On the Competitive Effect of Informative Advertising

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Abstract

This paper challenges the robustness of the pro-competitive effect of informative advertising. In their seminal work Grossman and Shapiro (1984) find that informative advertising results in lower prices, and that firms may benefit from advertising restrictions. A crucial assumption in their model is that demand in the monopoly segment is perfectly price inelastic. Replicating their model in a Hotelling duopoly version, we show that results are in fact reversed if we allow for price elastic monopoly demand. The reason is that the marginal consumer informed about only one product is more price responsive than the marginal consumer informed about both products. We then use general demand (and advertising cost) functions and derive exact conditions for when informative advertising is pro-competitive.

Keywords: Informative Advertising; Price Competition; Product differentiation

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1 Introduction

Informative advertising is, as opposed to persuasive advertising, generally perceived to promote competition (Bagwell, 2007). When a firm advertises, consumers receive (at low costs) information about products, prices, etc. This information is claimed to make the firm's demand curve more elastic and competition more intense, resulting in lower prices and profits.[1] In this paper, we challenge the robustness of the pro-competitive effect of informative advertising.

Butters (1977) offers a first formal analysis of informative advertising in a multi-firm setting. Firms produce a homogeneous product (at constant unit costs). Consumers learn of a firm's existence and price only by receiving an ad from that firm, and ads are distributed randomly across consumers at some costs. There are three kinds of consumers: (i) *uniformed* consumers that receive no ads, and therefore do not buy any of the products; (ii) *captive* consumers that have received ads from only one firm and buy this product provided that the price is below their reservation price; and (iii) *selective* consumers that receive ads from more than one firm, and buys the product with the lowest price.

Grossman and Shapiro (1984) extend the work by Butters (1977) to include horizontally differentiated products using a Salop-type model. In this setting, advertising informs not just about existence and price, but also the advertising firm's (product's) *location*. Selective consumers trade-off price differences against travelling costs, and buys the product that yields the higher net utility. A striking result from their analysis is that informative advertising triggers competition resulting in *lower* prices. In the same line, Grossman and Shapiro (1984) show that firms may benefit from advertising restrictions. More costly advertising lowers advertising and reduces profits, for given prices. However, less advertising *softens* competition, resulting in

[1] It is also argued that informative advertising can faciliate entry, as it provides a means through which a new entrant can inform potential buyers (Bagwell, 2007).

higher prices. The net effect on profits depends on the strength of the direct cost effect relative to the strategic price effect. Obviously, a complete ban (infinite advertising costs) on advertising would be harmful to the firms.

A crucial assumption in Grossman and Shapiro (1984), and also Butters (1977), is that the demand from captive consumers (who only know about one of the products) is *perfectly price inelastic*. The reservation price (or gross utility) is assumed to be sufficiently high, such that *all* captive consumers buy (one unit of) the product they are informed about irrespective of the price. Consequently, only demand from selective consumers (informed about more than one product) is elastic with respect to prices. Thus, advertising will *by assumption* lead to lower demand elasticity.

We find this assumption quite restrictive. In the current paper, we therefore revisit the Grossman and Shapiro (1984) model by allowing for demand from captive consumers to be price elastic. In the first part, we replicate their model by using the familiar Hotelling version (see, Tirole, 1988: 292-4). In the second part, we generalise this model by using general demand (and advertising cost) functions. In both parts, we first derive the price equilibrium for given levels of information (advertising). Afterwards, we endogenise the degree of information by allowing for this to be a choice variable for the firms, as in the informative advertising models, and derive the symmetric price-advertising equilibrium.

In the Hotelling setting, we show that the pro-competitive results derived in GS is in fact reversed once we allow for demand in the monopoly segment to respond to prices. The reason is that the marginal consumer trading off whether or not to buy the advertised product is more price responsive than the marginal consumer in the competitive segment deciding to buy either product. Since products are differentiated, the transport costs for consumers in the monopoly segment is higher than in the competitive segment (on average), which explain the higher price responsiveness in the monopoly segment. However, if we impose the assumption that consumers informed about only one product, buys this with certainty, then informative advertising becomes pro-competitive again.

In the generalised version, we derive exact conditions for the competitive effects of informative advertising. Here, we show that informative advertising is pro-competitive if demand is more price elastic in the competitive segment than in the monopoly segment, provided that prices are strategic complements.

