# Oligopolistic Competition Among Inequity Averse Firms†

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**Abstract**

This papers studies how competition in oligopolistic markets is affected when firms are averse to inequity in the sense of Fehr and Schmidt (1999). We find that if firms compete in quantities and have similar costs, then inequity aversion between firms gives rise to a continuum of equilibria. The equilibria can be ranked in terms of welfare for firms and for consumers. An increase in compassion between firms moves the set of equilibria closer to the collusive outcome. By contrast, an increase in envy between firms moves the of set of equilibria closer to the perfectly competitive outcome. These results also hold under price competition in differentiated products. However, when firms are Bertrand competitors, inequity aversion between firms either has no impact on the set of equilibria or it can lead firms to charge higher prices. Finally, the paper shows that even when all firms are strictly averse to inequality, the impact of inequity aversion on equilibrium outcomes vanishes with an increase in the number of firms in the market.

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# 1 Introduction

Many experiments show that individuals are not only motivated by material self-interest, but also care about the well-being of others. More precisely, evidence from experiments with a small number of players shows that many individuals are willing to give up some material payoff to move in the direction of more equitable distributions of payoffs.

The impact of interdependent preferences on equilibrium outcomes has been studied in different economic contexts. Researchers have focused mainly on the implications of interdependent preferences in labor market outcomes and behavior in bargaining games.[1] More recently, several papers look at the implications of interdependent preferences in optimal contracts. For example, Englmaier and Wambach (2002) and Biel (2003) study the implications of inequity aversion in moral hazard problems.[2] Sappington (2004) and Choi (2004) study inequity aversion in adverse selection contexts. Closely related to this paper, is the work by Segal and Sobel (2004) that focuses on the implications of social preferences in equilibrium outcomes in perfectly competitive markets. However, as far as we know, the impact of social preferences on equilibrium outcomes in oligopolistic markets has not been studied before. This paper is the first step in that direction.

The paper incorporates inequity aversion between firms in the basic microeconomic textbook models of strategic interaction between firms: the Cournot model of quantity competition and the Bertrand model of price competition.[3] The novelty here it that we assume that a firm cares about her own monetary payoff and, in addition, would like to reduce the inequality in payoffs between her and the competitors. Firms' interdependent preferences are modelled according to Fehr and Schmidt's (1999) approach.[4] That is, firms are assumed to dislike advantageous inequity (firms feel compassion) and also to dislike disadvantageous inequity (firms feel envy). In this paper a firm feels compassion towards her competitors when the average profits of the competitors are smaller than a firm's own profits. Similarly, a firm fells envy towards her competitors when the average profits of the competitors are greater than a firm's own profits. The preferences of firms are assumed to be common knowledge.

The paper finds that if there is quantity competition and firms' production costs are similar, then inequity aversion between firms gives rise to a continuum of symmetric equilibria. The intuition for this result is simple. Suppose that a firm knows that her opponent will produce the Cournot-Nash quantity. If the firm is averse to inequity, then her best response is to produce also the Cournot-Nash quantity. Producing an output different from the Cournot-Nash quantity reduces the firm's material payoff and increases inequity costs. Now,

[1] Sobel (2005) provides a review of this literature.

[2] Englmaier and Wambach (2002) study optimal contracts when the agent suffers from being better off or worse off than the principal. Biel (2003) studies how the optimal incentive contract in team production is affected when workers are averse to inequity.

[3] We also consider the implications of inequity aversion for quantity and price competition in markets with differentiated products.

[4] For a discussion of alternative approaches see Sobel (2005).

suppose that a firm knows that her opponent will produce somewhat less than the Cournot-Nash quantity. If the firm is averse to advantageous inequity, then her best response is to produce exactly the same quantity as the opponent. Producing a higher quantity than the opponent increases the firm's material payoff by less than the cost from advantageous inequity. Similarly, if a firm knows that her opponent will produce somewhat more than the Cournot-Nash quantity, then her best response is also to produce the same quantity as the opponent. Producing a lower quantity than the opponent increases the firm's material payoff by less than the cost from disadvantageous inequity.[5]

When there is price competition in homogeneous products, inequity aversion between firms either has no impact on the set of equilibria or it can raise firms' prices. This happens because under Bertrand competition only compassion between firms has an impact on equilibrium outcomes. Envy between firms has effect on equilibrium outcomes of Bertrand competition since the lowest equilibrium price, in the absence of inequity aversion, is equal to marginal cost. Additionally, the paper shows that only under very restrictive conditions on preferences will compassion raise prices under symmetric Bertrand competition. For example, when there is Bertrand competition between two firms and marginal costs are constant, only if both firms are willing to give up more than one dollar of their profit to raise the average profit of their opponents by a dollar, can there be an equilibrium where price is above marginal cost.[6]

The paper also considers oligopolistic markets where products are differentiated. When there is quantity competition in differentiated products firms' best responses are downward sloping just like in the homogeneous products case. So, the impact of inequity aversion between firms on quantity competition in differentiated products is analogous to that in the homogeneous products case. When there is price competition in differentiated products firms' best responses are upward sloping, that is, prices are strategic complements. The paper shows that inequity aversion between firms also leads to a continuum of equilibria when there is price competition in differentiated products. The equilibria can also be ranked in terms of welfare for firms and consumers.

We find that compassion between firms hurts consumers whereas envy is either beneficial or innocuous for consumers. We show that this result is valid for both quantity and price competition, and for competition in homogeneous as well as differentiated products. We state results that show that the set of Nash equilibria of oligopolistic markets where firms feel inequity aversion changes monotonically with compassion and envy. For example, if there is quantity competition and firms' degree of envy increases, then the largest Nash equilibria of the Cournot game moves closer to the perfectly competitive quantities.[7]

[5] The paper shows that a necessary condition for the continuum of equilibria to exist is that the game is not too asymmetric. In fact, when there are large cost asymmetries between firms the result no longer holds and there is a unique asymmetric equilibrium.

[6] We show that this result also extends to Bertrand competition between two firms with increasing marginal costs.

[7] A similar result has also been found in a different context. Demougin and Fluet (2003) show that in a rank order tournament the principal is better off when agents are envious than

By contrast, if there is quantity competition and compassion between firms increases, then the smallest Nash equilibria of the Cournot game moves closer to the collusive quantities.[8]

Finally, the paper shows that as the number of firms grows the impact of inequity aversion on the set of Nash equilibria of a $n$-firm game vanishes. In other words, even when all firms in a given oligopolistic market are averse to inequity, the equilibrium outcome is almost the same as if firms had no such concerns. This happens because it takes only one selfish firm to destroy the continuum of equilibria generated by inequity aversion. This point has been made before in papers that study the implications of interdependent preferences in ultimatum games. For example, Bolton and Ockenfels (2000) and Fehr and Schmidt (1999) show how the competitive prediction of the ultimatum game with many proposers and one responder studied by Prasnikar and Roth (1992) continues to hold under the assumption that some individuals in the population care about inequity aversion. Segal and Sobel (2004) also show that interdependent preferences have no impact on equilibria in perfectly competitive markets. In the same line, and consistent with the theory developed in this paper, experimental Cournot games, show that when there are only two firms in the market collusive outcomes are frequent. However, as the number of firms increases output approaches the Nash-equilibrium.[9]

The reminder of the paper proceeds as follows. Section 2 introduces inequity aversion between firms in the standard Cournot game and derives its implications. Section 3 studies the impact of inequity aversion between firms in the Bertrand model of price competition. Section 4 studies inequity aversion between firms in oligopolistic markets with differentiated products. Section 5 concludes the paper. All proofs are in the Appendix.

