

# THE CONSUMPTION-WEALTH RATIO UNDER ASYMMETRIC ADJUSTMENT\*

VASCO J. GABRIEL<sup>†</sup>

Department of Economics, University of Surrey, UK and NIPE-UM

FERNANDO ALEXANDRE

Department of Economics and NIPE, University of Minho, Portugal

PEDRO BAÇÃO

GEMF and Faculty of Economics of the University of Coimbra, Portugal

23rd April 2007

## Abstract

This paper argues that nonlinear adjustment may provide a better explanation of fluctuations in the consumption-wealth ratio. The nonlinearity is captured by a Markov-switching vector error-correction model that allows the dynamics of the relationship to differ across regimes. Estimation of the system suggests that these states are related to the behaviour of financial markets. In fact, estimation of the system suggests that short-term deviations in the consumption-wealth ratio will forecast either asset returns or consumption growth: the first when changes in wealth are transitory; the second when changes in wealth are permanent. Our approach uncovers a richer and more complex dynamics in the consumption-wealth ratio than previous results in the literature, whilst being in accordance with theoretical predictions of standard models of consumption under uncertainty.

*JEL Classification:* C32; C5; E21; E44; G10

*Keywords:* Consumption; Financial markets; Uncertainty; Forecast; Markov switching

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\*The authors acknowledge financial support provided by the Portuguese Foundation for Science and Technology under research grant POCI/EGE/56054/2004 (partially funded by FEDER). We are grateful to Maximo Camacho for help with his GAUSS procedure. We are also grateful for the comments received from participants at University of Surrey and University of Minho seminars, and at the International Atlantic Economic Society 2005 conference, held in New York.

<sup>†</sup>Corresponding author. Address: Department of Economics, University of Surrey, Guildford, Surrey, GU2 7XH, UK. Email: v.gabriel@surrey.ac.uk. Tel: + 44 1483 682769. Fax: + 44 1483 689548.

# 1 Introduction

There has been a renewed interest in the literature concerning the linkages between asset wealth and consumption. Indeed, the preceding decade has witnessed remarkable changes in households' wealth, particularly due to stock market valuations, which may have had implications for the pattern of consumer spending. On the other hand, movements in aggregate macroeconomic relationships, such as the consumption-wealth ratio, may provide some guidance on the future performance of asset markets. (Lettau and Ludvigson 2001) and (Lettau and Ludvigson 2004) (L&L henceforth), as well as (Ludvigson and Steindel 1999) and (Poterba 2000), for example, provide recent accounts of the subject.

L&L start from a fairly standard model of consumer behaviour involving consumption, asset wealth and labour income, which implies that fluctuations in the consumption wealth-ratio forecast changes in one of these variables. In order to disentangle this question, these authors estimate a vector error-correction model (VECM) and conclude that adjustment from shocks distorting the long-run equilibrium takes place mainly through asset returns. This, in turn, means that deviations from the common trend embody agents' expectations of future returns on the market portfolio and, therefore, are a useful predictor of stock and excess returns.

However, given the nature of the variables, it is likely that these adjustments occur in different ways, depending on the state of economy and, in particular, on the phase of the stock market. Indeed, asset wealth displays a more volatile behaviour than consumption or labour income, a feature that is clearly linked with the state of asset markets. Several papers document the existence of different regimes in financial markets; see (Cecchetti, Lam, and Mark 1990), (Bonomo and Garcia 1994) and (Driffill and Sola 1998), for example. Therefore, in this paper, we argue that regime switching may provide a better explanation for fluctuations in the consumption-wealth ratio. We explicitly allow for different states, by postulating that the dynamics of the equilibrium errors follow a Markov-switching process. This, in turn, leads to a Markov-switching VECM (MS-VECM) representation of the trivariate relationship, which we use to investigate the possibility of nonlinear adjustment in the consumption-wealth ratio.

Estimation of this MS-VECM suggests that the mechanism through which deviations

from the long-run relationship are eliminated depends on the state of the economy. Thus, we find a regime whereby wealth does most of the error-correction in the system, coinciding with periods of “bullish” markets. However, we also identify a more “tranquil” state, where it is consumption growth that drives the system back to long-run equilibrium. Therefore, and unlike L&L, our findings suggest that short-term deviations in the trivariate relationship (consumption, labour income and non-human wealth) will forecast either asset returns or consumption growth, depending on the state of the economy.

These results seem to provide a more accurate description of the dynamics of the consumption-wealth ratio than the standard, linear specification, while being consistent with the theoretical framework employed by L&L. Our results also help to explain why other researchers — (Davis and Palumbo 2001), or (Mehra 2001), among others — found consumption to adjust sluggishly to shocks in income and wealth. In fact, single-equation error-correction models with consumption growth as the dependent variable will partly detect the adjustments in consumption that occur in the regime where markets are less volatile, although the main driving force of the system is the behaviour of asset wealth.

A Markov-switching type of asymmetric adjustment in cointegrated systems has been suggested by (Psaradakis, Sola, and Spagnolo 2004) and (Camacho 2005). These papers form the basis of the methodology employed in this study. (Paap and van Dijk 2003) employ a similar method, using a Bayesian approach to estimate possible Markov trends in the consumption-income relationship. However, they do not include asset wealth in their model and therefore they do not capture the dynamic features present in the cointegrated system studied by L&L.

Our paper is organised as follows. The next section briefly reviews the model employed by L&L, reassesses their results and argues that the characteristics of the data calls for the estimation of a nonlinear specification. Section 3 presents a possible account of the switching nature of consumption-wealth adjustment. In section 4 we discuss econometric tests for nonlinear adjustment and apply them to the L&L data. System estimation is carried out in section 5. Section 6 summarises and concludes.

