Complementarities and Costly Investment in a One-Sector Growth Model*

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Summary. We develop a growth model which combines the assumptions of complementarities between capital goods in the production function, and of internal costly investment in capital. Building on Evans, Honkapohja and Romer's (1998) two-sector model, we adopt instead a one-sector framework and replace their analytically-non-observable external cost of investment with an analytically-observable internal investment cost function. Hence we aim to provide microfoundations to the convex adjustment costs in Evans et al. (1998), while introducing internal investment costs in the R&D-based growth literature. We find that, whereas the combination of complementarities and costly investment generates multiple equilibria in Evans et al.'s (1998), in our model such combination of assumptions generates a unique equilibrium.

Keywords and Phrases: Complementarities, Costly Investment, Economic Growth.

JEL Classification Numbers: O30, O40, O41

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1 Introduction

With this paper we propose a growth model whose key aspect is the combination of the assumptions of complementarities between capital goods in the production function and of internal costly investment in capital. This second assumption of internal costs of investment, that we introduce, is new to the R&D-based growth literature.

Bryant [3] stresses the importance of complementarity between capital goods in production. Additionally, as Romer [8] writes, investment decisions are better captured by a standard theory which emphasises the existence of costs to accumulating capital.

Our proposed model builds on the strongly motivated multiple equilibria model by Evans, Honkapohja and Romer [5] in assuming complementarities between capital goods in the production function.

Personal computers, printers and communication networks are examples of capital goods that are complements. If the number of its complementary goods increases, the production of a capital good will increase. In turn, by increasing its output, a producer of a capital good is raising the demand for its complementary goods. Evans et al. [5] embed this self-reinforcing process in a standard model of monopolistic competition.

Building on their model, we replace Evans et al.'s [5] analytically-non-observable external cost of investment with an analytically-observable internal investment cost function due to Hayashi [6]. Hence we aim to contribute to growth literature by providing microfoundations to the convex adjustment costs in Evans et al. [5], and by introducing internal investment costs in the R&D-based growth literature.

Evans et al. [5] generate a nonlinear Technology curve through an analytically-non-observable mechanism. The authors aggregate physical capital and inventions into a single variable called general-purpose-capital and make the two-sector assumption that there is a non-linear trade-off between consumption and investment in general-purpose-capital. The price of general-purpose-capital in terms of consumption varies positively with the growth rate, through an analytically-non-observable function.

In contrast, we generate a nonlinear Technology curve through an analytically-observable mechanism. We assume that our model has a one-sector structure; in that consumption, investment in physical capital and inventions are all undertaken with the same technology. We then introduce the assumption that final-good producers incur an internal investment cost when accumulating total capital. Total capital is used for both the invention and the production of capital goods.

Our main finding is that, whereas the combination of the assumptions of complementarities between capital goods and of costly investment in capital generates multiple equilibria in Evans et al.'s [5] model, in our model such combination of assumptions generates a unique equilibrium.

The paper is organised as follows. After this Introduction, Section 2 provides motivation for the introduction of the assumption of internal costly investment.

Section 3 presents the specification of our proposed general equilibrium growth model, and its main results. Section 4 closes the present study with Concluding Remarks.

2 Motivation for Internal Costly Investment

Consider the baseline model of investment in which firms maximise the present discounted value of their cash flows, facing zero capital investment costs. Assume also that capital depreciation is zero, for simplicity. The *current-value Hamiltonian* is, then:

$$H(t) = F(K(t), \overline{L}) - I(t) + q(t)(I(t) - \dot{K}(t)), \tag{1}$$

where q is the *current-value* of capital.

The solution to this maximisation problem is the standard condition:

$$\frac{dF(K, \overline{L})}{dK} = r \tag{2}$$

As Hayashi [6] analyses, in this model the rate of optimal investment is indeterminate and the optimal level of capital stock can be determined for a given level of output and a linearly homogeneous production function.

This means that if, for instance, the initial level of capital $K(0)$ is lower than the optimal capital level K^*, investment will be infinitely positive. Or, if the interest rate falls, the stock of capital that satisfies the standard condition increases, and this requires an infinite rate of investment. However, as investment is limited by aggregate output, it cannot be infinite.