# 2 Quantity Competition

Let $N = \{1, 2, \ldots, n\}$ denote the set of firms. Consider the standard Cournot model of quantity competition between $n$ firms. Price is determined according to the inverse demand function $P(Q)$, where $Q = \sum q_i$. We make the standard assumption that $P(Q)$ is strictly positive on some bounded interval $(0, \bar{Q})$ on which it is twice continuously differentiable, strictly decreasing, and concave, that si $P' < 0$ and $P'' \leq 0$, with $P(Q) = 0$ for $Q \geq \bar{Q}$. Firms have costs of production given by $C_i(q_i)$, which are increasing and convex, that is, $C_i' \geq 0$ and $C_i'' \geq 0$, $i = 1, \ldots, n$. Firms choose outputs simultaneously to maximize profits, which are given by

$$\pi_i = P\left(Q\right) q_i - C_i(q_i),\ i = 1, \ldots, n.$$

when they are compassionate.

[8] If there is price competition in differentiated products and firms' degree of compassion increases, then the largest Nash equilibria of the price game moves closer to the collusive prices. If there is price competition in differentiated products and firms' degree of envy increases, then the smallest Nash equilibria of the price game moves closer to the perfectly competitive prices.

[9] Hucka et al. (2004) review of the evidence on experimental oligopolistic markets.

For firm $i$, the best response function is defined as

$$R_i(q_{-i}) = \arg_{q_i} \max P(Q)\, q_i - C_i(q_i)\,,\ i = 1, \ldots, n, \tag{1}$$

where $q_{-i} = (q_1, \ldots, q_{i-1}, q_{i+1}, \ldots, q_n)$. Finally, we assume that $\left|\partial^2 \pi_i / \partial q_i^2\right| > \left|\sum_{j \neq i} \partial^2 \pi_i / \partial q_i \partial q_j\right|$. It is a well know result that this last condition is sufficient to guarantee that there is a unique symmetric Cournot-Nash equilibrium in the $n$ firm game.[10] Denote that equilibrium by $q^N$.[11]

To model inequity aversion, we make use of Fehr and Schmidt's (1999) approach. Thus, we assume that firm $i$'s payoff is given by

$$U_i(\pi) = \pi_i - \left[\frac{\alpha_i}{n-1} \sum_{j \neq i} \max(\pi_j - \pi_i, 0) + \frac{\beta_i}{n-1} \max \sum_{j \neq i} (\pi_i - \pi_j, 0)\right]. \tag{2}$$

The terms in the square bracket are the payoff effects of disadvantageous and advantageous inequity, respectively. We see that if firm $i$'s profit is larger than the average profit of its competitors then firm $i$ feels compassion towards its competitors, this is the advantageous inequity term. However, if firm $i$'s profit is smaller than the average profit of its competitors then firm $i$ feels envy towards its competitors, this is the disadvantageous inequity term.[12] Thus, firm $i$'s degree of inequity aversion towards it's competitors is characterized by the pair of parameters $(\alpha_i, \beta_i)$, $i = 1, 2, \ldots, n$.[13] We say that firm $i$ exhibits strict inequality aversion when both $\alpha_i$ and $\beta_i$ are strictly greater than zero. We say that a firm is not averse to inequity when $\alpha_i = \beta_i = 0$. In all other cases we say that a firm is (weakly) averse to inequity. We assume that $\alpha_i$ and $\beta_i$, $i = 1, \ldots, n$, are common knowledge. We let the vector $\beta$ denote firms' compassion degrees and we let the vector $\alpha$ denote firms' degrees of envy. Our first result characterizes a firm's best response in the presence of inequity aversion.

[10] See Tirole (1995).

[11] By this we mean, $-1 \leq \partial R^i(q^j)/\partial q^i < 0$. The second condition ensures the existence of a unique single-period Cournot-Nash equilibrium. A set of sufficient conditions for $R^i$ functions to be "well-behaved" is that $P(q^i + q^j)$ is strictly positive on some bounded interval $(0, Q)$ on which it is twice continuously differentiable, strictly decreasing, and concave, with $P(q^i + q^j) = 0$ for $q^i + q^j \geq Q$.

[12] When there are only two firms in the market firm $i$'s payoff becomes

$$U_i(\pi_i, \pi_j) = \pi_i - [\alpha_i \max(\pi_j - \pi_i, 0) + \beta_i \max(\pi_i - \pi_j, 0)]\,,\ i \neq j = 1, 2. \tag{3}$$

Fehr and Schmidt assume that the dislike of disadvantageous inequity is stronger than that of advantageous inequity, i.e. $\alpha_i > \beta_i$ and that $\beta_i$ is smaller than 1. We make no assumptions about the relation between $\alpha_i$ and $\beta_i$ but we assume, like Fehr and Schmidt, that $\beta_i$ is smaller than 1.

[13] Alternatively, we could have considered that firm $i$ has different feelings of compassion and envy towards each competitor. In this case we would have two inequity aversion parameters for each competitor per firm, that is, we would have $\alpha_{ij}$ and $\beta_{ij}$ for $i \neq j = 1, \ldots, n$ . We assume , like Ferh and Schmidt that firm $i$ feels the same degree of envy and compassion towards all competitors. This makes the analysis simpler.

**Proposition 1**: *The best response of firm $i$, $i = 1, \dots, n$, in the $n$-firm Cournot game with inequity aversion is defined by*

$$R_i(q_{-i}) = \begin{cases} s_i(q_{-i}), & 0 \leq \frac{1}{n-1}\sum_{j\neq i} q_j \leq q(\beta_i) \\ \frac{1}{n-1}\sum_{j\neq i} q_j, & q(\beta_i) \leq \frac{1}{n-1}\sum_{j\neq i} q_j \leq q(\alpha_i) \\ t_i(q_{-i}), & q(\alpha_i) \leq \frac{1}{n-1}\sum_{j\neq i} q_j \end{cases},$$

*where*

$$s_i(q_{-i}) = \arg_{q_i} \max \left(1-\beta_i\right)\left[P\left(Q\right)q_i - C_i(q_i)\right] + \frac{\beta_i}{n-1}\sum_{j\neq i}\left[P\left(Q\right)q_j - C_j(q_j)\right],$$

$$t_i(q_{-i}) = \arg_{q_i} \max \left(1+\alpha_i\right)\left[P\left(Q\right)q_i - C_i(q_i)\right] - \frac{\alpha_i}{n-1}\sum_{j\neq i}\left[P\left(Q\right)q_j - C_j(q_j)\right],$$

*$q(\beta_i)$ is the solution to*

$$\left(1-\beta_i\right)\left[P(nq) - C_i'(q)\right] + P'(nq)q = 0, \tag{4}$$

*and $q(\alpha_i)$ is the solution to*

$$\left(1+\alpha_i\right)\left[P(nq) - C_i'(q)\right] + P'(nq)q = 0. \tag{5}$$

Proposition 1 characterizes the impact of inequity aversion on a firm's optimal output choice for any output levels of its competitors. It tells that a firm's best response is continuous like in the standard Cournot game. However, by contrast with the standard Cournot game, a firm's best response function in the Cournot game with inequity aversion is no longer monotonic.

With inequity aversion the best response has three different segments. When the competitors produce low output levels the best response of a firm that feels inequity aversion has a negative slope and consists of a smaller level of output than the output level that the firm would produce if she felt no inequity aversion. When the competitors produce intermediate output levels the best response of a firm that feels inequity aversion has a positive slope and consists in producing the average output level of the competitors. Finally, when the competitors produce high output levels the best response of a firm that feels inequity aversion has a negative slope and consists of a larger level of output that the output level that the firm would produce if she felt no inequity aversion.

The intuition behind this result is straightforward. Consider, without loss of generality, Cournot competition between two firms. Suppose that firm $i$ knows that firm $j$ will produce a low output level (by comparison with the Nash equilibrium output level). If firm $i$ feels no compassion towards firm $j$, then firm $i$'s best response is to produce $R_i(q_j)$ given by (1). However, if firm $i$ feels compassion towards firm $j$, that is $\beta_i > 0$, then producing $R_i(q_j)$ is no longer the optimal choice. By producing somewhat less than $R_i(q_j)$ there is a second order loss in profits for firm $i$ but a first order gain in reduction of advantageous inequity.

