Optimal monetary policy with a regime-switching exchange rate in a forward-looking model*

Fernando Alexandre†‡, Pedro Bação§ and John Driffill¶

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Abstract

We evaluate the macroeconomic performance of different monetary policy rules when there is exchange rate uncertainty. We do this in the context of a non-linear rational expectations model. The exchange rate is allowed to deviate from its fundamental value and the persistence of the deviation is modelled as a Markov switching process. Our results suggest that taking into account the switching nature of the economy is important only in extreme cases.

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†NIPE and University of Minho.

‡Corresponding author: Fernando Alexandre, Departamento de Economia, Escola de Economia e Gestão, Universidade do Minho, Campus de Gualtar, 4710-Braga, Portugal. Tel.: +351-253-604510; fax: +351-253-676375; e-mail: falex@eeg.uminho.pt.

§GEMF and Faculty of Economics of the University of Coimbra.

¶Birkbeck College, University of London.

1 Introduction

There are examples throughout history of instances where dramatic changes in asset prices have coincided with prolonged booms and busts. Several papers have examined the optimal reaction of central banks when they face extreme changes in asset prices (see, among others, Bernanke and Gertler, 1999; Gilchrist and Saito, 2006). Among asset prices, the exchange rate, when driven by non-fundamental movements, has been an important issue in monetary policy debates, namely, for inflation targeting economies.

The exchange rate introduces additional channels for monetary policy, making it a potential policy target. However, it is well documented that exchange rates can experience sustained deviations from its long run equilibrium, followed by sudden corrections. The impact of these unwarranted exchange rate movements on macroeconomic performance has been a concern of central bankers and scholars. Results from the estimation of structural VARs, such as in Clarida and Gertler (1997), suggest that central banks indirectly try to influence the exchange rate through movements in the interest rate. However, evidence from simulated open-economy models on whether monetary policy should react to the exchange rate is mixed.

Ball (1999) shows that an interest rate policy rule that responds only to output and inflation, like a Taylor rule, is not optimal for an open economy. Svensson (2000) using a forward-looking model, concludes that the exchange rate can be a very useful instrument in stabilising Consumer Price Index (CPI) inflation. Ceccheti et al. (2000) believe that central banks can improve macroeconomic performance by responding to asset prices as well as to expected inflation and to the output gap. Batini and Nelson (2000) model a bubble in the exchange rate as an exogenous process that temporarily shifts the exchange rate away from its long-run equilibrium and conclude that responding to the exchange rate does not improve welfare in most cases, and may even lead to lower welfare.

Leitemo and Soderstrom (2005) analyse the impact of exchange rate uncertainty on the conduct of monetary policy and conclude that policy rules without an exchange rate term, namely a Taylor rule, are optimal for the stabilisation of a small open economy. However, Wollmershauser (2006) and Zampolli (2006) concluded that reacting to the exchange rate improves macroeconomic stability in models that incorporate exchange rate uncertainty. Wollmershauser (2006) concluded that monetary policy rules that include an exchange rate term are more robust to a high degree of exchange rate uncertainty, concerning the relationship between the nominal exchange rate and the nominal interest rate or other macroeconomic variables. Zampolli (2006) used a simple backward-looking model of the type defined in Ball (1999) with a regime-switching exchange rate, which aimed at capturing the complex behaviour of financial markets, to analyse the benefits from reacting to the exchange rate when there is uncertainty about the nature of the misalignment.

However, the absence of forward-looking behaviour prevents the model from capturing the essential role of expectations in monetary policy and in asset markets. Therefore, we extend Zampolli's (2006) analysis by considering an open-economy forward-looking model of the type used in Svensson (2000). In that sense we test the conjecture of Zampolli (2006, pp. 1530) that "the main insights and conclusions are likely to carry over to a forward-looking model".

This extension is of great importance as the exchange rate, being an asset price, is inherently forward-looking. Additionally, we assume regime-switching in the exchange rate, that is, the exchange rate may be in one of two states. In one regime it randomly oscillates around its equilibrium, defined by the real interest parity condition. In the other regime the deviations from equilibrium are persistent. We experiment a range of values for the transition probabilities and for the persistence coefficient. We assume that the transition probabilities are exogenous and observed by policymakers. We therefore abstract from the issue of imperfect information at this stage. Uncertainty in this context results from the policymaker not knowing in which regime the exchange rate will be in the next period.

We start our analysis by comparing the optimal welfare loss when the policymaker faces no uncertainty about the nature of the shock to the real exchange rate and when policymakers have to assign a given probability to a transitory shock and to a very persistent or bubble shock. Then we analyse the performance of simple policy rules, both with and without an exchange rate term, and evaluate their robustness in dealing with exchange rate uncertainty.

Finally, we evaluate the benefits from taking into account the switching nature of the economy by comparing the performance of time invariant rules to regime switching rules.

Section 2 describes our open-economy model and the monetary policy framework. Section 3 evaluates the welfare loss for a set of policy rules under exchange rate uncertainty. Section 4 concludes.

2 An open economy with a regime-switching exchange rate

The exchange rate introduces additional channels for monetary policy through its effects on aggregate demand and domestic inflation. In Ball (1999), the change in the exchange rate affects inflation because it is passed directly into import prices. Following Svensson (2000), the inclusion of the exchange rate in our model adds three channels for monetary policy to affect the Consumer Price Index (CPI). First, it can affect inflation with a lag through its effect on aggregate demand. Second, the exchange rate can affect domestic inflation, and therefore the CPI, by affecting domestic currency prices of imported intermediate goods and, more indirectly, through its effects on nominal wages that depend on the evolution of the CPI. Finally, the exchange rate affects CPI inflation through its effects on domestic currency prices of imported final goods. Therefore, the model that we describe below tries to capture all these three effects. The lag structure of our model is such that it captures the often mentioned fact (see, e.g., Svensson, 2000; Ball, 1999; Edwards, 2006) that monetary policy can affect the consumer price index with a shorter lag through the exchange rate channels.

2.1 The model

Our stylised system of macroeconomic equations is the following:

$$
\begin{aligned}
y_t &= E_t y_{t+1} - \alpha_1 \left(i_t - E_t \pi_{t+1}\right) + \alpha_2 y_{t-1}^* + \alpha_3 q_t + \varepsilon_t^d && (1)\\
\pi_t^d &= \beta_1 \pi_{t-1}^d + \left(1 - \beta_1\right) \beta E_t \pi_{t+1}^d + \beta_2 y_{t-1} + \beta_3 (q_t - q_{t-1}) + \varepsilon_t^s && (2)\\
q_t &= E_t q_{t+1} - i_t + E_t \pi_{t+1} + i_t^* - E_t \pi_{t+1}^* + \varepsilon_t^q && (3)\\
\pi_t &= \pi_t^d + \omega (q_t - q_{t-1}) && (4)\\
\varepsilon_t^s &= \rho^s \varepsilon_{t-1}^s + e_t^s && (5)\\
\varepsilon_t^d &= \rho^d \varepsilon_{t-1}^d + e_t^d && (6)\\
\varepsilon_t^q &= \rho_{s_t}^q \varepsilon_{t-1}^q + e_t^q && (7)\\
y_t^* &= \rho_{y^*} y_{t-1}^* + e_t^{y^*} && (8)\\
\pi_t^* &= \rho_{\pi^*} \pi_{t-1}^* + e_t^{\pi^*} && (9)\\
i_t^* &= \rho_{i^*} \pi_t^* + \rho_{i^*}' y_t^* + e_t^{i^*} && (10)
\end{aligned}
$$

Equation (1) is the aggregate demand equation for an open economy of the type used in Svensson (2000). Output depends on its own expected value, on the real interest rate, on the lagged foreign output, y_t^*, and on the real exchange rate, q. In this model the real exchange rate affects the aggregate demand because it affects the the relative price between domestic and foreign goods: a higher q means depreciation, that is, $q_t \equiv s_t + p_t^* - p_t$, where s is the price of foreign currency in terms of domestic money. Additionally, output depends on a demand shock that we assume to follow an AR(1) process, as in equation (6). Following Svensson (2000) we set the following values for the coefficients in the aggregate demand equation: $\alpha_1 = 0.6$, $\alpha_2 = 0.05$ and $\alpha_3 = 0.04$.

