The Specification of Monetary Policy Inertia in Empirical Taylor Rules

(preliminary and incomplete)

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Abstract

In the empirical literature on interest rate rules, central bank gradualism (or monetary policy inertia) is typically modeled by specifying an inertial Taylor rule with lagged policy rate. We introduce an alternative specification which implies inertia in the central bank's adjustment of the operating target for the policy rate. We find empirical evidence supporting the alternative specification against the standard specification. We will argue that, unlike the standard specification, the proposed specification is consistent with the low predictability of interest rates.

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1 Introduction

There is a conventional view that central banks adjust interest rates gradually in response to macroeconomic developments. Indeed, the em-

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pirical evidence on the behavior of the central banks in the last two decades has been summarized as an inertial Taylor (1993) rule, where the nominal interest rate adjusts only partially to inflation and the output gap, as there is an interest rate smoothing component.[1]

A typical rationale for gradualism is the concern for financial stability.[2] An alternative explanation is concern for the adverse reactions of financial markets to frequent changes in the direction of movement of short-term interest rates (see for example Goodfriend 1991).

Moreover, the optimality of central bank gradualism is supported by several recent analyses of monetary policy. First, gradualism may be optimal in presence of uncertainty about the structure of the macroeconomic model, about the values of the parameters of the model, or about measurement errors in contemporaneously reeased data.[3] Second, the linkage between future monetary policy and aggregate demand can be exploited by central banks in order to stabilize the economy optimally. This implies that in presence of forward-looking expectations on inflation it may be optimal to adjust the interest rate with some inertia.[4] Third, interest rate smoothing may emerge under a discretionary monetary policy regime, when it may be desirable to delegate policy to a

[1] See for instance Clarida, Galì and Gertler (2000), who enphasize the empirical importance of including a lagged interest rate in a monetary policy rule for the United States. For a similar result for other industrial countries see Clarida, Galì and Gertler (1998).

[2] Reviews on this literature are provided by Cukierman (1992), Goodhart (1996), Walsh (2003), Sack and Wieland (2000).

[3] The importance of such uncertainties for gradualism is examined by Sack (1998,2000), Startz (2003), Orphanides (2003), Rudebusch (2001), Wieland (1998), among others.

[4] See Woodford (1999).

central banker with an explicit interest rate smoothing objective.[5]

However, recently Rudebusch (2002) has challenged the conventional wisdom on central banks gradualism (or monetary policy inertia). In particular, by focusing on the apparent contradiction between interest rate smoothing and the low predictability of policy rates, he argues that monetary policy inertia is an illusion. He argues that given the large and highly significant estimates of the coefficient of the lagged policy rate found in empirical analyses, we should observe high predictability of interest rates.[6] This implies that empirical Taylor rules may be misspecified and that the inertia found may be actually related to the presence of serially correlated shocks faced by the central bank.[7]

Söderlind, Söderström and Vredin (2002) have found further evidence against the inertial Taylor rule. They argue that a high coefficient of the partial adjustment component is a necessary but not sufficient condition for having a highly predictable interest rate. Actually the predictability of the interest rate depends also on the other variables that enter the Taylor rule, namely output and inflation. They show that, while it is relatively easy to predict the variables that enter the Taylor rule, it is very difficult to predict interest rates. This result might be due to an

[5] See Woodford (2003a). The previous two arguments for the optimality of monetary inertia considered in the text do not presume a central bank's loss function trading off objectives related to macroeconomic stability with an interest rate smoothing objective (usually interpreted as a financial stability motive).

[6] In the empirical literature the estimated coefficient for the lagged policy rate is ranging from .7 to .9. See Rudebusch (2002) for a review of the estimates found in the literature.

[7] See also Lansing (2002) for a theorical support of the 'illusion of monetary inertia' hypothesis, based on real-time estimation of trend output.

omitted variable problem in the Taylor rule, with the potentially omitted variable being not easily predictable.

In his analysis, Rudebusch, using yield curve data, provides an indirect proof of the implausibility of the inertial Taylor rule. English, Nelson and Sack (2003) show that it is possible to test directly the null of serial correlation against the alternative of partial adjustment. They find that it is not possible to reject the presence of both serial correlation and partial adjustment.

In the present analysis we try to reconcile the empirical evidence on monetary policy inertia with the low predictability of short-term interest rates by examining an inertial Taylor rule, which is an alternative to the one considered in the literature. In particular, we postulate a different specification of the inertial component which implies inertia in the central bank's adjustment of the operating target for the interest rate. We will argue that, for given coefficient of partial adjustment, our alternative specification implies lower predictability of the interest rate than that implied by the standard specification of the inertial Taylor rule. In our empirical analysis we find support for the alternative specification against the standard specification. Moreover, in the alternative specification, the estimated coefficient of partial adjustment is below 0.5, which is lower than is usually found in the literature.

The structure of the presentation is the following. In section 2 we consider a simple empirical macro-model, along the lines of Svensson (1997), and derive the optimal interest rate rule for the central bank. This rule in general looks rather complicated. Thus we specify a simple inertial Taylor rule - with only three parameters - that might approximate the optimal rule. Section 3 discusses our empirical findings based on the alternative inertial Taylor rule. Section 4 will make some concluding observations and address future research.