## 2 Background discussion

In this section, we briefly review the model employed by L&L and point out why their results (and economic theory) suggest that a nonlinear framework may offer a better characterisation of the evolution of consumption and the components of wealth. We begin by considering a standard household budget constraint. Define  $W_t$  as the beginning of period aggregate wealth in period  $t$ , with a asset wealth component,  $A_t$ , and a human capital component,  $H_t$ . By letting  $C_t$  denote aggregate consumption in period  $t$  and  $R_{w,t+1}$  denote the net return on  $W_t$ , a simple wealth accumulation equation is given by

$$W_{t+1} = (1 + R_{w,t+1})(W_t - C_t). \quad (1)$$

Based on this equation, (Campbell and Mankiw 1989) derive an expression for the consumption-wealth ratio in logs. They take a first-order Taylor expansion of the equation, solve the difference equation forward and take expectations, resulting in

$$c_t - w_t = E_t \sum_{i=1}^{\infty} \rho_w^i (r_{w,t+i} - \Delta c_{t+i}), \quad (2)$$

where  $r = \log(1 + R)$ ,  $\rho_w = (W - C)/W$  is the steady-state ratio of new investment to total wealth, and lower case letters denote variables in logs.

Despite the fact that  $H_t$  is not observable, L&L show that an empirically valid approximation may be obtained by using labour income,  $Y_t$ , as a proxy for human capital,  $H_t$ , resulting in the following log consumption-wealth ratio

$$c_t - \alpha_a a_t - \alpha_y y_t \approx E_t \sum_{i=1}^{\infty} \rho_w^i ((1 - v)r_{at+i} - \Delta c_{t+i} + v\Delta y_{t+1+i}), \quad (3)$$

where  $(1 - v)$  and  $v$  represent the steady-state shares of the wealth components  $a_t$  and  $y_t$ , respectively, and  $r_{at+i}$  denotes the net returns on asset wealth. The L&L papers provide a detailed discussion of the assumptions employed in the approximation. L&L then show that  $c_t$ ,  $a_t$  and  $y_t$  share a common trend, with cointegration vector  $(1, -\alpha_a, -\alpha_y)$  and cointegration residual  $c_t - \alpha_a a_t - \alpha_y y_t$  ( $cay_t$  in brief). Importantly for our argument, equation (3) implies that fluctuations in the consumption-wealth ratio will reflect future changes in asset wealth, consumption or labour income.

L&L proceed with their analysis by testing for the number of cointegration vectors, which they conclude to be only one. The cointegrating vector is estimated by the Dynamic

OLS method of (Stock and Watson 1993) as  $(1, -0.3, -0.6)$ , but the results appear to be robust with respect to the estimation method; therefore, our analysis will also employ this estimate. Secondly, L&L estimate a vector error-correction model (VECM) of the trivariate system, with the estimated cointegration vector imposed as the long-run attractor. The authors conclude that when a shock occurs, it is asset wealth that does most of the subsequent adjustment in order to restore the common trend.

However, a closer look at the results of L&L seems to suggest that the dynamic structure of the system may be further explored.<sup>1</sup> Take, for instance, the estimated equilibrium error  $cay_t = c_t - 0.3a_t - 0.6y_t$  depicted in Figure 1. It suggests that the adjustment dynamics follows the cyclical patterns of asset markets, as recognised by (Lettau and Ludvigson 2004, p. 291). This is natural, given the presence of  $a_t$  in the long-run relationship. The “bull markets” of the late 1960s and late 1990s, for example, are clearly identified as periods where wealth seems to be above its equilibrium path. Notice also that these cycles are irregular, thus implying that equilibrium is most likely being restored in an asymmetric fashion.

On the other hand, a more detailed inspection of the results of the linear VECM reveals some potential specification problems. Table 1 reports results of maximum likelihood estimation of a first-order VECM, as well as of standard single and multi-equation specification tests. The order of the VECM was chosen to be 1 by all tests and information criteria employed. In addition, we report heteroskedastic and autocorrelation consistent (HAC) asymptotic standard errors, computed with the plug-in procedure and the Quadratic Spectral kernel, as suggested by (Andrews and Monahan 1992). This table is comparable to Table 1 in (Lettau and Ludvigson 2004). Analysing the results of the specification tests, it is clear that the estimated model appears to suffer from problems on all counts. Looking at individual equations, the LM test for autocorrelation up to 5 lags points to problems in the consumption equation, while heteroskedasticity (as revealed by a White test) and

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<sup>1</sup>In what follows, we resort to an updated version of the dataset used in (Lettau and Ludvigson 2004). A detailed description of the data can be found in their Appendix A. The data itself is available from Ludvigson’s webpage (<http://www.econ.nyu.edu/user/ludvigsons/>). The results do not change if the actual data in (Lettau and Ludvigson 2004) is used instead. The dataset comprises quarterly data on aggregate consumption, asset wealth and labour income, spanning from the fourth quarter of 1951 to the third quarter of 2003.

ARCH (LM statistic) mainly affects the wealth equation. Moreover, a Jarque-Bera test for normality indicates that the assumption of normal errors is violated. If the whole system is considered, the conclusions appear to be the same. Therefore, the use of HAC standard errors seems justified. Notice that, although the conclusions of L&L are not altered, the t-ratio (2.228) of the adjustment coefficient associated with wealth growth is significantly lower.

A possible explanation for these results lies in the stochastic properties of the variables in the system. Take, for example, consumption and wealth. It is clear from Figure 2, which represents the levels and growth rates of these variables, that a linear specification is hardly compatible with the exhibited dynamics. In particular, asset wealth displays not only a much more volatile path than consumption, but also the volatility seems to be changing over time. This fact is acknowledged by (Lettau and Ludvigson 2004, p. 277), but it is not explicitly accounted for. Indeed, a simple mean-variance switching representation for the first difference of log asset wealth illustrates this point. Figure 3 plots the estimated variance against asset wealth growth, revealing the time-varying nature of asset wealth growth volatility. We discuss possible ways to account for this feature in the next section.

### 3 Regime switching and consumption

The short-run variance of asset wealth is essentially driven by asset-price volatility. Financial markets are known to experience “changes of mood” , i.e., regime switching, probably derived from regime switching in dividends — see, e.g., (Driffill and Sola 1998). This fact has implications for consumption behaviour. For example, (Guidolin and Timmermann 2005) argue that the optimal consumption behaviour of an investor depends on the nature of the regime switches of asset returns. This implication is easily derived from simple, standard models of consumption behaviour under uncertainty, such as the example presented next.