Equation (2) is a "hybrid" Phillips curve where π_t^d, is domestic inflation. In face of the discussion and evidence provided in Gali and Gertler (1999), we have substituted lagged output for the marginal cost, and we also include some open-economy elements. Following the survey of empirical estimates presented in Rudebusch (2002), we consider the inflation persistence coefficient to be $\beta_1 = 0.4$. We set $\beta = 0.99$ as in Gali and Gertler (1999), and $\beta_2 = 0.13$ as in Rudebusch (2002). The inclusion of the change in the exchange rate in the domestic inflation equation aims at capturing its effect on domestic currency prices of imported intermediate goods. In our analysis, we follow Svensson (2000) and we set the pass-through parameter, that gives the impact of changes in the exchange rates on domestic inflation, $\beta_3 = 0.01$.

We assume that the uncovered interest parity condition holds, that is, $i_t - i_t^* = E_t s_{t+1} - s_t + \varepsilon_t^q$, where ε_t^q is the exchange-rate risk premium. Using this assumption and the definition of real interest rate given above, equation (3) defines the real interest parity condition.

Equation (7) specifies the process for the shock in the exchange rate. We assume that the exchange rate may be in one of two states. In state 1, $\rho_{s_t}^q > 0$ and therefore the exchange rate deviates persistently from its fundamental value. In state 2, $\rho_{s_t}^q = 0$ and thus the exchange rate is subject to random shocks that deviate it from its long-run equilibrium value, but without any persistence. The variance of the exogenous shock e_t^q is the same across regimes, which implies that, as seems reasonable, the variance of ε_t^q increases in the first regime, and the higher the persistence the more it increases.

State 1 represents times of instability, where the exchange rate is "disconnected" from fundamentals for long periods. Several authors have been providing rationales for the "disconnect puzzle" described in Obstfeld and Rogoff (2000). For example, De Grauwe and Grimaldi (2006) assume heterogeneous agents with different beliefs about the behaviour of the exchange rate, which results in persistent deviations from equilibrium and non-linear behaviour. The state is assumed to evolve as a Markov chain with the following probability transition matrix:

$$P = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix} \tag{11}$$

where $p_{ij} = 1 - p_{ii}$ (when $i \neq j$) and p_{ij} is the probability of moving from state i in the current period to state j in the next period. In our computations we use the values 0.25, 0.5 and 0.75 for p_{ii}. In the single state model, we have $\rho_1^q = \rho_2^q$ and thus the probability transition matrix becomes irrelevant. We also use a range of values for the autoregressive coefficient in the first regime: 0.5 (mild persistence), 0.9 (high persistence) and 1.1 (explosive).

Bordo and Jeanne (2002), in a three period model, assumed that monetary policy can affect the transition probabilities. Zampolli (2006) argues that assuming exogenous transition probabilities is not unreasonable given the high degree of uncertainty about the stochastic properties of an asset price and their relationship with monetary policy. We follow this author and in our computations we assume that the transition probabilities are exogenous and observed by policymakers. We therefore abstract from the issue of imperfect information at this stage. Uncertainty in this context results from the policymaker not knowing in which regime the exchange rate will be in the next period.

Clarida, Galí and Gertler (2001) show that in an open economy it is important to distinguish between domestic inflation and consumer price inflation, as measured by the Consumer Price Index. These authors concluded that for an economy with perfect exchange-rate pass-through the central bank should target domestic inflation and let the exchange rate to float. Equation (4) defines CPI inflation, π, as a function of domestic inflation and the change in the real exchange rate. This results from defining the CPI as a weighted average of domestic inflation, π_t, and domestic-currency inflation of imported foreign goods, π_t^f,[1] where ω is the share of imported goods in CPI. This equation shows that the exchange rate can affect the consumer price directly, which allows monetary policy to affect CPI inflation with a shorter lag than through the aggregate demand channel. The effect of the exchange rate on the CPI depends on the weight of the domestic-currency inflation of imported foreign goods. Svensson (2000) sets $\omega = 0.3$.

As in Svensson (2000) we assume that foreign output and foreign inflation follow stationary AR(1) processes as described in equations (8) and (9). We set $\rho_{y^*} = \rho_{\pi^*} = 0.8$. The foreign interest rate is assumed to follow a Taylor rule with $\rho_{i^*} = 1.5$, $\rho'_{i^*} = 0.5$, as in Svensson (2000).

[1] As described in Svensson (2000), π_t^f fulfils is given by $\pi_t^f = p_t^f - p_{t-1}^f = \pi_t^* + s_t - s_{t-1} = \pi_t + q_t - q_{t-1}$, where $p_t^f = p_t^* + s_t$ is the domestic-currency price of imported foreign goods and π_t^* is foreign inflation.

2.2 Policy rules and welfare

A Markov-switching rational expectations model requires adequate solution methods. Svensson and Williams (2005) and Farmer, Waggoner and Zha (2006) propose two such methods. Svensson and Williams' method uses an iterative procedure similar to the one used to solve simple optimal linear quadratic regulator problems. Farmer et al. rightly argue that Svensson and Williams' method does not guarantee that the solution is unique. Farmer et al. propose a modification of Sims (2001) method to deal with the case of Markov-switching rational expectations while maintaining the ability to analyse the uniqueness of the solution. In this paper we employ Svensson and Williams' method to compute the optimal loss, and base our numerical optimisation of simple rules on Farmer et al.' method, selecting only rules that correspond to unique and stable solutions.

Simple rules have been widely discussed among academics in monetary policy analysis. Several arguments have been used in its defense. On one hand, it has been argued that simple rules perform nearly as well as optimal rules (see, for example, Rudebusch and Svensson, 1999). On the other hand, it has been argued that simple rules are very robust to several types of uncertainty (see, for example, Levin, Wieland and Williams, 1999). We therefore use simple rules to see how they compare to the optimal policy rule and how robust they are in dealing with exchange rate uncertainty. The different rules are summarised in Table 1.

We compute the optimal parameters for Taylor-type and inflation-forecast based policy rules. In the Taylor-type policy rule the interest rate reacts to deviations of output and inflation from the target. Additionally, we look at the Taylor-type policy rule with an exchange rate term. As a benchmark, we also look at the performance of the Taylor rule as defined in Taylor (1993).

In the inflation-forecast based policy rule the interest rate responds to deviations of expected inflation from the target. We also consider an inflation-forecast based rule with an exchange rate term.

As in Zampolli (2006), we compute both time-invariant policy rules and regime-switching

Table 1: Simple policy rules

Rule	Formula
TRo	$i_t = 1.5\pi_t + 0.5y_t$
TR+q	$i_t = \delta^{\pi}_{s_t}\pi_t + \delta^{y}_{s_t}y_t + \delta^{q}_{s_t}q_t$
TR	$i_t = \delta^{\pi}_{s_t}\pi_t + \delta^{y}_{s_t}y_t$
TR+q R	$i_t = \delta^{\pi}_{s_t}\pi_t + \delta^{y}_{s_t}y_t + \delta^{q}_{s_t}q_t, \delta^{\pi}_{s_t}, \delta^{y}_{s_t} \in [0,5], \delta^{q}_{s_t} \in [-1,1]$
TR R	$i_t = \delta^{\pi}_{s_t}\pi_t + \delta^{y}_{s_t}y_t, \delta^{\pi}_{s_t}, \delta^{y}_{s_t} \in [0,5]$
TR+q RI	$i_t = \delta^{\pi}\pi_t + \delta^{y}y_t + \delta^{q}q_t, \delta^{\pi}, \delta^{y} \in [0,5], \delta^{q} \in [-1,1]$
TR RI	$i_t = \delta^{\pi}\pi_t + \delta^{y}y_t, \delta^{\pi}, \delta^{y} \in [0,5]$
TR+q I	$i_t = \delta^{\pi}\pi_t + \delta^{y}y_t + \delta^{q}q_t$
TR I	$i_t = \delta^{\pi}\pi_t + \delta^{y}y_t$
IFT+q	$i_t = \delta^{e}_{s_t}E_t\pi_{t+1} + \delta^{q}_{s_t}q_t$
IFT	$i_t = \delta^{e}_{s_t}E_t\pi_{t+1}$
IFT+q I	$i_t = \delta^{e}E_t\pi_{t+1} + \delta^{q}q_t$
IFT I	$i_t = \delta^{e}E_t\pi_{t+1}$

policy rules. The inclusion of time-invariant policy rules, where the switching nature of the exchange rate misalignments is not taken into consideration, is based on the argument that they could be a good option if the policymaker cannot observe the regime (Zampolli, 2006).

The values of the parameters in policy rules are chosen so as to minimise the following loss function (also used by, e.g., Rudebusch and Svensson, 1999):

$$Loss\ Function = V(\pi_t) + V(y_t) + 0.5V(i_t - i_{t-1})$$

where $V(x)$ represents the unconditional variance of variable x.