There are many papers on informative advertising in oligopoly markets (see Bagwell, 2007, for an overview).[2] However, the competitive effect of informative advertising has not been subject to closer investigation. The main purpose of this paper is therefore to shed more light on this issue.

The rest of the paper is organised as follows. In section 2 we present the Hotelling duopoly version of the GS model. In section 3 we apply general demand (and advertising cost) functions in the duopoly framework. In section 4 we conclude the paper with some remarks.

2 A Hotelling Duopoly Model

We start by replicating the duopoly version of Grossman and Shapiro (1984), henceforth GS, as presented in Tirole (1988: 292-4).[3] Consider a market with two firms, indexed by $i = 1, 2$, offering one product each at price p_i. The firms (or products) are located at either end of the unit interval $S = [0, 1]$, where $z_1 = 0$ and $z_2 = 1$ are the locations of firm 1 and 2, respectively.

In this market there is a uniform distribution of consumers on the interval S with mass 1. Each consumer demands one unit of either product or no product at all. The utility to an arbitrary consumer $x \in S$ of consuming

[2] The paper is also related to the literature on market transparency (see e.g., Schultz, 2000).

[3] The duopoly version has been used in numerous studies; e.g., Boyer and Moreaux (1999), Ishigaki (2000), Brekke and Kuhn (2006).

product i is given by

$$u_i = v - p_i - t\left|x - z_i\right|, \tag{1}$$

where v is the gross consumption benefit (or reservation price), and t is the travelling cost per unit distance between the consumer's location x and the location of product (or firm) i given by z_i.

Consumers are ex ante uninformed about the products available in the market. To generate demand, each firm must advertise its product to the consumers. We let $a_i \in (0, 1)$ be the advertising level of product i. Advertising is assumed to contain true information about existence, characteristics (location) and price of the advertised product.

Demand of firm i, with potential size a_i, can be decomposed into two parts: (i) a fraction $1 - a_j$ of consumers who are informed only about product i; and (ii) a fraction a_j of consumers who are informed about both products. The residual fraction $(1 - a_i)(1 - a_j)$ of consumers remains uninformed and does not demand either of the products. We refer to the first segment as the *monopoly* segment (of firm i), and the second segment as the *competitive* segment (for both firms).

Consumers informed about both products trade-off relative prices and distances, and choose the product that provides the higher net utility. The consumer that is exactly indifferent between product 1 and 2, i.e., $u_1(\widehat{x}) = u_2(\widehat{x})$, is located at

$$\widehat{x} = \begin{cases} 1 & if \quad p_1 \leq p_2 - t \\ \frac{1}{2} - \frac{p_1 - p_2}{2t} & if \quad p_1 \in (p_2 - t, p_2 + t) \\ 0 & if \quad p_1 \geq p_2 + t \end{cases}. \tag{2}$$

All (fully informed) consumers to the left of $\widehat{x}$ demand product 1, while the residual fraction demand product 2.

We assume existence of a competitive segment, which requires the following two conditions to be fulfilled (in equilibrium):

Assumption 1: $\widehat{x} \in (0,1) \quad \Leftrightarrow \quad t > |p_1 - p_2|$,

Assumption 2: $u_i(\widehat{x}) \geq 0 \quad \Leftrightarrow \quad v - \frac{t}{2} \geq \frac{p_1+p_2}{2}$.

While the first assumption requires that the transport cost (t) cannot be too high relative to the price difference, the second one requires that the net benefit of consuming the product must be sufficiently high relative to average price levels.

Consumers that are informed only about product i, demand this product provided that consumption yields non-negative utility. The consumer that is exactly indifferent between buying or not buying product i, i.e., $u_i(\widetilde{x}_i) = 0$ is located at:

$$\widetilde{x}_i = \begin{cases} 1 - z_i & if \quad v - t \geq p_i \\ \left|\frac{v-p_i}{t} - z_i\right| & if \quad v - t < p_i \\ z_i - 0 & if \quad v \leq p_i \end{cases} . \tag{3}$$

Note that Assumption 2 above implies that $v > p_i$, so that each firm faces a positive demand from the monopoly segment, i.e., $\widetilde{x}_2 \leq \widehat{x} \leq \widetilde{x}_1$.

