gamma}}$$

The price of this index, defined as the minimum expenditure required to purchase one unit is given by:

$$P_G = (P_H)^{\gamma} (P_N)^{1-\gamma}$$

The foreign government spending basket is similar with G_F^* substituting G_H and G_N^* substituting G_N. The price index is $P_G^* = (P_F^*)^{\gamma} (P_N^*)^{1-\gamma}$. Given these definitions, the representative consumer at home maximizes the following expected utility function:

$$E[U] = E\left[\frac{\left(C^i + \frac{P_G}{P} G^i\right)^{1-\rho}}{1-\rho} + \ln\left(\frac{M^i}{P}\right) - \frac{K}{v}\left(Y^i\right)^v\right]$$

where $\rho > 0$ is the coefficient of relative risk aversion, $\frac{M}{P}$ are real money balances and $\nu \geq 1$. It assumes a disutility of labour of the form $-\phi L$, where L is labour and ϕ a positive parameter, and a production function of the form $Y = AL^{\alpha}$, where $\alpha = 1/v$ and $K = v\phi A^{-1/v}$. The variable A is labour productivity (a rise in A is captured by a fall in K).[3] Notice that government spending enters utility in a non-separable way.[4] The maximization of the utility of the representative home consumer is subject to the following budget constraint:

$$PC^i + M^i = M_0^i + PT^i + W(i)L^i + \int_0^1 [\Pi_H(j) + \Pi_N(j)]\, dj$$

where Π stands for firms' profits and T for net transfers form the fiscal and monetary authorities:

$$\begin{aligned} PT^i &= PT^{CB,i} + PT^{G,i} \\ PT^{CB,i} &= M^i - M_0^i \\ PT^{G,i} &= -P_G G^i \end{aligned}$$

where T^{CB} denotes transfers from the monetary authorities and T^G are net transfers from the government (in this case they are negative).

[3] Canzoneri, Cumby and Diba (2002a) show that allowing for different sectoral productivity shocks can increase the potential gains for monetary policy cooperation. This is also likely to be the case for fiscal policy, but we leave this extension for further research.

[4] Ganelli (2003) entends Obstfeld and Rogoff (1995,1996)'s model in a similar way.

In this model, as in Obstfeld and Rogoff (2002) and Corsetti and Pesenti (2001), wages are set one period in advance and remain fixed for one period but prices are allowed to fluctuate. Solving the producers' maximisation problem, gives that in a symmetric equilibrium:

$$\begin{aligned} P_j &= \frac{\theta}{\theta-1}W \qquad j=H,N \\ P_j^* &= \frac{\theta}{\theta-1}W^* \qquad j=F,N \end{aligned} \tag{1}$$

Given the constant-elasticity of demand preferences, the law of one price holds, such that $P_H = \varepsilon P_H^*$ and $P_F = \varepsilon P_F^*$. Consumers set wages by maximizing their expected utility, before knowing the realization of shocks. The condition for optimality is:

$$W = \frac{\phi}{\phi-1}\frac{E\left\{K\left(Y_H+Y_N\right)^v\right\}}{E\left\{\frac{Y_H+Y_N}{P}\left(CF\right)^{-\rho}\right\}} \tag{2}$$

where $F \equiv Y/(Y-\frac{P_G}{P}G)$ is a useful reparameterization of the fiscal stance (F is increasing in G).[5]

A particular feature of this model, which appears also in Obstfeld and Rogoff (2002), is that consumption in traded goods will be equalized across countries, even if overall private and government consumption need not move together. To see this, notice that output market clearing requires:

$$\begin{aligned} P_H\left(Y_H-G_H\right) &= P_H C_H + \mathcal{E}P_H^* C_H^* \\ P_F\left(Y_F-G_F\right) &= P_F C_F + \mathcal{E}P_F^* C_F^* \end{aligned}$$

Given that the commodity demand functions resulting from cost minimisation are given by the following set of equations,

$$\begin{aligned} C_H &= \tfrac{1}{2}\left(\tfrac{P_H}{P}\right)^{-1}C \quad & C_H^* &= \tfrac{1}{2}\left(\tfrac{P_H^*}{P^*}\right)^{-1}C^* \\ C_F &= \tfrac{1}{2}\left(\tfrac{P_F}{P}\right)^{-1}C \quad & C_F^* &= \tfrac{1}{2}\left(\tfrac{P_F^*}{P^*}\right)^{-1}C^* \\ G_H &= \tfrac{1}{2}\left(\tfrac{P_H}{P}\right)^{-1}G \quad & G_H^* &= \tfrac{1}{2}\left(\tfrac{P_H^*}{P^*}\right)^{-1}G^* \\ G_F &= \tfrac{1}{2}\left(\tfrac{P_F}{P}\right)^{-1}G \quad & G_F^* &= \tfrac{1}{2}\left(\tfrac{P_F^*}{P^*}\right)^{-1}G^* \end{aligned}$$

the goods market clearing condition implies that:

$$\frac{P_H\left(Y_H-G_H\right)}{P_F\left(Y_F-G_F\right)} = 1$$

Using also the aggregated budget constraints for the home and foreign economies, it follows that:

$$\frac{P_T C_T}{\mathcal{E}P_T^* C_T^*} = \frac{P_H\left(Y_H-G_H\right)}{P_F\left(Y_F-G_F\right)} \Longrightarrow C_T = C_T^*$$

It will be useful to define home and foreign private consumption spending in units of tradables as Z_c and Z_c^*, such that

$$\begin{aligned} Z_c &\equiv C_T + \frac{P_N}{P_T}C_N \\ Z_c^* &\equiv C_T^* + \frac{P_N^*}{P_T^*}C_N^* \end{aligned}$$

[5] This reparametrisation is also used in Corseti and Pesenti (2001).

and it is possible to show that $Z_c = \frac{1}{\gamma}C_T = \frac{1}{\gamma}C_T^* = Z_c^*$, using the demand functions described above and the current account identity given by:

$$PC = P_H Y_H - P_H G_H + P_N Y_N \Leftrightarrow P_T C_T + P_N C_N = P_T \left[C_T + \frac{P_N}{P_T} C_N \right] \equiv P_T Z_c \tag{3}$$

Finally, from utility maximisation, the optimal condition for money demand is:

$$\frac{M}{P} = \chi (CF)^{\rho} \tag{4}$$

3 Model Solution

The solution method follows a similar strategy as Obstfeld and Rogoff (2002).[6] To simplify the algebra it will be assumed in this solution that $\nu = 1$. In Obstfeld and Rogoff (2002) the gains from coordination are reduced as v increases, and the same should happen in this extension of the model. Therefore by setting $v = 1$ we find an upper bound for such gains. In order to solve the model it is convenient to substitute the output market equilibrium and pricing conditions into the wage equation (2). This gives the optimal relative wage in the home country:

$$\left(\frac{W}{W^*} \right)^{\frac{\rho(1-\gamma)+\gamma}{2}} = \frac{\phi\theta}{(\phi-1)(\theta-1)} \frac{E\left\{ K\mathcal{E}^{1/2} F Z_c \right\}}{E\left\{ \mathcal{E}^{\frac{(1-\gamma)(1-\rho)}{2}} F^{1-\rho} Z_c^{1-\rho} \right\}} \tag{5}$$

which combined with its Foreign analog gives the following equilibrium relative wage equation:

$$\left(\frac{W}{W^*} \right)^{\rho(1-\gamma)+\gamma} = \frac{E\left\{ K\mathcal{E}^{1/2} F Z_c \right\} E\left\{ \mathcal{E}^{-\frac{(1-\gamma)(1-\rho)}{2}} {F^*}^{1-\rho} Z_c^{1-\rho} \right\}}{E\left\{ K^*\mathcal{E}^{-1/2} F^* Z_c \right\} E\left\{ \mathcal{E}^{\frac{(1-\gamma)(1-\rho)}{2}} F^{1-\rho} Z_c^{1-\rho} \right\}} \tag{6}$$

Notice that with no uncertainty:

$$\left(\frac{W}{\mathcal{E}W^*} \right)^{\rho(1-\gamma)+\gamma} = \frac{KF^{\rho}}{K^* {F^*}^{\rho}}$$

In this equation it is possible to observe that there is a positive relationship between fiscal spending and relative wages (recall that $0 \leq \gamma \leq 1$). This occurs because an increase in government spending shifts out the aggregate demand for labour, putting upward pressure on domestic wages. As in Lane and Perotti (2001) this can be identified as the cost channel in the transmission of fiscal policy.

As in Obstfeld and Rogoff (2002) we decompose productivity shocks into world or symmetric productivity shocks, k_w, and relative or asymmetric productivity shocks, k_d, such that:

$$k_w = \frac{k + k^*}{2}; k_d = \frac{k - k^*}{2}$$

[6] The calculations are explained in the Appendix. More details about the solutions can also be found in a sparate "Technical Appendix".

Lower case letters now and throughout the paper denote natural logarithms of the variable labelled with the corresponding upper case letter, e.g., $k = \ln K$. Assume, for simplicity, that $Ek = Ek^* = 0$ and $\sigma_k^2 = \sigma_{k^*}^2$. It follows that $Cov(k_w, k_d) = 0, \sigma_k^2 = \sigma_{kw}^2 + \sigma_{kd}^2$. In addition, it is also assumed that all shocks $\{m, m^*, f, f^*, k, k^*\}$ are jointly normally distributed. In the next step, the relationship between uncertainty and the expected levels of private spending and the terms of trade are derived using equations (5) and (6). Log-linearizing the latter, making use of the normality of shocks, gives the expected value of the logarithm of the terms of trade, defined as in Obstfeld and Rogoff (2002) as $E\tau \equiv Ee + w^* - w = Ee + p_F^* - p_H$:

$$E\tau = \frac{-1}{\rho(1-\gamma)+\gamma}\left\{\begin{array}{c}\rho\left(Ef - Ef^*\right) + \left(1-(1-\gamma)(1-\rho)^2\right)\sigma_{ez_c} \\ +\frac{1-(1-\gamma)(1-\rho)^2}{2}\left(\sigma_{ef}+\sigma_{ef^*}\right) + \left(1-(1-\rho)^2\right)\left(\sigma_{fz_c}-\sigma_{f^*z_c}\right) \\ +\frac{1-(1-\rho)^2}{2}\left(\sigma_f^2-\sigma_{f^*}^2\right)+\sigma_{k_we}+2\sigma_{k_dz_c}+\left(\sigma_{k_wf}-\sigma_{k_wf^*}\right)+\left(\sigma_{k_df}+\sigma_{k_df^*}\right)\end{array}\right\} \tag{7}$$

It can be shown that the logarithm of the expected real exchange rate $Ee + p^* - p$ is equal to $(1-\gamma)\tau$. Log-linearizing equation (5) gives instead the mean world private spending in terms of variances and fiscal spending:

$$Ez_c = -\frac{Ef + Ef^*}{2} + \frac{1}{\rho}\left\{\begin{array}{c}\omega + \lambda - \frac{1}{2\rho}\sigma_k^2 - \frac{1-(1-\gamma)^2(1-\rho)^2}{8}\sigma_e^2 - \frac{1-(1-\rho)^2}{2}\sigma_{z_c}^2 - \frac{1}{2}\sigma_{k_de} \\ -\frac{1-(1-\rho)^2}{4}\left(\sigma_f^2+\sigma_{f^*}^2\right) - \frac{1-(1-\gamma)(1-\rho)^2}{4}\left(\sigma_{ef}-\sigma_{ef^*}\right) \\ -\frac{1-(1-\rho)^2}{2}\left(\sigma_{fz_c}+\sigma_{f^*z_c}\right) - \sigma_{k_wz_c} - \frac{1}{2}\left(\sigma_{k_wf}+\sigma_{k_wf^*}\right) - \frac{1}{2}\left(\sigma_{k_df}-\sigma_{k_df^*}\right)\end{array}\right\} \tag{8}$$

where ω and λ are defined as follows:

$$\begin{aligned}\omega &\equiv \ln\frac{(\phi-1)(\theta-1)}{\phi\theta} + \frac{(1-\rho)}{2\rho}\sigma_k^2 - \lambda \\ \lambda &\equiv \frac{(1-\rho)\gamma\left[\left(1-\frac{\gamma}{2}\right)-(1-\gamma)(1-\rho)\right]}{\rho\left[\rho(1-\gamma)+\gamma\right]^2}\sigma_{k_d}^2\end{aligned}$$

The next step in the solution strategy is to express the variances of the endogenous variables in terms of the exogenous variables. This can be achieved by solving for the sticky-wage equilibrium levels of ex post private expenditure and ex post exchange rate, using the log-linearized version of the money demand equation (4) and its Foreign analog:

$$\begin{aligned}m - p &= \ln\chi + \rho\left(c+f\right) \\ m^* - p^* &= \ln\chi^* + \rho\left(c^*+f^*\right)\end{aligned}$$

Recall also results (1) and (3) and note that they imply:

$$P = \left(\frac{\theta}{\theta-1}\right)\left(\frac{\mathcal{E}W^*}{W}\right)^{\gamma/2} W \tag{9}$$

$$C = \left(\frac{\mathcal{E}W^*}{W}\right)^{\frac{1-\gamma}{2}} Z_c \tag{10}$$

Averaging the two log-linearized money demand equation, assuming that $\chi = \chi^*$ and using the logarithms of (9) and (10) to substitute for prices and consumption, it is possible to obtain:[7]

$$z_c = \frac{1}{2\rho}\left(m+m^*\right) - \frac{1}{2}\left(f+f^*\right) - \frac{1}{2\rho}\left(w+w^*\right) - \frac{1}{\rho}\left[\ln\chi - \ln\left(\frac{\theta}{\theta-1}\right)\right] \tag{11}$$

[7] All calculations are available in the Technical Appendix.

Taking instead the differences of the log-linearized equations and substituting again for c,c^*,p and p^* yields:

$$e = \frac{m - m^*}{\rho(1-\gamma)+\gamma} - \frac{(1-\gamma)(1-\rho)(w-w^*)}{\rho(1-\gamma)+\gamma} - \frac{\rho(f-f^*)}{\rho(1-\gamma)+\gamma} \tag{12}$$

Notice that an increase in government spending generates nominal appreciation. This would be the exchange rate, or terms-of-trade, channel of fiscal policy transmission identified in Lane and Perotti (2001).