The optimal policy rule aims at minimising a weighted sum of the unconditional variances of output, inflation and interest rate.

3 Monetary policy under exchange rate uncertainty

As mentioned in the introductory section, evidence from simulated open-economy models with exchange rate uncertainty on whether monetary policy should react to the exchange rate is mixed. Leitemo and Soderstrom (2005) analyse the impact of exchange rate uncertainty for the conduct of monetary policy and conclude that policy rules without an exchange rate term, namely a Taylor rule, are optimal at stabilising a small open economy. However, Wollmershauser (2006) and Zampolli (2006) concluded that reacting to the exchange rate improves macroeconomic stability in models that incorporate exchange rate uncertainty. Wollmershauser (2006) concluded that monetary policy rules that include an exchange rate term are more robust to a high degree of exchange rate uncertainty, concerning the relationship between the nominal exchange rate and the nominal interest rate or other macroeconomic variables.

Zampolli (2006) used a simple backward looking model of the type defined in Ball (1999) with a regime-switching exchange rate, which aimed at capturing the complex behaviour of financial markets. In his analysis policymakers are uncertain about the nature of the shock that hits the real exchange. Policymakers therefore have to assign given probabilities to a transitory shock and to a very persistent or bubble shock. Zampolli (2006) then investigates how that type of uncertainty affects the optimal reaction of policy instruments and how that reaction depends on the transition probabilities that characterise the shock. He concludes that an invariant Taylor rule performs significantly worse than the optimal policy when the probability of continuing in the bubble regime is high and the probability of continuing in the other regime is low. Zampolli also concludes that a time invariant Taylor rule that includes an exchange rate term performs noticeably better than a time invariant Taylor rule without an exchange rate term.

A drawback of Zampolli's analysis is the absence of forward-looking behaviour prevents the model from capturing the essential role of expectations in monetary policy and in asset markets. Therefore, we extend Zampolli's analysis by considering an open-economy forward-looking model of the type described above.

Following Zampolli's strategy, we started by computing, as a benchmark, the value of the optimal loss when policymakers face no uncertainty about the nature of the shock on the real exchange rate, that is, they know it to be white noise. In that case, we found the value of the loss to be 14.352 (see Table 2).

Table 2: Loss with 1 Regime

	$\rho_1^q = \rho_2^q$		
Policy	0.0	0.5	0.9
Optimal	14.352	14.536	17.123
TR+q	19.117	19.533	24.750
TR	19.142	19.543	24.785
TR+q R	19.233	19.648	24.775
TR R	19.253	19.657	24.877
TRo	20.721	21.098	27.933
IFT+q	28.423	28.509	32.227
IFT	30.534	30.770	37.636

Then we simulated the model and computed the optimal policy for different values of the transition probabilities and for different values of the autoregressive coefficient on the real exchange rate shock. We assumed the shock on the real exchange rate to be mildly persistent, very persistent or to be of the bubble type. The values for the transition probabilities and the autoregressive coefficients and the corresponding value of the central bank's loss are presented in Tables 2 and 3.

The results in Table 3 show that introducing uncertainty in the behaviour of the non-fundamental shock that affects the real exchange rate increases, as expected, the welfare loss. The welfare loss increase is higher when the persistence of the non-fundamental shock is higher. It also increases with the probability of being in the regime where the non-fundamental shock to the real exchange rate is persistent, i.e., the loss increases with p_{11} and decreases with p_{22}. The effect on welfare is nonlinear: the effect is magnified as persistence increases towards (and beyond) unity. In fact, the values in Table 3 that stand out are those associated with high

Table 3: Loss: Optimal policy and the original Taylor rule

		Optimal			TRo		
			p_{22}			p_{22}	
ρ_1^q	p_{11}	0.25	0.50	0.75	0.25	0.50	0.75
0.5	0.25	14.405	14.394	14.379	20.831	20.808	20.776
0.5	0.50	14.426	14.414	14.394	20.873	20.848	20.810
0.5	0.75	14.464	14.451	14.427	20.950	20.930	20.892
0.9	0.25	14.500	14.470	14.426	21.038	20.968	20.876
0.9	0.50	14.633	14.586	14.509	21.301	21.209	21.067
0.9	0.75	15.041	14.965	14.813	22.169	22.080	21.885
1.1	0.25	14.584	14.538	14.468	21.266	21.127	20.966
1.1	0.50	14.963	14.863	14.693	22.103	21.791	21.428
1.1	0.75	19.155	18.627	17.563	31.541	27.840	25.581

persistence ($\rho_1^q = 1.1$) and high duration of the "bubble" period ($p_{11} = 0.75$); the loss increases between 22% and 34% compared to the case with white noise deviations and without regime-switching in the exchange rate.

The Taylor and inflation-based forecast rules, described above and summarised in Table 1, perform much worse than the optimal policy. The difference exceeds 30% of the optimal loss. The optimised Taylor rule is always better than the corresponding inflation-based forecast rule, by a margin of at least 20%. The original Taylor rule is worse than an optimised Taylor rule by at least 7%. But it is usually better than an inflation-based forecast rule, except in our worst possible scenario: $\rho_1^q = 1.1, p_{11} = 0.75, p_{22} = 0.25$.

Optimised Taylor rules have coefficients that vary widely with the parameters of the model and tend to be extremely large, sometimes even exceeding 2000. However, restricting the coefficients not to exceed 5, so as not to be too far from the original coefficients and from the coefficients employed in other studies, does not affect the loss very much: the difference is below 0.7%. The optimised parameters of IFT rules are always between 1.5 and 2.5 for $E_t\pi_{t+1}$, and between 0 and 0.3 for q_t.

Reacting to the exchange rate does not yield much dividends in the case of the Taylor rule: the difference is less than 0.8%. The optimised Taylor rule without an exchange rate term seems to be robust in the context of regime-switching in the exchange rate. These results appear to reinforce the findings of Leitemo and Soderstrom (2005).

However, in the case of an inflation-based forecast rule, the benefit from reacting to the exchange rate is never below 7% and may even go beyond 20%. Similarly to the optimal policy case, the more significant benefits from reacting to the exchange rate arise when the shock and the bubble-regime are very persistent: $\rho_1^q = 1.1, p_{11} = 0.75$ (see Table 6). Welfare gains from the reaction to the exchange rate result from a more stable output, inflation and policy instrument.

In order to evaluate the benefits from switching the policy rule coefficients according to the exchange rate regime we compare the performance of time invariant rules to regime-switching rules. In the case of the inflation-based forecast rule, an optimised time invariant rule leads to an increase in welfare loss inferior to 0.2%, i.e., taking into account the switching nature of the economy does not bring significant benefits, both when the policy rule includes an exchange rate term and it does not.

In the case of the Taylor rule, use of an optimised time invariant rule leads to an increase in welfare loss below 0.5%, in general. However, the difference goes up to 6% in our worst scenario ($\rho_1^q = 1.1, p_{11} = 0.75, p_{22} = 0.25$). It appears that taking into account the switching nature of the economy is important only in extreme cases. The same applies to the case where a restricted, optimised, time invariant Taylor rule is used, though the difference in welfare loss is slightly bigger. These results seem to corroborate the findings of Zampolli (2006) in the context of a backward-looking model.

4 Conclusion

Evidence from simulated open-economy models with exchange rate uncertainty on whether monetary policy should react to the exchange rate is mixed. Zampolli (2006) concluded that policy rules that include an exchange rate term are more robust when the policymaker faces exchange rate uncertainty. In his analysis Zampolli used a backward-looking model. We extend his analysis by considering a forward-looking model. The computation of the optimal loss uses Svensson and William (2005) solution method and numerical optimisation of simple rules uses Farmer et al (2006) method.

As in Zampolli (2006), we consider that the exchange rate may be in one of two states: in one regime it randomly oscillates around its equilibrium; in the other regime the deviations from equilibrium are persistent. We experiment a range of values for the transition probabilities and for the autoregressive coefficient. We assume that the transition probabilities are exogenous and observed by policymakers.

We conclude that introducing uncertainty in the behaviour of the non-fundamental shock, that affects the real exchange rate, increases the welfare loss, compared to the case with white noise deviations and no regime-switching in the exchange rate. The welfare loss increases with the persistence of the non-fundamental shock and with the probability of being in the regime where the misalignment in the exchange rate is persistent.