2 A simple framework

2.1 The model

Here we use a simple framework for examining the optimal interest rate rule for a central bank, which is an extended version of the model used by Svensson (1997).[8] He argues that, even if there is no explicit role for private agents' expectations, the model has many similarities with more elaborate models used by central banks.[9]

Consider the following model[10]

$$\pi_{t+1} = \alpha_1 y_t + (1 - \alpha_2)\,\pi_t + \alpha_2 \pi_{t-1} + \epsilon_{t+1}, \tag{1}$$

and

$$y_{t+1} = \beta_1' y_t - \beta_2\,(i_t - E_t \pi_{t+1}) + \beta_3 y_{t-1} + \eta_{t+1}, \tag{2}$$

[8] In the litereature, Svensson's (1997) model has been extended in several directions: for examining nominal income targeting (Ball 1999); for studying the implications of monetary policy for the yield curve (Ellingsen and Söderström 2001; Eijffinger, Schaling and Verhagen 2000); for examining model uncertainty, interest rate smoothing and interest rate stabilization - i.e. for studying the optimality of a more gradual adjustment of the monetary instrument (Svensson 1999). Moreover, Rudebusch and Svensson (1999) provide empirical estimates for a model similar to Svensson (1997) and use a calibrated version of the model in order to evaluate a large number of interest rate rules.

[9] See for instance the discussions in Rudebusch and Svensson (1999) and Rudebusch (2001).

[10] We have used the same notation as in Svensson (1997).

where π_t is the inflation rate, y_t is the output gap, i_t is the nominal repo rate, i.e. the monetary instrument of the central bank, and ϵ_t, η_t are i.i.d. shocks.[11] All the variables are considered as deviations from their long-run average levels, which are normalized to zero for simplicity.

After substituting $E_t\pi_{t+1}$ with the expectation of expression (1), expression (2) becomes:

$$y_{t+1} = \beta_1 y_t - \beta_2 i_t + \beta_3 y_{t-1} + \beta_4 \pi_t + \beta_5 \pi_{t-1} + \eta_{t+1}, \tag{3}$$

with

$$\begin{aligned} \beta_1 &\equiv \beta_1' + \beta_2 \alpha_1; \\ \beta_4 &\equiv \beta_2 \left(1 - \alpha_2\right); \\ \beta_5 &\equiv \beta_2 \alpha_2. \end{aligned} \tag{4}$$

The coefficients in (1) and (3) are all assumed to be positive, with $0 < \alpha_2 < 1$. Equations (1) and (3) coincide with those considered in Svensson (1997) (equations 6.4 and 6.5 in his text) when $\alpha_2 = \beta_3 = 0$.[12] The restriction that the sum of the lag coefficients of inflation in (1) equals 1

[11] See Svensson (1997) for the details on the model and in particular for the implications of substituting the long-term nominal rate with the repo rate.

[12] Contrary to Svensson we have assumed that the coefficient of one-period lagged inflation in (1) is less than 1, instead of equal to 1. McCallum (1997) has shown that when the coefficient is equal to 1 we may have problems of instability of nominal income rules that would not arise if expectations of current or future inflation were included in the model considered. See also Rudebusch (2002) and Jensen (2002) for further analyses of the performance of nominal income rules for monetary policy when a forward-looking price-setting behaviour is explicitly included in the analytical framework.

is consistent with the empirical evidence.[13] An important feature of this model is the presence of lags in the transmission of monetary policy. In particular, the repo rate affects output with a one-period lag (where one period corresponds to one year), while affects inflation with a two-period lag. This feature is broadly consistent with the "stylized facts" of the impact of monetary policy on output and inflation.

Finally, monetary policy is conducted by a central bank with the following period loss function

$$L(\pi_t, y_t) = \frac{1}{2}\left[\pi_t^2 + \lambda y_t^2\right], \tag{5}$$

where $\lambda > 0$ is the relative weight on output stabilization. The intertemporal loss function is

$$E_t \sum_{\tau=t}^{\infty} \delta^{\tau-t} L(\pi_\tau, y_\tau). \tag{6}$$

The central bank minimizes the above intertemporal loss function by choosing a sequence of current and future repo rates $\{i_\tau\}_{\tau=t}^{\infty}$.

2.2 Optimal interest rate rule

Before solving the optimization problem we introduce first a convenient simplification for the choice variable. In the expression (3) of output the choice of i_τ affects y_{t+1}, but y_t, y_{t-1}, π_t and π_{t-1} are all predetermined. Thus we can write

$$y_{t+1} = \Delta_t + \eta_{t+1}, \tag{7}$$

[13] See for instance Rudebusch and Svensson (1999) for a test of this restriction in a model similar to the one considered here.

with

$$\Delta_t \equiv \beta_1 y_t - \beta_2 i_t + \beta_3 y_{t-1} + \beta_4 \pi_t + \beta_5 \pi_{t-1}. \tag{8}$$