At this stage it is possible to solve explicitly for the expected utility. Taking into account that expenditure on money services is small relative to that on other goods, the welfare implications of the different policy regimes will be evaluated in the limiting case of $\chi \to 0$, hence the welfare measure to be analysed reduces to:

$$EU = E\left\{\frac{(CF)^{1-\rho}}{1-\rho} - \frac{K}{v}L\right\}$$

Using the condition for the optimal choice of wages and also results (1) and (3), it is possible to write:

$$E\left\{\frac{K}{\nu}L\right\} = \frac{(\phi-1)(\theta-1)}{\nu\phi\theta}E\left\{(CF)^{1-\rho}\right\}$$

Substituting in the expected utility and recalling (10) yields:[8]

$$EU = \frac{\phi\theta-(1-\rho)(\phi-1)(\theta-1)}{\phi\theta(1-\rho)}\exp\left\{\begin{array}{c} \frac{(1-\gamma)(1-\rho)}{2}E\tau + (1-\rho)(Ef+Ez_c) + \frac{(1-\gamma)^2(1-\rho)^2}{8}\sigma_e^2 + \frac{(1-\rho)^2}{2}\left(\sigma_f^2+\sigma_{z_c}^2\right) \\ +\frac{(1-\gamma)(1-\rho)^2}{2}\sigma_{z_ce} + \frac{(1-\gamma)(1-\rho)^2}{2}\sigma_{fe} + (1-\rho)^2\sigma_{z_cf} \end{array}\right\}$$

The foreign analog being:

$$EU^* = \frac{\phi\theta-(1-\rho)(\phi-1)(\theta-1)}{\phi\theta(1-\rho)}\exp\left\{\begin{array}{c} -\frac{(1-\gamma)(1-\rho)}{2}E\tau + (1-\rho)(Ef^*+Ez_c) + \frac{(1-\gamma)^2(1-\rho)^2}{8}\sigma_e^2 + \frac{(1-\rho)^2}{2}\left(\sigma_{f^*}^2+\sigma_{z_c}^2\right) \\ -\frac{(1-\gamma)(1-\rho)^2}{2}\sigma_{z_ce} + \frac{(1-\gamma)(1-\rho)^2}{2}\sigma_{f^*e} + (1-\rho)^2\sigma_{z_cf^*} \end{array}\right\}$$

4 Levels of Welfare

In order to simplify the analysis and make the results more clear it important to solve first for the flexible price levels of utility in the Home and Foreign economies. It can be shown (the proof is in the Appendix) that under flexible wages the level of expected utility, denoted by a tilde is equal to:

$$E\widetilde{U} = E\widetilde{U}^* = \frac{\phi\theta-(1-\rho)(\phi-1)(\theta-1)}{\phi\theta(1-\rho)}\exp\left\{\left(\frac{(1-\rho)\omega}{\rho}\right)\right\}$$

Now Home utility under sticky wages can be written in terms of the flexible wage utility level, using (7) and (8):

$$\begin{aligned} EU &= E\widetilde{U}\exp\{(1-\rho)\Omega(\rho)\} \\ \Omega(\rho) &= \Omega_w(\rho)+\Omega_d(\rho) \end{aligned}$$

[8]The proofs are given in the Appendix.

where $\Omega_w(\rho)$ contains the terms that affect both countries in the same way (symmetric component of welfare), while $\Omega_d(\rho)$ contains only the terms that affect countries with opposite sign:

$$\begin{aligned}\Omega_w(\rho) &= -\frac{\left(\sigma^2_{k_w}+\sigma^2_{k_d}\right)}{2\rho^2}+\frac{\lambda}{\rho}-\frac{\sigma^2_{z_c}}{2}-\frac{\left[1-(1-\gamma)^2(1-\rho)\right]\sigma^2_e}{8\rho}-\frac{\sigma_{k_w z_c}}{\rho}-\frac{\sigma_{k_d e}}{2\rho}-\frac{[\rho(1-\gamma)+\gamma]\left(\sigma_{ef}-\sigma_{ef^*}\right)}{4\rho} \quad (13)\\ &\quad -\frac{\left(\sigma_{f z_c}+\sigma_{f^* z_c}\right)}{4}-\frac{\left(\sigma^2_f+\sigma^2_{f^*}\right)}{4}-\frac{\left(\sigma_{k_w f}+\sigma_{k_w f^*}\right)}{2\rho}-\frac{\left(\sigma_{k_d f}-\sigma_{k_d f^*}\right)}{2\rho}\\ \Omega_d(\rho) &= -\frac{\rho(1-\gamma)\sigma_{e z_c}}{2[\rho(1-\gamma)+\gamma]}-\frac{(1-\gamma)\sigma_{k_w e}}{2[\rho(1-\gamma)+\gamma]}-\frac{(1-\gamma)\sigma_{k_d z_c}}{[\rho(1-\gamma)+\gamma]}-\frac{\rho(1-\gamma)\left(\sigma_{ef}+\sigma_{ef^*}\right)}{4[\rho(1-\gamma)+\gamma)]}-\frac{(\rho-\gamma)\left(\sigma_{f z_c}-\sigma_{f^* z_c}\right)}{2[\rho(1-\gamma)+\gamma]} \quad (14)\\ &\quad -\frac{(\rho-\gamma)\left(\sigma^2_f-\sigma^2_{f^*}\right)}{4[\rho(1-\gamma)+\gamma)]}-\frac{(1-\gamma)\left(\sigma_{k_w f}-\sigma_{k_w f^*}\right)}{2[\rho(1-\gamma)+\gamma]}-\frac{(1-\gamma)\left(\sigma_{k_d f}+\sigma_{k_d f^*}\right)}{2[\rho(1-\gamma)+\gamma]}\end{aligned}$$

Here we assume that $Ef = E\widetilde{f}$ and $Ef^* = E\widetilde{f}^*$, since the aim is to focus on the stabilisation component of fiscal policy. The welfare in Foreign is given by:

$$\begin{aligned}EU^* &= E\widetilde{U}\exp\left\{(1-\rho)\,\Omega^*(\rho)\right\}\\ \Omega^*(\rho) &= \Omega_w(\rho)-\Omega_d(\rho)\end{aligned}$$

5 Fiscal Stabilisation under Flexible Exchange Rates

In the following discussion only policy rules will be considered, hence the authorities will not try to use surprises to raise employment and output systematically. Under these rules, money supplies and fiscal stances respond to productivity shocks, which the authorities observe after the wages are set:

$$\begin{aligned}\widehat{m} &= m-Em=-\delta_d\widehat{\kappa}_d-\delta_w\widehat{\kappa}_w\\ \widehat{m}^* &= m^*-Em^*=\delta^*_d\widehat{\kappa}_d-\delta^*_w\widehat{\kappa}_w\\ \widehat{f} &= f-Ef=-\eta_d\widehat{\kappa}_d-\eta_w\widehat{\kappa}_w\\ \widehat{f}^* &= f-Ef=\eta^*_d\widehat{\kappa}_d-\eta^*_w\widehat{\kappa}_w\end{aligned}$$

Hence, the ex post levels of private spending and the exchange rate will be given by:

$$\widehat{z}=\frac{1}{2\rho}\left(\widehat{m}+\widehat{m}^*\right)-\frac{1}{2}\left(\widehat{f}+\widehat{f}^*\right)=-\frac{\left(\delta_d-\delta^*_d\right)-\rho\left(\eta_d-\eta^*_d\right)}{2\rho}\widehat{k}_d-\frac{\left(\delta_w+\delta^*_w\right)-\rho\left(\eta_w+\eta^*_w\right)}{2\rho}\widehat{k}_w \quad (15)$$

$$\widehat{e}=\frac{\widehat{m}-\widehat{m}^*}{\rho(1-\gamma)+\gamma}-\frac{\rho\left(\widehat{f}-\widehat{f}^*\right)}{\rho(1-\gamma)+\gamma}=-\frac{\left(\delta_d+\delta^*_d\right)-\rho\left(\eta_d+\eta^*_d\right)}{\rho(1-\gamma)+\gamma}\widehat{k}_d-\frac{\left(\delta_w-\delta^*_w\right)-\rho\left(\eta_w-\eta^*_w\right)}{\rho(1-\gamma)+\gamma}\widehat{k}_w \quad (16)$$

From these expressions it is possible to calculate the variances and covariances needed to evaluate welfare in terms of the variances of exogenous shocks and policy parameters. These are presented in the Appendix.

5.1 The Cooperative Solution

The cooperative solution is defined as the one which maximizes the weighted sum of the utilities in both economies. It is possible to show that, as in Obstfeld and Rogoff (2002):

$$Max\left(\frac{1}{2}EU+\frac{1}{2}EU^*\right)\Rightarrow Max\ \Omega_w(\rho)$$

since

$$\Omega^w(\rho) \equiv \frac{1}{2}\Omega(\rho) + \frac{1}{2}\Omega^*(\rho) = \frac{1}{2}\Omega_w(\rho) + \frac{1}{2}\Omega_d(\rho) + \frac{1}{2}\Omega_w(\rho) - \frac{1}{2}\Omega_d(\rho) = \Omega_w(\rho)$$

Now all that is needed is to rewrite $\Omega_w(\rho)$ in terms of the variances and covariances of exogenous shocks and policy parameters and maximize it over the parameters of interest. Although it is clear from equations (15) and (16) that there are important interactions between fiscal and monetary policy, at this stage, for simplicity, the focus will be only on fiscal policy. Hence it is assumed that $\delta_j = \delta_j^* = 0, j = w, d$.

Proposition 1 *Fiscal Policy cannot increase the symmetric level of welfare by responding to symmetric shocks.*

In a symmetric equilibrium with $\eta_w = \eta_w^*$, and $\eta_d = \eta_d^* = 0$, it is possible to show that the symmetric component of utility is given by

$$\Omega_w(\rho) = -\frac{\left(\sigma_{k_w}^2 + \sigma_{k_d}^2\right)}{2\rho^2} + \frac{\lambda}{\rho} - \frac{\sigma_{z_c}^2}{2} - \frac{\sigma_{k_w z_c}}{\rho} - \frac{\left(\sigma_{f z_c} + \sigma_{f^* z_c}\right)}{4} - \frac{\left(\sigma_f^2 + \sigma_{f^*}^2\right)}{4} - \frac{\left(\sigma_{k_w f} + \sigma_{k_w f^*}\right)}{2\rho}$$

With a neutral monetary policy, inducing a correlation between the world fiscal stance and symmetric productivity shocks is counterweighted by a crowding-out effect on private spending. Since it can be shown that:

$$\sigma_{z_c}^2 = -\frac{\left(\sigma_{f z_c} + \sigma_{f^* z_c}\right)}{2}$$

$$\sigma_{k_w z_c} = -\frac{\left(\sigma_{k_w f} + \sigma_{k_w f^*}\right)}{2}$$

More formally, after rewriting $\Omega_w(\rho)$ in terms of the variances and covariances of exogenous shocks and policy parameters, it is possible to show that, in a symmetric equilibrium with $\eta_w = \eta_w^*$, the first order condition with respect to η_w simplifies to (see Appendix for details):

$$\frac{\partial \Omega_w(\rho)}{\partial \eta_w} = -\frac{1}{2}\eta_w \sigma_{k_w}^2 \tag{17}$$

Any value of η_w above zero reduces welfare, because the only effect of fiscal policy is to increase the variability of fiscal spending, which reduces welfare. Hence the maximum is reached when:

$$\eta_w^{coop} = 0$$

In the cooperative equilibrium it is optimal for the fiscal authorities not to respond to a symmetric shock.

Proposition 2 *When $\rho > 1$ fiscal policies that respond countercyclically to asymmetric shocks can improve world welfare.*

After rewriting $\Omega_w(\rho)$ in terms of the variances and covariances of exogenous shocks and policy parameters, it is possible to show that, in a symmetric equilibrium with $\eta_d = \eta_d^*$, the first order condition with respect to η_d simplifies to (see Appendix for details):

$$\frac{\partial \Omega_w(\rho)}{\partial \eta_d} = \frac{\gamma^2(1-\rho)\eta_d}{2\left[\rho(1-\gamma)+\gamma\right]^2}\sigma_{k_d}^2 + \frac{\gamma(1-\rho)}{2\left[\rho(1-\gamma)+\gamma\right]}\sigma_{k_d}^2 \tag{18}$$

and the maximum is reached when:

$$\eta_d^{coop} = -\frac{\rho\left(1-\gamma\right)+\gamma}{\gamma\rho} \text{ if } \rho \geq 1 \quad \vee \quad \eta_d^{coop} = 0 \text{ if } \rho < 1$$

Notice that the optimal cooperative response to a negative asymmetric productivity shock k_d is countercyclical when $\rho > 1$, and it is more countercyclical the larger the share of non-traded goods in consumption (in Lane and Perotti, 2001, fiscal policy transmission also depends on the share of non-traded goods). In order to understand the intuition behind this result, notice that in a symmetric equilibrium (see Appendix) with $\eta_d = \eta_d^*$, and $\eta_w = \eta_w^* = 0$, $\Omega_w\left(\rho\right)$ reduces to:

$$\Omega_w\left(\rho\right) = -\frac{\left(\sigma_{k_w}^2+\sigma_{k_d}^2\right)}{2\rho^2} + \frac{\lambda}{\rho} - \frac{\left[1-(1-\gamma)^2(1-\rho)\right]\sigma_e^2}{8\rho} - \frac{\left(\sigma_f^2+\sigma_{f^*}^2\right)}{4} - \frac{\sigma_{k_d e}}{2\rho} - \frac{\left[\rho+\gamma(1-\rho)\right]\left(\sigma_{ef}-\sigma_{ef^*}\right)}{4\rho} - \frac{\left(\sigma_{k_d f}-\sigma_{k_d f^*}\right)}{2\rho}$$

A countercyclical response to asymmetric shocks has two opposite effects on world welfare. Firstly it increases world welfare with the stabilization of the countries' terms-of-trade. It induces both a negative covariance between the relative fiscal stance and the nominal exchange rate and a negative covariance between the negative productivity shock and the nominal exchange rate, meaning that the exchange rate of the country that is hit appreciates restoring its terms-of-trade trade. Secondly it can reduce welfare through the "crowding-out" of aggregate demand, since a positive covariance between the fiscal stance and the negative productivity shock has a negative impact on expected private spending (see equation 8). In this case, the terms-of-trade effect is high enough to compensate the "crowding-out" effect. In addition, when $\rho > 1$, the terms-of-trade effect is also large enough to compensate for the negative effect that fiscal stabilization (either procyclical or countercyclical) has on welfare, through the increase in exchange rate and government expenditure variability. When $\rho < 1$ this is not the case and the best cooperative policy is no response also in the case of asymmetric shocks.