Now suppose that firm $i$ knows firm $j$ will produce an intermediate output level. If firm $i$ dislikes inequity aversion, then there will be a cost in advantageous inequity associated with producing a higher level of output than firm $j$ and there will be also a cost in disadvantageous inequity associated with producing a smaller output level than firm $j$. For intermediate values of output the loss in profits from not matching the opponent's output is small while the inequity costs are large. If that is the case then firm $i$ is better off by producing the same level of output as firm $j$.

Finally, if firm $i$ knows that firm $j$ will produce a high output level and firm $i$ feels no envy towards firm $j$, then firm $i$'s best response is to produce $R_i(q_j)$ given by (1). However, if firm $i$ feels envy towards firm $j$, that is, $\alpha_i > 0$, then producing $R_i(q_j)$ is no longer the optimal choice. By producing somewhat more than $R_i(q_j)$ there is a second order loss in profits for firm $i$ but a first order gain in reduction of disadvantageous inequity.

We can now ready to characterize the set of Nash equilibria of the $n$-firm symmetric Cournot game when firms are averse to inequity. We do that in the next two results.

**Proposition 2**: *The unique Nash equilibrium of the standard $n$-firm symmetric Cournot game is always an equilibrium of the $n$-firm symmetric Cournot game with inequity aversion.*

**Proposition 3**: *The set of Nash equilibria of the $n$-firm symmetric Cournot game with inequity aversion is given by*

$$N^{IA} = \{(q_1, \dots, q_n) : q_i = q_j,\ \forall i \neq j,\ \text{and } q(\beta) \leq q_i \leq q(\alpha),\ i = 1, \dots, n\}, \tag{6}$$

*where*

$$q(\beta) = \max\left[q\left(\beta_1\right), \dots, q(\beta_n)\right],$$

*and*

$$q(\alpha) = \min\left[q(\alpha_1), \dots, q(\alpha_n)\right].$$

These two results describe the implications of inequity aversion in quantity competition in symmetric Cournot games. Recall that, under the assumptions that we made, there is a unique equilibrium of the standard $n$-firm symmetric Cournot game. In that equilibrium firms produce the same amount and the market price is between the perfectly competitive market price and the monopolistic or collusive market price.

Proposition 2 shows that the unique Nash equilibrium of the standard $n$-firm symmetric Cournot game always belongs to the set of equilibria of the $n$-firm symmetric Cournot game with inequity aversion. Proposition 3 tells us that if all firms are strictly averse to inequity, then there is a continuum of equilibria in the $n$-firm symmetric Cournot game. In some of the equilibria of the Cournot game with inequity aversion, the market price is lower than the equilibrium market price in the standard Cournot game whereas in other equilibria the market price is higher. Thus, it is not clear whether inequity aversion between firms will be beneficial to consumers or not.

Proposition 3 also shows that if there is at least one firm that is not averse to inequity, then there is a unique equilibrium of the $n$-firm symmetric Cournot game with inequity aversion which is the equilibrium of the standard $n$-firm symmetric Cournot game. This point has been made before in papers that study the implications of interdependent preferences in ultimatum games and in papers that look at social preferences in perfectly competitive markets.[14]

Even though the model does not tell us whether inequity aversion between firms is generally good or bad for consumers (or for firms), we can state conditions that under which inequity aversion is better or worse for consumers. To do that we look at the impact of changes in the firms' degree of compassion and of envy.

**Proposition 4**: *The largest Nash equilibria of the $n$-firm symmetric Cournot game with inequity aversion is a nondecreasing function of $\alpha$. The smallest Nash equilibria is a nonincreasing function of $\beta$.*

This welfare result characterizes the impact of envy and compassion between firms on the set of Nash equilibria of the Cournot game. It tells us that there is a weak complementarity between the firms' degree of envy and their equilibrium output, that is, an increase in envy between firms increases the total output produced in the largest Nash equilibria of the Cournot model with inequity aversion. If that is the case, then an increase in the degree of envy between firms is likely to reduce firms' profits and increase consumer surplus. On the other hand, Proposition 4 tells us that an increase in compassion between firms reduces the total output produced in the smallest Nash equilibria of the Cournot model with inequity aversion. If that is the case, then an increase in the degree of compassion between firms is likely to increase firms' profits and decrease consumer surplus. This result is quite intuitive. In fact, Fehr and Schmidt's payoff function implies that if firm $i$ has a higher monetary payoff than the average payoff of her opponents and $\beta_i = 1/2$, then firm $i$ is just as willing to keep one dollar to herself as to give it to her competitors. Now, suppose that all firms have the same preferences as firm $i$. In this case firms are acting as if they are maximizing their joint profit, $\sum \pi_i$. So, if $\beta_i = 1/2$, with $i = 1, \ldots, n$, then compassion leads to the collusive outcome.

The next result studies the implications of an increase in the number of firms when there is quantity competition in markets where firms are averse to

[14] See Bolton and Ockenfels (2000), Fehr and Schmidt (1999), and, more recently, Segal and Sobel (2004).

inequity. To state this result we assume that $\alpha_i$ and $\beta_i$, $i = 1, \ldots, n$, are drawn from a uniform distribution with support on $[0,1]$.

**Proposition 5**: *As the number of firms increases the set of Nash equilibria of the $n$-firm symmetric Cournot game when all firms are strictly averse to inequity converges to the unique Nash equilibrium of the standard $n$-firm symmetric Cournot game.*

This result shows that increasing the number of firms reduces the impact of inequity aversion on the set of Nash equilibria of the $n$-firm Cournot game. This happens because when there are $n$ firms, the smallest Nash equilibria of the game is determined by the firm that has the lowest degree of compassion. Similarly, the largest Nash equilibria of the game is determined by the firm with the lowest degree of envy.[15] If the degree of compassion and of envy of each firm are drawn from a uniform distribution with support on $[0,1]$, then an increase in the number of firms makes it more likely that the lowest level of compassion is very close to zero and that the lowest level of envy is very close to zero. Thus, as the number of firms increases the smallest and the largest Nash equilibria of the $n$-firm symmetric Cournot game with inequity aversion converge to the Nash equilibrium of the standard $n$-firm symmetric Cournot game.

We will now relax the assumption that firms have identical costs. To make the analysis simple we consider only two firms and assume that $\alpha_1 = \alpha_2 = \alpha$, $\beta_1 = \beta_2 = \beta$. We also assume that demand is linear, and that marginal costs are constant.

**Proposition 6**: *The set of Nash equilibria of the Cournot duopoly game with inequity aversion, linear demand, $P = a - bQ$, and constant marginal costs, is given by*

$$\left\{ \begin{array}{cc} (q_1^r, q_2^r), & 0 \leq \frac{a-c_2}{a-c_1} < \frac{3+2\alpha}{1+\alpha}\frac{1-\beta}{3-2\beta} \\ N^{IA}, & \frac{3+2\alpha}{1+\alpha}\frac{1-\beta}{3-2\beta} \leq \frac{a-c_2}{a-c_1} \leq \frac{1+\alpha}{3+2\alpha}\frac{3-2\beta}{1-\beta} \\ \left(q_1^l, q_2^l\right), & \frac{1+\alpha}{3+2\alpha}\frac{3-2\beta}{1-\beta} < \frac{a-c_2}{a-c_1} \end{array} \right. ,$$

*where $N^{IA}$ is given by (6), $(q_1^r, q_2^r)$ is the solution to*

$$\left\{ \begin{array}{l} (1-\beta)(a - 2bq_1 - bq_2 - c_1) - \beta b q_2 = 0 \\ (1+\alpha)(a - 2bq_2 - bq_1 - c_2) + \alpha b q_1 = 0 \end{array} \right. , \tag{7}$$

*and $\left(q_1^l, q_2^l\right)$ is the solution to*

$$\left\{ \begin{array}{l} (1+\alpha)(a - 2bq_1 - bq_2 - c_1) + \alpha b q_2 = 0 \\ (1-\beta)(a - 2bq_2 - bq_1 - c_2) - \beta b q_1 = 0 \end{array} \right. . \tag{8}$$

Proposition 6 tells us that if the cost asymmetry between firms is not very large, then the Cournot duopoly game with inequity aversion still has a continuum of Nash equilibria. However, if the cost asymmetry between firms is

[15] The same intuition is present in the first model in Fehr and Schmidt (1999).

sufficiently large, then the equilibrium is unique. This shows that the existence of a continuum of equilibria is robust to small cost asymmetries between firms but not to large ones. The intuition for this result is simple. A large cost asymmetry implies that the loss in profits associated with producing symmetric quantities is larger than the gain obtained by reducing inequity aversion. In fact, each firm would have to reduce profits too much in order to produce as much as the other firm and this would not be compensated by a large enough gain from reduction in inequity aversion.