As observed above, the repo rate affects affects inflation with a two-period lag. This can be seen by rewriting the expression (1) for inflation in the following form

$$\pi_{t+2} = \alpha_1 \Delta_t + (1 - \alpha_2)\,\pi_{t+1} + \alpha_2 \pi_t + \alpha_1 \eta_{t+1} + \epsilon_{t+2}, \tag{9}$$

where we have considered inflation at time $t+2$ and inserted expression (7). By means of dynamic programming, we can derive the optimal rule as the solution to the following problem

$$V\left(E_t \pi_{t+1}, \pi_t\right) = \min_{\Delta_t} E_t \left\{ \frac{1}{2}\left[\pi_{t+1}^2 + \lambda y_{t+1}^2\right] + \delta V\left(E_{t+1}\pi_{t+2}, \pi_{t+1}\right) \right\}, \tag{10}$$

subject to (7) and (9). The value function $V\left(\pi_{t+1}, \pi_t\right)$ will be quadratic and in the present case, where constant terms are absent, it can be expressed without loss of generality as

$$V\left(\pi_{t+1}, \pi_t\right) = \frac{1}{2}\gamma_1 \pi_{t+1}^2 + \gamma_2 \pi_{t+1}\pi_t + \frac{1}{2}\gamma_3 \pi_t^2 + k, \tag{11}$$

where the coeffcients γ_1, γ_2 and γ_3 need to be determined. The remaining constant k is a function of the variances of the shocks.

Here we have two state variables and one control variable. In general, the optimization problem cannot be solved analytically by means of dynamic programming if there is more than one state variable. In the simpler case with only one state variable, considered by Svensson, it is possible to get an analytical solution for the optimization problem.

Nevertheless, we can make a qualitative assessment of the form of the optimal rule. Svensson has shown that in the simpler case considered by him the optimal rule takes the form of the Taylor (1993) rule

$i_t = \phi_1 \pi_t + \phi_2 y_t,$

with $\phi_1 > 1$ and $\phi_2 > 0$. What emerges in the present case?

The first order condition with respect to Δ_t is given by

$$y_{t+1|t} = -\frac{\alpha_1 \delta}{\lambda} \left(\gamma_1 \pi_{t+2|t} + \gamma_2 \pi_{t+1|t} \right),$$

where we have used (11).

The optimal interest rate can be derived by substituting (**??**) in (3) and using the inflation equation (1) to yield

$$\begin{aligned} i_t = \alpha_2 & \left[(1+C)\, \pi_{t-1} + \frac{\beta_3}{\alpha_2 \beta_2} y_{t-1} \right] + \\ (1-\alpha_2) & \left[(1+A)\, \pi_t + \left(\frac{\beta_1}{(1-\alpha_2)\, \beta_2} + B \right) y_t \right], \end{aligned} \tag{12}$$

with

$$\begin{aligned} A &\equiv \delta \alpha_1 \frac{(\gamma_1 + \gamma_2)(1-\alpha_2) + \gamma_1 \alpha_2^2}{(1-\alpha_2)\, \beta_2 \left(\lambda + \delta \alpha_1^2 \gamma_1 \right)}; \\ B &\equiv \delta \alpha_1^2 \frac{\gamma_1 (1-\alpha_2) + \gamma_2}{(1-\alpha_2)\, \beta_2 \left(\lambda + \delta \alpha_1^2 \gamma_1 \right)}; \\ C &\equiv \delta \alpha_1 \frac{\gamma_1 (1-\alpha_2) + \gamma_2}{\beta_2 \left(\lambda + \delta \alpha_1^2 \gamma_1 \right)}. \end{aligned} \tag{13}$$

In general, in a problem of this type, the optimal feedback rule can be represented as a linear function of the state variables, here $E_t \pi_{t+1}$, and

π_t. So we could represent the rule for Δ_t as $\Delta_t = f_1 E_t \pi_{t+1} + f_2 \pi_t$. Since $E_t \pi_{t+1}$ can be represented as a function of current values and the first lag of the output gap and inflation, when we solve for the interest rate, the policy rule also emerges as a linear function of the same variables. It would be useful to be able to sign the parameters in the feedback rule (13). Since the value function is a positive definite quadratic form, it must be the case that $\gamma_1 > 0$, $\gamma_3 > 0$, and $\gamma_1\gamma_3 - \gamma_2^2 > 0$, but it is not possible to sign γ_2. If the coefficients on the right hand side variables in (12) are all positive, and if the ratios of coefficients on the current variables (and) are the same as the ratios of coefficients on lagged variables (and), then the policy rule may have the form of a moving average of a simple Taylor rule. That is, (12) can be written as

$$\begin{aligned} i_t = {} & \alpha_2 \left[\mu_3 \pi_{t-1} + \mu_4 y_{t-1}\right] + \\ & (1-\alpha_2)\left[\mu_1 \pi_t + \mu_2 y_t\right], \end{aligned} \tag{14}$$

with $\mu_1 = (1 + A)$, $\mu_2 = \left(\frac{\beta_1}{(1-\alpha_2)\beta_2} + B\right)$, $\mu_3 = (1 + C)$, and $\mu_4 = \frac{\beta_3}{\alpha_2\beta_2}$. If the pattern of coefficient were such that $\mu_1/\mu_2 = \mu_3/\mu_4$ then the actual rule could be thought of as a moving average of a simple rule $\bar{i}_t = \mu_1 \pi_t + \mu_2 y_t$.