Simple policy rules perform much worse than the optimal policy. Optimised Taylor rules are always better than the corresponding inflation-based forecast rule. As found in Leitemo and Soderstrom (2005), the optimised Taylor rule without an exchange rate term seems to be robust in the context of exchange rate uncertainty. However, significant welfare gains arise from adding an exchange rate term to the inflation-based forecast rule. These gains are over 20% when the shock and the bubble-regime are very persistent.

Finally, we evaluate the benefits from taking into account the switching nature of the economy by comparing the performance of time invariant rules to regime switching rules. We conclude that taking into account the regime-switching in the exchange rate, both for the Taylor rule and for the inflation-based forecast rule, does not bring significant benefits. However, when the shock and the bubble-regime are very persistent an optimised time invariant Taylor rule can increase the welfare loss significantly.

Our results for a forward-looking model seem to corroborate the findings, in the context of a backward-looking model, of Zampolli (2006) that taking into account the switching nature of the economy is important only in extreme cases.

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Tables

Table 4: Loss: optimised Taylor rule

		TR+q			TR		
			p_{22}			p_{22}	
ρ_1^q	p_{11}	0.25	0.50	0.75	0.25	0.50	0.75
0.5	0.25	19.239	19.214	19.179	19.260	19.236	19.203
0.5	0.50	19.286	19.261	19.220	19.306	19.282	19.242
0.5	0.75	19.373	19.354	19.314	19.390	19.371	19.334
0.9	0.25	19.481	19.405	19.302	19.501	19.428	19.327
0.9	0.50	19.770	19.677	19.525	19.790	19.698	19.547
0.9	0.75	20.621	20.561	20.406	20.632	20.574	20.420
1.1	0.25	19.753	19.599	19.414	19.771	19.623	19.439
1.1	0.50	20.604	20.320	19.936	20.629	20.325	19.958
1.1	0.75	27.304	25.284	23.828	27.309	25.313	23.864

		TR+q R			TR R		
			p_{22}			p_{22}	
ρ_1^q	p_{11}	0.25	0.50	0.75	0.25	0.50	0.75
0.5	0.25	19.357	19.331	19.295	19.375	19.350	19.315
0.5	0.50	19.404	19.377	19.335	19.421	19.395	19.354
0.5	0.75	19.489	19.469	19.427	19.504	19.484	19.444
0.9	0.25	19.592	19.515	19.413	19.612	19.535	19.433
0.9	0.50	19.872	19.779	19.628	19.892	19.798	19.647
0.9	0.75	20.725	20.663	20.488	20.743	20.676	20.502
1.1	0.25	19.850	19.699	19.519	19.874	19.721	19.539
1.1	0.50	20.708	20.401	20.025	20.733	20.421	20.043
1.1	0.75	27.427	25.388	23.892	27.460	25.426	23.959

Table 5: Loss: invariant optimised Taylor rule

		TR+q I			TR I		
			p_{22}			p_{22}	
ρ_1^q	p_{11}	0.25	0.50	0.75	0.25	0.50	0.75
0.5	0.25	19.241	19.216	19.180	19.260	19.237	19.203
0.5	0.50	19.289	19.264	19.221	19.307	19.283	19.243
0.5	0.75	19.375	19.356	19.315	19.391	19.373	19.334
0.9	0.25	19.490	19.412	19.306	19.505	19.430	19.328
0.9	0.50	19.788	19.691	19.532	19.799	19.705	19.551
0.9	0.75	20.693	20.620	20.427	20.694	20.622	20.431
1.1	0.25	19.770	19.611	19.420	19.784	19.629	19.442
1.1	0.50	20.698	20.362	19.955	20.703	20.370	19.968
1.1	0.75	28.807	25.935	24.001	28.886	25.986	24.034

		TR+q RI			TR RI		
			p_{22}			p_{22}	
ρ_1^q	p_{11}	0.25	0.50	0.75	0.25	0.50	0.75
0.5	0.25	19.359	19.332	19.296	19.375	19.350	19.315
0.5	0.50	19.406	19.379	19.336	19.421	19.396	19.354
0.5	0.75	19.491	19.470	19.428	19.504	19.485	19.444
0.9	0.25	19.600	19.521	19.416	19.614	19.537	19.434
0.9	0.50	19.889	19.791	19.634	19.898	19.803	19.649
0.9	0.75	20.775	20.698	20.506	20.775	20.699	20.509
1.1	0.25	19.867	19.709	19.523	19.879	19.725	19.541
1.1	0.50	20.768	20.437	20.039	20.771	20.443	20.050
1.1	0.75	28.817	25.949	24.029	28.960	26.025	24.074

Table 6: Loss: inflation forecast targeting rule

		IFT+q			IFT		
			p_{22}			p_{22}	
ρ_1^q	p_{11}	0.25	0.50	0.75	0.25	0.50	0.75
0.5	0.25	28.422	28.418	28.418	30.586	30.573	30.559
0.5	0.50	28.428	28.425	28.424	30.610	30.597	30.579
0.5	0.75	28.451	28.448	28.446	30.661	30.651	30.632
0.9	0.25	28.519	28.485	28.454	30.799	30.733	30.654
0.9	0.50	28.629	28.584	28.529	31.031	30.937	30.808
0.9	0.75	29.121	29.060	28.940	31.868	31.729	31.495
1.1	0.25	28.652	28.571	28.496	31.083	30.920	30.751
1.1	0.50	29.079	28.894	28.704	31.934	31.536	31.141
1.1	0.75	34.528	32.421	30.971	41.639	37.125	34.543
		IFT+q I			IFT I		
			p_{22}			p_{22}	
ρ_1^q	p_{11}	0.25	0.50	0.75	0.25	0.50	0.75
0.5	0.25	28.422	28.418	28.418	30.586	30.573	30.559
0.5	0.50	28.428	28.425	28.424	30.610	30.597	30.579
0.5	0.75	28.451	28.448	28.446	30.661	30.651	30.632
0.9	0.25	28.520	28.486	28.454	30.799	30.733	30.655
0.9	0.50	28.629	28.584	28.530	31.031	30.937	30.808
0.9	0.75	29.123	29.063	28.942	31.868	31.729	31.496
1.1	0.25	28.654	28.572	28.496	31.084	30.921	30.752
1.1	0.50	29.079	28.894	28.705	31.936	31.537	31.141
1.1	0.75	34.576	32.472	31.008	41.712	37.162	34.562

Table 7: Variance of the exchange rate - 1

p_{11}	p_{22}	ρ_1^q	Optimal	TR+q	TR	IFT+q	IFT	TRo	TR+q R
0.25	0.25	0.5	44.263	46.725	45.423	44.597	56.045	43.301	46.255
0.25	0.5	0.5	44.143	46.657	45.31	44.421	55.79	43.156	46.172
0.25	0.75	0.5	43.983	46.565	45.163	44.182	55.446	42.964	46.047
0.5	0.25	0.5	44.522	46.924	45.652	44.92	56.547	43.6	46.465
0.5	0.5	0.5	44.374	46.815	45.506	44.712	56.239	43.418	46.344
0.5	0.75	0.5	44.152	46.663	45.295	44.393	55.774	43.148	46.17
0.75	0.25	0.5	45.058	47.287	46.129	45.547	57.529	44.212	46.895
0.75	0.5	0.5	44.895	47.178	45.957	45.324	57.2	44.003	46.752
0.75	0.75	0.5	44.594	46.936	45.649	44.911	56.594	43.627	46.481
0.25	0.25	0.9	45.23	47.623	46.431	45.99	58.254	44.652	47.263
0.25	0.5	0.9	44.866	47.298	45.986	45.399	57.351	44.088	46.864
0.25	0.75	0.9	44.404	46.874	45.49	44.7	56.285	43.444	46.398
0.5	0.25	0.9	47.182	49.355	48.319	48.264	61.863	47.023	49.066
0.5	0.5	0.9	46.503	48.512	47.359	47.14	60.157	45.869	48.171
0.5	0.75	0.9	45.513	47.507	46.227	45.734	58.003	44.467	47.086
0.75	0.25	0.9	56.875	57.495	57.402	58.652	76.441	57.776	57.587
0.75	0.5	0.9	55.237	54.9	54.356	55.389	72.107	54.417	54.67
0.75	0.75	0.9	52.23	50.999	50.341	50.873	65.956	49.79	50.693
0.25	0.25	1.1	46.12	49.013	47.834	47.745	61.068	46.474	48.744
0.25	0.5	1.1	45.532	48.056	46.754	46.417	59.018	45.131	47.691
0.25	0.75	1.1	44.791	47.196	45.789	45.143	57.031	43.884	46.735
0.5	0.25	1.1	52.247	56.288	55.391	56.39	74.257	55.592	55.981
0.5	0.5	1.1	50.603	52.048	51.07	51.619	67.225	50.584	51.811
0.5	0.75	1.1	48.166	48.768	47.689	47.606	61.13	46.413	48.436
0.75	0.25	1.1	207.04	210.14	211.03	226.71	277.5	225.5	209.36
0.75	0.5	1.1	188.17	115.68	116.73	126.74	159.42	125.11	115.34
0.75	0.75	1.1	151.52	70.244	71.464	77.02	99.965	75.595	70.084