This indeterminacy of investment led to modifications of the baseline model. Such modifications involve the introduction of costs to the accumulation of capital. Hayashi [6] defines the result of these changes as the modified neoclassical investment theory, where the representative firm maximises the present discounted value of its cash flows, subject to capital installation costs.

The specification for the capital installation cost function that we use in this paper is an application of Hayashi's [6] cost of investment framework to a continuous time context, as done by Benavie et al. [2], Cohen [4] and Van Der Ploeg [10], in models different from the one developed in this paper.

This investment cost specification assumes, then, that installing $I(t) = \dot{K}(t)$ new units of capital requires the firms to spend an amount given by:

$$J(t) = I(t) + \frac{1}{2}\theta\frac{I(t)^2}{K(t)} \tag{3}$$

where the installation cost is $C(I(t), K(t)) = \frac{1}{2}\theta\frac{I(t)^2}{K(t)}$.

The *current-value Hamiltonian* is, then:

$$H(t) = BK(t) - I(t) - \frac{1}{2}\theta\frac{I(t)^2}{K(t)} + q(t)(I(t) - \dot{K}(t)) \tag{4}$$

A one-unit increase in the firm's capital stock increases the present value of the firm's cash flow by q, and thus increases the value of the firm by q. Hence q is the market value of a unit of capital.

Since the purchase price of capital is assumed to be $P_K = 1$, the ratio of the market value of a unit of capital to its replacement cost, $\frac{q}{P_K}$, is equal to q. This ratio is known as Tobin's [9] *marginal q*.

In turn, the ratio of the market value of the firm to the replacement cost of its total capital stock, $\frac{V}{P_K K} = \frac{V}{K}$ is called *average q*.

It is *marginal q* that is relevant to investment. However only *average q* is observable. Thus empirical studies have relied on *average q* as an approximation to *marginal q*. Hayashi [6] solved this empirical issue because with his installation cost function, described above, *marginal q* and *average q* are equal.

After this motivation for the capital installation cost function adopted for the model developed in this paper, we proceed with the specification of the model and its main results.

3 Specification and Results of the Model

3.1 Consumption Side

The preferences structure adopted is the standard optimising one. Infinitely lived homogeneous consumers maximise, subject to a budget constraint, the discounted value of their representative utility:

$$\max \int_0^\infty e^{-\rho t} U(C(t))dt \qquad , \qquad U(C(t)) = \frac{C(t)^{1-\sigma}}{1-\sigma},$$

where variable $C(t)$ is consumption in period t, ρ is the rate of time preference and $\frac{1}{\sigma}$ is the elasticity of substitution between consumption at two periods in time.

Consumption decisions are given by the familiar Euler equation:

$$g_c = \frac{\dot{C}}{C} = \frac{1}{\sigma}(r - \rho), \tag{5}$$

so, a balanced growth path solution requires a constant interest rate.

3.2 Production Side

The technology in this economy is characterised by a combination of the effects of complementarities between capital goods in the production function and the effects of internal costly investment in capital.

The production side is composed by three productive activities: final good production, capital goods production and invention of new capital goods, that is, research and development (R&D) activities. We make the one-sector-model assumption that the same technology is used to undertake the three productive activities, as in Rivera-Batiz and Romer [7].

3.2.1 Complementarities between Capital Goods

For the specification of final good production activities, we follow Evans et al. [5] in assuming that the final good Y is produced using as inputs labour L, assumed constant, and a number A of differentiated durable capital goods i, each produced in quantity x_i. Capital goods enter complementarily in the production function. All this is captured by the following production function:

$$Y(t) = L(t)^{1-\alpha} \left(\int_0^{A(t)} x_i(t)^\gamma di \right)^\phi \quad , \quad \phi > 1 \quad , \quad \gamma\phi = \alpha, \tag{6}$$

where the assumption $\phi > 1$ is made so that capital goods are complementary to one another, that is, so that an increase in the quantity of one good increases the marginal productivity of the other capital goods. The restriction $\gamma\phi = \alpha$ is imposed to preserve homogeneity of degree one.