Assume that a consumer lives for two periods. In the first period there is a shock ( $\varepsilon_1$ ) to the consumer’s wealth as a result of an increase in asset prices. However, the consumer is unsure whether the shock is permanent or temporary, i.e., whether there will be an offsetting shock in the second period ( $\varepsilon_2$ ). The problem of the consumer is to maximise

expected life-time utility as of the first period:

$$\max E_1 [u(C_1) + u(C_2)], \quad (4)$$

subject to the budget constraints:

$$C_1 + A_1 = L_1 + A_0 + \varepsilon_1 \quad (5)$$

and

$$C_{2,s} = L_2 + A_1 + \varepsilon_{2,s}, \quad s = 1, 2, \quad (6)$$

where  $C_1$  is consumption in the first period,  $C_{2,s}$  is consumption in the second period when the second-period shock takes the value  $\varepsilon_{2,s}$ ,  $A_i$  is asset wealth at the end of period  $i$  (excluding the shock) and  $L_i$  is labour income in period  $i$ . The model incorporates several simplifications to allow the results to come through as clearly as possible; for instance, there is no time discounting and inflation is zero (all variables are in real terms). The consumer has to choose consumption and asset holdings in the first period, and consumption in the second period contingent on the second-period shock. The life-time budget constraint is:

$$C_1 + C_{2,s} = A_0 + L_1 + L_2 + \varepsilon_1 + \varepsilon_{2,s}, \quad s = 1, 2. \quad (7)$$

If the shock were temporary, call it state 1, then  $\varepsilon_2 = \varepsilon_{2,1} = -\varepsilon_1$  and therefore lifetime wealth would be what it would have been in the absence of any shock:  $A_0 + L_1 + L_2$ . If the shock to wealth were permanent, call it state 2, then  $\varepsilon_2 = \varepsilon_{2,2} = 0$ . Given the second-period values, equation (6) implies  $C_{2,2} = C_{2,1} + \varepsilon_1$ .

Letting  $u_i$  denote the marginal utility of consumption in period  $i$  (as usual, assumed to be a decreasing function), the first-order conditions of the maximisation problem imply:

$$u_1 = E_1(u_2) \quad (8)$$

Let  $P$  be the probability that the consumer assigns to the occurrence of state 2 and let  $u_{2,i}$  denote the marginal utility of consumption in the second period in state  $i$ . The previous equation can be written as:

$$u_1 = (1 - P)u_{2,1} + Pu_{2,2} \quad (9)$$

If the consumer correctly believes that the shock is permanent ( $\varepsilon_2 = 0$ ,  $P = 1$ ), then equation (9) becomes  $u_1 = u_{2,2}$  and therefore  $C_1 = C_{2,2}$ , i.e., consumption in the first

period will adjust fully to the new “long-run” value. In case the shock is wrongly believed to be permanent ( $\varepsilon_2 = -\varepsilon_1$ ,  $P = 1$ ), the consumer will first increase consumption and later, after the mistake is known, will decrease it. If the shock were correctly believed to be temporary ( $\varepsilon_2 = -\varepsilon_1$ ,  $P = 0$ ), then the consumer would not react to it. Instead, asset wealth would temporarily increase in the first period and then return to normal in the second period, i.e., wealth would be doing all of the adjustment. In the case where the consumer wrongly believes the shock to be temporary ( $\varepsilon_2 = 0$ ,  $P = 0$ ), the consumer will let wealth adjust in the first period. In the second period, after realising the true nature of the shock, the consumer will adjust consumption.

The message of this simple model is that the adjustment of consumption and wealth to shocks, and their relation with the consumption-wealth ratio, will depend on the nature of those shocks and on how they are perceived by the consumer. For instance, if an increase in wealth is temporary, and seen as such, the consumption-wealth ratio will initially decrease as a result of that increase in wealth. In this case, this change in the consumption-wealth ratio will signal a future decline in wealth, which will restore the long-run equilibrium, after the temporary nature of the shock reveals itself. On the contrary, if the shock is permanent, but viewed as temporary, then the consumption-wealth ratio will initially decrease (as a result of the increase in wealth), but subsequently it is consumption that will increase, i.e., in this case the movement in the consumption-wealth ratio would forecast the change in consumption.

If the nature of the shocks varies over time (probably accompanying changes in the state of financial markets), then the implications of the foregoing analysis are clear: the adjustment of consumption and wealth to shocks should be modelled with a nonlinear specification to accommodate changes in the dynamics, such as the ones described above.

In the next section, we consider a formal approach to testing for nonlinear adjustment. We also introduce a multivariate Markov-switching representation of the trivariate relationship studied by L&L. This representation will be estimated and tested in section 5.



## 4 Testing for asymmetric adjustment

Following the discussion above, in this section we investigate the possibility of asymmetric adjustment in the consumption-wealth linkage. There is a difficulty in casting the testing problem in the usual framework (null of no cointegration vs. null of nonlinear cointegration), as some parameters will not be identified under the null. We follow the multi-step approach suggested in (Psaradakis, Sola, and Spagnolo 2004) to detect nonlinear error-correction.

As a first step, conventional procedures to establish the “global” properties of the series (such as unit root and cointegration tests) remain valid, as long as regularity conditions are obeyed (even though the deviations from the long-run equilibrium ( $z_t$ ) may be nonlinear). Once cointegration between the variables is discovered, a second step follows, focusing on the potential nonlinear “local” characteristics of the system, by looking at either the equilibrium error (in our case  $cay_t = c_t - 0.3a_t - 0.6y_t$ ), or the associated error-correction model for signs of nonlinear adjustment. This task may be carried out by using a range of tests that include parameter instability tests (for example, those of (Hansen 1992b) or (Andrews and Ploberger 1994)), general tests for neglected nonlinearity (e.g., RESET, White, Neural Networks) or nonlinearity tests designed to test linear adjustment against nonlinear error-correction alternatives, such as Markov switching ((Hansen 1992a)) and threshold adjustment ((Hansen 1997) and (Hansen 1999)). Moreover, and as suggested by (Psaradakis, Sola, and Spagnolo 2004), we may also resort to conventional model selection criteria such as the AIC (or BIC and Hannan-Quinn criteria), which was found to perform well in these circumstances.