Table 8: Variance of the exchange rate - 2

p_{11}	p_{22}	ρ_1^q	TR R	TR+q RI	TR RI	TR+q I	TR I	IFT+q I	IFT I
0.25	0.25	0.5	45.081	46.252	45.081	46.659	45.404	44.597	56.045
0.25	0.5	0.5	44.967	46.165	44.964	46.607	45.295	44.421	55.79
0.25	0.75	0.5	44.825	46.045	44.812	46.564	45.157	44.181	55.446
0.5	0.25	0.5	45.324	46.466	45.321	46.853	45.635	44.919	56.547
0.5	0.5	0.5	45.172	46.349	45.171	46.81	45.494	44.711	56.239
0.5	0.75	0.5	44.96	46.165	44.952	46.64	45.289	44.393	55.774
0.75	0.25	0.5	45.822	46.894	45.821	47.252	46.108	45.547	57.529
0.75	0.5	0.5	45.644	46.753	45.643	47.179	45.943	45.323	57.2
0.75	0.75	0.5	45.328	46.482	45.326	46.901	45.648	44.91	56.593
0.25	0.25	0.9	46.154	47.256	46.151	47.579	46.413	45.994	58.257
0.25	0.5	0.9	45.687	46.864	45.685	47.239	45.978	45.401	57.353
0.25	0.75	0.9	45.172	46.404	45.165	46.874	45.49	44.7	56.286
0.5	0.25	0.9	48.124	49.052	48.115	49.347	48.307	48.267	61.867
0.5	0.5	0.9	47.126	48.171	47.121	48.534	47.362	47.139	60.159
0.5	0.75	0.9	45.947	47.093	45.947	47.52	46.225	45.733	58.004
0.75	0.25	0.9	57.418	57.501	57.37	57.663	57.281	58.642	76.44
0.75	0.5	0.9	54.353	54.64	54.34	54.868	54.335	55.369	72.106
0.75	0.75	0.9	50.222	50.726	50.239	50.993	50.359	50.849	65.955
0.25	0.25	1.1	47.644	48.744	47.636	49.012	47.827	47.761	61.081
0.25	0.5	1.1	46.511	47.703	46.506	48.09	46.762	46.425	59.026
0.25	0.75	1.1	45.49	46.745	45.489	47.182	45.805	45.145	57.035
0.5	0.25	1.1	55.403	55.957	55.361	56.029	55.262	56.396	74.278
0.5	0.5	1.1	51.004	51.815	50.992	52.01	51.047	51.617	67.235
0.5	0.75	1.1	47.44	48.45	47.446	48.785	47.647	47.602	61.135
0.75	0.25	1.1	212.44	204.28	210.55	204.46	208.83	227.02	278.16
0.75	0.5	1.1	117.68	113.75	117.45	113.85	116.8	126.91	159.74
0.75	0.75	1.1	71.928	69.819	72.216	69.97	72.023	77.009	100.12

Table 9: Variance of π^d - 1

p_{11}	p_{22}	ρ_1^q	Optimal	TR+q	TR	IFT+q	IFT	TRo	TR+q R
0.25	0.25	0.5	7.6157	8.6583	8.7011	10.763	11.407	10.25	8.7532
0.25	0.5	0.5	7.6154	8.6525	8.6972	10.77	11.411	10.25	8.7456
0.25	0.75	0.5	7.615	8.6438	8.6899	10.785	11.417	10.25	8.7348
0.5	0.25	0.5	7.6164	8.6738	8.7145	10.75	11.402	10.251	8.7678
0.5	0.5	0.5	7.616	8.6702	8.7104	10.758	11.406	10.251	8.7603
0.5	0.75	0.5	7.6154	8.6575	8.7026	10.775	11.413	10.25	8.7476
0.75	0.25	0.5	7.6177	8.7015	8.7385	10.734	11.395	10.252	8.7945
0.75	0.5	0.5	7.6172	8.6975	8.7358	10.741	11.398	10.252	8.7903
0.75	0.75	0.5	7.6165	8.6874	8.7288	10.758	11.406	10.251	8.7758
0.25	0.25	0.9	7.6181	8.7449	8.7762	10.751	11.439	10.253	8.8325
0.25	0.5	0.9	7.6172	8.7264	8.7663	10.753	11.432	10.252	8.8108
0.25	0.75	0.9	7.6161	8.6913	8.7375	10.769	11.428	10.251	8.7765
0.5	0.25	0.9	7.6227	8.8419	8.877	10.722	11.452	10.258	8.9201
0.5	0.5	0.9	7.6211	8.8171	8.8557	10.727	11.439	10.255	8.8945
0.5	0.75	0.9	7.6187	8.7699	8.8121	10.749	11.429	10.252	8.8463
0.75	0.25	0.9	7.6407	9.0913	9.0768	10.713	11.516	10.295	9.1639
0.75	0.5	0.9	7.6375	9.0695	9.0834	10.716	11.479	10.282	9.1588
0.75	0.75	0.9	7.6315	9.064	9.0896	10.733	11.438	10.267	9.1161
0.25	0.25	1.1	7.6203	8.8497	8.8714	10.757	11.505	10.256	8.9263
0.25	0.5	1.1	7.6189	8.7975	8.8446	10.75	11.47	10.253	8.876
0.25	0.75	1.1	7.617	8.7361	8.7822	10.762	11.444	10.251	8.8153
0.5	0.25	1.1	7.6337	9.0889	9.0964	10.715	11.616	10.28	9.184
0.5	0.5	1.1	7.6301	9.0483	9.0367	10.716	11.521	10.267	9.0979
0.5	0.75	1.1	7.6245	8.9199	8.9183	10.738	11.457	10.257	8.9787
0.75	0.25	1.1	7.8494	9.8478	9.8925	10.777	12.623	11.239	10.14
0.75	0.5	1.1	7.8223	10.159	10.464	10.771	11.806	10.649	10.268
0.75	0.75	1.1	7.7696	10.095	10.071	10.766	11.45	10.379	10.135

Table 10: Variance of π^d - 2

p_{11}	p_{22}	ρ_1^q	TR R	TR+q RI	TR RI	TR+q I	TR I	IFT+q I	IFT I
0.25	0.25	0.5	8.779	8.7541	8.7789	8.6565	8.7156	10.763	11.407
0.25	0.5	0.5	8.7702	8.7463	8.7722	8.6519	8.7082	10.77	11.411
0.25	0.75	0.5	8.753	8.7352	8.7618	8.6448	8.6941	10.785	11.417
0.5	0.25	0.5	8.7915	8.769	8.7931	8.6725	8.7278	10.75	11.402
0.5	0.5	0.5	8.7858	8.7613	8.7858	8.6701	8.7204	10.758	11.406
0.5	0.75	0.5	8.7688	8.748	8.774	8.6585	8.707	10.775	11.413
0.75	0.25	0.5	8.8172	8.798	8.8168	8.6998	8.7541	10.735	11.395
0.75	0.5	0.5	8.8118	8.7894	8.8124	8.6991	8.7467	10.741	11.398
0.75	0.75	0.5	8.8	8.7777	8.8015	8.6875	8.7324	10.758	11.406
0.25	0.25	0.9	8.8604	8.8373	8.8606	8.7496	8.7996	10.752	11.439
0.25	0.5	0.9	8.8368	8.8128	8.8373	8.7275	8.7718	10.753	11.432
0.25	0.75	0.9	8.7997	8.7791	8.8043	8.6974	8.736	10.768	11.428
0.5	0.25	0.9	8.9451	8.9319	8.9487	8.8597	8.8806	10.723	11.452
0.5	0.5	0.9	8.9189	8.9008	8.9218	8.8341	8.8495	10.727	11.439
0.5	0.75	0.9	8.8727	8.8502	8.8738	8.7749	8.8147	10.749	11.429
0.75	0.25	0.9	9.1745	9.2009	9.2023	9.1538	9.1644	10.713	11.516
0.75	0.5	0.9	9.1714	9.1796	9.1838	9.1297	9.151	10.717	11.479
0.75	0.75	0.9	9.1274	9.1204	9.1295	9.0767	9.0961	10.736	11.438
0.25	0.25	1.1	8.9526	8.9346	8.9563	8.8621	8.8904	10.759	11.506
0.25	0.5	1.1	8.9037	8.8813	8.9064	8.8169	8.8358	10.751	11.471
0.25	0.75	1.1	8.8436	8.8185	8.8445	8.7405	8.7705	10.762	11.444
0.5	0.25	1.1	9.2038	9.2284	9.2389	9.1938	9.2121	10.715	11.617
0.5	0.5	1.1	9.1206	9.1174	9.1337	9.0758	9.1037	10.716	11.522
0.5	0.75	1.1	9.0036	8.9864	9.007	8.9316	8.9661	10.738	11.457
0.75	0.25	1.1	10.059	11.516	11.462	11.593	11.628	10.777	12.597
0.75	0.5	1.1	10.21	10.755	10.651	10.792	10.761	10.785	11.79
0.75	0.75	1.1	10.08	10.209	10.149	10.24	10.182	10.793	11.44