The second productive activity concerns the production of the physical machines for each of the already invented types of capital goods. Assuming that it takes one unit of physical capital to produce one physical unit of any type of capital good, in each period physical capital K is related to the capital goods by the rule:

$$K(t) = \int_0^{A(t)} x_i(t) di, \tag{7}$$

Turning now to the R&D activities, new designs are invented with the same technology as that of the production of the final good and of capital goods. We assume that the invention of patent i requires $P_A\ i^\xi$ units of foregone output, where P_A is the fixed price of one new design in units of foregone output, and i^ξ represents an additional cost of patent i in terms of foregone output, meaning that there is a higher cost for designing goods with a higher index. This extra cost is introduced in order to avoid an explosive technological growth.

Total investment in each period $\dot{W}(t)$ is then given by:

$$\dot{W}(t) = \dot{K}(t) + P_A \dot{A}(t) A(t)^\xi, \tag{8}$$

where $\dot{K}(t)$ represents investment in physical capital, and $P_A \dot{A}(t) A(t)^\xi$ represents investment in the invention of new designs.

Variable W stands for total capital, and it is equal to:

$$W(t) = K(t) + P_A \frac{A(t)^{\xi+1}}{\xi + 1} \tag{9}$$

The accumulation equation for total capital is:

$$\dot{W}(t) = Y(t) - C(t) \tag{10}$$

In order to solve the model for a constant growth rate, we follow Evans et al. [5] in imposing the following restriction:

$$\xi = \frac{\phi - 1}{1 - \alpha} \tag{11}$$

Final good producers are price takers in the market for capital goods. In equilibrium they equate the rental rate on each capital good with its marginal productivity. So the demand curve faced by each capital good producer is:

$$R_j(t) = \frac{dY(t)}{dx_j(t)} = \phi\gamma L^{1-\alpha} x_j(t)^{\gamma-1} \left(\int_0^{A(t)} x_i(t)^\gamma di \right)^{\phi-1}, \tag{12}$$

which is equivalent to:

$$x_j(t) = \left[\frac{\alpha L^{1-\alpha} \left(\int_0^{A(t)} x_i(t)^\gamma di \right)^{\phi-1}}{R_j(t)} \right]^{\frac{1}{1-\gamma}} \tag{13}$$

Capital good firms face the same market conditions. So they produce the same quantities of their differentiated goods and sell them at the same price. That is, the symmetry of the model implies that $R_j(t) = R(t)$, and $x_j(t) = x(t)$. Hence the production function for aggregate output 6, can be rewritten as:

$$\begin{aligned} Y &= L^{1-\alpha} A^\phi x^\alpha \\ &= L A^{1+\xi} \left(\frac{\alpha}{R} \right)^{\frac{\alpha}{1-\alpha}}, \end{aligned}$$

Likewise, the expression for total capital 9, can be rewritten as:

$$W = K + P_A \frac{A^{\xi+1}}{\xi+1} = A^{1+\xi} \left[L \left(\frac{\alpha}{R} \right)^{\frac{1}{1-\alpha}} + \frac{P_A}{\xi+1} \right]$$

It follows that:

$$\begin{aligned} \frac{Y}{W} &= \frac{L A^{1+\xi} \left(\frac{\alpha}{R} \right)^{\frac{\alpha}{1-\alpha}}}{A^{1+\xi} \left[L \left(\frac{\alpha}{R} \right)^{\frac{1}{1-\alpha}} + \frac{P_A}{\xi+1} \right]} \\ &= \frac{L \left(\frac{\alpha\gamma}{r} \right)^{\frac{\alpha}{1-\alpha}}}{\left[L \left(\frac{\alpha\gamma}{r} \right)^{\frac{1}{1-\alpha}} + \frac{P_A}{\xi+1} \right]} = B \end{aligned} \tag{14}$$

That is, the production function of this economy can be expressed as a function of total capital in the following way:

$$Y = BW,$$

where B, the marginal productivity of total capital, is constant.