2.3 Simple rules

During the past decade, the research on monetary policy design has focused on simple rules - among which Taylor's (1993) rule is a prominent example - as opposed to more complicated or fully optimal rules.[14] As argued by Woodford (2003b, p. 507), a rationale for this choice can be found in the greater transparency provided by simple rules, which may

[14] For a review of this literature see for example Williams (2003).

increase central bankers' accountability in terms of their commitment to the given policy rule.[15] Typically this literature has focused on simple rules based on two or three parameters (and variables) which are optimized for the given preferences and the given form of the rule assumed. For example Rudebusch and Svensson (1999) estimate a model similar to that presented here, with more lagged variables and an interest rate smoothing argument added in the loss function. They derive numerically the optimal policy rule, which looks more complicated than ours. Moreover they use the model to evaluate a large number of simple rules for setting the interest rate.

Two main findings of this literature are that simple rules perform nearly as well as fully optimal rules and that simple rules are more robust than more complicated rules to model misspecification.

In this vein, we can simplify the optimal rule in a way that approximates the behaviour of the optimal rule. In particular it is straightforward to see that the optimal rule (12) could be approximated by a simple rule of the following form

$$i_t = \rho \bar{i}_{t-1} + (1-\rho)\,\bar{i}_t, \tag{15}$$

with

$$\bar{i}_t = \mu_1 \pi_t + \mu_2 y_t, \tag{16}$$

and $0 < \rho < 1$.

[15] See Svensson (2003) for a discussion of the problems associated to using judgements in monetary policy based on simple instrument rules or targeting rules.

In the empirical literature the standard inertial Taylor rule takes instead the following form

$$i_t = \rho i_{t-1} + (1-\rho)\,\bar{i}_t, \tag{17}$$

with $\bar{i}_t$ equal to (16) or to a forward-looking version of (16) with future expected inflation. The term $\bar{i}_t$ is usually interpreted as an operating target for the policy rate.

The crucial difference of (15) with respect to (17) is that the inertial component is proportional to the lagged operating target, instead of the lagged interest rate. Hence, our alternative specification of the inertial policy rule implies that the central bank gradually adjusts the operating target for the policy rate.[16]

In our framework, substituting the lagged operating target with the lagged interest rate in the simple rule could improve the approximation of the optimal rule only if we had the lagged interest rate in the optimal rule. This only happens if we have an interest rate smoothing objective in the central bank loss function.

By using a model with forward-looking private sector, Woodford (2003a) has shown that it may be optimal to delegate monetary policy to a central bank that has an objective function with an interest rate smoothing motive. This is an interesting result. However, while there exist examples in the real world of institutional arrangements that penalize central banks for not achieving given inflation targets, there is less evidence of central banks being penalized for interest rate changes. The

[16] See Woodford (2003b, p. 96) for a discussion of interest rate rules with partial adjustment on lagged operating target.

reference to a financial stability objective is very general and it is consistent also with an interest rate targeting objective without necessarily implying an interest rate smoothing objective.[17] Thus, to presume, as Woodford and others do, that central banks have preferences of this kind, which are unlike those specified in social loss functions, requires an explicit reference to an interest smoothing objective in the Law concerning central banks.

Sack (2000, pp. 230-231) provides a further argument against an explicit interest rate smoothing objective:

> "To describe this behaviour, which has been referred to as gradualism, many empirical studies of monetary policy incorporate an explicit interest-rate smoothing incentive in the objective function of the Fed. However, introducing this argument has little justification beyond matching the data. Furthermore, the above statistics provide evidence of gradualism only if the Fed would otherwise choose a random-walk policy in the absence of an interest-rate smoothing objective. Therefore, while establishing that the funds rate is not a random walk, these statistics do not necessarily provide evidence of gradualism in monetary policy".

Thus we can argue that it would be perfectly plausible to test empirically for alternative specifications of simple rules which do not necessarily include the lagged interest rate, but provide as well some degree of inertia reflecting the dynamic structure of the economy (and eventually the uncertainty surrounding that structure).

[17] See for example Goodfriend (1987).

3 Empirical evidence

3.1 Estimation of inflation and output equations

In order to gain some insights into the parameters of the inflation and output equations used in the previous theoretical analysis we have first estimated the following empirical model based on Rudebusch and Svensson (1999):

$$\pi_t = \kappa_{\pi 1}\pi_{t-1} + \kappa_{\pi 2}y_{t-1} + \kappa_{\pi 3}\pi_{t-2} + \omega_t, \tag{18}$$

and

$$y_t = \kappa_{y1}y_{t-1} + \kappa_{y2}y_{t-2} + \kappa_{y3}\left(\widetilde{i}_{t-1} - \widetilde{\pi}_{t-1}\right) + \psi_t, \tag{19}$$

where the variables were de-meaned prior to estimation. The data used here are ex post revised quarterly data. Inflation is defined using the GPD-chain weighted price index (P_t), with $\pi_t = 400 \cdot (\ln P_t - \ln P_{t-1})$. The output gap is defined as the percentage difference betweeen actual real GDP (Q_t) and potential output (Q^*) estimated by the Congressional Budget Office. The interest rate i_t is the quarterly average of the Fed Funds rate.[18]