In contrast to GS (and Tirole, 1988), we do not impose the restriction that all consumers in the monopoly segments demand the product they know about with certainty, i.e., $v \geq t + p_i$, such that $\widetilde{x}_1 = 1$ and $\widetilde{x}_2 = 0$. In other words, we do not restrict attention to the special case where demand from consumers informed about only one product is *perfectly price inelastic*. In the following, we allow for demand in the monopoly segment to respond to prices by also considering the parameter range $v - t < p_i$, though constrained by Assumption 1 and 2.

The demand for product 1 and 2 can now be written as:

$$D_1 = \int_0^{\widetilde{x}_1} a_1 (1 - a_2)\, sds + \int_0^{\widehat{x}} a_1 a_2 sds, \tag{4}$$

$$D_2 = \int_{\widetilde{x}_2}^1 a_2 (1 - a_1)\, sds + \int_{\widehat{x}}^1 a_1 a_2 sds. \tag{5}$$

To gain better understanding of the mechanisms of the model, it is useful to look at the demand properties. By differentiation (), we obtain (firm 2's demand has the same properties):

$$\frac{\partial D_1}{\partial a_1} = (1-a_2)\,\widetilde{x}_1 + a_2\widehat{x} > 0, \;\; \frac{\partial D_1}{\partial a_2} = -a_1\left(\widetilde{x}_1 - \widehat{x}\right) < 0$$

$$\frac{\partial D_1}{\partial p_1} = a_1\left(1-a_2\right)\frac{\partial \widetilde{x}_1}{\partial p_1} + a_1 a_2 \frac{\partial \widehat{x}}{\partial p_1} < 0, \;\; \frac{\partial D_1}{\partial p_2} = a_1 a_2 \frac{\partial \widehat{x}}{\partial p_2} > 0,$$

where

$$\frac{\partial \widetilde{x}_1}{\partial p_1} = \begin{cases} 0 & if \quad v-t \geq p_1 \\ -\frac{1}{t} & if \quad v-t < p_1 \end{cases},$$

$$\frac{\partial \widehat{x}}{\partial p_1} = -\frac{1}{2t} \quad \text{and} \quad \frac{\partial \widehat{x}}{\partial p_2} = \frac{1}{2t}.$$

We see that a higher price on own product reduces own demand, with the reduction coming from the competitive segment and potentially the monopoly segment, depending on whether or not partially informed consumers are price responsive or not. A higher price on rival product increases own the demand, with the increase coming solely from the competitive segment. Note also that the price responsiveness is either higher or lower in the monopoly segment than in the competitive segment; i.e., $0 > (-1/2t) > (-1/t)$.

Moreover, higher own advertising increases own demand, with the effect coming from both the monopoly and the competitive segment: (i) some uninformed consumers become informed about product 1 (market expansion); and (ii) some consumers informed about product 2 become fully informed *and* demand product 1 (business-stealing). Finally, more consumers informed about rival product reduces demand for product 1. The effect comes solely from the competitive segment: consumers partially informed about product 1 become fully informed and (some of them) demand product 2 (business stealing).

Following GS and Tirole (1988), we assume that the firms face constant marginal production costs ($c = 0$), while the cost of reaching a_i consumers with at least one ad is given by an increasing and strictly convex advertising cost function $C(a_i) = ka_i^2/2$.[4] We can now specify firm i's profit function as follows:

$$\pi_i = p_i D_i - \frac{k}{2} a_i^2. \qquad (6)$$

2.1 Price equilibrium with exogenous information

Let us start by assuming that the degree of product information among consumers is exogenous. Each firm i sets the price that maximises (gross) profits given by () simultaneously and independently, resulting in the following set of first-order conditions:

$$\frac{\partial \pi_i}{\partial p_i} = \begin{cases} a_i(1-a_j) + a_i a_j \left(\frac{1}{2} - \frac{2p_i - p_j}{2t}\right) = 0 & if \quad v \geq t + p_i \\ a_i(1-a_j)\left(\frac{v-2p_i}{t}\right) + a_i a_j \left(\frac{1}{2} - \frac{2p_i - p_j}{2t}\right) = 0 & if \quad v < t + p_i \end{cases}, \qquad (7)$$

depending on whether or not the demand in the monopoly segment is price elastic or not. The first equation refers to the case with price inelastic case, wheras the latter equation represent the price elastic case. Below we derive and characterise the price equilibrium for both cases separately.