The unique equilibrium of the Cournot duopoly game with inequity aversion and large cost asymmetries can be compared to the unique equilibrium of the standard Cournot duopoly game with asymmetric costs. First, the unique Nash equilibrium of the standard Cournot duopoly game with asymmetric costs is never an equilibrium of the Cournot duopoly game with large asymmetric costs and strict inequality aversion. Second, with or without inequity aversion, the low cost firm produces more than the high cost firm. Third, when there is inequity aversion the most efficient firm feels compassion towards the less efficient firm and the less efficient firm feels envy towards the most efficient firm.

# 3 Price Competition

We will now study the impact of inequity aversion between firms on price competition with homogeneous products. Consider the model of Bertrand competition, where firms select independently the prices they charge for the product and where every firm has the commitment to supply whatever demand is forthcoming at the price it sets. Let $N = \{1, 2, \ldots, n\}$ be the set of firms. Assume that demand is strictly downward-sloping when positive, cutting both axes, and that the $n$ firms have increasing cost functions $C_i(q_i)$.

As usual it is assumed that the firms that set the lowest price split the demand and that the remaining firms do not sell anything. That is, given a vector of prices $(p_i)_{i \in N}$ the sales of firm $i$ are

$$q_i = \begin{cases} \frac{D(p_i)}{l}, & \text{if } p_j \geq p_i, \forall j \in N \\ 0, & \text{otherwise} \end{cases},$$

where $l = \#\{j \in N : p_j = p_i\}$. As with the analysis of inequity aversion in Cournot competition with many firms, we make use of (2) to model inequity aversion in Bertrand competition. It is a well know result the equilibrium outcomes in Bertrand competition depend on the shape of the cost function. Therefore, we will analyze the impact of inequity aversion in Bertrand competition for different types of cost function.

## 3.1 Constant Marginal Costs

It is a well know result that, under the assumptions made here, if there is no inequity aversion, marginal costs are constant and identical, then the only equilibrium is one where all firms set price equal to marginal cost, have zero profits, and split the market demand equally. Our next result characterizes the equilibrium of the Bertrand game when firms feel inequity aversion and have constant marginal costs.

**Proposition 7**: *The set of Nash equilibria of the n-firm symmetric Bertrand game with inequity aversion and constant marginal costs is given by*

$$p = \begin{cases} a, & \text{if } 1 - \frac{1}{n} \leq \min(\beta_1, \dots, \beta_n) \\ c, & \text{otherwise} \end{cases},$$

*where* $a \in (c, \bar{p}]$, *with* $\bar{p}$ *being the choke-off price for demand.*

This result shows that if marginal cost are constant and there is at least one firm with a degree of compassion smaller than $1-1/n$, then the only equilibrium is for all firms to charge price equal to marginal cost. By contrast, if marginal costs are constant and all firms have a degree of compassion greater than $1-1/n$, then there is a continuum of symmetric equilibria where firms charge a price between marginal cost and the price that leads to zero market demand.

There are two interpretations for Proposition 7. For a fixed number of firms, this result tells us that inequity aversion can only raise price above marginal cost in Bertrand competition between firms with constant marginal costs when all firms have a very high level of compassion.[16] For a fixed level of compassion, say $\beta$, with $\beta \in (1/2, 1)$, this result tells us that an increase in the number of firms makes it is harder for inequity aversion to lead firms to set price above marginal cost. Of course, if we assume that $\beta_i$, $i = 1, \dots, n$, has a uniform distribution on $[0, 1]$, then an increase in $n$ raises $1 - \frac{1}{n}$ and reduces $\min(\beta_1, \dots, \beta_n)$ which makes it even harder to satisfy the condition that allows firms to charge price above marginal cost.

## 3.2 Decreasing Returns

Dastidar (1995) shows that in symmetric Bertrand competition between firms with increasing marginal costs (decreasing returns), there is a continuum of symmetric equilibria where firms set a price in the interval $[p_L, p_H]$, and this interval contains the perfectly competitive price. To study Bertrand competition with decreasing returns, $n$ firms, and with inequity aversion, we make two simplifying assumptions. We take demand to be linear–$D(p) = a - bp$–and we assume that firms' costs are given by $C_i(q_i) = cq_i^2/2$. If that is the case, then

$$p_L = \frac{ac}{2n + bc}, \tag{9}$$

and

$$p_H = \frac{ac}{2(1 - n^{-1})(1 - n^{-2})^{-1} + bc},$$

[16] Recall that if $\beta = 1/2$ implies that a firm is just indifferent between keeping one dollar to heself and giving this dollar to her competitors.

and the perfectly competitive price is given by $p = ac/(n+bc)$.[17] Our next result states the impact of inequity aversion between firms on the $n$-firm symmetric Bertrand game with linear demand and quadratic costs.

**Proposition 8**: *The set of Nash equilibria of the $n$-firm symmetric Bertrand game with inequity aversion, linear demand, and quadratic costs, is given by the price interval $\left[p_L^{IA}, p_H^{IA}\right]$, where $p_L^{IA}$ is equal to (9) and*

$$p_H^{IA} = \min\left[p_H(\beta_1), \dots, p_H(\beta_n)\right],$$

*where*

$$p_H(\beta_i) = \frac{ac}{2\left(1 - n^{-1} - \beta_i\right)\left(1 - n^{-2} - \beta_i\right)^{-1} + bc}, \; i = 1, \dots, n, \tag{10}$$

*and with $\beta_i \leq 1/2$, $i = 1, \dots, n$.*

Proposition 8 tells us that inequity aversion between firms may lead to a higher range of equilibrium prices when there is Bertrand competition and firms have increasing marginal costs. This happens because the upper bound of the equilibrium price interval in the Bertrand game with inequity aversion is greater than the upper bound of the equilibrium price interval in the Bertrand game without inequity aversion whereas the lower bound stays the same.[18] Dastidar (1995) shows that the upper bound price is determined by a condition that makes a firm indifferent between playing the symmetric equilibrium and being the single producer in the market. The fact that a firm feels compassion for other firms reduces the payoff of being the single producer in the market and does not change a firm's profit when all firms are in the market. This implies that it is possible to have a larger range of higher prices in the symmetric equilibrium.

However, as we can see, the conditions for this to happen are quite restrictive. Like in the $n$-firm Cournot model with inequity aversion, increasing the number of firms in the $n$-firm Bertrand game with increasing marginal costs reduces the impact of inequity aversion on the set of equilibria.[19] Furthermore, like in the $n$-firm Cournot model, it is enough for one firm to be selfish for the effect to go away.

# 4 Differentiated Products

We will now study the implications of inequity aversion in price competition in differentiated products.[20] Let $N = \{1, 2\}$. Assume that the demand for

[17] See Vives (2001).

[18] Since $\beta_i$ belongs to $[0, 1/2]$ for $i = 1, \dots, n$, it follows that the denominator in (10) is smaller than the denominator in (9). If that is the case, then $p_H(\beta_i) > p_H$, for all $i$. If that is the case then $p_H^{IA} > p_H$, that is, the upper bound of the equilibrium price interval in the Bertrand game with inequity aversion is greater than the upper bound of the equilibrium price interval in the Bertrand game without inequity aversion.

[19] If one assumes that $\beta_i$ is drawn from a uniform distribution on $[0, 1/2]$, then an increase in $n$ makes it more likely that $\min(\beta_1, \dots, \beta_n)$ is close to zero, which in turn implies that it is more likely that $p_H^{IA}$ is close to $p_H$.