In table 1 we report Ordinary Least Squares estimates of the above two equations over the period 1961 Q1 - 2004 Q2, with robust standard errors for the inflation equation. Following Rudebusch and Svensson the equations were estimated individually. In the output equation $\widetilde{i}_t = (1/4)\sum_{j=0}^{3} i_{t-j}$ and $\widetilde{\pi}_t = (1/4)\sum_{j=0}^{3} \pi_{t-j}$. The inflation equation is

[18] While real GDP and the GPD-chain weighted price index were taken from FRED of the Federal Reserve of San Louis, the (effective) Fed Funds rate was taken from Datastream.

somewhat simpler compared to that estimated by Rudebusch and Svensson. According to the Wald test the null hypothesis that $\kappa_{\pi 3} = (1 - \kappa_{\pi 1})$ has a p-value of .15, therefore we have imposed this restriction in the estimation.

3.2 Findings on policy rules

We examine the two policy rules considered above. The standard inertial Taylor rule

$$i_t = \rho i_{t-1} + (1 - \rho)\, \bar{i}_t + \xi_t, \tag{20}$$

and the alternative inertial Taylor rule

$$i_t = \rho \bar{i}_{t-1} + (1 - \rho)\, \bar{i}_t + \xi_t, \tag{21}$$

with

$$\bar{i}_t = \mu_0 + \overline{\mu}_\pi \widetilde{\pi}_t + \overline{\mu}_y y_t, \tag{22}$$

and $0 < \rho < 1$. ξ_t is an i.i.d. error term. Following Taylor (1993) and Rudebusch (2002) the policy rate reacts to four-quarter inflation $\widetilde{\pi}_t$.

As argued by Rudebusch (2002), the evidence on the near-observational equivalence of partial adiustment and serially correlated shocks for monetary policy rules provides a motivation for testing whether rules like (20) and (21) are misspecified. In fact the omission of a persistent, serially correlated variable that influences monetary policy could yield the spurious appearance of partial adjustment in the estimated rule. Indirect testing of these two alternative hypotheses, based on the evidence on the low predictability of policy rates, leads him to the conclusion that monetary inertia is an illusion and the lagged interest rate is not a fundamental component of the U.S. policy rule. However English, Nelson and Sack (2003) show that, by testing these two alternative hypotheses directly in the estimation of the policy rule, both play an important role in describing the behaviour of the federal funds rate.

Following English, Nelson and Sack (2003), our estimations are based on re-specifications of rules (20) and (21) that allow for both partial adjustment and serially correlated errors. We assume that the shock ξ_t follows an AR(1) process:

$$\xi_t = \theta\xi_{t-1} + \varepsilon_t. \tag{23}$$

It then follows that the combination of rule (20) with (23) yields the following expression for the first difference of the interest rate:

$$\Delta i_t = (1-\rho)\Delta\bar{i}_t - (1-\rho)(1-\theta)(i_{t-1} - \bar{i}_{t-1}) + \rho\theta\Delta i_{t-1} + \varepsilon_t. \tag{24}$$

This expression corresponds to that used by English, Nelson and Sack (2003) for distinguishing the two alternative hypotheses, monetary inertia versus omission of serially correlated variables, directly in the estimation of the policy rule. Similarly, the combination of rule (21) with (23) yields the following expression for the first difference of the interest rate:

$$\Delta i_t = (1-\rho)\Delta\bar{i}_t - (1-\theta)(i_{t-1} - \bar{i}_{t-1}) + \rho\theta\Delta\bar{i}_{t-1} + \varepsilon_t. \tag{25}$$

Nonlinear Least Squares estimates of specifications (24) and (25) are reported respectively in tables 2 and 3, for the period 1987 Q4 -

2004 Q2, and for two subsamples of it. The point estimates of ρ and θ are both highly significant for all rules, suggesting that both partial adjustment and serially correlated errors are present. The coefficients on the output gap and inflation are largely consistent with other estimates from the literature, with a significant coefficient on the output gap and a coefficient on inflation greater than one. Moreover, it is possible to observe that both rules appear to fit the data relatively well.

Interestingly, the degree of inertia implied by the alternative inertial Taylor rule is systematically lower than that implied by the standard specification, with an estimated coefficient of partial adjustment ρ for the whole sample of .60 against one of .77. Meanwhile, the coefficient θ is systematically higher in the case of the alternative specification than in the standard specification. However, we have not tested whether these differences are significant statistically.

Thus, as in English, Nelson and Sack (2003), we find empirical evidence supporting specifications (24) and (25) of the policy rules against specifications (20) and (21). But the alternative specification features a reduced importance of monetary inertia and a greater importance of serially correlated variables compared to the standard specification.