5.2 The Nash Solution

In the alternative scenario, the authorities do not cooperate and undertake policy stabilisation by playing Nash. Hence the fiscal authority in the Home economy maximizes EU taking the policy parameters of Foreign as given. Notice that:

$$MaxEU \Rightarrow Max\left\{\underbrace{\Omega_w\left(\rho\right)}_{global\ component} + \underbrace{\Omega_d\left(\rho\right)}_{country-specific\ component}\right\}$$

Hence the Nash solutions for η_j can be found by equalizing the following sum to zero (provided that the second order condition for a maximum hold):

$$\frac{\partial\Omega_w\left(\rho\right)}{\partial\eta_j} + \frac{\partial\Omega_d\left(\rho\right)}{\partial\eta_j}$$

Proposition 3 *The asymmetric component of welfare cannot be improved by the fiscal stabilisation of asymmetric shocks. Therefore the Nash responses to asymmetric shocks do not deviate from the cooperative solution.*

Stabilizing asymmetric shocks using fiscal policy cannot induce real exchange rate changes that would result in expenditure switching. To see this notice that in a symmetric equilibrium with $\eta_d = \eta_d^*$, and $\eta_w = \eta_w^* = 0$, $\Omega_d(\rho)$ reduces to:

$$\Omega_d(\rho) = -\frac{\rho(1-\gamma)\sigma_{ez_c}}{2[\rho(1-\gamma)+\gamma]} - \frac{(1-\gamma)\sigma_{k_d z_c}}{[\rho(1-\gamma)+\gamma]} - \frac{\rho(1-\gamma)\left(\sigma_{ef}+\sigma_{ef^*}\right)}{4[\rho(1-\gamma)+\gamma]} - \frac{(\rho-\gamma)\left(\sigma_{fz_c}-\sigma_{f^*z_c}\right)}{2[\rho(1-\gamma)+\gamma]} - \frac{(\rho-\gamma)\left(\sigma_f^2-\sigma_{f^*}^2\right)}{4[\rho(1-\gamma)+\gamma]} - \frac{(1-\gamma)\left(\sigma_{k_d f}+\sigma_{k_d f^*}\right)}{2[\rho(1-\gamma)+\gamma]}$$

Asymmetric shocks have an impact in the real exchange rate. It can be shown that responding to these shocks using an asymmetric policy tool such as fiscal policy offsets that impact, such that:

$$\begin{aligned} \sigma_{k_d z_c} &= -\frac{(\sigma_{k_d f} + \sigma_{k_d f^*})}{2} \\ \sigma_{fz_c} - \sigma_{f^* z_c} &= -\frac{\left(\sigma_f^2 - \sigma_{f^*}^2\right)}{2} \\ \sigma_{ez_c} &= -\frac{(\sigma_{ef} + \sigma_{ef^*})}{2} \end{aligned}$$

It is immediate to see that $\Omega_d(\rho)$ turns out to be independent of η_d, hence $\eta_d^{nash} = \eta_d^{*nash} = \eta_d^{coop}$. The Nash and cooperative solutions to an asymmetric shock coincide.

Proposition 4 *Domestic fiscal stabilization of symmetric shocks can increase home welfare by increasing the asymmetric component of welfare, at the cost of a lower welfare abroad.*

Responding to asymmetric shocks using fiscal policy can instead affect the real exchange rate. It can be shown that when $\eta_w \neq 0$, the asymmetric component of utility is given by:

$$\Omega_d(\rho) = -\frac{(1-\gamma)\sigma_{k_w e}}{2[\rho(1-\gamma)+\gamma]} - \frac{(1-\gamma)\left(\sigma_{k_w f} - \sigma_{k_w f^*}\right)}{2[\rho(1-\gamma)+\gamma]}$$

which in terms of the exogenous shocks and policy parameters can be written as:

$$\Omega_d(\rho) = \frac{\gamma(1-\gamma)(1-\rho)}{2[\rho(1-\gamma)+\gamma]^2}(\eta_w - \eta_w^*)\sigma_{kw}^2$$

Notice that the first derivative of this expression with respect to η_w is given by:

$$\frac{\partial \Omega_d(\rho)}{\partial \eta_w} = \frac{\gamma(1-\gamma)(1-\rho)}{2[\rho(1-\gamma)+\gamma]^2} \tag{19}$$

The Nash solution can now be obtained by equalizing the sum of this derivative with the first derivative of $\Omega_w(\rho)$ relative to η_w, given in (17), to zero. Combining (17) and (19) gives:

$$\frac{\partial \Omega_w(\rho)}{\partial \eta_w} + \frac{\partial \Omega_d(\rho)}{\partial \eta_w} = -\frac{1}{2}\eta_w + \frac{\gamma(1-\gamma)(1-\rho)}{2[\rho(1-\gamma)+\gamma]^2}$$

And this expression takes the value zero when:

$$\eta_w^{nash} = \eta_w^{*nash} = -\frac{\gamma(1-\gamma)(\rho-1)}{[\rho(1-\gamma)+\gamma]^2}$$

In the Nash solution, the optimal response to a symmetric negative productivity shock is countercyclical if $\rho > 1$ and procyclical if $\rho < 1$. The intensity of the response depends on the size of the relative size of the tradable sector relatively to the parameter of risk aversion. When $\rho > 1$, for instance, $\left|\partial\eta_w^{nash}/\partial\gamma\right|$ is positive when $\rho > \gamma/(1-\gamma)$, since:

$$\left|\frac{\partial\eta_w^{nash}}{\partial\gamma}\right| = \frac{(\rho-1)(\rho(1-\gamma)-\gamma)}{[\rho(1-\gamma)+\gamma]^3}$$

When $\rho = 1$ the Nash solution coincides with the optimal cooperative solution, as in Obstfeld and Rogoff in the case of monetary policy. Notice in equation (7) that a countercyclical fiscal policy by the home country has to effects of opposite direction on the expected real exchange rate. First it increases (depreciates) the real exchange rate by inducing a negative correlation between the negative productivity shock and the exchange rate. This occurs because a negative correlation between the productivity shock and the exchange rate means that demand will be shifted away from home goods (because of the appreciating nominal exchange rate) when home workers are less productive and have a higher disutility of labour, and as a consequence they can lower their pre-set wages and improve home's competitiveness. On the other hand, a positive correlation between the negative productivity shock and the fiscal stance means that the government increases its demand for home goods precisely when the disutility from labour is high, which will lead workers to increase their pre-set wages and hamper home's competitiveness. When $\rho > 1$, the transmission of fiscal policy to the nominal exchange rate is stronger, as can be seen in equation (16), hence the first channel dominates and the optimal policy is countercyclical. When $\rho < 1$, however, the second channel dominates and the optimal policy is procyclical instead (so that the government's demand for home goods is low when the disutility of labour is high). Notice that in either case the optimal policy will always be beggar-thy-neighbour because the opposite effect will prevail in the foreign country. To see this, notice that home and foreign welfare as a function of the home fiscal policy response to symmetric shocks ($\eta_w^* = 0$) are given by:

$$\Omega(\rho) = -\frac{1}{2\rho^2}\left(\sigma_{k_w}^2 + \sigma_{k_d}^2\right) + \frac{\lambda}{\rho} - \frac{1}{4}\eta_w^2\sigma_{k_w}^2 - \frac{\gamma(1-\gamma)(1-\rho)}{2[\rho(1-\gamma)+\gamma]^2}\eta_w\sigma_{kw}^2$$

$$\Omega^*(\rho) = -\frac{1}{2\rho^2}\left(\sigma_{k_w}^2 + \sigma_{k_d}^2\right) + \frac{\lambda}{\rho} - \frac{1}{4}\eta_w^2\sigma_{k_w}^2 + \frac{\gamma(1-\gamma)(1-\rho)}{2[\rho(1-\gamma)+\gamma]^2}\eta_w\sigma_{kw}^2$$

5.3 The Gains from Fiscal Cooperation under Flexible Exchange rates

At this stage it is possible to calculate the extra welfare gains that can be achieved through policy cooperation as a percentage of the potential stabilisation gains. First notice that in a symmetric equilibrium welfare in the flexible exchange rate regime can be written as:

$$\Omega(\rho)^{FLEX} = \underbrace{-\frac{1}{2\rho^2}\left(\sigma_{k_w}^2 + \sigma_{k_d}^2\right) + \frac{\lambda}{\rho}}_{no\ response} \underbrace{-\frac{1}{2}\eta_{w,FLEX}^2\sigma_{k_w}^2}_{loss\ from\ fiscal\ competition} + \underbrace{\left(\frac{\gamma(1-\rho)}{\rho(\rho(1-\gamma)+\gamma)}\eta_{d,FLEX} - \gamma^2\frac{(\rho-1)}{2[\rho(1-\gamma)+\gamma]^2}\eta_{d,FLEX}^2\right.}_{gains\ from\ asymmetric\ stabilization} \quad (20)$$

Overall, responding to symmetric shocks using fiscal policy makes both countries always worse of at the end. The resulting (potential) loss will be labelled "*loss from fiscal competition*". Under cooperation the value of this loss is zero, since the cooperative solution implies $\eta_{w,FLEX}^{nash} = 0$. The potential loss, in the absence of cooperation, can be calculated by substituting he policy parameter by the optimal Nash solution estimated in the previous section:

$$\textit{loss from fiscal competition} = \frac{\gamma^2 (1-\gamma)^2 (\rho - 1)^2}{2 (\rho (1-\gamma) + \gamma)^4}$$

On the other hand, stabilizing asymmetric shocks has a positive effect on the welfare of both countries, when $\rho > 1$. The resulting gains will be referred to as "*gains from asymmetric stabilization.* We can quantify these gains by substituting the policy parameter by the optimal value estimated in the previous section. Recall that this value is the same with and without coordination:

$$\textit{gains from asymmetric stabilization} = \frac{(\rho - 1)}{2\rho^2}$$

In order to compare the magnitude of the gains from fiscal stabilization with the gains from cooperation, it is common in the literature to quantify the gains from stabilization as the gains from moving from a no-response solution to the Nash solution, while the net gains from cooperation are simply the gains from moving from the Nash solution to the cooperative solution. In this model, the gains from stabilization as a percentage of the mean flexible-wage output level, can be written as:

$$GS \equiv \Omega(\rho)_{Nash}^{FLEX} - \Omega(\rho)_{NR}^{FLEX} = \textit{gains from asymmetric stabilization} - \textit{loss from fiscal competition}$$

where $\Omega(\rho)_{Nash,FLEX}$ is the value of $\Omega(\rho)_{FLEX}$ when the Nash policies are implemented, and $\Omega(\rho)_{NR,FLEX}$ is the value of $\Omega(\rho)_{FLEX}$ when the authorities do not respond to any shocks, that is, when $\eta_j = 0, j = w, d$. Similarly, we can write the gains from cooperation as a percentage of the mean flexible-wage output level as:

$$GC \equiv \Omega(\rho)_{Coop}^{FLEX} - \Omega(\rho)_{Nash}^{FLEX} = \textit{gains from asymmetric stabilization}$$

where $\Omega^{Coop}(\rho)$ is the value of $\Omega(\rho)$ when the optimal cooperative policies are implemented. Then, the ratio between the gain from moving from "no response" to the Nash solution to the gain from moving from the Nash solution to the cooperative solution, the R-ratio, measures the extra welfare gains of policy cooperation relatively to the Nash equilibrium:

$$R^{FLEX} = \frac{\Omega(\rho)_{Coop}^{FLEX} - \Omega(\rho)_{Nash}^{FLEX}}{\Omega(\rho)_{Nash}^{FLEX} - \Omega(\rho)_{NR}^{FLEX}}$$

To evaluate the maximum potential gains from fiscal policy coordination that could be achieved in this version of the model, the gains GS and GC are simulated for different values of ρ. The variance of shocks is set to 0.01 and γ to 0.6, exactly as in Obstfeld and Rogoff (2002). In this calibration, the gains from fiscal cooperation are at most 14% of the gains from fiscal stabilization. Table 1 summarizes the results. In this version of the model the size of the gains in comparable to that of the gains found by Obstfeld and Rogoff (2002) for monetary policy. In this case, the gains from cooperation appear to be only of second order when compared to the gains from independent fiscal stabilisation policies.

Table 1: Gains from fiscal policy stabilization and coordination under flexible exchange rates (percent of output), for different values of the coefficient of risk aversion ρ, and $\gamma = 0.6$.

	$\rho = 1$	$\rho = 2$	$\rho = 3$	$\rho = 5$	$\rho = 8$
(i) Stabilization gain	0.0000	0.1175	0.1001	0.0699	0.0479
(ii) Coordination gain	0	0.0075	0.011	0.0101	0.0068
(iii) Ratio 100x(ii)/(i)	0	6.3802	10.9589	14.4225	14.1235

However, as mentioned earlier the results are not independent of the relative size of the non-tradable sector. To show that results vary with the choice of the parameter γ, but not independently of the choice of ρ, the results for $\gamma = 0.75$ are shown in Table 2. As γ increases the gains from cooperation are higher when $\rho > 2$, because the Nash response to a symmetric shock become stronger in that case, deviating more from the cooperative solution.

Table 2: Gains from fiscal policy stabilization and coordination under flexible exchange rates (percent of output), for different values of the coefficient of risk aversion ρ, and $\gamma = 0.75$.

	$\rho = 1$	$\rho = 2$	$\rho = 3$	$\rho = 5$	$\rho = 8$
(i) Stabilization gain	0.4200	0.0842	0.0309	0.0082	0.0023
(ii) Coordination gain	0.0800	0.0408	0.0247	0.0118	0.0055
(iii) Ratio 100x(ii)/(i)	19.0476	48.4848	80.0000	144.9275	243.8095

Notice that, both in Table 1 and in Table 2, as ρ increases, the absolute value of the gains from cooperation increase when $\rho < \frac{2-\gamma}{1-\gamma}$, and decrease when $\rho > \frac{2-\gamma}{1-\gamma}$ (the threshold is equal to 3.5 when $\gamma = 0.6$ and to 5 when $\gamma = 0.75$). This occurs because, in the first case, governments will choose to respond more to symmetric shocks when they do not cooperate, increasing the potential loss from fiscal competition, while the opposite will occur in the second case. On the other hand, in these examples, the gains from stabilization always decline as ρ increases. This is due to the fact that as ρ increases the stabilization of asymmetric shocks is lower. Even when ρ is sufficiently high, so that an increase in ρ may lead to a fall in the loss from fiscal competition, the fall in the gains from the stabilization of asymmetric shocks still dominates (assuming that the variances of symmetric and asymmetric shocks are of the same magnitude). The rates at which both types of gains (stabilization and coordination) fall as ρ increases is not very different when $\gamma = 0.6$, hence the ratio R^{FLEX} does not change much with ρ in Table 1. When $\gamma = 0.75$, the rate at which the fiscal competition decreases is lower than the rate at which the gains from the stabilization of asymmetric shocks decline, therefore, the gains from coordination gain more importance as ρ increases, reaching 38% of the stabilization gains when $\rho = 8$. In the next section we test whether these results remain true under a monetary union.