Table 11: Variance of y - 1

p_{11}	p_{22}	ρ_1^q	Optimal	TR+q	TR	IFT+q	IFT	TRo	TR+q R
0.25	0.25	0.5	7.3605	8.0302	8.0691	10.18	10.199	8.1065	8.0881
0.25	0.5	0.5	7.3589	8.0288	8.0677	10.177	10.192	8.1039	8.0897
0.25	0.75	0.5	7.3566	8.0281	8.0676	10.166	10.182	8.1002	8.0934
0.5	0.25	0.5	7.3636	8.0254	8.0651	10.188	10.212	8.1113	8.0828
0.5	0.5	0.5	7.3617	8.0216	8.0638	10.184	10.204	8.1086	8.0844
0.5	0.75	0.5	7.3587	8.0234	8.0625	10.173	10.19	8.1041	8.0876
0.75	0.25	0.5	7.3691	8.0212	8.0586	10.194	10.234	8.12	8.0745
0.75	0.5	0.5	7.3672	8.0174	8.0568	10.192	10.226	8.1178	8.0727
0.75	0.75	0.5	7.3636	8.0148	8.0547	10.183	10.21	8.1132	8.0771
0.25	0.25	0.9	7.3741	7.9906	8.0418	10.199	10.215	8.1297	8.0473
0.25	0.5	0.9	7.3696	7.988	8.0313	10.201	10.204	8.122	8.0529
0.25	0.75	0.9	7.3631	8.001	8.04	10.188	10.189	8.1115	8.0681
0.5	0.25	0.9	7.3936	7.9587	7.9913	10.213	10.27	8.1588	8.0173
0.5	0.5	0.9	7.3865	7.9583	7.9936	10.214	10.249	8.1487	8.0203
0.5	0.75	0.9	7.3751	7.9672	8.0058	10.201	10.218	8.1322	8.0363
0.75	0.25	0.9	7.4587	7.9656	7.9855	10.176	10.47	8.25	7.9895
0.75	0.5	0.9	7.4466	7.945	7.9688	10.194	10.419	8.24	7.9616
0.75	0.75	0.9	7.4229	7.8825	7.8969	10.212	10.34	8.2158	7.9542
0.25	0.25	1.1	7.3861	7.9315	7.9963	10.215	10.217	8.1556	7.9921
0.25	0.5	1.1	7.3791	7.9499	7.9847	10.217	10.206	8.1401	8.0144
0.25	0.75	1.1	7.369	7.9724	8.0135	10.201	10.191	8.1216	8.0432
0.5	0.25	1.1	7.4426	7.8971	7.9518	10.225	10.372	8.2477	7.9258
0.5	0.5	1.1	7.4271	7.8578	7.9537	10.232	10.314	8.2123	7.9338
0.5	0.75	1.1	7.402	7.9001	7.9751	10.221	10.252	8.1703	7.9744
0.75	0.25	1.1	8.1493	8.4969	8.3885	10.076	12.448	9.152	8.3701
0.75	0.5	1.1	8.0602	7.9925	7.5688	10.127	11.367	8.7933	7.9977
0.75	0.75	1.1	7.8829	7.8024	7.6659	10.237	10.753	8.5607	7.8268

Table 12: Variance of y - 2

p_{11}	p_{22}	ρ_1^q	TR R	TR+q RI	TR RI	TR+q I	TR I	IFT+q I	IFT I
0.25	0.25	0.5	8.1404	8.0872	8.1406	8.0356	8.0534	10.18	10.199
0.25	0.5	0.5	8.1452	8.0892	8.1431	8.0318	8.056	10.177	10.192
0.25	0.75	0.5	8.1572	8.093	8.1477	8.0268	8.0631	10.166	10.182
0.5	0.25	0.5	8.1358	8.0814	8.134	8.0306	8.0506	10.188	10.212
0.5	0.5	0.5	8.1369	8.0827	8.1368	8.0218	8.0527	10.184	10.204
0.5	0.75	0.5	8.1471	8.087	8.1414	8.0223	8.0576	10.173	10.191
0.75	0.25	0.5	8.1242	8.0703	8.1246	8.0244	8.0418	10.194	10.234
0.75	0.5	0.5	8.1258	8.0733	8.1251	8.0148	8.0447	10.192	10.226
0.75	0.75	0.5	8.1299	8.0744	8.1282	8.0156	8.0497	10.183	10.21
0.25	0.25	0.9	8.0933	8.0429	8.0935	7.988	8.012	10.199	10.215
0.25	0.5	0.9	8.1049	8.0505	8.1045	7.9899	8.0251	10.201	10.204
0.25	0.75	0.9	8.1264	8.0644	8.1212	7.9926	8.0414	10.188	10.189
0.5	0.25	0.9	8.0557	8.006	8.0525	7.9392	7.9884	10.213	10.27
0.5	0.5	0.9	8.0658	8.0126	8.0623	7.9358	7.9995	10.214	10.249
0.5	0.75	0.9	8.0855	8.0299	8.0837	7.9581	8.0019	10.201	10.218
0.75	0.25	0.9	7.9925	7.9484	7.9559	7.8726	7.8883	10.176	10.47
0.75	0.5	0.9	7.9709	7.9391	7.9562	7.8651	7.8811	10.192	10.419
0.75	0.75	0.9	7.9723	7.9407	7.968	7.8589	7.8864	10.207	10.34
0.25	0.25	1.1	8.0391	7.9851	8.0362	7.9198	7.9684	10.213	10.217
0.25	0.5	1.1	8.0645	8.0081	8.0617	7.9258	7.994	10.216	10.207
0.25	0.75	1.1	8.0965	8.0385	8.0953	7.967	8.0248	10.202	10.191
0.5	0.25	1.1	7.948	7.8765	7.9039	7.7906	7.824	10.225	10.373
0.5	0.5	1.1	7.9658	7.9124	7.9513	7.8274	7.8642	10.232	10.315
0.5	0.75	1.1	8.0155	7.9609	8.0099	7.879	7.9213	10.22	10.252
0.75	0.25	1.1	8.283	7.4809	7.2296	7.3573	7.0552	10.08	12.524
0.75	0.5	1.1	7.8862	7.6211	7.4576	7.5363	7.3295	10.121	11.406
0.75	0.75	1.1	7.6775	7.7331	7.5636	7.6381	7.4911	10.215	10.771