If the analysis of the “local” features of the data points to nonlinearity, then a third step ensues, in which one should fit a MS model, either to  $z_t$  or to the error-correction representation. However, in the case considered here, the results in L&L indicate that wealth does most of the adjustment towards equilibrium, meaning that a single-equation ECM with consumption as the dependent variable would be misspecified. Thus, one needs to analyse the whole system, which implies that a Markov-switching vector error-correction model should be employed instead.

(Camacho 2005) shows that if the equilibrium errors of a cointegrated system for the

$m \times m$  vector  $x_t$  follow a MS-(V)AR,

$$z_t = c_{s_t} + A_{s_t}(L)z_{t-1} + \theta_{st}\varepsilon_t \quad (10)$$

then there is a corresponding MS-VECM representation

$$\Delta x_t = \mu_{s_t} + \Gamma_{s_t}z_{t-1} + \Pi_{s_t}(L)\Delta x_{t-1} + \sigma_{st}u_t \quad (11)$$

where  $\Pi_i$ 's are  $m \times m$  coefficient matrices and  $\Gamma_{s_t}$  is a regime-dependent long-run impact matrix. Indeed, the nonlinear dynamics of the equilibrium errors  $z_t$  may lead to a switching adjustment matrix  $\Gamma$  and to short-run dynamics of the endogenous variables (given by  $\Pi$ ) that vary across regimes. Several possibilities may arise, including one where cointegration switches on and off, for example. The system may be estimated by a multi-equation version of the Hamilton filter and estimates of the (possibly different) adjustment coefficients obtained.

The second panel of Table 1 revisits the results in (Lettau and Ludvigson 2004) regarding the long-run properties of the system, confirming that there is indeed cointegration among consumption, labour income and asset wealth, judging by the results of Johansen cointegration tests. Next, we focus on the local properties of the system. Using the estimated equilibrium error  $z_{t-1}$ , we fit an over-parameterised linear  $AR(p)$  for  $z_{t-1}$  (initially with 4 lags, then tested down to 1), which was found to be an  $AR(1)$  with autoregressive coefficient  $\hat{\phi} = 0.851$ . Then, we test for neglected instability and nonlinearity in this specification. The statistics include the  $L_c$  test of (Hansen 1992b) against martingale parameter variation, (Andrews and Ploberger 1994) sequential tests, the White test and the RESET test. Furthermore, (Carrasco 2002) shows that tests for threshold effects will also detect MS behaviour, so we employ (Hansen 1997) threshold tests. As recommended by (Hansen 1999), we use bootstrapped p-values.

Results are presented in Table 2. Some procedures fail to reveal mis-specifications, namely the RESET test, the  $L_c$  test for joint stability and the avg  $F$  test. However, all other tests reject their respective nulls at the 5% or 10% significance levels, so, overall, the evidence for nonlinear behaviour is sufficiently compelling.

Due to computational difficulties, we do not use the (Hansen 1992a) test. Nevertheless, the standard likelihood ratio (LR) of linear specification against the estimated MS-AR(1)

model favours the latter (although the usual asymptotic distribution for the LR statistic is not strictly valid). Thus, we compute the upper bound on the significance level of the test using the approach in (Davies 1987), which confirms the initial result. Alternatively, using (Garcia 1998) critical values (Table 3, for the case  $\phi = 0.8$ ) as an approximation for the distribution LR test, the same conclusion emerges. The bottom panel of Table 2 reports results on the estimation of a MS-AR(1) with changes in mean and variance for  $z_{t-1}$ , while Figure 4 depicts the corresponding regime probabilities against  $z_{t-1}$ . It is apparent that the MS model is picking up distinguished periods of large and volatile deviations from equilibrium. Thus, and following (Camacho 2005), one should investigate the error-correction representation of the system, which is likely to offer a more complete description of the dynamics of the relationship.

## 5 A MS-VECM for the Consumption-Wealth Ratio

In order to estimate a Markov-switching vector error-correction model for the consumption-wealth ratio, one must consider carefully the dimension of the model. Indeed, even in a simple trivariate system, if all parameters are allowed to switch, identification problems may occur and estimation will be intractable. Hence, we opt to restrict matrix  $\Pi$  in (11) to be constant across regimes. Additionally, we follow L&L in estimating a first-order VAR system. More importantly, we specify  $\Gamma_{s_t}$  in (11) as a regime-dependent long-run impact matrix defined as

$$\Gamma_{s_t} = \alpha_{s_t}\beta$$

with cointegration vector  $\beta$  and adjustment matrix  $\alpha_{s_t}$ . Note that we assume an invariant long-run relationship, following the evidence in the previous section, while allowing the adjustment towards equilibrium to be state-dependent. This implies that shocks to any of the three variables can have different effects across regimes through  $\alpha_{s_t}$ . For example, shocks to asset wealth can have different effects on consumption depending on whether markets are in a boom or in a recession, or, alternatively, whether these shocks are permanent or temporary. In addition, the coefficients in  $\alpha_{s_t}$  can also capture the speed at which agents learn the nature of the shocks.

Thus, we initially allow  $\mu$  and  $\Gamma$  in (11) to be state-dependent (as well as the variance

of the error term), and then exploit potential parameter restrictions in order to achieve a more parsimonious MS-VECM specification. The model to be estimated is therefore

$$\Delta x_t = \mu_{s_t} + \gamma_{s_t} z_{t-1} + \pi(L)\Delta x_{t-1} + \sigma_{s_t} u_t, \quad (12)$$

where  $x_t = \{c_t, a_t, y_t\}$ , with 35 parameters. Estimation is carried out in GAUSS, using the multi-equation version of the Hamilton filter, as explained in (Camacho 2005).

Table 3 displays results of the estimation of (12), using heteroskedasticity-robust standard errors based on the Outer-Product-Gradient matrix. We begin by noting that the model is able to identify two regimes, whereby the mechanism through which deviations from the long-run relationship are eliminated depends on the state of the economy. One state corresponds to high asset wealth growth (0.9% per quarter) and higher volatility, where asset growth does the adjustment, albeit at a faster rate than in the linear model (0.458 against 0.33). However, a second regime of “calm” periods and lower asset wealth growth is instead associated with adjustments in consumption (negative coefficient of  $-0.136$ ), since now it is the adjustment coefficient on consumption growth that is significant. This, of course, contrasts with the results for the linear model, which does not allow for switching adjustment. On the other hand, note that the estimated  $\Pi$  matrix presents values similar to those found for the linear model, which suggests that the restrictions imposed may be valid.