3.2.2 Internal Costly Investment

Final good producers own total capital W and incur an internal investment cost. We assume that installing $I(t) = \dot{W}(t)$ new units of total capital requires the final good firms to spend an amount given by:

$$J(t) = I(t) + \frac{1}{2}\theta\frac{I(t)^2}{W(t)}, \tag{15}$$

where $C(I(t), W(t)) = \frac{1}{2}\theta\frac{I(t)^2}{W(t)}$ represents the Hayashi's [6] installation cost.

Final good firms choose their investment rate so as to maximise the present discounted value of their cash flows. Their profit maximisation problem is then:

$$\max_{I(t)} V(t) = \int_0^\infty \left(Y(t) - I(t) - \frac{1}{2}\theta\frac{I(t)^2}{W(t)}\right) e^{-rt} dt \tag{16}$$

subject to:

$$\dot{W}(t) = I(t)$$

The *current-value Hamiltonian* is:

$$H(t) = Y(t) - I(t) - \frac{1}{2}\theta\frac{I(t)^2}{W(t)} + q(t)(I(t) - \dot{W}(t)), \tag{17}$$

where $q(t)$ is the market value of capital.

The transversality condition of this optimization problem is:

$$\lim_{t\to\infty} e^{-rt} q(t)W(t) = 0, \tag{18}$$

the first-order condition is equivalent to:

$$\frac{I}{W} = \frac{q-1}{\theta}, \tag{19}$$

and the co-state equation is equivalent to:

$$\frac{\dot{q}}{q} = r - \frac{B + \frac{1}{2}\theta\left(\frac{I}{W}\right)^2}{q}, \tag{20}$$

The problem is solved for its balanced growth path solution. Recalling the production function $Y = BW$, the growth rate of output is:

$$g = \frac{I}{W}$$

This means that equation 19 can be rewritten as:

$$q = 1 + \theta g \tag{21}$$

In a balanced growth path, the growth rate must be constant, which implies that q must be constant. Therefore equation 20 becomes:

$$q = \frac{B + \frac{1}{2}\theta g^2}{r} \tag{22}$$

Continuing with the description of the model, we turn now to the capital good firms production decisions. Once invented, the physical production of each unit of the specialised capital good requires one unit of capital. So, in each period the monopolistic capital good producer maximises its profits, taking as given the demand curve 12 for its good:

$$\max_{x(t)} \pi(t) = R(t)x(t) - rqx(t)$$

This leads to the markup rule:

$$R = \frac{rq}{\gamma} \tag{23}$$

At time t, in order to enter the market and produce the Ath capital good, a firm must spend upfront an amount given by $P_A A(t)^{\xi}$, where, as mentioned earlier, P_A is the fixed price of one new design in units of foregone output, and i^{ξ} represents an additional cost of patent i in terms of foregone output. Hence, the dynamic zero-profit/free-entry condition is:

$$P_A A(t)^{\xi} = \int_t^{\infty} e^{-r(\tau - t)} \pi(\tau) d\tau,$$

which is equivalent to:

$$\xi g_A = r - \frac{\pi}{P_A A^{\xi}}, \tag{24}$$

In a balanced growth path, x is growing at the rate:

$$\frac{\dot{x}}{x} = \frac{\xi \dot{A} A^{\xi-1} L \left(\frac{\alpha}{R}\right)^{\frac{1}{1-\alpha}}}{L A^{\xi} \left(\frac{\alpha}{R}\right)^{\frac{1}{1-\alpha}}} = \xi g_A$$

Consequently physical capital is growing at the rate:

$$g_k = (1 + \xi) g_A,$$

and output is growing at the rate:

$$g_y = \phi g_A + \alpha \xi g_A = (1 + \xi) g_A$$

It follows, from equation 8, that total capital W grows at the same rate as output:

$$g_w = \left[\frac{(1 + \xi)K + P_A A^{\xi+1}}{W}\right] g_A = (1 + \xi) g_A$$

Then equation 24 leads us to the equation that describes the decisions made on the production side:

$$g_y = \frac{1+\xi}{\xi}\left[r - \frac{\pi}{P_A A^{\xi}}\right],$$

which, recalling and rearranging the expression for profits:

$$\pi = (R - rq)\,x = \left(\frac{1-\gamma}{\gamma}\right) rqLA^{\xi}\left(\frac{\alpha\gamma}{rq}\right)^{\frac{1}{1-\alpha}},$$

becomes:

$$g = \frac{1+\xi}{\xi}\left[r - \frac{\Omega}{(rq)^{\frac{\alpha}{1-\alpha}}}\right], \qquad \Omega = \frac{\left(\frac{1-\gamma}{\gamma}\right) L\,(\alpha\gamma)^{\frac{1}{1-\alpha}}}{P_A} \tag{25}$$