2.1.1 Price inelastic monopoly demand

Assuming that $v - t \geq p_i$, and solving the corresponding set of first-order conditions, results in the following price equilibrium:

$$p_i^A = t\left(\frac{4a_i + 2a_j}{3a_i a_j} - 1\right), \quad i, j = 1, 2 \text{ and } i \neq j. \qquad (8)$$

Using () we get the relative prices and market shares in the competitive segment:

$$p_1^A - p_2^A = \frac{2t}{3}\frac{a_1 - a_2}{a_1 a_2} \quad \text{and} \quad \widehat{x} = \frac{1}{2} - \frac{a_1 - a_2}{3a_1 a_2}.$$

[4] For details about the advertising technology, see Grossman and Shapiro (1984).

Thus, () constitutes an equilibrium if:

1. $\widehat{x}^A \in (0,1) \Leftrightarrow t > \left|p_i^A - p_j^A\right| \Leftrightarrow \left|\frac{a_i - a_j}{a_i a_j}\right| < \frac{3}{2}$

2. $\widetilde{x}_i^A = 1 \quad \Leftrightarrow \quad v \geq t\left(\frac{4a_i + 2a_j}{3a_i a_j}\right).$

Note that the latter restriction implies that $u\left(\widehat{x}\left(p_1^A, p_2^A\right)\right) > 0$.

From () we see that if $a_i = a_j = 1$, then $p_i^A = t$, which is the standard price equilibrium under full consumer information: the equilibrium price is above marginal costs ($c = 0$), and is increasing in the degree of product differentiation (measured by t).

The impact of information on prices is obtained by taking the partial derivatives.

$$\frac{\partial p_i^A}{\partial a_i} = -\frac{2t}{3a_i^2} < 0, \; \frac{\partial p_i^A}{\partial a_j} = -\frac{4t}{3a_j^2} < 0$$

Thus, equilibrium prices are decreasing in the degree of information of own and rival products – a result that is consistent with the pro-competitive effect of informative advertising identified by GS. Here, we show that the effect on prices holds for any level of product information of both own and rival products (within the boundaries defined above).

Considering the price difference, we see that in fact the firm with higher advertising levels has the higher prices, i.e., $p_i^A > p_j^A$ if $a_i > a_j$. This might seem counterintuitive considering the negative impact of advertising on prices. However, the firm with more informed consumers has a larger monopoly segment with price inelastic consumers. Since the optimal price is balancing marginal revenues from the two segments, the firm with a larger monopoly segment would have a higher price. This is also the reason why the firm with more advertising captures less consumers in the competitive segment.

Inserting () into (), we obtain the equilibrium demand

$$D_i^A = \frac{4a_i + 2a_j - 3a_i a_j}{6}, \tag{9}$$

Observe that if $a_i = a_j = 1$, then each firm receives half of the market, i.e., $D_i^A = 1/2$. Comparing demand levels, it is easily verified that $D_i^A > D_j^A$ if $a_i > a_j$. Thus, the firm with more informed consumers has the larger demand. This is not trivial because the firm with more informed consumers has lower market share in the competitive segment due to higher price. However, the monopoly segment is larger.

The impact of information on demand is found by taking the partial derivatives:

$$\frac{\partial D_i^A}{\partial a_i} = \frac{4 - 3a_j}{6} > 0 \quad \text{and} \quad \frac{\partial D_i^A}{\partial a_j} = \frac{2 - 3a_i}{6}.$$

An increase in a_i has three effects on the demand for product i: (i) it expands demand by informing some uninformed consumers; (ii) it steals consumers that are better off with product i from the rival firm; and (iii) it changes relative prices and thus the demand from fraction of fully informed consumers (competitive segment). While the latter effect is negative on demand, as shown above, the two first effects are positive, and dominates the change in relative prices.