As in the previous section, it is not straightforward to test the appropriateness of the MS-VECM over the linear model. A likelihood ratio test of a linear vs Markov specification is clearly favourable to the MS model, producing 77.224 with an upper-bound p-value of 0.000. This test is not usually valid, since the regularity conditions that justify the usual  $\chi^2$  approximation do not hold. However, the very large value of the statistic seems to offer support to the MS model. In addition, all of the model selection criteria favour the MS-VECM specification. Although the transition probabilities ( $p_{11} = 0.927$ ,  $p_{22} = 0.952$ ) are estimated imprecisely (standard errors of 0.631 and 0.60), a multi-equation version of a Hamilton-White test of Markov specification (see (Hamilton 1996)) with a p-value of 0.70 reveals that the Markov assumption should not be rejected. Nevertheless, there seems to be scope for simplification through the imposition of restrictions on redundant parameters.

Thus, we employ a sequence of LR tests on model (12), arriving at a more parsimonious specification without redundant adjustment coefficients and with constant intercepts (28 parameters in total), with a LR test supporting these restrictions (p-value of 0.16). Estimates for this model are presented in Table 4. Notice that both the regime probabilities and the adjustment coefficients are now estimated more precisely. The short-run matrix displays practically the same values, as well as the consumption adjustment coefficient, while the wealth adjustment parameter is now closer to the value in the linear model. Again, the Hamilton-White Markov specification test produces a p-value of 0.68, confirming the superiority relatively to the linear model. All model selection criteria continue to favour the restricted model. Furthermore, the smoothed probabilities<sup>2</sup> depicted in Figure 5 pick up very well the phases that one usually associates with “bullish” and “bearish” markets. Indeed, the associated regimes are comparable to those of the univariate MS model for returns, implicit in Figure 3. This fact appears to indicate that the regime switching in the system is being driven by asset wealth (and, therefore, by financial markets).

Overall, it seems that the MS-VECM captures the main dynamic features in the trivariate system, and does that better than a linear VECM. Our findings also suggest that short-term deviations in the relationship will forecast either asset returns or consumption growth, depending on the state of the economy. These results differ from the conclusions of L&L, but note that the theoretical relationship in (3) does not preclude our findings. Indeed, fluctuations in *cay* may be related to future values of either  $r_t$ ,  $\Delta c_t$  or  $\Delta y_t$ . We believe our results allow us to make an empirical point: if we allow for nonlinear adjustment, the data reveals two possible channels to restore equilibrium, that will be “switched on/off” according to the phase of the business cycle.

A possible interpretation of regime 1 is that in this state consumers are able to recognise periods of transitory growth in wealth and, in accordance with the theoretical models discussed in sections 2 and 3, consumers let wealth vary until it eventually returns to its equilibrium path and the long-run equilibrium is restored. In state 2, consumption does adjust: variations in wealth are recognised as permanent and therefore, as the theory predicts, agents adjust their consumption paths accordingly. Thus, the results derived from the MS-VECM seem to be interpretable in the light of standard models of consumption,

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<sup>2</sup>These are very similar those obtained with the unrestricted MS-VECM, not reported here.

such as the one in section 2, which predict varying adjustment dynamics.

## 6 Concluding remarks

The behaviour of consumption is one of the most studied issues in economics. It is a matter of importance to policy-making, especially in an era in which a consensus appears to have emerged concerning the desirability of keeping inflation low. The extraordinary movement in asset prices in the late 1990s raised the problem of knowing whether it heralded a new period of high inflation, due to demand pressures fuelled by the “wealth effect” of asset prices on consumption. In face of this, the traditional linear model of consumption and wealth, as the one discussed at length by Lettau and Ludvigson, reveals an intriguing picture: a picture in which consumption appears not to adjust to deviations of the consumption-wealth ratio from its long-run trend; instead, wealth does all the adjustment.

Theoretical models of consumption suggest that consumption should react to movements in wealth. We have shown that the reaction depends on whether the shocks are viewed as more likely temporary or more likely permanent, which in turn should depend on the state of financial markets. Based on this insight, we estimated a Markov-switching vector error-correction model of consumption, labour income and asset wealth.

Our theoretical and empirical models deliver results consistent with those of the reference papers, such as (Lettau and Ludvigson 2001) and (Lettau and Ludvigson 2004), provided one takes into account the fact that the financial markets seem to go through different regimes. L&L conclude that most of the variation in wealth is transitory and unrelated to variations in consumption. The theoretical model discussed in this paper leads to the same conclusion: when the shock to wealth is transitory, the consumption-wealth ratio should forecast the subsequent change in wealth. However, when the change in wealth is permanent, the theoretical model predicts that consumption could be forecast by the consumption-wealth ratio. Our empirical model allows for these different adjustment dynamics and therefore nests that of L&L. Unsurprisingly, our model provides a better description of the data than the traditional linear model. Namely, as mentioned above, it helps to explain recent controversial results, concerning the adjustment of the variables to

deviations from the long-run equilibrium and the forecasting ability of the system.