Equation 25 unites the equilibrium balanced growth path pairs (g, r) on the production side of this economy. We call it Technology curve, after Rivera-Batiz and Romer [7].

3.3 General Equilibrium

The capital accumulation equation 10 tells us that a constant growth rate of W implies that consumption grows at the same rate as output. Which means that the per-capita economic growth rate is:

$$g_c = g_y = g_k = g_w = g = (1+\xi)g_A$$

The general equilibrium solution is obtained by solving the system of the two equations 5 and 25 in the two unknowns, r and g. Recalling equation 21, the system to be solved is:

$$\left\{\begin{array}{c} g = \frac{1}{\sigma}(r-\rho) \\ g = \frac{1+\xi}{\xi}\left[r - \frac{\Omega}{(r+r\theta g)^{\frac{\alpha}{1-\alpha}}}\right] \end{array}\right. , \qquad r > g > 0, \tag{26}$$

where $\Omega = \frac{\left(\frac{1-\gamma}{\gamma}\right)L(\alpha\gamma)^{\frac{1}{1-\alpha}}}{P_A}$ and the restriction $r > g$ is imposed so that present values will be finite. Also our solution(s) must have positive values for the interest rate and the growth rate.

The Euler equation 5 is linear and positively sloped in the space (r, g).

In order to analyse the shape of the Technology curve 25, and as it is impossible to isolate r on one side of the equation, we rewrite it as $F(r, g) = 0$ and apply the implicit function theorem, so as to obtain, in the neighbourhood of an interior point of the function, the derivative $\frac{dr}{dg}$ as:

$$\frac{dr}{dg} = -\frac{\frac{dF(r,g)}{dg}}{\frac{dF(r,g)}{dr}}$$

So, we have:

$$F(r,g) = \xi g - (1+\xi) r + (1+\xi) \Omega r^{\frac{-\alpha}{1-\alpha}} (1+\theta g)^{\frac{-\alpha}{1-\alpha}} = 0$$

which leads to:

$$\frac{dr}{dg} = \frac{\xi - \left(\frac{\alpha}{1-\alpha}\right) \theta (1+\xi) \Omega r^{\frac{-\alpha}{1-\alpha}} (1+\theta g)^{\frac{-1}{1-\alpha}}}{(1+\xi) + \left(\frac{\alpha}{1-\alpha}\right) (1+\xi) \Omega r^{\frac{-1}{1-\alpha}} (1+\theta g)^{\frac{-\alpha}{1-\alpha}}} \tag{27}$$

Hence, our nonlinear Technology curve is positively sloped when:

$$r^{\frac{-\alpha}{1-\alpha}} (1+\theta g)^{\frac{-1}{1-\alpha}} < \frac{\xi}{\left(\frac{\alpha}{1-\alpha}\right) \theta (1+\xi) \Omega},$$

and negatively sloped when:

$$r^{\frac{-\alpha}{1-\alpha}} (1+\theta g)^{\frac{-1}{1-\alpha}} > \frac{\xi}{\left(\frac{\alpha}{1-\alpha}\right) \theta (1+\xi) \Omega}$$

Replacing the expression for g given by the Euler equation 5 in the Technology curve 25, we obtain the equilibrium expression for r:

$$r\left(\frac{1}{\sigma} - \frac{1+\xi}{\xi}\right) + \frac{\left(\frac{1+\xi}{\xi}\right)\Omega}{\left[\frac{\theta}{\sigma} r^2 + \left(1 - \frac{\theta\rho}{\sigma}\right) r\right]^{\frac{\alpha}{1-\alpha}}} = \frac{\rho}{\sigma} \tag{28}$$