Table 13: Variance of π - 1

p_{11}	p_{22}	ρ_1^q	Optimal	TR+q	TR	IFT+q	IFT	TRo	TR+q R
0.25	0.25	0.5	6.7138	7.9531	7.94	11.577	12.904	9.6091	8.0766
0.25	0.5	0.5	6.7052	7.9432	7.9303	11.572	12.886	9.601	8.0636
0.25	0.75	0.5	6.6921	7.9282	7.9144	11.571	12.863	9.5898	8.0441
0.5	0.25	0.5	6.7317	7.9796	7.9654	11.585	12.935	9.624	8.1029
0.5	0.5	0.5	6.7215	7.9719	7.9552	11.58	12.917	9.6155	8.0894
0.5	0.75	0.5	6.7045	7.9506	7.9371	11.578	12.889	9.6017	8.0667
0.75	0.25	0.5	6.7623	8.0246	8.0112	11.608	12.994	9.6518	8.1499
0.75	0.5	0.5	6.7522	8.0185	8.0036	11.605	12.98	9.6446	8.1418
0.75	0.75	0.5	6.7318	8.0004	7.9864	11.601	12.953	9.6301	8.1168
0.25	0.25	0.9	6.7926	8.0969	8.0776	11.668	13.118	9.6792	8.2174
0.25	0.5	0.9	6.7683	8.0655	8.0475	11.635	13.048	9.6551	8.178
0.25	0.75	0.9	6.7317	8.006	7.9926	11.605	12.962	9.6232	8.1173
0.5	0.25	0.9	6.9021	8.2656	8.2539	11.772	13.349	9.7727	8.3781
0.5	0.5	0.9	6.8642	8.2207	8.2082	11.731	13.257	9.7396	8.3297
0.5	0.75	0.9	6.8	8.1364	8.1229	11.678	13.124	9.6878	8.2404
0.75	0.25	0.9	7.2299	8.7392	8.7099	12.248	14.058	10.096	8.8486
0.75	0.5	0.9	7.1693	8.7032	8.691	12.185	13.957	10.055	8.821
0.75	0.75	0.9	7.0457	8.6336	8.632	12.061	13.764	9.9698	8.7169
0.25	0.25	1.1	6.8629	8.2727	8.2454	11.787	13.381	9.7568	8.3838
0.25	0.5	1.1	6.8248	8.1852	8.176	11.712	13.225	9.709	8.2941
0.25	0.75	1.1	6.7672	8.0801	8.0654	11.643	13.056	9.6529	8.185
0.5	0.25	1.1	7.1737	8.7678	8.7268	12.192	14.167	10.058	8.8813
0.5	0.5	1.1	7.0915	8.6236	8.5709	12.018	13.817	9.9421	8.7036
0.5	0.75	1.1	6.9523	8.3859	8.3272	11.839	13.447	9.8083	8.4739
0.75	0.25	1.1	10.5	12.63	12.724	17.522	21.363	13.901	12.947
0.75	0.5	1.1	10.081	11.563	11.995	15.444	18.184	12.246	11.688
0.75	0.75	1.1	9.2335	10.628	10.701	13.98	16.307	11.264	10.682

Table 14: Variance of π - 2

p_{11}	p_{22}	ρ_1^q	TR R	TR+q RI	TR RI	TR+q I	TR I	IFT+q I	IFT I
0.25	0.25	0.5	8.0492	8.0772	8.0489	7.9471	7.957	11.577	12.904
0.25	0.5	0.5	8.0332	8.0639	8.0354	7.9395	7.9432	11.573	12.886
0.25	0.75	0.5	8.0054	8.0443	8.0154	7.9292	7.9194	11.571	12.863
0.5	0.25	0.5	8.0742	8.1039	8.076	7.974	7.9809	11.585	12.935
0.5	0.5	0.5	8.0616	8.0906	8.0615	7.9713	7.9668	11.58	12.917
0.5	0.75	0.5	8.0322	8.067	8.0381	7.9502	7.9423	11.578	12.889
0.75	0.25	0.5	8.1234	8.1537	8.1228	8.02	8.0295	11.608	12.994
0.75	0.5	0.5	8.1123	8.1405	8.1128	8.0202	8.0162	11.605	12.98
0.75	0.75	0.5	8.0884	8.1192	8.0901	7.9985	7.9901	11.601	12.953
0.25	0.25	0.9	8.1943	8.221	8.1938	8.0983	8.1014	11.669	13.118
0.25	0.5	0.9	8.1499	8.1795	8.1499	8.0627	8.0533	11.635	13.048
0.25	0.75	0.9	8.0838	8.1204	8.089	8.0127	7.9905	11.604	12.962
0.5	0.25	0.9	8.3587	8.3874	8.3609	8.2822	8.2555	11.772	13.349
0.5	0.5	0.9	8.3055	8.3351	8.3075	8.2396	8.1986	11.731	13.257
0.5	0.75	0.9	8.2147	8.2453	8.2155	8.1426	8.1253	11.678	13.124
0.75	0.25	0.9	8.8484	8.8715	8.8669	8.791	8.7856	12.25	14.058
0.75	0.5	0.9	8.8188	8.8351	8.825	8.7516	8.7512	12.187	13.957
0.75	0.75	0.9	8.7078	8.7239	8.7086	8.6478	8.6374	12.062	13.764
0.25	0.25	1.1	8.3581	8.3897	8.3606	8.2839	8.259	11.79	13.383
0.25	0.5	1.1	8.2671	8.2989	8.2692	8.2082	8.164	11.713	13.226
0.25	0.75	1.1	8.1565	8.1889	8.1572	8.0842	8.0506	11.643	13.056
0.5	0.25	1.1	8.8706	8.9129	8.8972	8.846	8.8329	12.192	14.169
0.5	0.5	1.1	8.6879	8.719	8.6972	8.6459	8.6314	12.018	13.817
0.5	0.75	1.1	8.4534	8.4839	8.4563	8.4	8.3828	11.839	13.448
0.75	0.25	1.1	12.988	13.813	13.97	13.924	14.141	17.577	21.394
0.75	0.5	1.1	11.749	11.969	12.024	12.012	12.12	15.488	18.195
0.75	0.75	1.1	10.768	10.735	10.819	10.758	10.818	14	16.311

Table 15: Variance of i - 1

p_{11}	p_{22}	ρ_1^q	Optimal	TR+q	TR	IFT+q	IFT	TRo	TR+q R
0.25	0.25	0.5	2.0443	11.144	11.115	21.285	22.959	12.154	11.374
0.25	0.5	0.5	2.0437	11.126	11.096	21.287	22.96	12.142	11.357
0.25	0.75	0.5	2.0429	11.101	11.068	21.296	22.963	12.125	11.331
0.5	0.25	0.5	2.0458	11.183	11.154	21.273	22.942	12.177	11.409
0.5	0.5	0.5	2.045	11.165	11.134	21.276	22.946	12.164	11.39
0.5	0.75	0.5	2.0438	11.132	11.1	21.287	22.953	12.143	11.359
0.75	0.25	0.5	2.0489	11.252	11.224	21.264	22.911	12.221	11.472
0.75	0.5	0.5	2.048	11.237	11.207	21.266	22.917	12.21	11.458
0.75	0.75	0.5	2.0464	11.204	11.174	21.275	22.93	12.187	11.424
0.25	0.25	0.9	2.0493	11.336	11.322	21.283	23.014	12.262	11.544
0.25	0.5	0.9	2.0475	11.281	11.255	21.273	22.995	12.224	11.491
0.25	0.75	0.9	2.0451	11.198	11.168	21.278	22.977	12.175	11.416
0.5	0.25	0.9	2.0609	11.566	11.55	21.265	22.929	12.411	11.75
0.5	0.5	0.9	2.0571	11.49	11.469	21.255	22.927	12.357	11.68
0.5	0.75	0.9	2.0516	11.364	11.338	21.261	22.934	12.275	11.564
0.75	0.25	0.9	2.1208	12.199	12.201	21.356	22.76	12.962	12.355
0.75	0.5	0.9	2.1109	12.144	12.135	21.323	22.776	12.881	12.297
0.75	0.75	0.9	2.0928	11.989	11.983	21.288	22.819	12.728	12.139
0.25	0.25	1.1	2.054	11.565	11.558	21.296	23.126	12.382	11.74
0.25	0.5	1.1	2.051	11.436	11.418	21.27	23.049	12.306	11.627
0.25	0.75	1.1	2.0472	11.286	11.26	21.268	22.995	12.22	11.492
0.5	0.25	1.1	2.0908	12.233	12.218	21.311	22.933	12.874	12.357
0.5	0.5	1.1	2.0813	11.98	11.963	21.268	22.901	12.679	12.121
0.5	0.75	1.1	2.0673	11.662	11.631	21.254	22.904	12.462	11.834
0.75	0.25	1.1	3.002	15.414	15.492	21.699	24.751	20.001	15.52
0.75	0.5	1.1	2.8913	14.716	14.923	21.585	23.597	16.741	14.822
0.75	0.75	1.1	2.6759	14.029	14.151	21.431	23.062	14.888	14.107