The effect of a_j on demand for product i is two-fold: (i) it steals consumers from firm i that are better off with product j; and (ii) it changes relative prices and thus demand in the competitive segment. The first effect is negative, while the second is positive, on firm i's demand. For low levels of information about product i, i.e., $a_i < 2/3$, the strategic price effect dominates the business-stealing effect, and vice versa for $a_i > 2/3$.

Defining gross profit as $V_i := p_i D_i$, and inserting () and () into (), we obtain:

$$V_i^A = t\frac{(4a_i + 2a_j - 3a_i a_j)^2}{18 a_i a_j}. \tag{10}$$

We see that if $a_i = a_j = 1$, then $V_i^A = t/2$, which is the standard outcome under full information. Comparing profit levels, we get

$$V_i^A - V_j^A = \frac{2t\,(a_i + a_j - a_i a_j)\,(a_i - a_j)}{3a_i a_j} > (<)\,0 \text{ iff } a_i > (<)\,a_j.$$

The impact of information on gross profit is found by differentiation of ():

$$\frac{\partial V_i^A}{\partial a_i} = t\frac{(4a_i - 2a_j - 3a_i a_j)\,(4a_i + 2a_j - 3a_i a_j)}{18a_i^2 a_j} \gtrless 0, \tag{11}$$

$$\frac{\partial V_i^A}{\partial a_j} = t\frac{(-4a_i + 2a_j - 3a_i a_j)\,(4a_i + 2a_j - 3a_i a_j)}{18a_i a_j^2} < 0. \tag{12}$$

Thus, more information about own product (a_i) would decrease prices, but increase demand, with the net effect on gross profits being indeterminate. However, more information about rival product (a_j) would depress prices and demand (unless a_i is high). The net effect on gross profits is, however, always negative.

We can sum up the results in the following proposition.

Proposition 1 *Assume a Hotelling model with price inelastic monopoly segment demand ($\widetilde{x}_i = 1$), then in equilibrium:*
(i) $\frac{\partial p_i^A}{\partial a_i} < 0$, $\frac{\partial p_i^A}{\partial a_j} < 0$, $\frac{\partial D_i^A}{\partial a_i} > 0$, $\frac{\partial D_i^A}{\partial a_j} \gtrless 0$, $\frac{\partial V_i^A}{\partial a_i} \gtrless 0$, $\frac{\partial V_i^A}{\partial a_j} < 0$.
(ii) $p_i^A > p_j^A$, $D_i^A > D_i^A$ *and* $V_i^A > V_j^A$ *if* $a_i > a_j$, *and vice versa.*

2.1.2 Price elastic monopoly demand

Assuming that $v - t < p_i$ and solving the corresponding set of first-order conditions, results in the following price equilibrium:

$$p_i^B = \frac{v\,(8 - 4a_i - 6a_j + 2a_i a_j) + ta_j\,(4 - a_i)}{16 - 8a_i - 8a_j + 3a_i a_j}, \quad i, j = 1, 2;\ i \neq j. \tag{13}$$

Inserting () into () and (), we get the following demand in the monopoly and competitive segments

$$\widetilde{x}_i^B = \frac{v\left(8 - 4a_i - 2a_j + a_i a_j\right) - t a_j \left(4 - a_i\right)}{t\left(16 - 8a_i - 8a_j + 3a_i a_j\right)},$$

$$\widehat{x}^B = \frac{1}{2} - \left(\frac{2t - v}{t}\right)\left(\frac{a_1 - a_2}{16 - 8a_1 - 8a_2 + 3a_1 a_2}\right).$$

The equilibrium prices given by () constitute an equilibrium if the following three assumptions are fulfilled:

1. $\widetilde{x}_i^B = \frac{v - p_i^B}{t} < 1 \quad \Leftrightarrow \quad v < 2t,$
2. $\widehat{x}^B \in (0,1) \quad \Leftrightarrow \; t > \left|\frac{2(2t-v)(a_i - a_j)}{16 - 8a_i - 8a_j + 3a_i a_j}\right|,$
3. $u\left(\widehat{x}^B\right) \geq 0 \quad \Leftrightarrow \quad v > \frac{t}{2}\frac{(4-a_i)(4-a_j)}{8 - 3a_i - 3a_j + a_i a_j} \in \left(t, \frac{3t}{2}\right).$