In the literature there exists also empirical evidence supporting the importance of forward-looking policy rules - see for instance Orphanides (2001) and Clarida, Galì and Gertler (2000). Thus it might be useful to compare our estimated backward-looking policy rules with the estimates obtained from the standard specification of the operative target with expectations of future inflation. In table 4 are reported Generalized Method of Moments (GMM) estimates of rule (20) with (23) and for the case when

$$\bar{i}_t = \mu_0 + \overline{\mu}_\pi E_{t-1}\widetilde{\pi}_{t+4} + E_{t-1}\overline{\mu}_y y_t. \tag{26}$$

The instruments chosen were four lags each of inflation, the funds rate and the output gap. As it is possible to see from table 4, the goodness-of-fit is not improved compared to the case of backward-looking policy rules.

3.3 Testing for robustness

In order to check for the robustness of our findings we estimate specifications (24) and (25) based on real-time data, instead of ex post revised data.[19] The consideration of real-time data might be relevant in order to account for the presence of measurement errors. In fact, we might have measurement errors in the estimates presented in table 2 and 3 due to the fact that they are based on data that were not available to the Federal Reserve at the time of its policy decisions.[20] The real-time measures of the output gap and inflation used are based the given quarter's releases of data for the previous quarter.[21]

In table 5 and 6 are reported Nonlinear Least Squares estimates of specifications (24) and (25) for the period from 1987 Q4 to 2001 Q2. The

[19] For easing the comparison with the findings of English, Nelson and Sack (2003), we have used the same real-time data considered in their work. We thank Brian sack for having kindly provided us the data.

[20] See Orphanides (2001) for an analysis of the informational problems related with the estimation of simple monetary policy rules. In particular he shows that estimates derived from ex post revised data differ remarkably from estimates derived from real-time data.

[21] The real-time data set is made available by the Federal Reserve Bank of Philadelphia.

results obtained confirm the presence of both partial adjustment and serially correlated errors in the estimated interest rate rules. However, like the findings of Orphanides (2001), in the case of rule (24) the coefficient of inflation falls below one and is not statistically different from zero.[22] This is unfortunate! As Henderson and McKibbin (1993) and Clarida, Galì and Gertler (2000) show, a coefficient on inflation greater than one is required a value greater than one for stability in macroeconomic models with policy rules of this type.[23]

On the contrary in the case of rule (25) the coefficient of inflation is statistically different from zero. This suggests that, on the basis of real-time data, rule (24) is misspecified, while the correct specification is more likely to be rule (25).

Notice that the coefficient of inflation in rule (25) is greater than one only for the subsample 1987 Q4 - 1993 Q4, where the estimated coefficient is equal to 1.04. The fact that the coefficient of inflation is

[22] In the working paper version of their analysis, of 2002, also English, Nelson and Sack report a not significant coefficient for inflation in the standard inertial Taylor rule in the estimates based on real-time data (see table 3 in their text).

[23] The principle that interest rate rules should respond more than one for one to changes in inflation is called "Taylor principle": see for instance Walsh (2003). However, Bullard and Mitra (2002) and Woodford (2003b) have shown that in general the necessary and sufficient condition required for stability may have a more complex form than that expressed by the Taylor principle. In particular it is possible to show that $\overline{\mu}_\pi > 1$ is only a necessary condition for the determinacy of the rational expectations equilibrium, and even values of $0 < \overline{\mu}_\pi < 1$ can be consistent with stability. However, as argued by Woodford (2003b, p. 254) the Taylor principle continues to be a crucial condition for determinacy if it is reformulated as: "[...] *At least in the long run, nominal interest rates should rise by more than the increase in the inflation rate*".

not greater than one for the period 1987 Q4 - 2001 Q2 could be due to the possibility that the Federal Reserve is reacting to more timely information than the lagged GDP deflator. Forecasts from surveys or alternative indicators of inflation might be included in the information set available for the policy maker. Anyway, the Wald test fails to reject the null that the coefficient of inflation in rule (25) is equal to 1.04 also for period 1987 Q4 - 2001 Q2.

4 Conclusions

APPENDIX

The equation for inflation can be written as

$$\pi_{t+2} = \alpha_1 \Delta_t + (1 - \alpha_2)\,\pi_{t+1} + \alpha_2 \pi_t + \alpha_1 \eta_{t+1} + \epsilon_{t+2}, \tag{27}$$

and this can be converted to a system of first order difference equations so that it can be written as a standard dynamic programming problem. The choice of Δ_t is made at time t knowing π_t, $E_t(\pi_{t+1})$, y_t and so on. So we write the equation as

$$E_{t+1}(\pi_{t+2}) = \alpha_1 \Delta_t + (1 - \alpha_2)\,E_t(\pi_{t+1}) + \alpha_2 \pi_t + \alpha_1 \eta_{t+1} + (1 - \alpha_2)\epsilon_{t+1}$$

and we supplement the system with

$$\pi_{t+1} = E_t(\pi_{t+1}) + \epsilon_{t+1}$$

then we have a first order system in the two variables $E_t(\pi_{t+1})$ and π_t. It can be written as

$$z_{t+1} = Az_t + Bu_t + \nu_{t+1}$$

where we have defined $z_t \equiv \begin{bmatrix} E_t(\pi_{t+1}) \\ \pi_t \end{bmatrix}$, $u_t \equiv [\Delta_t]$ and $\nu_{t+1} \equiv \begin{bmatrix} \alpha_1 \eta_{t+1} + (1 - \alpha_2)\epsilon_{t+1} \\ \epsilon_{t+1} \end{bmatrix}$ and the parameter vectors and matrices are $A = \begin{bmatrix} 1 - \alpha_2 & \alpha_2 \\ 1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} \alpha_1 \\ 0 \end{bmatrix}$.