[20] When there is quantity competition in differentiated products firms' best responses are downward sloping and quantities are strategic substitutes just like in the homogeneous prod-

product $i$, $D_i(p_1, p_2)$, is a function of the prices charge by both firms. We make the standard assumptions that $D_i(p_1, p_2)$ is decreasing in own price, increasing in the price of the competitor's product, and concave in own price. Furthermore, assume that costs are given by $C_i = cq_i$. The next result characterizes a firm's best response when there is price competition in differentiated products and firms display inequity aversion.

**Proposition 9**: *The best response of firm $i$, $i = 1, \ldots, n$, in the price competition in differentiated products duopoly game with inequity aversion and constant marginal costs is defined by*

$$R_i(p_j) = \begin{cases} s_i(p_j), & 0 \leq p_j \leq p(\alpha_i) \\ p_j, & p(\alpha_i) \leq p_j \leq p(\beta_i) \\ t_i(p_j), & p(\beta_i) \leq p_j \end{cases},$$

*where*

$$s_i(p_j) = \arg_{p_i} \max (1 + \alpha_i)(p_i - c_i) D_i(p_1, p_2) - \alpha_i (p_j - c_j) D_j(p_1, p_2),$$

$$t_i(p_j) = \arg_{p_i} \max (1 - \beta_i)(p_i - c_i) D_i(p_1, p_2) + \beta_i (p_j - c_j) D_j(p_1, p_2),$$

*$p(\alpha_i)$ is the solution to*

$$(1 + \alpha_i)\left[D_i(p, p) + (p - c_i)\frac{\partial D_i}{\partial p_i}(p, p)\right] - \alpha_i (p - c_j)\frac{\partial D_j}{\partial p_i}(p, p) = 0,$$

*and $p(\beta_i)$ is the solution to*

$$(1 - \beta_i)\left[D_i(p, p) + (p - c_i)\frac{\partial D_i}{\partial p_i}(p, p)\right] + \beta_i (p - c_j)\frac{\partial D_j}{\partial p_i}(p, p) = 0.$$

In the standard duopoly game of price competition in differentiated products the firms best responses are positively sloped, that is, prices are strategic complements. Proposition 7 tells us that when firms are inequity averse, prices continue to be strategic complements but now the best response of a firm has three different segments.

Proposition 9 says that if a firm is averse to inequity aversion and knows that her opponent is going to set a low price–by comparison with the Nash equilibrium price of the standard game–, then the best response of that firm is to set a price that is higher than the price set by the opponent but lower than the price that would be set by a firm who is not averse to inequality. Recall that, in the Stackelberg solution to the standard price competition in differentiated product game, the profits of the leader–the firm that charges the higher price–are lower than those of the follower–the firm that the lower price. Thus, when

ucts case. So, the impact of inequity aversion between firms on quantity competition in differentiated products is analogous to the homogeneous products case.

a firm that is averse to inequity charges a higher price than the opponent she ends up feeling envy towards the opponent. Proposition 9 shows us that in the margin, a firm that is averse to inequity chooses her optimal price such that it equalizes the unfavorable marginal impact on profits to the favorable marginal impact on reduction of envy towards the competitor.

By contrast, when a firm know that her opponent is going to set a high price–by comparison with the Nash equilibrium price of the standard game–, then the best response in the presence of inequity aversion is to set a price that is lower than the price set by the opponent but higher than the price that would be set by a firm that is not averse to inequality. This happens because charging a higher price than the opponent leads to a greater profit than the one of the opponent and this generates compassion.

**Proposition 10**: *The set of Nash equilibria in the symmetric price competition in differentiated products duopoly game with inequity aversion and constant marginal costs is given by*

$$N^{IA} = \{(p_1, p_2) : p_1 = p_2, \text{ and } p(\alpha) \leq p_i \leq p(\beta),\ i = 1, 2\}, \tag{11}$$

*where*

$$p(\alpha) = \max\left[p(\alpha_1), p(\alpha_2)\right].$$

*and*

$$p(\beta) = \min\left[p\left(\beta_1\right), p(\beta_2)\right],$$

**Proposition 11**: *The largest Nash equilibria of in the symmetric price competition in differentiated products duopoly game with inequity aversion and constant marginal costs is a nondecreasing function of* $\beta$*. The smallest Nash equilibria is a nonincreasing function of* $\alpha$*.*

This result says that when there is price competition in differentiated products an increase in the level of compassion between firms may lead to higher equilibrium prices. By contrast, an increase in the level of envy between firms may lead to lower equilibrium prices.

# 5 Conclusion

This paper studies the impact of inequity aversion between firms in market outcomes. We consider quantity and price competition. We consider competition in homogeneous and in differentiated products. The paper finds that, under quantity competition and price competition in differentiated products, if firms cost are similar then strict inequity aversion gives rise to a continuum of equilibria. The equilibria are symmetric and can be ranked in terms of welfare for both firms and consumers. By contrast, when there is price competition in homogeneous products, inequity aversion between firms either has no impact on the equilibrium outcome or raises the range of equilibrium prices. This happens because envy between firms has no impact on price competition in homogeneous products. The paper shows that across all types of competition considered, compassion between firms tends to reduce consumer surplus whereas envy between firms tends to raise it.

The paper also finds that inequity aversion between firms has no impact on equilibrium outcomes as long as at least one firm in the market only cares about her own material payoff. Furthermore, even if all firms strictly care about inequality, the paper shows that as the number of firms in the market increases, the impact of inequity aversion on equilibrium outcomes vanishes. Propositions 5, 7 and 8 show that this result is valid both all types of competition studied in the paper. An implication of this result is that inequity aversion is only likely to play a role in oligopolistic markets with a very small number of firms. Also, if an increase in monetary stakes reduces the impact of social preferences in equilibrium outcomes, as it is reported in some experiments, then the findings of this paper only apply to markets with a small number of small firms (e.g., neighborhood stores). As Joel Sobel (2005) points out (pp. 419): "The existence of markets may not change preferences, but it may remove incentives for reciprocal behavior."

As it was mentioned in the introduction, this paper is a first step in studying the impact of social preferences in oligopolistic markets. Our starting point was that firms only feel inequity aversion concerns among themselves. A natural extension of this paper would be to consider that firms' preferences also include consumers' welfare. It could also be interesting to find out what happens to market outcomes when some firms appeal to consumers' fairness concerns.[21] It is well known that in certain markets some firms donate a large share of their profits for charitable purposes while others do not.[22] It would be fruitful to explore the implications of these and related phenomena on equilibrium outcomes in oligopolistic markets.

[21] Rabin (1993) considers a market where consumers perceive the price charged by a monopolist as unfair. He finds that the highest fairness equilibrium price is lower than the standard monopoly price.

[22] One of the most famous examples being Paul Newman's brand Newman's Own, Inc. which has given more than $175 million.dollars to thousands of charities since 1982.