From () we see that if $a_i = a_j = 1$, then $p_i^B = p_j^B = t$, and if $a_i = a_j = 0$, then $p_i^B = p_j^B = v/2$. From equilibrium condition 1 ($v < 2t$) it follows that $p_i^B(1,1) > p_i^B(0,0)$. Furthermore, differentiating () with respect to own and rival product information yields

$$\frac{\partial p_i^B}{\partial a_i} = \frac{2a_j\left(4 - a_j\right)\left(2t - v\right)}{\left(16 - 8a_i - 8a_j + 3a_i a_j\right)^2} > 0, \tag{14}$$

$$\frac{\partial p_i^B}{\partial a_j} = \frac{4\left(4 - a_i\right)\left(2 - a_i\right)\left(2t - v\right)}{\left(16 - 8a_i - 8a_j + 3a_i a_j\right)^2} > 0. \tag{15}$$

In contrast to the case with price inelastic monopoly demand, prices now increase as more consumers become informed about own and/or rival product. Notably, this result is *not* consistent with the pro-competitive result in GS. To understand the result, consider the price elasticities of the two demand segments

$$\left|\varepsilon_{\widetilde{x}_i}\right| > \left|\varepsilon_{\widehat{x}_i}\right| \Longleftrightarrow \frac{p_i}{v - p_i} > \frac{p_i}{t - p_i + p_j} \Longleftrightarrow v - t - p_j < 0.$$

Thus, the monopoly segment is more price elastic than the competitive segment if $v < t + p_i$, which is always true for $\widetilde{x}_i < 1$. The reason is that the marginal consumer informed about only one product is in fact more price responsive than the marginal consumer that is fully informed due to being located further away on average. A marginal increase in price reduces demand in the monopoly segment with $-1/t$, while in the competitive segment the effect is only $-1/2t$.

Comparing the prices, we get

$$p_i^B - p_j^B = \frac{2(2t - v)(a_j - a_i)}{16 - 8a_i - 8a_j + 3a_i a_j} > 0 \text{ if } a_i < a_j.$$

Thus, in contrast to the case with price inelastic monopoly demand, we now find that the price of product i is higher if *less* consumers are informed about this product relative to product j. The reason is due to demand being more more price elastic in the monopoly segment than in the competitive segment, as shown above. Thus, the firm with a lower monpoly segment will charge higher prices.

Inserting () into (), we get the following equilibrium demand

$$D_i^B = \frac{a_i(2 - a_j)}{2t} \cdot p_i^B. \tag{16}$$

Comparing demand, it is easily verified that

$$D_i^B > (\leq) D_j^B \text{ iff } a_i > (\leq) a_j.$$

If $a_i > a_j$, then firm i has a larger monopoly segment than firm j. Moreover, the market share in the competitive segment is higher due $p_i < p_j$.

The impact of information on demand is obtained by differentiation

$$\frac{\partial D_i^B}{\partial a_i} = \left(\frac{2 - a_j}{2t}\right)\left(p_i^B + a_i \frac{\partial p_i^B}{\partial a_i}\right) > 0, \tag{17}$$

$$\frac{\partial D_i^B}{\partial a_j} = -\frac{a_i}{2t}\left(p_i^B - (2-a_j)\frac{\partial p_i^B}{\partial a_j}\right) < 0. \tag{18}$$

More information of own product (a_i) has a positive direct demand due to (i) some uninformed become informed about product i (market expansion); and (ii) some consumers informed about product j become fully informed and demand product i (business-stealing). However, a higher a_i has also indirect demand effects due to price changes. In the monopoly segment, a higher a_i leads to less demand due to price increases. In the competitive segment, the net effect on demand depends on the change in relative prices. We can show that

$$\frac{\partial \widehat{x}^B}{\partial a_i} = \frac{(2t-v)(4-3a_j)(4-a_j)}{t(16-8a_i-8a_j+3a_ia_j)^2} > 0.$$

As shown in (), the direct demand effects and the indirect demand effect in the competitive segment dominate the negative indirect demand effect in the monopoly segment.