The period loss function is

$$L_t = (1/2)(\pi_t^2 + \lambda y_t^2)$$

and we try to minimize $E_t\left(\sum_{s=t}^{\infty} \delta^{s-t} L_s\right)$ by choosing a sequence of $\Delta_t, \Delta_{t+1}, \Delta_{t+2}, \ldots.$ So we can write the period loss function (for period t+1) as

$$L_{t+1} = (1/2)\left((E_t(\pi_{t+1}) + \epsilon_{t+1})^2 + \lambda(\Delta_t + \eta_{t+1})^2\right)$$

In terms of expected values as of date t, we have

$$E_t(L_{t+1}) = (1/2)\left\{z_t'Rz_t + u_t'\lambda u_t + (\sigma_\epsilon^2 + \lambda\sigma_\eta^2)\right\}$$

Now the problem can be written in a standard form. We choose Δ_t so that

$$V_t(E_t(\pi_{t+1}, \pi_t) = \min_{\Delta_t} E_t\left\{z_t'Rz_t + u_t'\lambda u_t + (\sigma_\epsilon^2 + \lambda\sigma_\eta^2) + \delta V_{t+1}(E_{t+1}(\pi_{t+2}), \pi_{t+1})\right\},$$

subject to the equation of motion of the system given above. The cost-to-go function $V_t(E_t(\pi_{t+1}, \pi_t)$ has the form

$$V_t(E_t(\pi_{t+1}, \pi_t) = z_t'v_tz_t + k_t$$

where k_t is a constant (whose value depends on the variance terms). So we can write the problem as

$$z_t'v_tz_t + k_t = \min_{u_t} E_t\left\{z_t'Rz_t + u_t'\lambda u_t + (\sigma_\epsilon^2 + \lambda\sigma_\eta^2) + \delta(z_{t+1}'v_{t+1}z_{t+1} + k_{t+1})\right\},$$

This is the standard textbook formulation of the dynamic programming problem. The first order condition gives

$$E_t\left[\lambda u_t + \delta B v_{t+1} z_{t+1}\right] = 0$$

or

$$\lambda u_t + \delta B' v_{t+1}(Az_t + Bu_t) = 0$$

hence the feedback rule

$$u_t = -(\lambda + \delta B' v_{t+1} B)^{-1} \delta B' v_{t+1} A z_t$$

which is conventionally written as

$$u_t = F_t z_t$$

with

$$F_t \equiv -(\lambda + \delta B' v_{t+1} B)^{-1} \delta B' v_{t+1} A$$

Putting the feedback rule back into the expression for the cost-to-go function above gives

$$v_t = R + F_t' \lambda F_t + \delta (A + BF_t)' v_{t+1} (A + BF_t)$$

In the infinite horizon case, assuming the system can be controlled and we have convergence, $v_t = v_{t+1} = v$, and

$$v = R + A'[\delta v - \delta v B(\lambda + B'\delta v B)^{-1} B'\delta v] A$$

and

$$F \equiv -(\lambda + \delta B' v B)^{-1} \delta B' v A$$

What does all this imply for the interest rate rule? We have from the above that

$$\Delta_t = f_1 E_t(\pi_{t+1}) + f_2 \pi_t$$

where $F = [f_1\, f_2]$. Since the control variable Δ_t is defined as

$$\Delta_t \equiv \beta_1 y_t - \beta_2 i_t + \beta_3 y_{t-1} + \beta_4 \pi_t + \beta_5 \pi_{t-1}$$

and since

$$E_t(\pi_{t+1}) = \alpha_1 y_t + (1 - \alpha_2)\, \pi_t + \alpha_2 \pi_{t-1}$$

the rule for the interest rate becomes

$$i_t = \frac{\beta_1 - f_1 \alpha_1}{\beta_2} y_t + \frac{\beta_4 - f_1(1 - \alpha_2) - f_2}{\beta_2} \pi_t + \frac{\beta_3}{\beta_2} y_{t-1} + \frac{f_1 \alpha_2 + \beta_5}{\beta_2} \pi_{t-1}$$

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Table 1 - Inflation and output equations with ex post revised data

	Inflation		Output
$\kappa_{\pi 1}$	0.72 (7.80)	κ_{y1}	1.19 (16.52)
$\kappa_{\pi 2}$	0.09 (3.35)	κ_{y2}	-0.27 (-3.72)
		κ_{y3}	-0.06 (-2.12)
$\overline{R}^2$	0.81	$\overline{R}^2$	0.91
SE	1.08	*SE*	0.77

Notes: Ordinary Least Squares estimates. T-statistics shown in parentheses. $\overline{R}^2$ and standard errors (SE) of the residuals also are reported. For the inflation equation T-statistics are based on standard errors that have been corrected for heteroskedasticity and serial correlation using the method of Newey and West (1987). Variables were de-meaned prior to estimation. The sample period is 1961 Q1 – 2004 Q2.