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# 6 Appendix

**Proof of Proposition 1**: To prove this result we will start by showing that $q(\alpha_i)$ is an increasing function of $\alpha_i$ and that $q(\beta_i)$ is a decreasing function of $\beta_i$ for $i = 1, \dots, n$. From (5) we have

$$h(q, \alpha_i) = (1 + \alpha_i)\left[P(nq) - C_i'(q)\right] + P'(nq)q = 0,$$

which implies

$$\frac{\partial q}{\partial \alpha_i} = -\frac{\partial h / \partial \alpha_i}{\partial h / \partial q} = -\frac{P(Q) - C_i'(q)}{(1 + n(1 + \alpha_i))\, P'(Q) + nP''(Q)q - C_i''(q)} > 0,$$

since we have assumed that $P'(Q) < 0$, $P'(Q) \leq 0$, and $C_i''(q_i) \geq 0$. From (4) we have

$$g(q, \beta_i) = (1 - \beta_i)\left[P(nq) - C_i'(q)\right] + P'(nq)q = 0,$$

which implies

$$\frac{\partial q}{\partial \beta_i} = -\frac{\partial g / \partial \beta_i}{\partial g / \partial q} = -\frac{-\left[P(Q) - C_i'(q)\right]}{(1 + n(1 - \beta_i))\, P'(Q) + nP''(Q)q - C_i''(q)} < 0,$$

since we have assumed that $P'(Q) < 0$, $P'(Q) \leq 0$, and $C_i''(q_i) \geq 0$.
We will now show that $q_i = \frac{1}{n-1}\sum_{j \neq i} q_j$ is a best response for firm $i$ when the competitors produce

$$q_i^N \leq \bar{q}_j \leq q(\alpha_i), \tag{12}$$

where

$$\bar{q}_j = \frac{1}{n-1}\sum_{j \neq i} q_j.$$

To do that we will show that firm $i$ can not gain from deviating from $q_i = \bar{q}_j$ when (12) holds. Suppose, that (12) holds and that firm $i$ produces $q_i = \bar{q}_j + \varepsilon$, with $\varepsilon > 0$. In this case firm $i$'s payoff is given by

$$U_i = (1 - \beta_i)\left[P\left(Q\right) q_i - C_i(q_i)\right] + \frac{\beta_i}{n-1}\sum_{j \neq i}\left[P\left(Q\right) q_j - C_j(q_j)\right]$$

and the change in firm $i$'s payoff from producing $q_i = \bar{q}_j + \varepsilon$, $\varepsilon > 0$, instead of $\bar{q}_j$ is approximately equal to

$$\begin{aligned} dU_i &\approx \left.(1 - \beta_i)\left[P'\left(Q\right) q_i + P(Q) - C_i'(q_i)\right] + \frac{\beta_i}{n-1}\sum_{j \neq i} P'\left(Q\right) q_j\right|_{q_i = \bar{q}_j} (\varepsilon) \\ &= \left[\left(P'\left(n\bar{q}_j\right)\bar{q}_j + P(n\bar{q}_j) - C_i'(\bar{q}_j)\right) - \beta_i\left(P(n\bar{q}_j) - C_i'(\bar{q}_j)\right)\right]\varepsilon. \end{aligned}$$

The square brackets are negative since $q_i = \bar{q}_j > \arg\max\left[P\left(Q\right)q_i - C_i(q_i)\right]$ and $P(n\bar{q}_j) - C_i'(\bar{q}_j) > 0$. So, when (12) holds, firm $i$ can not gain by producing more than $\bar{q}_j$. Now, suppose that (12) holds and that firm $i$ produces $q_i = \bar{q}_j + \varepsilon$, with $\varepsilon < 0$. In this case firm $i$'s payoff is given by

$$U_i = (1+\alpha_i)\left[P\left(Q\right)q_i - C_i(q_i)\right] - \frac{\alpha_i}{n-1}\sum_{j\neq i}\left[P\left(Q\right)q_j - C_j(q_j)\right],$$

and the change in firm $i$'s payoff from producing $q_i = \bar{q}_j + \varepsilon$, $\varepsilon < 0$, instead of $\bar{q}_j$ is approximately equal to

$$\begin{aligned}
dU_i &\approx \left.(1+\alpha_i)\left[P'\left(Q\right)q_i + P(Q) - C_i'(q_i)\right] - \frac{\alpha_i}{n-1}\sum_{j\neq i}P'\left(Q\right)q_j\right|_{q_i=\bar{q}_j}(\varepsilon) \\
&= \left[(1+\alpha_i)\left[P(n\bar{q}_j) - C_i'(\bar{q}_j)\right] + P'(n\bar{q}_j)\bar{q}_j\right]\varepsilon \\
&= \left.h(q,\alpha_i)\right|_{q=\bar{q}_j}(\varepsilon).
\end{aligned}$$

So, since $\varepsilon < 0$, we have that

$$sign\ dU_i = -sign\ \left.h(q,\alpha_i)\right|_{q=\bar{q}_j}.$$

If $\bar{q}_j = q(\alpha_i)$ we have that $sign\ dU_i = 0$. If $q_i^N \leq \bar{q}_j < q(\alpha_i)$, the fact $h(q,\alpha_i)$ is a decreasing function of $q$ implies that $\left.h(q,\alpha_i)\right|_{q=\bar{q}_j} > 0$, which in turn implies that $sign\ dU_i < 0$. So, when (12) holds, firm $i$ can not gain by producing less than $\bar{q}_j$. From this result is follows immediately that if firm $i$'s competitors produce

$$q(\alpha_i) < \frac{1}{n-1}\sum_{j\neq i}q_j,$$

then the best response of firm $i$ is given by $t_i(q_{-i})$.
We will now show that $q_i = \frac{1}{n-1}\sum_{j\neq i}q_j$ is a best response for firm $i$ when the competitors produce

$$q(\beta_i) \leq \bar{q}_j \leq q_i^N, \tag{13}$$

To do that we will show that firm $i$ can not gain from deviating from $q_i = \bar{q}_j$ when (13) holds. Suppose, that (13) holds and that firm $i$ produces $q_i = \bar{q}_j + \varepsilon$, with $\varepsilon < 0$. In this case firm $i$'s payoff is given by

$$U_i = (1+\alpha_i)\left[P\left(Q\right)q_i - C_i(q_i)\right] - \frac{\alpha_i}{n-1}\sum_{j\neq i}\left[P\left(Q\right)q_j - C_j(q_j)\right],$$

and the change in firm $i$'s payoff from producing $q_i = \bar{q}_j + \varepsilon$, $\varepsilon < 0$, instead of $\bar{q}_j$ is approximately equal to

$$\begin{aligned}
dU_i &\approx \left.(1+\alpha_i)\left[P'\left(Q\right)q_i + P(Q) - C_i'(q_i)\right] - \frac{\alpha_i}{n-1}\sum_{j\neq i}P'\left(Q\right)q_j\right|_{q_i=\bar{q}_j}(\varepsilon) \\
&= \left[(1+\alpha_i)\left[P'(n\bar{q}_j)\bar{q}_j + P(n\bar{q}_j) - C_i'(\bar{q}_j)\right] - \alpha_i P'(n\bar{q}_j)\bar{q}_j\right]\varepsilon.
\end{aligned}$$

The square brackets are positive since $q_i = \bar{q}_j < \arg\max\left[P\left(Q\right)q_i - C_i(q_i)\right]$ and $P'(n\bar{q}_j) < 0$. So, when (13) holds, firm $i$ can not gain by producing less than $\bar{q}_j$. Now, suppose that (13) holds and that firm $i$ produces $q_i = \bar{q}_j + \varepsilon$, with $\varepsilon > 0$. In this case firm $i$'s payoff is given by

$$U_i = (1-\beta_i)\left[P\left(Q\right)q_i - C_i(q_i)\right] + \frac{\beta_i}{n-1}\sum_{j\neq i}\left[P\left(Q\right)q_j - C_j(q_j)\right]$$

and the change in firm $i$'s payoff from producing $q_i = \bar{q}_j + \varepsilon$, $\varepsilon > 0$, instead of $\bar{q}_j$ is approximately equal to

$$\begin{aligned} dU_i &\approx \left.(1-\beta_i)\left[P'\left(Q\right)q_i + P(Q) - C_i'(q_i)\right] + \frac{\beta_i}{n-1}\sum_{j\neq i}P'\left(Q\right)q_j\right|_{q_i=\bar{q}_j}(\varepsilon) \\ &= \left[(1-\beta_i)\left[P(n\bar{q}_j) - C_i'(\bar{q}_j)\right] + P'\left(n\bar{q}_j\right)\bar{q}_j\right]\varepsilon \\ &= \left.g(q,\beta_i)\right|_{q=\bar{q}_j}(\varepsilon)\,. \end{aligned}$$