The direct demand effect of more consumers informed about rival product is negative; some consumers informed only about product i shift to product j as they become aware of this product (business-stealing). The indirect demand effects due to price changes are two-fold: (i) a lower demand in the monopoly segment due to a higher p_i, and (ii) a lower demand in the competitive segment due to changes in relative prices:

$$\frac{\partial \widehat{x}^B}{\partial a_j} = -\frac{(2t-v)(4-3a_i)(4-a_i)}{t(16-8a_i-8a_j+3a_ia_j)^2} < 0.$$

Thus, it follows that $\partial D_i^B/\partial a_j < 0$ must be true.

Inserting () and () into (), we obtain the following equilibrium gross profit:

$$V_i^B = \frac{a_i(2-a_j)}{2t}\cdot\left(p_i^B\right)^2. \tag{19}$$

Comparing profit levels, we can show that

$$V_i^B - V_i^B = \frac{(a_i - a_j)\left(2v^2\left(2 - a_i - a_j\right) + a_i a_j t\left(2v - t\right)\right)}{t\left(3a_1a_2 - 8a_2 - 8a_1 + 16\right)} > 0 \;\Leftrightarrow\; a_i > a_j$$

Thus, if consumers are more informed about product i than product j, then firm i sets higher prices, receives more demand, and in turn obtains higher gross profits than firm j.

The impact of more information about product i and j is obtained by differentiation of ():

$$\frac{\partial V_i^B}{\partial a_i} = p_i^B \cdot \left(\frac{2 - a_j}{2t}\right)\left(p_i^B + 2a_i \frac{\partial p_i^B}{\partial a_i}\right) > 0,$$

$$\frac{\partial V_i^B}{\partial a_j} = -p_i^B \cdot \frac{a_i}{2t}\left(p_i^B - 2\left(2 - a_j\right)\frac{\partial p_i^B}{\partial a_j}\right) \gtrless 0.$$

More consumers informed about product i increase firm i's gross profits due to higher price and demand. More consumers informed about product j induce higher prices but lower demand for firm i. The net effect on firm i's gross profit is ambiguous.

We can sum up the results from this section in the following proposition:

Proposition 2 *Assuming a Hotelling model with price elastic monopoly segment demand, i.e., $\widetilde{x}_i < 1$, then in equilibrium:*
(i) $\frac{\partial p_i^B}{\partial a_i} > 0, \frac{\partial p_i^B}{\partial a_j} > 0, \frac{\partial D_i^B}{\partial a_i} > 0, \frac{\partial D_i^B}{\partial a_j} < 0, \frac{\partial V_i^B}{\partial a_i} > 0$ *and* $\frac{\partial V_i^B}{\partial a_j} < 0$.
(ii) $p_i^B < p_j^B, D_i^B > D_i^B$ *and* $V_i^B > V_j^B$ *if* $a_i > a_j$*, and vice versa.*

Comparing the results reported in Proposition 1 and 2 it is evident that the impact of product information on prices, demand and profits is highly sensitive to the assumption of whether or not demand in the monopoly segment is price elastic or not. If consumers informed about only one product buys the product with certainty, then information (advertising) has a pro-competitive effect. However, if these consumers trade-off whether or not to buy the product, then the pro-competitive effect vanish and prices increase in the degree of information.

2.2 Advertising and price competition

Let us now endogenise the degree of product information by assuming this is a choice variable for the firms. Employing the standard informative advertising model, as introduced by Butters (1977) and developed further by GS, we let the cost of reaching a fraction a_i of consumers with ads be given by $C_i(a_i)$, which is assumed to be increasing and strictly convex in a_i. To enable explicit solutions, we follow Tirole (1988) by assuming a quadratic advertising cost function, i.e., $C(a_i) = ka_i^2/2$. Firm i's profit function can now be written as

$$\pi_i = p_i \cdot D_i - \frac{k}{2}a_i^2. \tag{20}$$

As in GS and Tirole (1988), the firms set prices and advertising simultaneously and independently in order to maximise profits. The profit maximising price is given by (), wheras the profit maximising advertising level is defined by the following first-order condition

$$\frac{\partial \pi_i}{\partial a_i} = \begin{cases} p_i\left[1 - a_j + a_j\left(\frac{1}{2} - \frac{p_i - p_j}{2t}\right)\right] - ka_i = 0 & if \quad v - t \geq p_i \\ p_i\left[(1 - a_j)\left(\frac{v - p_i}{t}\right) + a_j\left(\frac{1}{2} - \frac{p_i - p_j}{2t}\right)\right] - ka_i = 0 & if \quad v - t < p_i \end{cases}.$$