Unable to derive analytically the expression for the equilibrium solution(s), we resort to solving the system through a numerical example. The chosen values for our parameters are:

$$\begin{aligned} \sigma &= 2; \qquad \rho = 0.02; \qquad \alpha = 0.4; \qquad \gamma = 0.1; \\ \xi &= 5; \qquad L = 1; \qquad \theta = 3; \qquad P_A = 5, \end{aligned}$$

where the values for α, γ and consequently $\phi = \frac{\alpha}{\gamma}$ and $\xi = \frac{\phi - 1}{1-\alpha}$ are the same as those used by Evans et al. [5] in their numerical example. The values for the preference parameters σ and ρ are in agreement with those found in empirical studies such as Barro and Sala-i-Martin [1]. Population is often chosen to have unity value. And the values for θ and P_A were chosen to give us realistic values for the equilibrium growth rate and interest rate.

Although the Technology curve is nonlinear, for the adopted parameter values, a unique solution is found to be:

$$g = 0.024; \qquad r = 0.068$$

Figure 1, with r on the horizontal axis, helps us visualise this economy's balanced growth path general equilibrium solution.

We conclude that, as opposed to its inspirational multiple equilibria model by Evans, Honkapohja and Romer [5], in the model here developed, the combinations of the assumptions of complementarities between capital goods in the production function and of costly investment in capital generate a unique balanced growth path equilibrium.

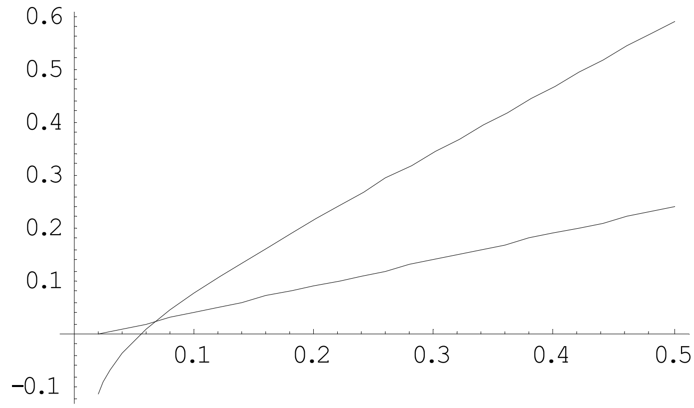

Figure 1:

4 Concluding Remarks

For this paper we have developed a growth model which we have used to provide microfoundations to the convex adjustment costs in Evans, Honkapohja and Romer's [5] model, while introducing the assumption of internal costly investment to R&D-based growth theory.

Our proposed model builds on Evans et al.'s [5] model in assuming complementarities between capital goods in the production function. However, we replace their analytically-non-observable external cost of investment with an analytically-observable internal investment cost function due to Hayashi [6].

Our model is structurally distinct from Evans et al.'s [5] model in two key aspects. Firstly, Evans et al. [5] aggregate physical capital and inventions into a single variable called general-purpose-capital and make the two-sector assumption that there is a non-linear trade-off between consumption and investment in general-purpose-capital. The price of general-purpose-capital in terms of consumption varies positively with the growth rate (through an analytically-non-observable function).

In comparison, our model has a one-sector structure, as it assumes that consumption, investment in physical capital and new designs are all produced under the same technology. We then assume that final-good producers incur an internal investment cost when accumulating capital which is used for both the invention and the production of capital goods.

Secondly, our introduced investment cost function is analytically-observable and hence allows us to derive analytically the nonlinear Technology curve. This contrasts with the analytically-non-observable mechanism to generate the nonlinear Technology curve in Evans et al.'s [5] model.

The paper proposes a two-fold contribution to growth theory. Firstly, the introduced assumption of internal costly investment, with which we give microfoundations to the convex adjustment costs in Evans et al.'s [5] model, is new to R&D-based growth theory.

Our second proposed contribution to growth theory is our finding that while the combination of complementarities between capital goods and costly installation of capital generates multiple equilibria in Evans et al.'s [5] model, in the model here developed such combination of assumptions generates a unique equilibrium.

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