6 Fiscal Stabilization in a Monetary Union

Under this regime the two countries join a monetary union. In this case equation (12) becomes irrelevant, because at all times:

$$\widehat{e} = 0$$

The union has a common monetary policy given by:

$$\widehat{m}^w = \widehat{m} + \widehat{m}^* = -\delta_w^w \widehat{\kappa}_w$$

These assumptions imply that in a monetary union ex post private consumption will be determined by fiscal policy in the following way:

$$\widehat{z} = \frac{1}{2\rho}\widehat{m}^w - \frac{1}{2}\left(\widehat{f} + \widehat{f}^*\right) = \frac{(\eta_d - \eta_d^*)}{2}\widehat{k}_d - \frac{(\delta_w^w) - \rho\,(\eta_w + \eta_w^*)}{2\rho}\widehat{k}_w$$

It is now possible to calculate the variances and covariances needed to evaluate welfare under this regime. These are shown in the Appendix.

6.1 The Cooperative Solution

In the next step, we calculate the cooperative solution, which requires maximizing $\Omega_w(\rho)$. Once more, in order to focus on fiscal policy, it will be assumed that $\delta_w^w = 0$. As show in the Appendix, the expression for $\Omega_w(\rho)$ in this case is given by:

$$\Omega_w(\rho) = -\frac{\left(\sigma_{k_w}^2 + \sigma_{k_d}^2\right)}{2\rho^2} + \frac{\lambda}{\rho} - \frac{\left(\eta_d^2 + \eta_d^{2*}\right)\sigma_{k_d}^2 + \left(\eta_w^2 + \eta_w^{2*}\right)\sigma_{k_w}^2}{4} + \frac{1}{2\rho}\left(\eta_d + \eta_d^*\right)\sigma_{k_d}^2$$

Proposition 5 *In a monetary union it remains true that fiscal policy cannot increase the symmetric level of welfare by responding to symmetric shocks*

Under a monetary union, inducing a correlation between fiscal policy and symmetric productivity shocks is counterweighted by the crowding-out effect on private spending, in the same way as in the flexible exchange rate case. When monetary policy is neutral, the exchange rate regime does not change the correlation between private spending and global shocks and fiscal spending and global shocks. More formally, the optimal cooperative response to a symmetric shock is again found by calculating the derivative of $\Omega_w(\rho)$ with respect to η_w, which under symmetry is equal to:

$$\frac{\partial \Omega_w(\rho)}{\partial \eta_w} = -\frac{v}{2}\eta_w \sigma_{k_w}^2$$

implying that the optimum is achieved when:

$$\eta_w^{coop} = 0$$

Proposition 6 *In a monetary union, fiscal policies responding procyclically to asymmetric shocks can improve welfare.*

The optimal cooperative response to an asymmetric shock can be found by calculating the derivative of $\Omega_w(\rho)$ with respect to η_d. Recalling that in the symmetric equilibrium $\eta_d = \eta_d^*$, it is possible to write:

$$\frac{\partial \Omega_w(\rho)}{\partial \eta_d} = -\frac{1}{2}\eta_d \sigma_{k_d}^2 + \frac{1}{2\rho}\sigma_{k_d}^2$$

implying that the maximum us achieved at

$$\eta_d^{coop} = \frac{1}{\rho}$$

Notice that in this case, in the symmetric equilibrium, with $\eta_w = 0$, $\Omega_w(\rho)$ reduces to:

$$\Omega_w(\rho) = -\frac{\left(\sigma_{k_w}^2 + \sigma_{k_d}^2\right)}{2\rho^2} + \frac{\lambda}{\rho} - \frac{1}{4}\eta_d^2\sigma_{k_d}^2 + \frac{1}{2\rho}\eta_d\sigma_{k_d}^2$$

This is a concave parabola whatever the level of the parameter ρ. In this case the "terms-of-trade" channel of fiscal policy, which calls for counter-cyclical policies, is shut, because the exchange rate is kept fixed, and only the "crowding-out" channel, which calls for procyclical policies operates. Hence, the optimal cooperative response to asymmetric shocks is a procyclical response.

6.2 The Nash Solution

The Nash solutions can also be found in the same way as before, by calculating the derivatives of $\Omega_d(\rho)$ with respect to η_d and η_w respectively, and combining those with the derivatives of $\Omega_w(\rho)$ with respect to η_d and η_w.

Proposition 7 *Under a monetary union it remains true that the asymmetric component of welfare cannot be improved by the fiscal stabilisation of asymmetric shocks. Therefore, also in this regime, the Nash responses to asymmetric shocks do not deviate from the cooperative solution.*

In this case, it remains true that the stabilization of asymmetric shocks using fiscal policy cannot influence the real exchange rate. When monetary policy is neutral, the exchange rate regime does not change the correlation between private spending and asymmetric shocks and fiscal spending and asymmetric shocks, and the correlation between the exchange rate and fiscal and private spending are zero. In this case $\Omega_d(\rho)$ is given by:

$$\Omega_d(\rho) = \frac{(1-\gamma)}{2\left[\rho(1-\gamma)+\gamma\right]}\left(\eta_w - \eta_w^*\right)\sigma_{kw}^2$$

and is independent of η_d. Hence:

$$\eta_d^{nash} = \eta_d^{*nash} = \eta_d^{coop}$$

Proposition 8 *A procyclical fiscal policy response to symmetric shocks can increase home welfare by increasing the asymmetric component of welfare, at the cost of a lower welfare abroad:*

Calculating the derivative of $\Omega(\rho)$ with respect to η_w gives:

$$\frac{\partial\Omega(\rho)}{\partial\eta_w} \equiv \frac{\Omega_w(\rho)}{\partial\eta_w} + \frac{\Omega_d(\rho)}{\partial\eta_w} = -\frac{\nu}{2}\eta_w + \frac{1-\gamma}{2[\rho(1-\gamma)+\gamma]}$$

Hence the optimum is achieved when:

$$\eta_w^{nash} = \eta_w^{*nash} = \frac{1-\gamma}{\rho(1-\gamma)+\gamma}$$

which is equal to $(1-\gamma)$ when $\rho = 1$. Under a monetary union (or more generally under a symmetric fixed exchange rate) the Nash solution differs from the cooperative solution even when $\rho = 1$.[9] The optimal Nash response to a symmetric shock is procyclical, independently of the value of ρ, and in this case the intensity of the response is stronger the lower the value of γ:

$$\left|\frac{\partial\eta_w^{nash}}{\partial\gamma}\right| = -\frac{1}{[\rho(1-\gamma)+\gamma]^2}$$

Notice also that this policy is at the expense of the other country because, in this case, home and foreign welfare as a function of the home fiscal policy response to symmetric shocks is given by:

$$\Omega(\rho) = -\frac{1}{2\rho^2}\left(\sigma_{k_w}^2 + \sigma_{k_d}^2\right) + \frac{\lambda}{\rho} - \frac{1}{4}\eta_w^2\sigma_{k_w}^2 + \frac{1-\gamma}{2[\rho(1-\gamma)+\gamma]}\eta_w\sigma_{kw}^2$$

$$\Omega^*(\rho) = -\frac{1}{2\rho^2}\left(\sigma_{k_w}^2 + \sigma_{k_d}^2\right) + \frac{\lambda}{\rho} - \frac{1}{4}\eta_w^2\sigma_{k_w}^2 - \frac{1-\gamma}{2[\rho(1-\gamma)+\gamma]}\eta_w\sigma_{kw}^2$$

In this case, the "nominal exchange rate" channel of fiscal policy, which calls for countercyclical policies, is shut, therefore the Nash response to a symmetric shock is procyclical. A procyclical response by the home country has a positive effect on the competitiveness of the home country (home minus foreign) by inducing a negative correlation between expected relative wages (home minus foreign) and the negative world productivity shock, via the wage channel of fiscal policy. This policy has a negative effect on the foreign country, by inducing positive correlation between a negative world shock to productivity and expected relative wages in the foreign country (foreign minus domestic).

[9] In a fixed exchange rate regime relative monetary policy must adjust in order to keep the exchange rate fixed. In this model that would require:

$$m - m^* = \rho\left(\widehat{f} - \widehat{f}^*\right)$$

In a symmetric regime the burden of the adjustment would fall equally on the two countries such that the world money supply would remain constant:

$$\begin{aligned} m &= \frac{\rho}{2}\left(\widehat{f} - \widehat{f}^*\right) \\ m^* &= \frac{\rho}{2}\left(\widehat{f}^* - \widehat{f}\right) \\ m + m^* &= 0 \end{aligned}$$

6.3 The Gains from Fiscal Cooperation under a Monetary Union

Notice that, in a symmetric equilibrium, welfare in a monetary union can be written as:

$$\Omega^{MU}(\rho) = \underbrace{-\frac{1}{2\rho^2}\left(\sigma_{k_w}^2 + \sigma_{k_d}^2\right) + \frac{\lambda}{\rho}}_{\text{no response}} \underbrace{-\frac{1}{2}\eta_{w,MU}^2\sigma_{k_w}^2}_{\text{loss from fiscal competition}} \underbrace{+\left(\frac{1}{\rho}\eta_{d,MU}\sigma_{k_d}^2 - \frac{1}{2}\eta_{d,MU}^2\sigma_{k_d}^2\right)\sigma_{k_d}^2}_{\text{gains from asymmetric stabilization}} \tag{21}$$

Substituting the policy parameters for the optimal solutions found in the previous section, allows to estimate the potential "*loss from fiscal competition*" and the "*gains from asymmetric stabilization*", in a monetary union. It is possible to show (see Appendix) that this loss is larger than the loss under flexible exchange rates when $\rho < \frac{2\gamma}{2\gamma-1}$ (this threshold is equal to 6 when $\gamma = 0.6$ and to 3 when $\gamma = 0.75$):

$$\textit{loss from fiscal competition} = \frac{(1-\gamma)^2}{(\rho(1-\gamma)+\gamma)^2}$$

On the other hand, the gains from asymmetric stabilization will be higher in the flexible exchange rate regime for any value of ρ above 2. This occurs because in the flexible exchange rate regime the stabilization gains are obtained through the *nominal exchange rate channel*, which in this case dominates the *crowding-out channel*, through which the gains are achieved in a monetary union.

$$\textit{gains from asymmetric stabilization} = \frac{1}{2\rho^2}$$

Table 3 summarizes the simulations for the potential gains under this regime when $\gamma = 0.6$. In a monetary union, the gains from coordination gain relative importance. Notice again that in this case the Nash and Cooperative solutions differ even when $\rho = 1$. For $\rho > 1$, the gains from fiscal stabilisation are smaller under this regime. This occurs because in a monetary union the *gains from asymmetric stabilization* tend to be lower than under flexible exchange rates for the reasons stated above, while the potential the loss from fiscal competition tends to be higher. Nevertheless, the stabilization gains calculated for this regime are still comparable in size to other results in the literature (see Kim and Kim , 2003). On the other hand, the gains from cooperation gain absolute (for $\rho < \frac{2\gamma}{2\gamma-1}$) and relative importance in this regime. In a monetary union, in order to reap the potential gains from fiscal stabilization, member countries must cooperate; if they do not, fiscal policy will be excessively procyclical (because it will respond both to asymmetric and to symmetric shocks) and, as a result, most what can be achieved is lost through "*fiscal competition*".

Table 3: Gains from fiscal policy stabilization and coordination under a monetary union (percent of output), for different values of the coefficient of risk aversion ρ and $\gamma = 0.6$.

	$\rho = 1$	$\rho = 2$	$\rho = 3$	$\rho = 5$	$\rho = 8$
(i) Stabilization gain	0.4200	0.0842	0.0309	0.0082	0.0023
(ii) Coordination gain	0.0800	0.0408	0.0247	0.0118	0.0055
(iii) Ratio 100x(ii)/(i)	19.0476	48.4848	80.0000	144.9275	243.8095

As before, the results change with the relative size of the non-tradable sector, but in this regime (under which the Nash responses are always procyclical) we observe the opposite from what we observed under flexible exchange rates. As γ increases the gains from cooperation are lower, because, in this case, the Nash response to a symmetric shock become less strong as the size of the tradable sector increases, deviating less from the cooperative solution. Table 4 shows how the results change when γ is increased from 0.6 to 0.75.

Table 4: Gains from fiscal policy stabilization and coordination under a monetary union (percent of output), for different values of the coefficient of risk aversion ρ and $\gamma = 0.75$.

	$\rho = 1$	$\rho = 2$	$\rho = 3$	$\rho = 5$	$\rho = 8$
(i) Stabilization gain	0.4688	0.105	0.0417	0.0122	0.0037
(ii) Coordination gain	0.0313	0.02	0.0139	0.0078	0.0041
(iii) Ratio 100x(ii)/(i)	6.6667	19.0476	33.3333	64.1026	112.2807

Under this regime, as ρ increases, the absolute value of the gains from cooperation always decreases. This occurs because as ρ increases governments will always choose to respond less to symmetric shocks when they do not cooperate, decreasing the potential loss from fiscal competition. As before, the gains from stabilization always decline as ρ increases. This is due to the fact that as ρ increases the stabilization of asymmetric shocks is lower and the decline in the "*gains from asymmetric stabilization*" is enough to offset the decline is the "*loss from fiscal competition*". The rate at which the fiscal competition decreases is much lower than the rate at which the gains from the stabilization of asymmetric shocks decline, therefore, the ratio R^{MU} increases significantly as ρ increases, and when $\rho = 8$ it reaches 243% if $\gamma = 0.6$ and 112% if $\gamma = 0.75$. Hence, in this case, the results depend more significantly on the value chosen for the coefficient of risk aversion ρ and unfortunately empirical estimates vary considerably. Some like Eichenbaum et al. (1988) found a range between 0.5 and 3, while others such as Hall (1988) find values greater than 5. While there is more support for values greater than 1, it is difficult to agree on a more precise magnitude.

7 Conclusions

This paper has extended the model by Obstfeld and Rogoff (2002) to include fiscal policy, in order to quantify the potential gains from the coordination of fiscal stabilization policies. The model assumes home bias in government spending, in order to avoid beggar-thyself fiscal expansions. It is also assumed that government spending is valued by consumers in a non-separable way. This ensures that fiscal policy affects private expenditure decisions.