Table 2 - Standard inertial Taylor rule with ex post revised data

	1987 Q4 – 1993 Q4	1987 Q4 – 2001 Q2	1987 Q4 – 2004 Q2
μ_0	0.15 (0.12)	1.10 (0.94)	1.28 (0.89)
$\overline{\mu}_\pi$	2.31 (7.12)	1.85 (4.31)	1.66 (2.41)
$\overline{\mu}_y$	0.92 (5.61)	0.77 (3.94)	0.94 (3.49)
ρ	0.51 (7.58)	0.61 (7.34)	0.72 (6.49)
θ	0.34 (2.09)	0.80 (5.52)	0.77 (5.41)
$\overline{R}^2$	0.99	0.97	0.98
SE	0.26	0.31	0.33

Notes: Nonlinear Least Squares estimates. T-statistics shown in parentheses are based on standard errors that have been corrected for heteroskedasticity and serial correlation using the method of Newey and West (1987). $\overline{R}^2$ and standard errors (SE) of the residuals are reported for the level of the funds rate.

Table 3 - Alternative inertial Taylor rule with ex post revised data

	1987 Q4 – 1993 Q4	1987 Q4 – 2001 Q2	1987 Q4 – 2004 Q2
μ_0	0.41 (0.41)	1.70 (1.07)	-4.08 (-0.17)
$\overline{\mu}_\pi$	2.15 (9.69)	1.40 (5.04)	1.10 (3.50)
$\overline{\mu}_y$	0.78 (5.56)	0.65 (4.56)	0.67 (4.56)
ρ	0.48 (6.62)	0.59 (6.49)	0.60 (8.64)
θ	0.70 (6.18)	0.94 (17.29)	0.99 (28.68)
$\overline{R}^2$	0.99	0.96	0.98
SE	0.28	0.35	0.36

Notes: Nonlinear Least Squares estimates. T-statistics shown in parentheses are based on standard errors that have been corrected for heteroskedasticity and serial correlation using the method of Newey and West (1987). $\overline{R}^2$ statistic and standard errors (SE) of the residuals are reported for the level of the funds rate.

Table 4 – Forward-looking inertial Taylor rule with ex post revised data

	1987 Q4 – 1993 Q4	1987 Q4 – 2001 Q2	1987 Q4 – 2004 Q2
μ_0	-3.35 (-1.28)	0.64 (0.50)	-0.87 (-0.55)
$\overline{\mu}_\pi$	2.57 (3.51)	2.16 (4.16)	2.58 (3.83)
$\overline{\mu}_y$	-0.30 (-0.59)	0.62 (3.73)	0.74 (4.20)
ρ	0.79 (11.69)	0.66 (5.32)	0.68 (6.51)
θ	0.07 (0.37)	0.62 (3.71)	0.67 (5.08)
$\overline{R}^2$	0.94	0.95	0.97
SE	0.60	0.37	0.38

Notes: Generalized Method of Moments estimates. Instruments are four lags each of inflation, the funds rate, and the output gap. T-statistics shown in parentheses are based on standard errors that have been corrected for heteroskedasticity and serial correlation using the method of Newey and West (1987). $\overline{R}^2$ and standard errors (SE) of the residuals are reported for the level of the funds rate.

Table 5 - Standard inertial Taylor rule with real-time data

	1987 Q4 – 1993 Q4	1987 Q4 – 2001 Q2
μ_0	3.61 (1.98)	3.66 (4.57)
$\overline{\mu}_\pi$	0.47 (0.73)	0.47 (1.36)
$\overline{\mu}_y$	0.95 (3.50)	0.64 (1.90)
ρ	0.67 (3.95)	0.65 (2.71)
θ	0.26 (1.63)	0.73 (2.17)
$\overline{R}^2$	0.98	0.96
SE	0.37	0.34

Notes: Nonlinear Least Squares estimates. T-statistics shown in parentheses are based on standard errors that have been corrected for heteroskedasticity and serial correlation using the method of Newey and West (1987). $\overline{R}^2$ and standard errors (SE) of the residuals are reported for the level of the funds rate.

Table 6 - Alternative inertial Taylor rule with real-time data

	1987 Q4 – 1993 Q4	1987 Q4 – 2001 Q2
μ_0	2.05 (2.21)	2.81 (1.65)
$\overline{\mu}_\pi$	1.04 (4.17)	0.71 (3.92)
$\overline{\mu}_y$	0.69 (6.39)	0.54 (5.43)
ρ	0.43 (2.99)	0.37 (3.62)
θ	0.65 (3.80)	0.94 (10.15)
$\overline{R}^2$	0.97	0.95
SE	0.41	0.38

Notes: Nonlinear Least Squares estimates. T-statistics shown in parentheses are based on standard errors that have been corrected for heteroskedasticity and serial correlation using the method of Newey and West (1987). $\overline{R}^2$ and standard errors (SE) of the residuals are reported for the level of the funds rate.