Table 16: Variance of i - 2

p_{11}	p_{22}	ρ_1^q	TR R	TR+q RI	TR RI	TR+q I	TR I	IFT+q I	IFT I
0.25	0.25	0.5	11.329	11.376	11.33	11.142	11.121	21.285	22.959
0.25	0.5	0.5	11.309	11.357	11.31	11.125	11.101	21.287	22.96
0.25	0.75	0.5	11.28	11.332	11.283	11.102	11.07	21.296	22.963
0.5	0.25	0.5	11.365	11.411	11.366	11.182	11.159	21.273	22.942
0.5	0.5	0.5	11.345	11.392	11.346	11.167	11.139	21.276	22.946
0.5	0.75	0.5	11.31	11.36	11.312	11.131	11.103	21.287	22.953
0.75	0.25	0.5	11.432	11.475	11.432	11.252	11.231	21.264	22.911
0.75	0.5	0.5	11.415	11.459	11.416	11.241	11.213	21.266	22.917
0.75	0.75	0.5	11.38	11.426	11.381	11.204	11.174	21.276	22.93
0.25	0.25	0.9	11.51	11.551	11.512	11.343	11.324	21.284	23.014
0.25	0.5	0.9	11.451	11.495	11.452	11.285	11.258	21.273	22.995
0.25	0.75	0.9	11.369	11.418	11.371	11.201	11.169	21.278	22.977
0.5	0.25	0.9	11.726	11.768	11.734	11.59	11.561	21.265	22.929
0.5	0.5	0.9	11.648	11.692	11.655	11.511	11.475	21.255	22.926
0.5	0.75	0.9	11.525	11.57	11.527	11.374	11.344	21.261	22.934
0.75	0.25	0.9	12.366	12.428	12.422	12.308	12.293	21.358	22.76
0.75	0.5	0.9	12.295	12.345	12.332	12.22	12.203	21.329	22.776
0.75	0.75	0.9	12.129	12.159	12.138	12.021	12	21.297	22.819
0.25	0.25	1.1	11.713	11.753	11.72	11.583	11.558	21.3	23.127
0.25	0.5	1.1	11.591	11.635	11.596	11.454	11.419	21.27	23.049
0.25	0.75	1.1	11.449	11.496	11.451	11.292	11.256	21.268	22.995
0.5	0.25	1.1	12.353	12.431	12.413	12.333	12.317	21.311	22.931
0.5	0.5	1.1	12.103	12.161	12.133	12.035	12.012	21.269	22.901
0.5	0.75	1.1	11.803	11.848	11.811	11.687	11.66	21.255	22.904
0.75	0.25	1.1	15.702	17.739	18.327	17.767	18.176	21.717	24.716
0.75	0.5	1.1	14.988	15.723	16.032	15.744	15.994	21.637	23.589
0.75	0.75	1.1	14.278	14.294	14.473	14.296	14.441	21.508	23.061

Table 17: Variance of $i - i_{-1}$ - 1

p_{11}	p_{22}	ρ_1^q	Optimal	TR+q	TR	IFT+q	IFT	TRo	TR+q R
0.25	0.25	0.5	0.66116	6.5116	6.5014	13.329	14.965	6.2309	6.3843
0.25	0.5	0.5	0.66062	6.4842	6.4768	13.338	14.99	6.2053	6.3552
0.25	0.75	0.5	0.65984	6.4461	6.4417	13.362	15.027	6.1713	6.3154
0.5	0.25	0.5	0.66241	6.5629	6.5513	13.31	14.927	6.2747	6.4355
0.5	0.5	0.5	0.66174	6.5356	6.5261	13.32	14.954	6.2489	6.4068
0.5	0.75	0.5	0.66067	6.4917	6.4856	13.346	14.999	6.2089	6.3611
0.75	0.25	0.5	0.66475	6.6541	6.6396	13.296	14.867	6.3558	6.5284
0.75	0.5	0.5	0.66404	6.6356	6.6222	13.303	14.891	6.3362	6.508
0.75	0.75	0.5	0.66265	6.5971	6.5854	13.324	14.939	6.2979	6.4668
0.25	0.25	0.9	0.66578	6.7864	6.7626	13.303	14.931	6.4592	6.6537
0.25	0.5	0.9	0.66424	6.703	6.6984	13.3	14.961	6.3812	6.5682
0.25	0.75	0.9	0.66205	6.5907	6.5879	13.324	15.008	6.2823	6.4549
0.5	0.25	0.9	0.67411	7.0906	7.0903	13.286	14.824	6.7385	6.9536
0.5	0.5	0.9	0.67134	6.9961	6.9919	13.279	14.862	6.6411	6.858
0.5	0.75	0.9	0.66697	6.8424	6.8364	13.301	14.931	6.4947	6.7027
0.75	0.25	0.9	0.70407	7.8317	7.8726	13.393	14.68	7.6457	7.7743
0.75	0.5	0.9	0.69879	7.8258	7.829	13.361	14.705	7.5694	7.7607
0.75	0.75	0.9	0.68857	7.7792	7.7822	13.334	14.783	7.3999	7.6341
0.25	0.25	1.1	0.66998	7.0974	7.0593	13.3	14.97	6.7079	6.949
0.25	0.5	1.1	0.66752	6.927	6.925	13.285	14.978	6.5562	6.7809
0.25	0.75	1.1	0.66405	6.7232	6.7204	13.305	15.009	6.3827	6.5808
0.5	0.25	1.1	0.69417	7.8791	7.9015	13.323	14.79	7.5961	7.8019
0.5	0.5	1.1	0.68788	7.6781	7.6002	13.287	14.812	7.2723	7.5267
0.5	0.75	1.1	0.67787	7.3008	7.3107	13.288	14.883	6.8979	7.1527
0.75	0.25	1.1	1.011	12.354	12.393	13.86	15.657	16.975	12.219
0.75	0.5	1.1	0.97144	11.457	11.499	13.699	15.147	13.601	11.403
0.75	0.75	1.1	0.89295	10.794	10.993	13.509	14.966	11.511	10.765

Table 18: Variance of $i - i_{-1}$ - 2

p_{11}	p_{22}	ρ_1^q	TR R	TR+q RI	TR RI	TR+q I	TR I	IFT+q I	IFT I
0.25	0.25	0.5	6.3708	6.3882	6.3715	6.5166	6.4999	13.329	14.965
0.25	0.5	0.5	6.3423	6.3583	6.3428	6.4889	6.4752	13.338	14.99
0.25	0.75	0.5	6.3039	6.3173	6.3033	6.4488	6.4415	13.362	15.027
0.5	0.25	0.5	6.4214	6.4404	6.4224	6.5691	6.5511	13.31	14.927
0.5	0.5	0.5	6.3929	6.411	6.3943	6.5409	6.5268	13.32	14.954
0.5	0.75	0.5	6.3484	6.3635	6.3485	6.4969	6.486	13.346	14.999
0.75	0.25	0.5	6.5117	6.5333	6.5136	6.662	6.6392	13.296	14.867
0.75	0.5	0.5	6.4917	6.5128	6.4935	6.6425	6.6236	13.303	14.891
0.75	0.75	0.5	6.4513	6.4695	6.4519	6.6026	6.5885	13.324	14.94
0.25	0.25	0.9	6.6485	6.6724	6.6534	6.8069	6.7836	13.305	14.931
0.25	0.5	0.9	6.5606	6.5817	6.5651	6.7191	6.7041	13.3	14.961
0.25	0.75	0.9	6.4455	6.4613	6.4473	6.6011	6.5921	13.324	15.007
0.5	0.25	0.9	6.9548	6.9917	6.9701	7.1331	7.1097	13.286	14.823
0.5	0.5	0.9	6.8525	6.8864	6.8659	7.0318	7.0144	13.279	14.861
0.5	0.75	0.9	6.6927	6.7166	6.6988	6.8635	6.8465	13.301	14.931
0.75	0.25	0.9	7.8031	7.9095	7.904	8.0589	8.0406	13.394	14.68
0.75	0.5	0.9	7.7723	7.8481	7.8358	8.0056	7.9791	13.367	14.705
0.75	0.75	0.9	7.6437	7.6831	7.6639	7.8396	7.8135	13.346	14.784
0.25	0.25	1.1	6.9529	6.9835	6.9653	7.1324	7.1137	13.305	14.968
0.25	0.5	1.1	6.7782	6.8042	6.7879	6.9536	6.9415	13.285	14.977
0.25	0.75	1.1	6.5728	6.5911	6.5775	6.738	6.7327	13.304	15.009
0.5	0.25	1.1	7.8279	7.9565	7.9388	8.1237	8.0924	13.323	14.787
0.5	0.5	1.1	7.5342	7.6103	7.5886	7.7771	7.7492	13.288	14.81
0.5	0.75	1.1	7.149	7.1889	7.1676	7.3515	7.3277	13.29	14.882
0.75	0.25	1.1	12.378	15.046	15.521	15.052	15.379	13.836	15.588
0.75	0.5	1.1	11.581	12.717	13.086	12.774	13.072	13.727	15.12
0.75	0.75	1.1	11.026	11.121	11.382	11.21	11.45	13.586	14.961