One main conclusion is drawn regarding the use of fiscal policies for stabilisation purposes. It is not optimal from the perspective of a central planner to use fiscal policy in response to symmetric shocks. Using fiscal policy to stabilize symmetric shocks is beggar-thy-neighbour. When countries play Nash, they will suboptimally choose to respond to symmetric shocks using fiscal policy because they do not internalize the negative spillover they impose on the other country via a change in the real exchange rate. In the Nash solution, countries will choose to stabilize symmetric shocks countercyclically under flexible exchange rates and procyclically in a monetary union. This occurs because under flexible exchange rates the nominal exchange rate channel of fiscal policy dominates for reasonable parameter values (when the parameter of risk aversion is greater than one), and the fiscal authority can induce a negative covariance between a negative global productivity shock and its nominal exchange rate, such that demand shifts away from home goods when home workers are less productive and their disutility of labour is high. This policy is beggar-thy-neighbour because the opposite will be felt in the home country, since the shock is symmetric. On the other hand, in a monetary union, this channel is shut and the dominating channel is instead the cost channel of fiscal policy, which calls for procyclical policies. A positive correlation between a negative global shock and the fiscal stance induces a negative correlation between the negative shock and relative wages, improving the competitiveness of the stabilizing country at the expense of the other country which will see their relative wages rise when the country is hit by a global shock.

On the other hand, fiscal policy is useful for stabilising asymmetric shocks even from a global perspective. Interestingly, in this model the optimal cooperative and Nash responses to an asymmetric shock do not differ. For reasonable values of the parameter of risk aversion (most empirical estimates point to values greater than one), the optimal fiscal policy response to asymmetric shocks should be countercyclical under flexible exchange rates and procyclical under a monetary union. This occurs because under a flexible exchange rate the nominal exchange rate channel of fiscal policy dominates once more, and by inducing a negative covariance between negative asymmetric productivity shocks and the nominal exchange rate (the nominal exchange rate of the country that is hit appreciates restoring its terms of trade), fiscal policy can increase expected private demand. Under a monetary union, this channel is shut and the dominant channel is the crowding-out channel of fiscal spending which calls for procyclical fiscal policies, since planned private expenditure increases when there is a negative correlation between negative asymmetric shocks and the fiscal stance (the government liberates resources to the private sector when they are reduced).

When calibrating the model to estimate the potential gains from fiscal policy coordination, we conclude that under a flexible exchange rate regime they are small, reaching at most 14% of the stabilization gains for the most reasonable parameterizations. Under a monetary union, the gains from unilateral fiscal stabilization are lower but can be improved significantly through cooperation. Under this regime, the gains from coordination reach as much as 48% of the stabilization gains for the parameter of risk aversion as low as 2. Therefore, if countries forming a monetary union are to use fiscal policy as stabilization tools, they must cooperate, otherwise most of what they can achieve is lost through relative wage competition. Relaxing some of the assumptions of the model could increase the coordination gains under a flexible exchange rate regime. However that will imply loosing the log-linearity of the welfare measure, requiring a second order approximation of the model to obtain an accurate measure of welfare (see Kim and Kim, 2002). Many authors have found a way around the problem by calculating only the cooperative solutions (in the cooperative solution the first order terms drop from the welfare function and a first order solution to the model is enough to measure the changes in the joint utility). Such a strategy allows to estimate the overall gains from fiscal stabilisation, but not distinguish between the Nash outcomes and the cooperative solution. This distinction requires a second-order accurate solution to the model to calculate the levels of welfare in the Nash solution. Kim and Kim (2003) estimate gains from international tax policy cooperation using a second-order accurate solution method but use a cashless model as a means of simplification. They find that in a cashless model the optimal tax policies are procyclical, a result which is consistent with our findings under a monetary union.

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A Appendix: Mathematical Solutions

A.1 Price setting

Firm j sets prices by maximizing profits its Π_j:

$$Max\Pi_j \equiv P_j Y_j - W L_j \qquad j = H, N$$

subject to a labour demand equation and to the demand curves faced by the firm (the labour demand curve is found by maximizing the production function subject to the wage cost constraint, while the product demand curves are found by maximizing the consumption bundles subject to an expenditure constraint):

$$L(i) = \left[\frac{W(i)}{W}\right]^{-\phi} Y_j(i) \tag{22}$$

$$\begin{array}{lll} C_H(i) = \left[\frac{P_H(i)}{P_H}\right]^{-\theta} C_H & C_H^*(i) = \left[\frac{P_H(i)}{P_H}\right]^{-\theta} C_H^* & G_H(i) = \left[\frac{P_H(i)}{P_H}\right]^{-\theta} G_H \\ C_N(i) = \left[\frac{P_H(i)}{P_H}\right]^{-\theta} C_N & G_N(i) = \left[\frac{P_H(i)}{P_H}\right]^{-\theta} G_N & \end{array}$$

Substituting for the constraints, this implies maximizing:

$$\int_0^1 P_j(i) \left[\frac{P_j(i)}{P_j}\right]^{-\theta} \left(C_j^w + G_j\right) - W \left[\frac{W(i)}{W}\right]^{-\phi} \left[\frac{P_j(i)}{P_j}\right]^{-\theta} \left(C_j^w + G_j\right) di$$

Calculating the first order derivative with respect to $P(i)_j$ and equalizing it to zero gives the following first order condition:

$$(1-\theta) \left[\frac{P_j(i)}{P_j}\right]^{-\theta} \left(C_j^w + G_j\right) + \theta W \left[\frac{W(i)}{W}\right]^{-\phi} \left[\frac{P_j(i)}{P_j}\right]^{-\theta} P_j(i)^{-1} \left(C_j^w + G_j\right) = 0$$

In a symmetric equilibrium $W(i) = W, \forall_i$ and $P(i) = P, \forall_i$, hence this first order condition implies that:

$$P_j = \frac{\theta}{\theta - 1} W \qquad j = H, N$$

$$P_j^* = \frac{\theta}{\theta - 1} W^* \qquad j = F, N$$

A.2 Pre-set wages

Workers pre-set their wages in order to maximize their expected utility:

$$MaxE\left[U^i\right] = E\left[\frac{\left(C^i + \frac{P_G}{P} G\right)^{1-\rho}}{1-\rho} + \ln\left(\frac{M^i}{P}\right) - \frac{K}{v}\left(L^i\right)^v\right]$$

subject to the labour demand given by (22). Equalizing the first order derivative with respect to W^i to zero gives the following first order condition:

$$E\left\{\frac{\left[C^i \frac{C^i + \frac{P_G}{P} G}{C^i}\right]^{-\rho}}{P}(1-\phi)\left[\frac{W(i)}{W}\right]^{-\phi} Y(j)\right\} + E\left\{\phi K \left(\left[\frac{W(i)}{W}\right]^{-\phi} Y(j)\right)^v W(i)^{=1}\right\} = 0 \tag{23}$$

which can be simplified to

$$E\left\{\frac{\left[C^iF^i\right]^{-\rho}}{P}(1-\phi)\left[\frac{W(i)}{W}\right]^{-\phi}L^i\right\}+E\left\{\phi K\left(L^i\right)^v W(i)^{=1}\right\}=0$$

using $F \equiv \frac{C+\frac{P_G}{P}G}{C} = \frac{PC+P_GG}{PC} = \frac{P_HY}{P_H(Y-G)} = \frac{Y}{Y-G}$, the national income identity and $P_H = P_N$. By symmetry $W(i) - W$ and $L = Y_H + Y_N$: Hence the optimum pre-set wage will be given by:

$$W=\frac{\phi}{\phi-1}\frac{E\left\{K\left(Y_H+Y_N\right)^v\right\}}{E\left\{\frac{Y_H+Y_N}{P}\left(CF\right)^{-\rho}\right\}} \tag{24}$$

In the sections that follow it will be assumed, as in the text that $v = 1$.

A.3 Goods market equilibrium

The goods market equilibrium requires that:

$$\begin{aligned} P_H\left(Y_H-G_H\right) &= P_HC_H+\mathcal{E}P_H^*C_H^* \\ P_F\left(Y_F-G_F\right) &= P_FC_F+\mathcal{E}P_F^*C_F^* \end{aligned} \tag{25}$$

Recall that the demand curves for each good are given by:

$$\begin{array}{ll} C_H=\frac{1}{2}\left(\frac{P_H}{P}\right)^{-1}C & C_H^*=\frac{1}{2}\left(\frac{P_H^*}{P^*}\right)^{-1}C^* \\ C_F=\frac{1}{2}\left(\frac{P_F}{P}\right)^{-1}C & C_F^*=\frac{1}{2}\left(\frac{P_F^*}{P^*}\right)^{-1}C^* \\ G_H=\frac{1}{2}\left(\frac{P_H}{P}\right)^{-1}G & G_H^*=\frac{1}{2}\left(\frac{P_H^*}{P^*}\right)^{-1}G^* \\ G_F=\frac{1}{2}\left(\frac{P_F}{P}\right)^{-1}G & G_F^*=\frac{1}{2}\left(\frac{P_F^*}{P^*}\right)^{-1}G^* \end{array}$$

Notice that substituting these demand curves into (25) and dividing the first equation by the second gives the following result:

$$\frac{P_H\left(Y_H-G_H\right)}{P_F\left(Y_F-G_F\right)}=1$$

Notice also that using the private and government budget constraints, it is possible to write:

$$\frac{P_TC_T}{\mathcal{E}P_T^*C_T^*}=\frac{P_H\left(Y_H-G_H\right)}{P_F\left(Y_F-G_F\right)}\Longrightarrow C_T=C_T^*$$

Using now the national income identity, it also follows that:

$$PC=P_HY_H-P_HG_H+P_NY_N=P_TC_T+P_NC_N=P_T\left[C_T+\frac{P_N}{P_T}C_N\right]=P_TZ_c \tag{26}$$

$$P_GG=P_HG_H+P_NG_N=P_T\left[\frac{P_H}{P_T}G_H+\frac{P_N}{P_T}G_N\right]=P_TZ_g$$

Additionally, from the demand curves for traded and non-traded goods it also follows that:

$$\left\{\begin{array}{c} C_T=\gamma\left(\frac{P_T}{P}\right)^{-1}C \\ C_N=(1-\gamma)\left(\frac{P_N}{P}\right)^{-1}C \end{array}\right. \Rightarrow \frac{P_N}{P_T}C_N=\frac{1-\gamma}{\gamma}C_T$$

Substituting this result in (26) also allows to conclude that $Z_c = Z_c^*$.

$$Z_c = \frac{1}{\gamma} C_T = \frac{1}{\gamma} C_T^* = Z_c^*$$

A.4 Equilibrium pre-set wages and market equilibrium

The price setting conditions $P_N = P_H = \left(\frac{\theta}{\theta-1}\right) W$, derived before, allow us to write the following expressions, which will be useful to calculate the equilibrium pre-set wages:

$$\begin{aligned} P_T &= (P_H)^{1/2} (P_F)^{1/2} = \left(\frac{\theta}{\theta-1}\right) W^{1/2} (\mathcal{E} W^*)^{1/2} \\ P_G &= (P_H)^{\gamma} (P_N)^{1-\gamma} = \left(\frac{\theta}{\theta-1}\right) W \end{aligned}$$

$$\frac{P_T}{P} = \frac{P_T}{(P_T)^{\gamma} (P_N)^{1-\gamma}} = \left(\frac{P_T}{P_N}\right)^{1-\gamma} = \left(\frac{\mathcal{E} W^*}{W}\right)^{\frac{1-\gamma}{2}}$$

$$P = (P_T)^{\gamma} (P_N)^{1-\gamma} = \left(\left(\frac{\theta}{\theta-1}\right) W^{1/2} (\mathcal{E} W^*)^{1/2}\right)^{\gamma} \left(\left(\frac{\theta}{\theta-1}\right) W\right)^{1-\gamma} = \left(\frac{\theta}{\theta-1}\right) \left(\frac{\mathcal{E} W^*}{W}\right)^{\gamma/2} W$$

$$C = \frac{P_T}{P} Z_c = \left(\frac{\mathcal{E} W^*}{W}\right)^{\frac{1-\gamma}{2}} Z_c$$

In addition, using the the national income identity, it is also possible to find that:

$$\begin{aligned} P_N Y_N &= P_N C_N + P_N G_N \Leftrightarrow Y_N = (1-\gamma) \frac{PC}{P_N} + (1-\gamma) \frac{P_G G}{P_N} \\ P_H Y_H &= P_T C_T + P_T G_T \Leftrightarrow Y_H = \gamma \frac{PC}{P_H} + \gamma \frac{P_G G}{P_H} \end{aligned}$$

Since $P_N = P_H = \left(\frac{\theta}{\theta-1}\right) W$ the two previous expressions can be combined to yield:

$$(Y_H + Y_N) = \frac{1}{P_H} (PC + P_G G) = \left(\frac{\mathcal{E} W^*}{W}\right)^{1/2} F Z_c \tag{27}$$

where we have used the definition $F \equiv Y/(Y - \frac{P_G}{P} G)$. Notice that:

$$F \equiv \frac{Y}{Y - \frac{P_G}{P} G} = \frac{C + \frac{P_G}{P} G}{C} = 1 + \frac{P_G G}{PC} = 1 + \frac{P_G G}{Z_c}$$

Now, substituting these market equilibrium conditions into the expression for the equilibrium pre-set wages (24) allows us to obtain the following equilibrium condition:

$$\left(\frac{W}{W^*}\right)^{\frac{1-(1-\gamma)(1-\rho)}{2}} = \frac{\phi\theta}{(\phi-1)(\theta-1)} \frac{E\left\{K \mathcal{E}^{1/2} F Z_c\right\}}{E\left\{\mathcal{E}^{\frac{(1-\gamma)(1-\rho)}{2}} F^{1-\rho} Z_c^{1-\rho}\right\}} \tag{28}$$

and its foreign analog.

$$\left(\frac{W^*}{W}\right)^{\frac{1-(1-\gamma)(1-\rho)}{2}} = \frac{\phi\theta}{(\phi-1)(\theta-1)} \frac{E\left\{K^*\mathcal{E}^{-1/2}F^*Z_c\right\}}{\left\{\mathcal{E}^{-\frac{(1-\gamma)(1-\rho)}{2}}F^{*^{1-\rho}}Z_c^{1-\rho}\right\}}$$

Dividing the two previous expressions gives a third condition.

$$\left(\frac{W}{W^*}\right)^{1-(1-\gamma)(1-\rho)} = \frac{E\left\{K\mathcal{E}^{1/2}FZ_c\right\}E\left\{\mathcal{E}^{-\frac{(1-\gamma)(1-\rho)}{2}}F^{*^{1-\rho}}Z_c^{1-\rho}\right\}}{E\left\{K^*\mathcal{E}^{-1/2}F^*Z_c\right\}E\left\{\mathcal{E}^{\frac{(1-\gamma)(1-\rho)}{2}}F^{1-\rho}Z_c^{1-\rho}\right\}} \tag{29}$$