So, since $\varepsilon > 0$, we have that

$$sign\ dU_i = sign\ \left.g(q,\beta_i)\right|_{q=\bar{q}_j}.$$

If $\bar{q}_j = q(\beta_i)$ we have that $sign\ dU_i = 0$. If $q(\beta_i) < \bar{q}_j \leq q_i^N$, the fact $g(q,\beta_i)$ is a decreasing function of $q$ implies that $\left.g(q,\beta_i)\right|_{q=\bar{q}_j} < 0$, which in turn implies that $sign\ dU_i < 0$. So, when (13) holds, firm $i$ can not gain by producing more than $\bar{q}_j$. From this result is follows immediately that if firm $i$'s competitors produce

$$0 \leq \frac{1}{n-1}\sum_{j\neq i}q_j < q(\beta_i),$$

then the best response of firm $i$ is given by $s_i(q_{-i})$. *Q.E.D.*

**Proof of Proposition 2**: We wish to show that $q_i = q_i^N$ is the best response to $q_{-i}^N = \left(q_1^N,\ldots,q_{i-1}^N,q_{i+1}^N,\ldots q_n^N\right)$ in the $n$-firm symmetric Cournot game with inequity aversion. The welfare of firm 1 under outcome $q^N$ is given by $\pi_1(q^N) = \left[P\left(nq_i^N\right) - C_i(q_i^N)\right]q_i^N$, where $q_i^N = \arg_{q_1}\max\left[P\left(q_i + \sum_{j\neq i}q_j^N\right) - C_i(q_i)\right]q_i$. If firm $i$ produces $q_i^N + \varepsilon$, with $\varepsilon > 0$, and all other firms produce $q_{-i}^N$, then the change in firm $i$'s profit is approximately equal to

$$\begin{aligned} d\pi_i &\approx \varepsilon\left.\partial\pi_i/\partial q_i\right|_{q_i=q_i^N} + \frac{1}{2}\varepsilon^2\left.\partial^2\pi_i/\partial q_i^2\right|_{q_i=q_i^N} \\ &= \frac{1}{2}\varepsilon^2\left[2P'(Q^N) + P''(Q^N)q_i^N - C''(q_i^N)\right]. \end{aligned} \tag{14}$$

The assumption that $P' < 0$, $P'' \leq 0$, and $C'' \geq 0$ imply that $d\pi_i < 0$. The change in the profit in one of firm $i$'s competitors, say firm $j$, is approximately equal to

$$\begin{aligned}
d\pi_j &\approx \varepsilon\, \partial\pi_j/\partial q_i\big|_{q_i=q_i^N} + \frac{1}{2}\varepsilon^2\, \partial^2\pi_j/\partial q_i^2\big|_{q_i=q_i^N} \\
&= \varepsilon P'(Q^N)q_j^N + \frac{1}{2}\varepsilon^2 P''(Q^N)q_j^N.
\end{aligned}$$

Note that the change in the average profit of firm $i$'s competitors is the same as the change in the profit of a single competitor since

$$\begin{aligned}
\frac{1}{n-1}\sum_{j\neq i} d\pi_j &\approx \frac{1}{n-1}\varepsilon P'(Q^N)\sum_{j\neq i} q_j^N + \frac{1}{2}\varepsilon^2 P''(Q^N)\sum_{j\neq i} q_j^N \\
&= \varepsilon P'(Q^N)q_j^N + \frac{1}{2}\varepsilon^2 P''(Q^N)q_j^N. \qquad (15)
\end{aligned}$$

The assumption that $P' < 0$ and $P'' \leq 0$ imply that $\frac{1}{n-1}\sum_{j\neq i} d\pi_j < 0$. We see from (14) and (15) that if firm $i$ produces $q_i^N + \varepsilon$, with $\varepsilon > 0$, and all other firms produce $q_{-i}^N$, then there is a first order decrease in profits of firm $i$ and a second order decrease in the average profit of firm $i$'s competitors. Thus, if firm $i$ produces $q_i^N + \varepsilon$, with $\varepsilon > 0$, it suffers a loss in profits and also a loss from an increase in inequity aversion given that the average profit of the competitors becomes smaller than firm $i$'s profit. If that is the case, then firm $i$ can not gain by producing $q_i^N + \varepsilon$, with $\varepsilon > 0$, instead of producing $q_i^N$.
If firm $i$ produces $q_i^N + \varepsilon$, with $\varepsilon < 0$, and all other firms produce $q_{-i}^N$, then the change in firm $i$'s profit is given by (14) and we have that $d\pi_i < 0$. The change in the average profit of firm $i$'s competitors is given by (15) and we have that $\frac{1}{n-1}\sum_{j\neq i} d\pi_j > 0$ since $\varepsilon < 0$ and the first term is of first order while the second term is of second order. Thus, if firm $i$ produces $q_i^N + \varepsilon$, with $\varepsilon < 0$, it suffers a loss in profits and also a loss from an increase in inequity aversion given that the average profit of the competitors becomes greater than firm $i$'s profit. If that is the case, then firm $i$ can not gain by producing $q_i^N + \varepsilon$, with $\varepsilon < 0$, instead of producing $q_i^N$. This proves that $q_i = q_i^N$ is the best response to $q_{-i}^N = \left(q_1^N, \dots, q_{i-1}^N, q_{i+1}^N, \dots q_n^N\right)$ in the $n$-firm symmetric Cournot game with inequity aversion. But, this in turn implies that $q^N$ is a Nash equilibrium of the $n$-firm symmetric Cournot game with inequity aversion. *Q.E.D.*

**Proof of Proposition 3**: We know from Proposition 2 that the set $N^{IA}$ is non-empty since it contains at least the Nash equilibrium of the standard $n$-firm symmetric Cournot game. We will now show that if all firms display strict inequity aversion, then $q(\beta) < q(\alpha)$, that is, $N^{IA}$ is an interval. We know from Proposition 1 that $q(\alpha_i)$ is an increasing function of $\alpha_i$ and that $q(\beta_i)$ is a decreasing function of $\beta_i$ for $i = 1, \dots, n$. It is obvious that if at least one firm does not feel inequity aversion then $q(\beta) = q(\alpha)$, and $N^{IA}$ is a singleton. To see this suppose that firm $i$ does not feel inequity aversion, that is, $\alpha_i = \beta_i = 0$. If this is the case, then (4) and (5) imply that $q(0) = q^N$. If $q(\alpha_i)$ is an increasing function of $\alpha_i$ and $q(0) = q^N$, then $q(\alpha) = q^N$. Similarly, if $q(\beta_i)$ is a decreasing function of $\beta_i$ and $q(0) = q^N$, then $q(\beta) = q^N$. So, if at least one firm does feel inequity aversion we have that $q(\beta) = q(\alpha) = q^N = N^{IA}$. We will now show that if all firms display strict inequity aversion, then $q(\beta) < q(\alpha)$, that is, $N^{IA}$ is an interval. If all firms display strict inequity aversion, $q(\alpha_i)$ is an increasing function of $\alpha_i$ and $q(0) = q^N$, then $q(\alpha) > q^N = q(0)$. Also, if all firms display strict inequity aversion, $q(\alpha_i)$ is an decreasing function of $\beta_i$ and $q(0) = q^N$, then $q(\beta) < q^N = q(0)$. This shows that $q(\beta) < q(\alpha)$ when all firms display strict inequity aversion, that is the set $N^{IA}$ is an interval. All outcomes in the set $N^{IA}$ are equilibria of the symmetric Cournot game with inequity aversion since for any profile of quantities, $q_{-i}$, the quantity $q_i$ belongs to the best response of firm $i$, $i = 1, \ldots n$. *Q.E.D.*

**Proof of Proposition 4**: The quantity produced by each firm in the largest Nash equilibria of $N^{IA}$ is given by $q(\alpha) = \min[q(\alpha_1), \ldots, q(\alpha_n)]$. The largest Nash equilibria of $N^{IA}$ is nondecreasing in $\alpha$ since $\min[q(\alpha_1), \ldots, q(\alpha_n)]$ is nondecreasing in $\alpha$. Similarly, the quantity produced by each firm in the smallest Nash equilibria of $N^{IA}$ is given by $q(\beta) = \max[q(\beta_1), \ldots, q(\beta_n)]$. The smallest Nash equilibria of $N^{IA}$ is nonincreasing in $\beta$ since $\max[q(\beta_1), \ldots, q(\beta_n)]$ is nonincreasing in $\beta$. *Q.E.D.*