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Table 1: Linear VECM

| Equation                                | $\Delta c_t$        | $\Delta a_t$        | $\Delta y_t$       |
|---|---------------------|---------------------|--------------------|
| $\hat{z}_{t-1}$                         | -0.0211<br>(-0.955) | 0.3337<br>(2.228)   | 0.0117<br>(0.326)  |
| $\Delta c_{t-1}$                        | 0.1996<br>(2.953)   | 0.0458<br>(0.141)   | 0.4957<br>(3.82)   |
| $\Delta a_{t-1}$                        | 0.0456<br>(3.219)   | 0.0924<br>(1.085)   | 0.0918<br>(2.44)   |
| $\Delta y_{t-1}$                        | 0.0763<br>(1.726)   | -0.0656<br>(-0.369) | -0.1222<br>(-0.97) |
| (t-ratios based on HAC standard errors) |                     |                     |                    |
| Tests [p-values]                        |                     |                     |                    |
| AR 1-5                                  | 3.039<br>[0.012]    | 0.718<br>[0.611]    | 0.923<br>[0.467]   |
| Normality                               | 5.822<br>[0.054]    | 25.532<br>[0.000]   | 48.653<br>[0.000]  |
| ARCH                                    | 0.323<br>[0.863]    | 6.352<br>[0.000]    | 1.725<br>[0.146]   |
| Heteroskedasticity                      | 0.948<br>[0.478]    | 5.439<br>[0.000]    | 1.531<br>[0.149]   |
| Vector AR                               | Vector Norm.        | Vector Het.         | Vector Het.        |
| 1.374<br>[0.058]                        | 70.828<br>[0.000]   | 1.744<br>[0.002]    | 1.653<br>[0.000]   |
| Log likelihood                          | AIC                 | BIC                 | HQ                 |
| -2630.909                               | -5219.819           | -5149.934           | -5191.555          |
| Johansen cointegration tests [p-values] |                     |                     |                    |
| $H_0 : r =$                             | Trace               | Max                 |                    |
| 0                                       | 52.861<br>[0.000]   | 35.526<br>[0.478]   |                    |
| 1                                       | 17.335<br>[0.121]   | 13.726<br>[0.106]   |                    |
| 2                                       | 3.609<br>[0.473]    | 3.609<br>[0.473]    |                    |

Table 2: Stability and linearity tests

| Instability                      |                              | Threshold                   |                                   | RESET                         |
|----------------------------------|------------------------------|-----------------------------|-----------------------------------|-------------------------------|
| $L_c$ (joint)                    | 0.843                        | sup $LM$                    | 9.943<br>[0.061]                  | 1.755<br>[0.187]              |
| $L_c$ (var.)                     | 0.541**                      | avg $LM$                    | 2.825<br>[0.064]                  |                               |
| avg $F$                          | 3.305                        | exp $LM$                    | 4.977<br>[0.043]                  | White                         |
| sup $F$                          | 14.554**                     | $F_{12}$                    | 10.51<br>[0.061]                  | 3.59<br>[0.029]               |
| exp $F$                          | 3.465**                      | [bootstrapped p-values]     |                                   |                               |
| Results from MS-AR(1) estimation |                              |                             |                                   |                               |
| $\mu_1 = 0.001$                  | $\phi_1 = 0.754$<br>(12.699) | $\phi_2 = 0.826$<br>(8.374) | $\sigma_1 = 0.059$<br>(7.315)     | $\sigma_2 = 0.101$<br>(3.302) |
| $p_{11} = 0.981$<br>(60.91)      | $p_{22} = 0.931$<br>(13.652) | LogL: 918.4                 | sup $LR$<br>[p-value upper bound] | : 17.225<br>[0.022]           |
| AIC: -1820.7                     |                              | AIC linear: -1813.5         |                                   |                               |

Table 3: MS-VECM(1) estimates

| Equation  | State 1           |                   |                    | State 2             |                   |                   |
|---|-------------------|-------------------|--------------------|---------------------|-------------------|-------------------|
|   | $\Delta c_t$      | $\Delta a_t$      | $\Delta y_t$       | $\Delta c_t$        | $\Delta a_t$      | $\Delta y_t$      |
| $\mu$   | 0.0037<br>(6.569) | 0.0089<br>(2.205) | 0.0048<br>(2.871)  | 0.0041<br>(5.428)   | 0.0049<br>(1.978) | 0.0023<br>(1.738) |
| $\widehat{cay}_{t-1}$   | 0.0129<br>(0.618) | 0.4784<br>(2.263) | 0.0722<br>(1.059)  | -0.1361<br>(-2.011) | 0.3021<br>(1.293) | 0.0328<br>(0.266) |
| Short run dynamics  |                   |                   |                    |                     |                   |                   |
|   | $\Delta c_{t-1}$  | 0.2206<br>(3.048) | -0.186<br>(-0.196) | 0.50<br>(3.346)     |                   |                   |
|   | $\Delta a_{t-1}$  | 0.0424<br>(3.256) | 0.1287<br>(1.893)  | 0.093<br>(2.672)    |                   |                   |
|   | $\Delta y_{t-1}$  | 0.0485<br>(1.295) | 0.0172<br>(0.427)  | -0.1056<br>(-1.039) |                   |                   |
| (t-ratios based on heteroskedasticity-robust standard errors) |                   |                   |                    |                     |                   |                   |
| LogL  | AIC               | BIC               | HQ                 |                     |                   |                   |
| -2669.521   | -5269.043         | -5151.567         | -5221.936          |                     |                   |                   |

Table 4: Restricted MS-VECM(1) estimates

| Equation  | State 1      |                   |                    | State 2             |                  |              |
|---|--------------|-------------------|--------------------|---------------------|------------------|--------------|
|   | $\Delta c_t$ | $\Delta a_t$      | $\Delta y_t$       | $\Delta c_t$        | $\Delta a_t$     | $\Delta y_t$ |
| $\widehat{cay}_{t-1}$   | —            | 0.3662<br>(2.166) | —                  | -0.1328<br>(-3.523) | —                | —            |
| Intercepts and short run dynamics                             |              |                   |                    |                     |                  |              |
| $\mu$   |              | 0.0038<br>(8.43)  | 0.0071<br>(5.667)  | 0.003<br>(3.467)    |                  |              |
| $\Delta c_{t-1}$  |              | 0.2137<br>(3.007) | -0.100<br>(-0.238) | 0.506<br>(3.439)    |                  |              |
| $\Delta a_{t-1}$  |              | 0.041<br>(3.100)  | 0.0981<br>(1.572)  | 0.0823<br>(2.577)   |                  |              |
| $\Delta y_{t-1}$  |              | 0.0459<br>(1.115) | 0.0172<br>(0.425)  | -0.108<br>(-1.032)  |                  |              |
|   | $p_{11} =$   | 0.9174<br>(2.415) |                    | $p_{22} =$          | 0.9475<br>(1.99) |              |
| (t-ratios based on heteroskedasticity-robust standard errors) |              |                   |                    |                     |                  |              |
| LogL  | AIC          | BIC               | HQ                 |                     |                  |              |
| -2664.319   | -5272.637    | -5179.457         | -5223.952          |                     |                  |              |