A.5 Solutions for mean private spending and terms of trade

Now we log-linearize the equilibrium wage equation (29):

$$\left\{\begin{array}{c} [\gamma(1-\rho)+\gamma](w-w^*) + E\left(k^*-\frac{1}{2}e+f^*+z_c\right) + \frac{1}{2}V\left(k^*-\frac{1}{2}e+f^*+z_c\right) \\ +E\left(\frac{(1-\gamma)(1-\rho)}{2}e+(1-\rho)f+(1-\rho)z_c\right) + \frac{1}{2}V\left(\frac{(1-\gamma)(1-\rho)}{2}e+(1-\rho)f+(1-\rho)z_c\right) \end{array}\right\}$$

$$= \left\{\begin{array}{c} E\left(k+\frac{1}{2}e+f+z_c\right)+\frac{1}{2}V\left(k+\frac{1}{2}e+f+z_c\right)+E\left(-\frac{(1-\gamma)(1-\rho)}{2}e+(1-\rho)f^*+(1-\rho)z_c\right) \\ +\frac{1}{2}V\left(-\frac{(1-\gamma)(1-\rho)}{2}e+(1-\rho)f^*+(1-\rho)z_c\right) \end{array}\right\}$$

Applying the expectations and variance operators allows to further write:

$$\left\{\begin{array}{c} [\gamma(1-\rho)+\gamma](w-w^*-Ee)-\rho(Ef-Ef^*) \\ +\frac{1}{2}\left(\begin{array}{c} \sigma_{k^*}^2+\frac{1^2}{4}\sigma_e^2+\sigma_{f^*}^2+\sigma_{z_c}^2 \\ -\sigma_{k^*e}+2\sigma_{k^*f^*}+2\sigma_{k^*z_c} \\ -\sigma_{ef^*}-\sigma_{ez_c}+2\sigma_{f^*z_c} \end{array}\right) + \frac{1}{2}\left(\begin{array}{c} \frac{(1-\gamma)^2(1-\rho)^2}{4}\sigma_e^2+(1-\rho)^2\sigma_f^2+(1-\rho)^2\sigma_{z_c}^2 \\ +(1-\gamma)(1-\rho)^2\sigma_{ef}+(1-\gamma)(1-\rho)^2\sigma_{ez_c} \\ +2(1-\rho)^2\sigma_{fz_c} \end{array}\right) \end{array}\right\}$$

$$= \left\{\begin{array}{c} \frac{1}{2}\left(\sigma_k^2+\frac{1}{4}\sigma_e^2+\sigma_f^2+\sigma_{z_c}^2+\sigma_{ke}+2\sigma_{kf}+2\sigma_{kz_c}+\sigma_{ef}+\sigma_{ez_c}+2\sigma_{fz_c}\right) \\ +\frac{1}{2}\left(\begin{array}{c} \frac{(1-\gamma)^2(1-\rho)^2}{4}\sigma_e^2+(1-\rho)^2\sigma_{f^*}^2+(1-\rho)^2\sigma_{z_c}^2 \\ -(1-\gamma)(1-\rho)^2\sigma_{ef^*}-(1-\gamma)(1-\rho)^2\sigma_{ez_c}+2(1-\rho)^2\sigma_{f^*z_c} \end{array}\right) \end{array}\right\}$$

This expression can be simplified to yield:

$$E\tau = \frac{-1}{v-(1-\gamma)(1-\rho)}\left[\begin{array}{c} \rho(Ef-Ef^*)+\left(1-(1-\gamma)(1-\rho)^2\right)\sigma_{ez_c}+\frac{1-(1-\gamma)(1-\rho)^2}{2}(\sigma_{ef}+\sigma_{ef^*}) \\ +\left(1-(1-\rho)^2\right)(\sigma_{fz_c}-\sigma_{f^*z_c})+\frac{1-(1-\rho)^2}{2}\left(\sigma_f^2-\sigma_{f^*}^2\right) \\ +\sigma_{k_we}+2\sigma_{k_dz_c}+(\sigma_{k_wf}-\sigma_{k_wf^*})+(\sigma_{k_df}+\sigma_{k_df^*}) \end{array}\right] \tag{30}$$

using the definition $E\tau \equiv Ee + w^* - w$,

Log-linearizing instead the Home's wage equation (28) gives:

$$\begin{aligned} & \frac{1}{2}[\gamma(1-\rho)+\gamma](w-w^*-Ee)-\rho Ez_c-\rho Ef+ \\ & +\frac{1}{2}\left(\frac{(1-\gamma)^2(1-\rho)^2}{4}\sigma_e^2+(1-\rho)^2\sigma_f^2+(1-\rho)^2\sigma_{z_c}^2+(1-\gamma)(1-\rho)^2\sigma_{ef}+(1-\gamma)(1-\rho)^2\sigma_{ez_c}+2(1-\rho)^2\sigma_{fz_c}\right) \\ = \; & \ln\frac{\phi\theta}{(\phi-1)(\theta-1)}+Ek+\frac{1}{2}\left(\sigma_k^2+\frac{1}{4}\sigma_e^2+\sigma_f^2+\sigma_{z_c}^2+\sigma_{ke}+2\sigma_{kf}+2\sigma_{kz_c}+\sigma_{ef}+\sigma_{ez_c}+2\sigma_{fz_c}\right) \end{aligned}$$

that is

$$
\begin{aligned}
& \frac{1}{2}\left[\gamma(1-\rho)+\gamma\right](w-w^*-Ee)-\rho E z_c \\
= \ & \ln\frac{\phi\theta}{(\phi-1)(\theta-1)}+Ek+\rho Ef+\frac{1}{2}\left(\begin{array}{c}\sigma_k^2+\frac{1-(1-\gamma)^2(1-\rho)^2}{4}\sigma_e^2+\left(1-(1-\rho)^2\right)\sigma_f^2+\left(1-(1-\rho)^2\right)\sigma_{z_c}^2 \\ +\left(1-(1-\gamma)(1-\rho)^2\right)\sigma_{ef}+\left(1-(1-\gamma)(1-\rho)^2\right)\sigma_{ez_c} \\ +2\left(1-(1-\rho)^2\right)\sigma_{fz_c}+\sigma_{ke}+2\sigma_{kf}+2\sigma_{kz_c}\end{array}\right)
\end{aligned}
$$

This expression can be further simplified using () and the assumption $Ek = Ek^* = 0$ to yield:

$$
Ez_c = -\frac{Ef+Ef^*}{2}+\frac{1}{\rho}\left\{\begin{array}{c}\omega+\lambda-\frac{1}{2\rho}\sigma_k^2-\frac{1-(1-\gamma)^2(1-\rho)^2}{8}\sigma_e^2-\frac{1-(1-\rho)^2}{2}\sigma_{z_c}^2-\frac{1}{2}\sigma_{k_d e} \\ -\frac{1-(1-\rho)^2}{4}\left(\sigma_f^2+\sigma_{f^*}^2\right)-\frac{1-(1-\gamma)(1-\rho)^2}{4}\left(\sigma_{ef}-\sigma_{ef^*}\right) \\ -\frac{1-(1-\rho)^2}{2}\left(\sigma_{fz_c}+\sigma_{f^*z_c}\right)-v\sigma_{k_w z_c}-\frac{1}{2}\left(\sigma_{k_w f}+\sigma_{k_w f^*}\right)-\frac{1}{2}\left(\sigma_{k_d f}-\sigma_{k_d f^*}\right)\end{array}\right\} \tag{31}
$$

where:

$$
\begin{aligned}
\omega &\equiv \ln\frac{(\phi-1)(\theta-1)}{\phi\theta}+\frac{(1-\rho)}{2\rho}\sigma_k^2-\lambda \\
\lambda &= \frac{(1-\rho)\gamma\left[\left(1-\frac{\gamma}{2}\right)-(1-\gamma)(1-\rho)\right]}{\rho\left[\gamma(1-\rho)+\gamma\right]^2}\sigma_{k_d}^2
\end{aligned}
$$

A.6 Solutions for ex-post spending and ex post exchange rate

Taking the logs of the money demand equations gives the following linearized conditions:

$$
\begin{aligned}
m-p &= \ln\chi+\rho(c+f) \\
m^*-p^* &= \ln\chi^*+\rho(c^*+f^*)
\end{aligned}
$$

Averaging the two assuming that $\chi=\chi^*$ yields:

$$
\frac{1}{2}(m+m^*)=\ln\chi+\frac{\rho}{2}(c+c^*)+\frac{\rho}{2}(f+f^*)+\frac{1}{2}(p+p^*) \tag{32}
$$

Recall that

$$
\begin{aligned}
C &= \frac{P_T}{P}Z_c=\left[\frac{W}{\mathcal{E}W^*}\right]^{-\frac{1-\gamma}{2}}Z_c \\
C^* &= \left[\frac{W}{\mathcal{E}W^*}\right]^{\frac{1-\gamma}{2}}Z_c
\end{aligned}
$$

Given that $P_H = P_N = \left(\frac{\theta}{\theta-1}\right)W$. and $z_c = z_c^*$, these conditions imply that $(c+c^*) = 2z_c$. Now to obtain $p+p^*$ notice that:

$$
PP^* = \left(\frac{\theta}{\theta-1}\right)^2 WW^*
$$

Hence:

$$
p+p^* = 2\ln\left(\frac{\theta}{\theta-1}\right)+w+w^*
$$

Substituting these results into (32) allows to obtain the expost level of private spending:

$$z_c = \frac{1}{2\rho}(m+m^*) - \frac{1}{2}(f+f^*) - \frac{1}{2\rho}(w+w^*) - \frac{1}{\rho}\left[\ln\chi - \ln\left(\frac{\theta}{\theta-1}\right)\right] \tag{33}$$

Taking instead the differences of the money demand equations allows to write:

$$(m-m^*) = \rho(c-c^*) + \rho(f-f^*) + (p-p^*) \tag{34}$$

In addition, from previous results it is also possible to write:

$$c - c^* = -(1-\gamma)(w-w^*-e)$$

$$p - p^* = (1-\gamma)(w-w^*) + \gamma e$$

Substituting these expressions into (34) allow to obtain the ex-post level of the nominal exchange rate:

$$e = \frac{m-m^*}{\rho(1-\gamma)+\gamma} - \frac{(1-\gamma)(1-\rho)(w-w^*)}{\rho(1-\gamma)+\gamma} - \frac{\rho(f-f^*)}{\rho(1-\gamma)+\gamma} \tag{35}$$

A.7 Expected utility

Recall the optimal pre-set wage condition (24), assuming $v = 1$:

$$W = \frac{\phi}{\phi-1}\frac{E\{KL\}}{E\left\{\frac{L}{P}(CF)^{-\rho}\right\}}$$

This can be re-written in the following form:

$$E\{KL\} = \frac{\phi-1}{\phi}WE\left\{\frac{L}{P}(CF)^{-\rho}\right\} \tag{36}$$

Using the equilibrium condition that $L = Y$, the market clearing condition (27) and the price setting conditions allow to rewrite L/P, such that:

$$\frac{L}{P} = \frac{\left(\frac{\mathcal{E}W^*}{W}\right)^{1/2} FZ_c}{\left(\frac{\theta}{\theta-1}\right)\left(\frac{\mathcal{E}W^*}{W}\right)^{\gamma/2} W} = \left(\frac{\theta}{\theta-1}\right)^{-1} W^{-1}\left(\frac{\mathcal{E}W^*}{W}\right)^{\frac{1-\gamma}{2}} Z_cF = \frac{\theta-1}{\theta}W^{-1}CF$$

Substituting the expression for L/P into (36) gives the following condition:

$$E\{KL\} = \frac{(\phi-1)(\theta-1)}{\phi\theta}E\left\{(CF)^{1-\rho}\right\}$$

This can be used to simplify the welfare measure (excluding utility derived from money balances):

$$\begin{aligned} EU &= E\left\{\frac{C^{1-\rho}}{1-\rho} - KL\right\} = \left[\frac{1}{1-\rho} - \frac{(\phi-1)(\theta-1)}{\phi\theta}\right]E\left\{(CF)^{1-\rho}\right\} \\ &= \frac{\phi\theta-(1-\rho)(\phi-1)(\theta-1)}{\phi\theta(1-\rho)}E\left\{\left(\frac{\mathcal{E}W^*}{W}\right)^{\frac{(1-\gamma)(1-\rho)}{2}} Z_c^{1-\rho}F^{1-\rho}\right\} \\ &= \frac{\phi\theta-(1-\rho)(\phi-1)(\theta-1)}{\phi\theta(1-\rho)}E\left\{\exp\left[\frac{(1-\gamma)(1-\rho)}{2}\tau + (1-\rho)f + (1-\rho)z_c\right]\right\} \end{aligned}$$

Solving for the expectations yields:

$$EU = \frac{\phi\theta-(1-\rho)(\phi-1)(\theta-1)}{\phi\theta(1-\rho)} \exp\left\{ \begin{array}{c} \frac{(1-\gamma)(1-\rho)}{2}E\tau + (1-\rho)\,Ef + (1-\rho)\,Ez_c + \frac{(1-\gamma)^2(1-\rho)^2}{8}\sigma_e^2 + \frac{(1-\rho)^2}{2}\sigma_f^2 \\ +\frac{(1-\rho)^2}{2}\sigma_{z_c}^2 + \frac{(1-\gamma)(1-\rho)^2}{2}\sigma_{z_c e} + \frac{(1-\gamma)(1-\rho)^2}{2}\sigma_{fe} + (1-\rho)^2\sigma_{z_c f} \end{array} \right\} \tag{37}$$

The expression for foreign welfare will be parallel, with the coefficient on $E\tau$ and $(\sigma_{z_c e} + \sigma_{f^* e})$ being of opposite sign and f^* replacing f:

$$EU^* = \frac{\phi\theta-(1-\rho)(\phi-1)(\theta-1)}{\phi\theta(1-\rho)} \exp\left\{ \begin{array}{c} -\frac{(1-\gamma)(1-\rho)}{2}E\tau + (1-\rho)\,Ef^* + (1-\rho)\,Ez_c + \frac{(1-\gamma)^2(1-\rho)^2}{8}\sigma_e^2 + \frac{(1-\rho)^2}{2}\sigma_{f^*}^2 \\ +\frac{(1-\rho)^2}{2}\sigma_{z_c}^2 - \frac{(1-\gamma)(1-\rho)^2}{2}\sigma_{z_c e} + \frac{(1-\gamma)(1-\rho)^2}{2}\sigma_{f^* e} + (1-\rho)^2\sigma_{z_c f^*} \end{array} \right\}$$