**Proof of Proposition 5**: Propositions 2 and 3 imply that when all firms feel strict inequity aversion it must be that

$$q(\beta) < q^N < q(\alpha).$$

Since $\alpha_i$ is drawn from a uniform distribution with support on $[0, 1]$, the larger is $n$ the most likely it becomes that $\min(\alpha_1, \ldots \alpha_n)$ is closer to zero, this in turn implies that the larger is $n$ the most likely is that $N(\alpha)$ is closer to $q^N$. Similarly, since $\beta_i$ is drawn from a uniform distribution with support on $[0, 1]$, the larger is $n$ the most likely it becomes that $\min(\beta_1, \ldots, \beta_n)$ is closer to zero, this in turn implies that the larger is $n$ the most likely is that $N(\beta)$ is closer to $q^N$. *Q.E.D.*

**Proof of Proposition 6**: If demand is $P = a - bQ$ and costs are $C_i(q_i) = c_i q_i$, then we have

$$R_i(q_j) = \begin{cases} \frac{a-c_i}{2b} - \frac{1}{2(1-\beta)} q_j, & 0 \leq q_j \leq q(\beta_i) \\ q_j, & q(\beta_i) \leq q_j \leq q(\alpha_i) \\ \frac{a-c_i}{2b} - \frac{1}{2(1+\alpha)} q_j, & q(\alpha_i) \leq q_j \end{cases},$$

where

$$q_i(\alpha) = \frac{(1+\alpha)(a-c_i)}{(3+2\alpha)b}, \quad i = 1, 2,$$

and

$$q_i(\beta) = \frac{(1-\beta)(a-c_i)}{(3-2\beta)b}, \quad i = 1, 2.$$

Suppose that $c_1 < c_2$. This implies that $q_1(\alpha) > q_2(\alpha)$ and $q_1(\beta) > q_2(\beta)$. If that is the case then $N^{IA}$ is given by

$$N^{IA} = \{(q_1, q_2) : q_1 = q_2, \text{ and } q_1(\beta) \leq q_i \leq q_2(\alpha), \ i = 1, 2\}.$$

If $\frac{a-c_2}{a-c_1} < \frac{3+2\alpha}{1+\alpha}\frac{1-\beta}{3-2\beta}$, then $q_2(\alpha) < q_1(\beta)$ and the set $N^{IA}$ is empty. In this case, the Nash equilibrium is obtained by solving (7). If $\frac{a-c_2}{a-c_1} \geq \frac{3+2\alpha}{1+\alpha}\frac{1-\beta}{3-2\beta}$, then $q_2(\alpha) > q_1(\beta)$ and the set $N^{IA}$ is non-empty. Now suppose that $c_1 > c_2$. This implies that $q_1(\alpha) < q_2(\alpha)$ and $q_1(\beta) < q_2(\beta)$. If that is the case then $N^{IA}$ is given by

$$N^{IA} = \{(q_1, q_2) : q_1 = q_2, \text{ and } q_2(\beta) \leq q_i \leq q_1(\alpha), \ i = 1, 2\}.$$

If $\frac{1+\alpha}{3+2\alpha}\frac{3-2\beta}{1-\beta} < \frac{a-c_2}{a-c_1}$, then $q_1(\alpha) < q_2(\beta)$ and the set $N^{IA}$ is empty. In this case the Nash equilibrium is obtained by solving (8). If $\frac{1+\alpha}{3+2\alpha}\frac{3-2\beta}{1-\beta} < \frac{a-c_2}{a-c_1}$, then $q_1(\alpha) > q_2(\beta)$ and the set $N^{IA}$ is non-empty. *Q.E.D.*

**Proof of Proposition 7**: If marginal costs are constant, then we have $C_i(q_i) = cq_i$, $i = 1, \ldots, n$. The payoff of firm $i$ in the presence of inequity aversion is given by

$$U_i(p_i, p_j) = \begin{cases} (1-\beta_i)(p_i - c) D(p_i), & \text{if } p_i < p_j^{\min} \\ \left(1 - \beta_i + \beta_i \frac{l-1}{n-1}\right) \frac{(p_i - c) D(p_i)}{l}, & \text{if } p_j \geq p_i, \ \forall j \in N \\ -\alpha_i \left(p_j^{\min} - c\right) D(p_j^{\min}), & \text{if } p_i > p_j^{\min} \end{cases},$$

where $p_j^{\min} = \min(p_1, \ldots, p_{i-1}, p_{i+1}, \ldots, p_n)$ and $l = \#\{j \in N : p_j = p_i\}$. For firm $i$ not to deviate from an equilibrium where firm $i$ plus $l-1$ firms charge a price $p \in (c, \bar{p}]$ and the remaining firms charge a higher price than $p$ it must be that

$$(1-\beta_i)(p_i - c) D(p_i) \leq \left(1 - \beta_i + \beta_i \frac{l-1}{n-1}\right) \frac{(p_i - c) D(p_i)}{l}$$

or

$$n - 1 \leq \frac{\beta_i}{1 - \beta_i}$$

or

$$1 - \frac{1}{n} \leq \beta_i.$$

For all firms not to deviate, the case when $l = n$, from such an equilibrium we need that

$$1 - \frac{1}{n} \leq \min\left(\beta_1, \ldots, \beta_n\right).$$

If this condition does not hold, then there is at least one firm that is always willing to undercut a price $p \in (c, \bar{p}]$. If that is the case, then the only equilibrium is for all firms to charge price equal to marginal cost. *Q.E.D.*

**Proof of Proposition 8**: If all firms produce in the market and all charge the same price the payoff of each firm is given by $\pi_n(p) = pD(p)/n - C(D(p)/n)$. If demand is $D(p) = a - bp$ and $C(q) = cq^2/2$ we have that $\pi_n(p) = p(a - bp)/n - c(a - bp)^2/2n^2$. Dastidar (1995) shows that the lower bound of the set of equilibrium prices, $p_L$, is given by the solution to $\pi_n(p) = 0$. In this case, our assumptions imply that $p_L = ac/(2n + b)$. If a firm is the single producer in the market, then all the other firms must have zero profit and therefore the profit of the single producer is given by $\pi_1(p) = pD(p) - C(D(p))$. However, with inequity aversion, the payoff of this firm is given by $U_i(p) = (1-\beta_i)\pi_1(p)$. If demand is $D(p) = a-bp$ and $C(q) = cq^2/2$ we have that $U_i(p) = (1 - \beta_i)\left[p(a - bp) - c(a - bp)^2/2\right]$. Applying Dastidar (1995) to our model with inequity aversion, we have that the upper bound of the set of equilibrium prices with inequity aversion, $p_H$, is the given by (10), where $p_H(\beta_i)$ solution to $U_i(p) = \pi_n(p)$. That is, $p_H(\beta_i)$ is the solution to

$$(1 - \beta_i)\left[p(a - bp) - c(a - bp)^2/2\right] = p(a - bp)/n - c(a - bp)^2/2n^2.$$

Solving this equation for $p$ we obtain

$$\begin{aligned} p_H(\beta_i) &= \frac{ac\left(n^2 - 1 - \beta_i n^2\right)}{2(n^2 - n - \beta_i n^2) + bc(n^2 - 1 - \beta n^2)} \\ &= \frac{ac}{2\left(n^2 - n - \beta_i n^2\right)\left(n^2 - 1 - \beta_i n^2\right)^{-1} + bc} \\ &= \frac{ac}{2\left(1 - n^{-1} - \beta_i\right)\left(1 - n^{-2} - \beta_i\right)^{-1} + bc}. \end{aligned}$$

*Q.E.D.*

**Proof of Proposition 9**: An application of the method of proof used in Proposition 1. *Q.E.D.*

**Proof of Proposition 10**: An application of the method of proof used in Proposition 3. *Q.E.D.*

**Proof of Proposition 11**: An application of the method of proof used in Proposition 4. *Q.E.D.*