Figure 1: Estimated equilibrium deviations for *cay*

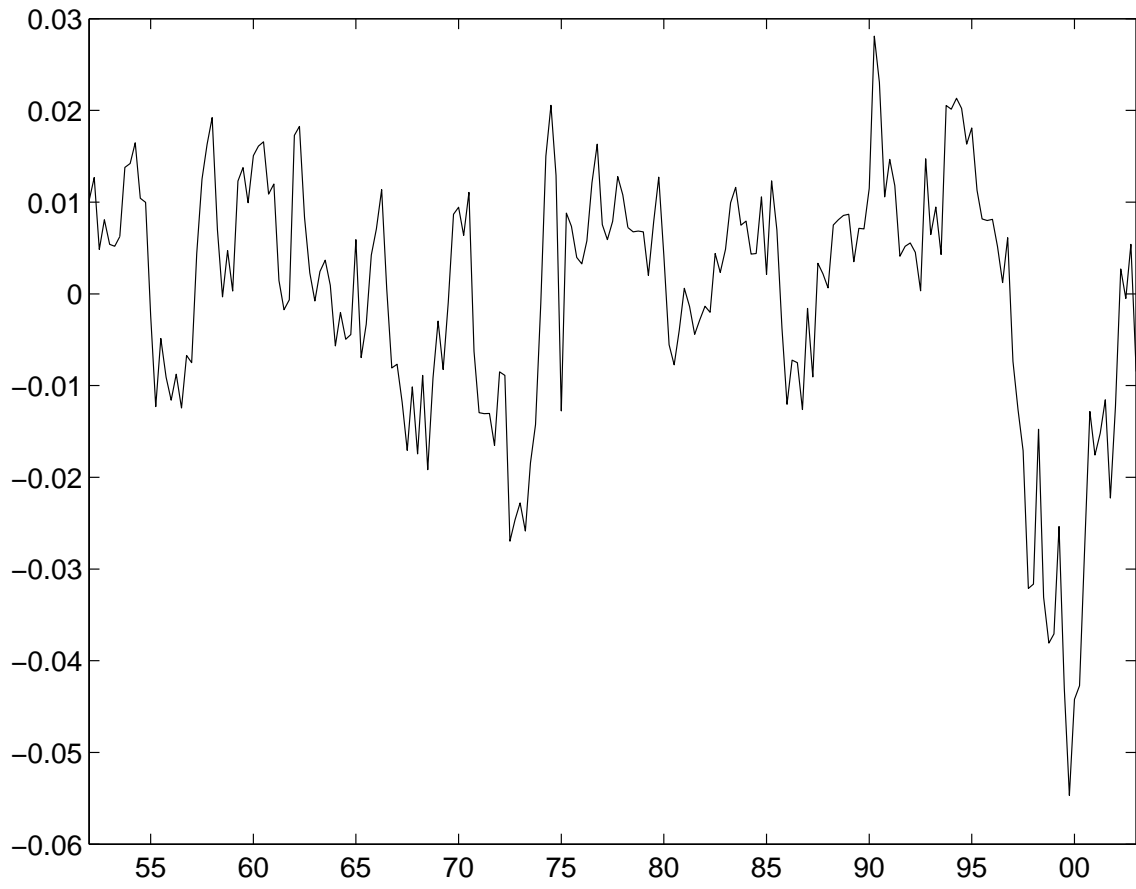


Figure 2: Consumption and asset wealth

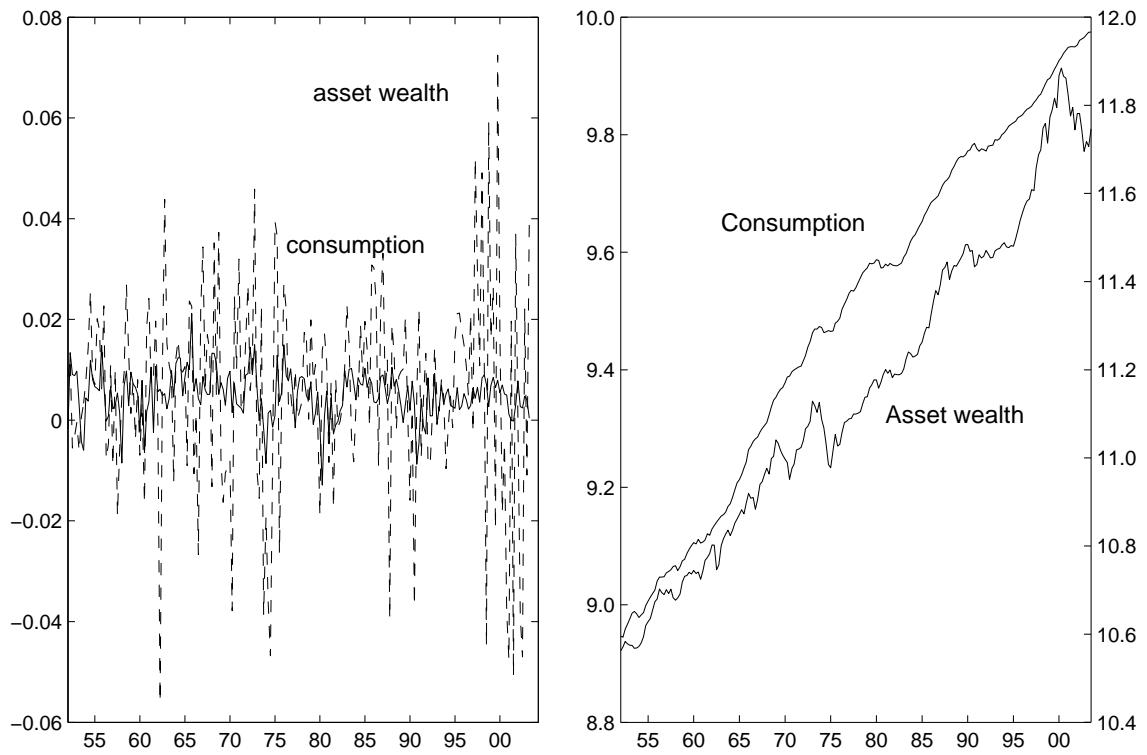


Figure 3: Asset wealth growth volatility