A.8 Flexible wage allocation

Under flexible wages, the first order condition given in (23) becomes (assuming $\nu = 1$):

$$\begin{aligned} \frac{\partial U^i}{\partial W^i} &= 0 \Leftrightarrow \frac{1}{\widetilde{P}}\left(\widetilde{C}^i\widetilde{F}^i\right)^{-\rho}(1-\phi)\left[\frac{\widetilde{W}(i)}{\widetilde{W}}\right]^{-\phi}\widetilde{Y}(j) + \phi K\left(\left[\frac{\widetilde{W}(i)}{\widetilde{W}}\right]^{-\phi}\widetilde{Y}(j)\right)\widetilde{W}(i)^{=1} = 0 \\ &\Leftrightarrow \frac{1}{\widetilde{P}}\left(\widetilde{C}^i\widetilde{F}^i\right)^{-\rho}(1-\phi)\widetilde{L}^i + \phi K\widetilde{L}^i\widetilde{W}(i)^{=1} = 0 \end{aligned}$$

Invoking symmetry allows to obtain the flexible wage solution:

$$\widetilde{W} = \frac{\phi}{\phi-1}K\widetilde{P}\left(\widetilde{C}^i\widetilde{F}^i\right)^{\rho}$$

Notice that it also follows that:

$$\begin{aligned} \widetilde{P} &= \left(\frac{\theta}{\theta-1}\right)\left(\frac{\widetilde{\mathcal{E}W^*}}{\widetilde{W}}\right)^{\gamma/2}\widetilde{W} \\ \widetilde{C} &= \left(\frac{\widetilde{\mathcal{E}W^*}}{\widetilde{W}}\right)^{\frac{1-\gamma}{2}}\widetilde{Z}_c \end{aligned}$$

and replacing these in the flexible wage equation yields:

$$\left(\frac{\widetilde{\mathcal{E}W^*}}{\widetilde{W}}\right)^{\frac{\rho(1-\gamma)+\gamma}{2}} = \frac{(\phi-1)(\theta-1)}{\theta\phi K}\left(\widetilde{F}\widetilde{Z}_c\right)^{-\rho} \tag{38}$$

The foreign analog being:

$$\left(\frac{\widetilde{\mathcal{E}W^*}}{\widetilde{W}}\right)^{-\frac{\rho(1-\gamma)+\gamma}{2}} = \frac{(\phi-1)(\theta-1)}{\theta\phi K^*}\left(\widetilde{F}^*\widetilde{Z}_c\right)^{-\rho}$$

Dividing the two and solving for the relative wage gives:

$$\left(\frac{\widetilde{\mathcal{E}W^*}}{\widetilde{W}}\right) = \left(\frac{K^*}{K}\right)^{\frac{1}{\rho(1-\gamma)+\gamma}}\left(\frac{\widetilde{F}}{\widetilde{F}^*}\right)^{-\frac{\rho}{\rho(1-\gamma)+\gamma}} \tag{39}$$

Substituting this result in the utility when $\chi \to 0$ yields, following similar steps as before, allows to obtain:

$$E\widetilde{U} = \frac{\phi\theta - (1-\rho)(\phi-1)(\theta-1)}{\phi\theta(1-\rho)} \exp\left\{ \begin{array}{c} (1-\rho) E\widetilde{z}_c + (1-\rho) E\widetilde{f} - \frac{(1-\gamma)(1-\rho)\rho}{2[\rho(1-\gamma)+\gamma]} \left(E\widetilde{f} - E\widetilde{f}^*\right) \\ +\frac{1}{2}\left[\frac{(1-\gamma)^2(1-\rho)^2}{[\rho(1-\gamma)+\gamma]^2}\sigma_{k_d}^2 + (1-\rho)^2 \sigma_{\widetilde{z}_c}^2 + 2\frac{(1-\gamma)(1-\rho)^2}{\rho(1-\gamma)+\gamma}\widetilde{\sigma}_{k_d z_c}\right] \end{array} \right\}$$

Notice that it is possible to solve for $\widetilde{Z}_c$ using both (38) and (39).

$$\left(\frac{\mathcal{E}\widetilde{W}^*}{\widetilde{W}}\right)^{\frac{\rho(1-\gamma)+\gamma}{2}} = \frac{(\phi-1)(\theta-1)}{\theta\phi K}\left(\widetilde{F}\widetilde{Z_c}\right)^{-\rho}$$

$$\left(\left(\frac{K^*}{K}\right)^{\frac{1}{\rho(1-\gamma)+\gamma}}\left(\frac{\widetilde{F}}{\widetilde{F}^*}\right)^{-\frac{\rho}{\rho(1-\gamma)+\gamma}}\right)^{\frac{\rho(1-\gamma)+\gamma}{2}} = \frac{(\phi-1)(\theta-1)}{\theta\phi K}\left(\widetilde{F}\widetilde{Z_c}\right)^{-\rho}$$

Rewriting the terms:

$$\widetilde{Z}_c = \left[\frac{(\phi-1)(\theta-1)}{\theta\phi K}\right]^{\frac{1}{\rho}}\left(\frac{K}{K^*}\right)^{\frac{1}{2\rho}}\left(\widetilde{F}\widetilde{F}^*\right)^{-\frac{1}{2}}$$

Taking logs and using the definition of "world" shock gives:

$$\widetilde{z}_c = \frac{1}{\rho}\left\{\ln\left[\frac{(\phi-1)(\theta-1)}{\theta\phi}\right] - k_w - \rho\left(\frac{\widetilde{f}+\widetilde{f}^*}{2}\right)\right\}$$

Now it is possible to calculate the mean, variance and covariances of the flexible wage level of private expenditure:

$$E\widetilde{z}_c = \frac{1}{\rho}\left\{\ln\left[\frac{(\phi-1)(\theta-1)}{\theta\phi}\right] - Ek_w - \rho\left(\frac{E\widetilde{f}+E\widetilde{f}^*}{2}\right)\right\}$$

$$\widetilde{\sigma}_z^2 = E\left[(z-Ez)^2\right] = \frac{1}{\rho^2}E\left(-(k_w - Ek_w)\right)^2 = \frac{1}{\rho^2}\sigma_{k_w}^2$$

$$\widetilde{\sigma}_{k_d z} = E\left[k_d\left(-\frac{1}{\rho}(k_w - Ek_w)\right)\right] = 0$$

which can be substituted into the flexible wage utility to yield:

$$E\widetilde{U} = \frac{\phi\theta - (1-\rho)(\phi-1)(\theta-1)}{\phi\theta(1-\rho)}\exp\left\{\left(\frac{(1-\rho)\omega}{v-(1-\rho)}\right) + \frac{\gamma}{2[\rho(1-\gamma)+\gamma]}\left(E\widetilde{f} - E\widetilde{f}^*\right)\right\}$$

In a symmetric model $E\widetilde{f} = E\widetilde{f}^*$, hence:

$$E\widetilde{U} = E\widetilde{U}^* = \frac{\phi\theta - (1-\rho)(\phi-1)(\theta-1)}{\phi\theta(1-\rho)}\exp\left\{\left(\frac{(1-\rho)\omega}{\rho}\right)\right\}$$

A.9 Sticky-wage welfare levels

In order to calculate the welfare levels under sticky wages we substitute (30) and (31) into the welfare expression (37):

$$EU = E\widetilde{U}\exp\left\{(1-\rho)\left(\begin{array}{c} -\frac{(1-\gamma)}{2[\rho(1-\gamma)+\gamma]}\left[\begin{array}{c} \left(1-(1-\gamma)(1-\rho)^2\right)\sigma_{ez_c} \\ +\left(\frac{1-(1-\gamma)(1-\rho)^2}{2}\right)\left(\sigma_{ef}+\sigma_{ef^*}\right) \\ +\left(1-(1-\rho)^2\right)\left(\sigma_{fz_c}-\sigma_{f^*z_c}\right)+ \\ +\frac{1-(1-\rho)^2}{2}\left(\sigma_f^2-\sigma_{f^*}^2\right)+\sigma_{k_we}+2\sigma_{k_dz_c} \\ +\left(\sigma_{k_wf}-\sigma_{k_wf^*}\right)+\left(\sigma_{k_df}+\sigma_{k_df^*}\right) \end{array}\right]+ \\ +\left[\begin{array}{c} \frac{\lambda}{\rho}-\frac{v}{2\rho^2}\sigma_k^2-\frac{1-(1-\gamma)^2(1-\rho)^2}{8[\rho]}\sigma_e^2-\frac{1-(1-\rho)^2}{2\rho}\sigma_{z_c}^2 \\ -\frac{1-(1-\rho)^2}{4\rho}\left(\sigma_f^2+\sigma_{f^*}^2\right)-\frac{1-(1-\gamma)(1-\rho)^2}{4\rho}\left(\sigma_{ef}-\sigma_{ef^*}\right) \\ -\frac{1-(1-\rho)^2}{2\rho}\left(\sigma_{fz_c}+\sigma_{f^*z_c}\right)-\frac{1}{2\rho}\sigma_{k_de}-\frac{1}{\rho}\sigma_{k_wz_c} \\ -\frac{1}{2\rho}\left(\sigma_{k_wf}+\sigma_{k_wf^*}\right)-\frac{1}{2\rho}\left(\sigma_{k_df}-\sigma_{k_df^*}\right) \end{array}\right]+ \\ +\frac{(1-\gamma)^2(1-\rho)}{8}\sigma_e^2+\frac{(1-\rho)}{2}\left(\sigma_f^2+\sigma_{z_c}^2\right)+\frac{(1-\gamma)(1-\rho)}{2}\left(\sigma_{z_ce}+\sigma_{fe}\right) \\ +(1-\rho)\sigma_{z_cf}+\frac{\gamma}{2[\rho(1-\gamma)+\gamma]}\left(\left[Ef-E\widetilde{f}\right]-\left[Ef^*-E\widetilde{f}^*\right]\right) \end{array}\right)\right\}$$

Assuming that $Ef = E\widetilde{f}$ and $Ef^* = E\widetilde{f}^*$, this expression can be written in the following form:

$$\begin{aligned} EU &= E\widetilde{U}\exp\{(1-\rho)\,\Omega(\rho)\} \\ \Omega(\rho) &= \Omega_w(\rho)+\Omega_d(\rho) \end{aligned}$$

where we combine the terms that affect both countries in the same way in $\Omega_w(\rho)$ and the terms that affect countries with opposite sign in $\Omega_d(\rho)$, such that:

$$\begin{aligned} \Omega_w(\rho) &= -\frac{\left(\sigma_{k_w}^2+\sigma_{k_d}^2\right)}{2\rho^2}+\frac{\lambda}{\rho}-\frac{\sigma_{z_c}^2}{2}-\frac{\left[1-(1-\gamma)^2(1-\rho)\right]\sigma_e^2}{8\rho}-\frac{\sigma_{k_wz_c}}{\rho}-\frac{\sigma_{k_de}}{2\rho}-\frac{[\rho(1-\gamma)+\gamma]\left(\sigma_{ef}-\sigma_{ef^*}\right)}{4\rho} \\ &\quad -\frac{\left(\sigma_{fz_c}+\sigma_{f^*z_c}\right)}{4}-\frac{\left(\sigma_f^2+\sigma_{f^*}^2\right)}{4}-\frac{\left(\sigma_{k_wf}+\sigma_{k_wf^*}\right)}{2\rho}-\frac{\left(\sigma_{k_df}-\sigma_{k_df^*}\right)}{2\rho} \\ \Omega_d(\rho) &= -\frac{\rho(1-\gamma)\sigma_{ez_c}}{2[\rho(1-\gamma)+\gamma]}-\frac{(1-\gamma)\sigma_{k_we}}{2[\rho(1-\gamma)+\gamma]}-\frac{(1-\gamma)\sigma_{k_dz_c}}{[\rho(1-\gamma)+\gamma]}-\frac{\rho(1-\gamma)\left(\sigma_{ef}+\sigma_{ef^*}\right)}{4[\rho(1-\gamma)+\gamma]}-\frac{(\rho-\gamma)\left(\sigma_{fz_c}-\sigma_{f^*z_c}\right)}{2[\rho(1-\gamma)+\gamma]} \\ &\quad -\frac{(\rho-\gamma)\left(\sigma_f^2-\sigma_{f^*}^2\right)}{4[\rho(1-\gamma)+\gamma]}-\frac{(1-\gamma)\left(\sigma_{k_wf}-\sigma_{k_wf^*}\right)}{2[\rho(1-\gamma)+\gamma]}-\frac{(1-\gamma)\left(\sigma_{k_df}+\sigma_{k_df^*}\right)}{2[\rho(1-\gamma)+\gamma]} \end{aligned}$$

Foreign welfare is given by a parallel expression with the asymmetric component of utility entering with opposite sign.

$$EU^* = E\widetilde{U}\exp\{(1-\rho)\,\Omega^*(\rho)\}$$

$$\Omega^*(\rho) = \Omega_w(\rho)-\Omega_d(\rho)$$

A.10 Solutions for variances and covariances under flexible exchange rates

Substituting the policy rules into (33) and (35) gives:

$$z_c = \frac{1}{2\rho}\left(-\left(\delta_d-\delta_d^*\right)\widehat{k}_d-\left(\delta_w+\delta_w^*\right)\widehat{k}_w\right)-\frac{1}{2}\left(-\left(\eta_d-\eta_d^*\right)\widehat{k}_d-\left(\eta_w+\eta_w^*\right)\widehat{k}_w\right)$$

$$e=\frac{1}{\rho\left(1-\gamma\right)+\gamma}\left(-\left(\delta_{d}+\delta_{d}^{*}\right)\widehat{k}_{d}-\left(\delta_{w}-\delta_{w}^{*}\right)\widehat{k}_{w}\right)-\frac{\rho}{\rho\left(1-\gamma\right)+\gamma}\left(-\left(\eta_{d}+\eta_{d}^{*}\right)\widehat{k}_{d}-\left(\eta_{w}-\eta_{w}^{*}\right)\widehat{k}_{w}\right)$$

Using these expressions it is possible to calculate the ex-post variances and covariances of e and z_c, which will be useful to calculate welfare in terms of the policy parameters and exogenous shocks. These are listed bellow.