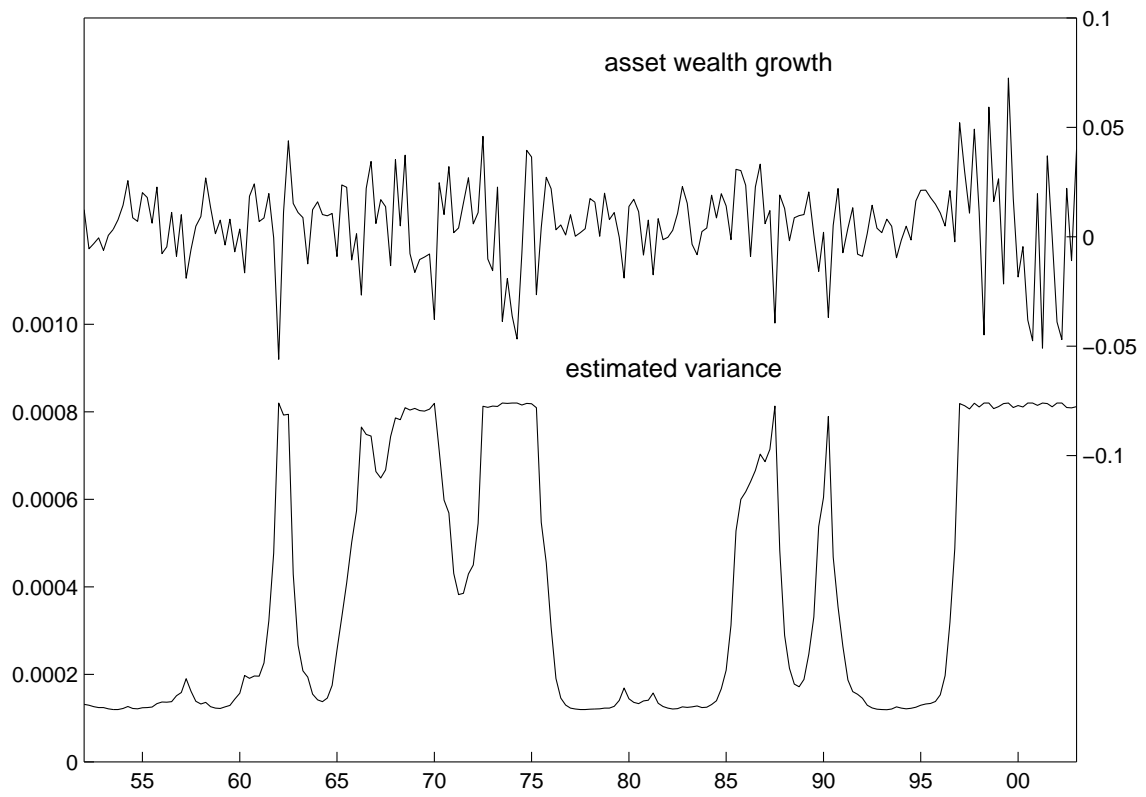




Figure 4: Smoothed probabilities and  $z_{t-1}$

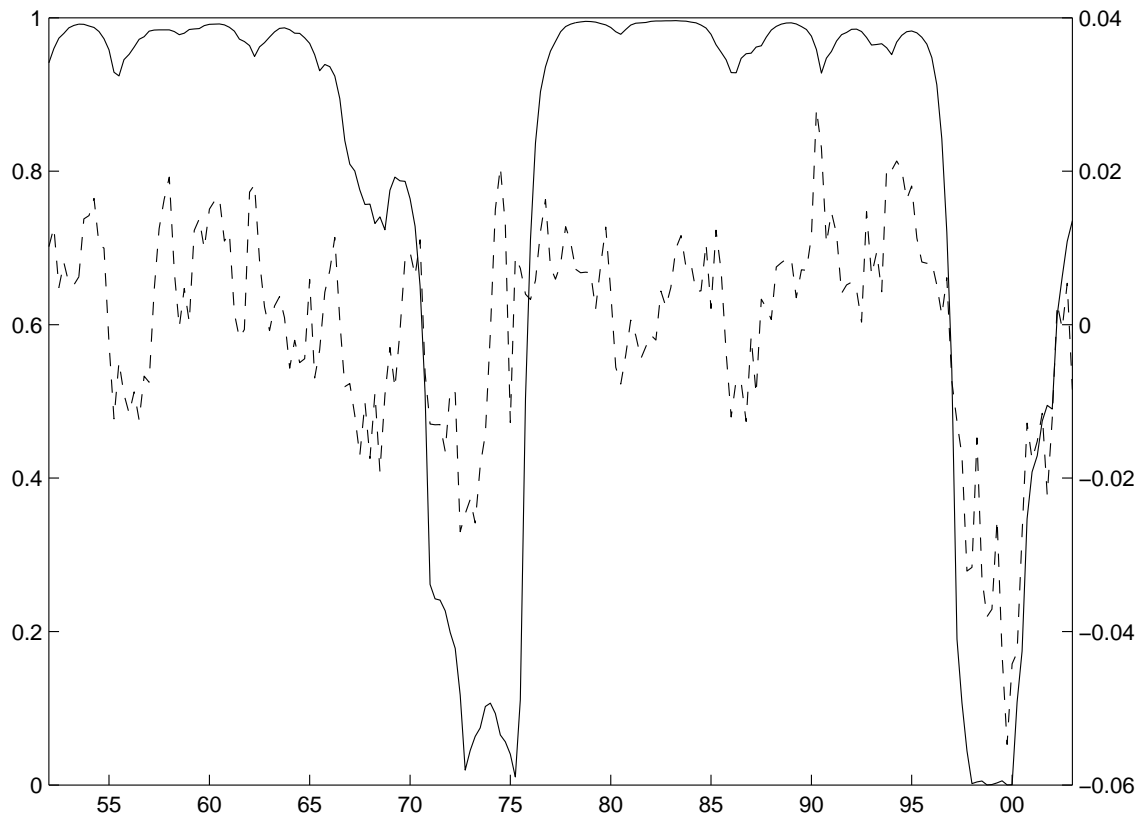


Figure 5: MS-VECM smoothed probabilities and  $z_{t-1}$