$$\begin{aligned}
\sigma_{z_c}^2 &= \left(\frac{\left(\delta_d-\delta_d^*\right)-\rho\left(\eta_d-\eta_d^*\right)}{2\rho}\right)^2\sigma_{k_d}^2+\left(\frac{\left(\delta_w+\delta_w^*\right)-\rho\left(\eta_w+\eta_w^*\right)}{2\rho}\right)^2\sigma_{k_w}^2\\
\sigma_e^2 &= \left(\frac{\left(\delta_d+\delta_d^*\right)-\rho\left(\eta_d+\eta_d^*\right)}{\rho\left(1-\gamma\right)+\gamma}\right)^2\sigma_{k_d}^2+\left(\frac{\left(\delta_w-\delta_w^*\right)-\rho\left(\eta_w-\eta_w^*\right)}{\rho\left(1-\gamma\right)+\gamma}\right)^2\sigma_{k_w}^2\\
\sigma_{k_w z} &= -\frac{\left(\delta_w+\delta_w^*\right)-\rho\left(\eta_w+\eta_w^*\right)}{2\rho}\sigma_{k_w}^2\\
\sigma_{k_d e} &= -\frac{\left(\delta_d+\delta_d^*\right)-\rho\left(\eta_d+\eta_d^*\right)}{\rho\left(1-\gamma\right)+\gamma}\sigma_{k_d}^2\\
\sigma_{e z_c} &= \left(\frac{\left(\delta_d-\rho\eta_d\right)^2-\left(\delta_d^*-\rho\eta_d^*\right)^2}{2\rho\left[\rho\left(1-\gamma\right)+\gamma\right]}\right)\sigma_{k_d}^2+\left(\frac{\left(\delta_w-\rho\eta_w\right)^2-\left(\delta_w^*-\rho\eta_w^*\right)^2}{2\rho\left[\rho\left(1-\gamma\right)+\gamma\right]}\right)\sigma_{k_w}^2\\
\sigma_{k_w e} &= -\left(\frac{\left(\delta_w-\delta_w^*\right)-\rho\left(\eta_w-\eta_w^*\right)}{\rho\left(1-\gamma\right)+\gamma}\right)\sigma_{k_w}^2\\
\sigma_{k_d z_c} &= -\left(\frac{\left(\delta_d-\delta_d^*\right)-\rho\left(\eta_d-\eta_d^*\right)}{2\rho}\right)\sigma_{k_d}^2\\
\sigma_{ef}-\sigma_{ef^*} &= \frac{\left(\eta_d+\eta_d^*\right)\left(\delta_d+\delta_d^*\right)-\rho\left(\eta_d+\eta_d^*\right)^2}{\rho\left(1-\gamma\right)+\gamma}\sigma_{k_d}^2+\frac{\left(\eta_w+\eta_w^*\right)\left(\delta_w-\delta_w^*\right)-\rho\left(\eta_d^2-\eta_d^{*2}\right)}{\rho\left(1-\gamma\right)+\gamma}\sigma_{k_w}^2\\
\sigma_{ef}+\sigma_{ef^*} &= \frac{\left(\eta_d-\eta_d^*\right)\left(\delta_d+\delta_d^*\right)-\rho\left(\eta_d^2-\eta_d^{*2}\right)}{\rho\left(1-\gamma\right)+\gamma}\sigma_{k_d}^2+\frac{\left(\eta_w+\eta_w^*\right)\left(\delta_w-\delta_w^*\right)-\rho\left(\eta_d^2-\eta_d^{*2}\right)}{\rho\left(1-\gamma\right)+\gamma}\sigma_{k_w}^2\\
\sigma_{fz_c}+\sigma_{f^*z_c} &= \frac{\left(\eta_d-\eta_d^*\right)\left(\delta_d-\delta_d^*\right)-\rho\left(\eta_d-\eta_d^*\right)^2}{2\rho}\sigma_{k_d}^2+\frac{\left(\eta_w+\eta_w^*\right)\left(\delta_w+\delta_w^*\right)-\rho\left(\eta_w+\eta_w^*\right)^2}{2\rho}\sigma_{k_w}^2\\
\sigma_{fz_c}-\sigma_{f^*z_c} &= \frac{\left(\eta_d+\eta_d^*\right)\left(\delta_d-\delta_d^*\right)-\rho\left(\eta_d^2-\eta_d^{*2}\right)}{2\rho}\sigma_{k_d}^2+\frac{\left(\eta_d-\eta_d^*\right)\left(\delta_w+\delta_w^*\right)-\rho\left(\eta_d^2-\eta_d^{*2}\right)}{2\rho}\sigma_{k_w}^2\\
\sigma_f^2+\sigma_{f^*}^2 &= \left(\eta_d^2+\eta_d^{2*}\right)\sigma_{k_d}^2+\left(\eta_w^2+\eta_w^{2*}\right)\sigma_{k_w}^2\\
\sigma_f^2-\sigma_{f^*}^2 &= \left(\eta_d^2-\eta_d^{2*}\right)\sigma_{k_d}^2+\left(\eta_w^2-\eta_w^{2*}\right)\sigma_{k_w}^2\\
\sigma_{k_w f}+\sigma_{k_w f^*} &= -\left(\eta_w+\eta_w^*\right)\sigma_{kw}^2\\
\sigma_{k_d f}-\sigma_{k_d f^*} &= -\left(\eta_d+\eta_d^*\right)\sigma_{k_d}^2\\
\sigma_{k_w f}-\sigma_{k_w f^*} &= -\left(\eta_w-\eta_w^*\right)\sigma_{kw}^2\\
\sigma_{k_d f}+\sigma_{k_d f^*} &= -\left(\eta_d-\eta_d^*\right)\sigma_{k_d}^2
\end{aligned}$$

A.11 Fiscal policy and welfare under flexible exchange rates

Substituting the variances and covariances calculated above in the expression for $\Omega_w(\rho)$, assuming that $\delta_j = 0, j = w, d$, and eliminating the terms that cancel out gives:

$$\begin{aligned}\Omega_w(\rho) &= -\frac{1}{2\rho^2}\left(\sigma_{k_w}^2 + \sigma_{k_d}^2\right) + \frac{\lambda}{\rho} - \frac{\rho\left[1-(1-\gamma)^2(1-\rho)\right]}{8\left[\rho(1-\gamma)+\gamma\right]^2}\left(\left(\eta_d+\eta_d^*\right)^2\sigma_{k_d}^2 + \left(\eta_w-\eta_w^*\right)^2\sigma_{k_w}^2\right)\\ &\quad -\frac{1}{2\left[\rho(1-\gamma)+\gamma\right]}\left(\eta_d+\eta_d^*\right)\sigma_{k_d}^2 + \frac{1}{4}\left(\left(\eta_d+\eta_d^*\right)^2\sigma_{k_d}^2 + \left(\eta_w-\eta_w^*\right)^2\sigma_{k_w}^2\right)\\ &\quad -\frac{1}{4}\left(\left(\eta_d^2+\eta_d^{2*}\right)\sigma_{k_d}^2 + \left(\eta_w^2+\eta_w^{2*}\right)\sigma_{k_w}^2\right) + \frac{1}{2\rho}\left(\eta_d+\eta_d^*\right)\sigma_{k_d}^2\end{aligned}$$

Substituting the variances and covariances calculated above in the expression for $\Omega_d(\rho)$, assuming that $\delta_j = 0, j = w, d$, and eliminating the terms that cancel out yields:

$$\Omega_d(\rho) = -\frac{(1-\gamma)\rho}{2\left[\rho(1-\gamma)+\gamma\right]^2}\left(\eta_w-\eta_w^*\right)\sigma_{k_w}^2 + \frac{(1-\gamma)}{2\left[\rho(1-\gamma)+\gamma\right]}\left(\eta_w-\eta_w^*\right)\sigma_{k_w}^2$$

This expression can be further simplified to:

$$\Omega_d(\rho) = \frac{\gamma(1-\gamma)(1-\rho)}{2\left[\rho(1-\gamma)+\gamma\right]^2}\left(\eta_w-\eta_w^*\right)\sigma_{kw}^2$$

A.12 Fiscal policy and welfare in a monetary union

Given ex-post private expenditure under the fixed exchange rate regime

$$\widehat{z} = \frac{1}{2\rho}\widehat{m}^w - \frac{1}{2}\left(\widehat{f}+\widehat{f}^*\right) = \frac{\left(\eta_d-\eta_d^*\right)}{2}\widehat{k}_d - \frac{\delta_w^w - \rho\left(\eta_w+\eta_w^*\right)}{2\rho}\widehat{k}_w$$

and the fiscal policy rules, it is possible to obtain the variances and covariances needed to evaluate welfare:

$$\begin{aligned}\sigma_{z_c}^2 &= \left(\frac{\left(\eta_d-\eta_d^*\right)}{2}\right)^2\sigma_{k_d}^2 + \left(\frac{\delta_w^w-\rho\left(\eta_w+\eta_w^*\right)}{2\rho}\right)^2\sigma_{k_w}^2\\ \sigma_{k_w z} &= -\frac{\delta_w^w-\rho\left(\eta_w+\eta_w^*\right)}{2\rho}\sigma_{k_w}^2\\ \sigma_{k_d z_c} &= \frac{\left(\eta_d-\eta_d^*\right)}{2}\sigma_{k_d}^2\\ \sigma_{f z_c} + \sigma_{f^* z_c} &= -\frac{\left(\eta_d-\eta_d^*\right)^2}{2}\sigma_{k_d}^2 + \frac{\left(\eta_w+\eta_w^*\right)\delta_w^w - \rho\left(\eta_w+\eta_w^*\right)^2}{2\rho}\sigma_{k_w}^2\\ \sigma_{f z_c} - \sigma_{f^* z_c} &= -\frac{\left(\eta_d^2-\eta_d^{*2}\right)}{2}\sigma_{k_d}^2 + \frac{\left(\eta_d-\eta_d^*\right)\delta_w^w - \rho\left(\eta_d^2-\eta_d^{*2}\right)}{2\rho}\sigma_{k_w}^2\\ \sigma_f^2 + \sigma_{f^*}^2 &= \left(\eta_d^2+\eta_d^{2*}\right)\sigma_{k_d}^2 + \left(\eta_w^2+\eta_w^{2*}\right)\sigma_{k_w}^2\\ \sigma_f^2 - \sigma_{f^*}^2 &= \left(\eta_d^2-\eta_d^{2*}\right)\sigma_{k_d}^2 + \left(\eta_w^2-\eta_w^{2*}\right)\sigma_{k_w}^2\\ \sigma_{k_w f} + \sigma_{k_w f^*} &= -\left(\eta_w+\eta_w^*\right)\sigma_{kw}^2\\ \sigma_{k_d f} - \sigma_{k_d f^*} &= -\left(\eta_d+\eta_d^*\right)\sigma_{k_d}^2\\ \sigma_{k_w f} - \sigma_{k_w f^*} &= -\left(\eta_w-\eta_w^*\right)\sigma_{kw}^2\\ \sigma_{k_d f} + \sigma_{k_d f^*} &= -\left(\eta_d-\eta_d^*\right)\sigma_{k_d}^2\end{aligned}$$

Replacing these variances in the expression for $\Omega_w(\rho)$, eliminating redundant allows to obtain:

$$\begin{aligned}\Omega_w(\rho) &= -\frac{1}{2\rho^2}\left(\sigma_{k_w}^2+\sigma_{k_d}^2\right)+\frac{\lambda}{\rho}-\frac{1}{2\rho}\left(\eta_w+\eta_w^*\right)\sigma_{k_w}^2-\frac{1}{4}\left(\left(\eta_d^2+\eta_d^{2*}\right)\sigma_{k_d}^2+\left(\eta_w^2+\eta_w^{2*}\right)\sigma_{k_w}^2\right)\\ &+\frac{1}{2\rho}\left(\eta_w+\eta_w^*\right)\sigma_{kw}^2+\frac{1}{2\rho}\left(\eta_d+\eta_d^*\right)\sigma_{k_d}^2\end{aligned}$$

Substituting instead the variances and covariances calculated in this section in the expression for $\Omega_d(\rho)$, eliminating the redundant terms gives:

$$\Omega_d(\rho)=\frac{(1-\gamma)}{2\left[\rho(1-\gamma)+\gamma\right]}\left(\eta_w-\eta_w^*\right)\sigma_{kw}^2$$

A.13 Gains

To analyse the potential gains, we first calculate the derivatives of the policy functions with respect to ρ:

$$\frac{\partial\left|\eta_{d,FLEX}\right|}{\partial\rho}=-\frac{\gamma^2}{(\gamma\rho)^2}<0$$

$$\frac{\partial\left|\eta_{w,FLEX}^{nash}\right|}{\partial\rho}=-\frac{\gamma(1-\gamma)}{\left[\rho(1-\gamma)+\gamma\right]^2}\left(1-\frac{2}{\left[\rho(1-\gamma)+\gamma\right]}\right)<0 \text{ if } \rho>\frac{2-\gamma}{1-\gamma}$$

$$\frac{\partial\left|\eta_{d,MU}\right|}{\partial\rho}=-\frac{1}{\rho^2}<0$$

$$\frac{\partial\left|\eta_{w,MU}^{nash}\right|}{\partial\rho}=-\frac{(1-\gamma)^2}{(\rho(1-\gamma)+\gamma)^2}<0$$

We also calculate the differences in the *losses from fiscal competition*:

$$\frac{1}{2}\left[\left(\eta_{w,MU}^{nash}\right)^2-\left(\eta_{w,FLEX}^{nash}\right)^2\right]=\frac{(1-\gamma)^2}{(\rho(1-\gamma)+\gamma)^2}\left[1-\frac{\gamma^2(\rho-1)^2}{(\rho-(\rho-1)\gamma)^2}\right]$$

This expression is positive when $\rho<\frac{2\gamma}{2\gamma-1}$since:

$$\begin{aligned}\gamma^2(\rho-1)^2 &< \gamma^2(\rho-1)^2+\rho^2-2\rho(\rho-1)\gamma\\ &\Rightarrow \rho<\frac{2\gamma}{2\gamma-1}\end{aligned}$$

Notice in addition that the *gains from asymmetric stabilization* under flexible exchange rates are given by:

$$-\gamma^2\frac{(\rho-1)}{2\left[\rho(1-\gamma)+\gamma\right]^2}\left(\eta_{d,FLEX}\right)^2-\frac{\gamma(\rho-1)}{\rho(\rho(1-\gamma)+\gamma)}\eta_{d,FLEX}=-\frac{(\rho-1)}{2\rho^2}+\frac{(\rho-1)}{\rho^2}=\frac{(\rho-1)}{2\rho^2}$$

In a monetary union, these gains are instead:

$$-\frac{1}{2}\left(\eta_{d,MU}\right)^2+\frac{1}{\rho}\eta_{d,MU}=-\frac{1}{2\rho^2}+\frac{1}{\rho^2}=\frac{1}{2\rho^2}$$

Comparing the two gives the difference in welfare across regimes, when there is cooperation:

$$\Omega^{flex}(\rho)-\Omega^{MU}(\rho)\big|_{Coop}=\frac{\rho-2}{2\rho^2}\sigma_{k_d}^2$$

This expression is positive for values of ρ above 2.