2.2.1 Price inelastic monopoly demand

Starting out with the GS assumption ($v - t \geq p_i$) and solving the corresponding set of first-order conditions, we obtain the symmetric price-advertising equilibrium is as in Tirole (1988: 292-4)[5]

$$p^C = t\left(\frac{2 - a^C}{a^C}\right) = \sqrt{2kt}, \tag{21}$$

[5] Note that we use a slightly different notation than Tirole (1988), where a_i is Φ_i and k is a. Otherwise, the set-up is identical.

$$a^C = \frac{2p^C}{2k + p^C} = \frac{2}{1 + \sqrt{2k/t}}, \tag{22}$$

which constitute an equilibrium if and only if

1. $\widetilde{x}^C = 1 \quad \Leftrightarrow \quad v \geq t + \sqrt{2kt}$.
2. $a^C \leq 1 \Leftrightarrow k \geq t/2$.

Note that $\partial p^C/\partial a^C < 0$ whereas $\partial a^C/\partial p^C > 0$. Hence, greater levels of advertising stimulate price competition (i.e. lower prices) and higher prices stimulate advertising competition (i.e. higher levels of advertising). We also see that price and advertising levels are increasing in product differensiation (t), while more costly advertising technology (k) induces less advertising but higher prices. In the limit case, where $k = t/2$, so that $a^c = 1$, we get the full information outcome, with $p^C = t$.

Inserting () and () into (), we obtain firm i's equilibrium profits:

$$\pi^C = \frac{2k}{\left(1 + \sqrt{2k/t}\right)^2}. \tag{23}$$

Expectedly, profit increases in the degree of product differentiation, reflecting higher prices and a greater level of demand due to additional advertising. Somewhat unexpectedly, however, profit also increases in the costliness of advertising. As firms engage in less advertising, the corresponding decrease in price competition overcompensates the direct tendency towards higher advertising costs.

This is precisely the result found by GS and Tirole (1988). We can summarise in the following proposition:

Proposition 3 *If monopoly demand segment is price inelastic, then*

(i) a higher advertising cost (k)*, lowers advertising, increases prices, and increases profits.*

(ii) more product differentiation (t)*, increases prices, advertising and profits.*

2.2.2 Price elastic monopoly segment

Assuming $v-t < p_i$, the symmetric price-advertising equilibrium is implicitly defined by the following set of equations:[6]

$$Z^p \;:\; = 2(1-a)v + at - (4-3a)p = 0 \tag{24}$$

$$Z^a \;:\; = (2-a)p^2 - 2akt = 0. \tag{25}$$

From this we can express the equilibrium price and advertising level as

$$p^D = \frac{2v\left(1-a^D\right) + ta^D}{4-3a^D}, \tag{26}$$

$$a^D = \frac{2\left(p^D\right)^2}{2kt + \left(p^D\right)^2}, \tag{27}$$

which constitute an equilibrium if and only if

1. $v \in \left[\underline{v}^D, 2t\right]$, with $\underline{v}^D = \frac{t\left(4-a^D\right)}{2\left(2-a^D\right)} \in \left(t, \frac{3}{2}t\right)$.
2. $\left(p^D\right)^2 \leq 2kt$.

Note that

$$\frac{\partial p^D}{\partial a^D} = \frac{2(2t-v)}{\left(4-3a^C\right)^2} > 0 \quad \text{and} \quad \frac{\partial a^D}{\partial p^D} = \frac{8p^D kt}{\left(2kt + \left(p^D\right)^2\right)^2} > 0.$$

Thus, in contrast to the previous case, advertising *relaxes* price competition, whereas higher prices continue to promote advertising competition.

[6] We obtain the expression in (25) when substituting $2(1-a)v + at = (4-3a)p$ into the foc with respect to a, $p[2(1-a)(v-p) + at] - 2akt = 0$, and rearranging.

Let us now look at the properties of the price-advertising equilibrium, $p^D(v,t,k), a^D(v,t,k)$, by performing comparative statics on the system (24) and (25):

$$\begin{aligned}
\frac{dp^D}{dv} &= -\frac{Z_v^p Z_a^a - Z_a^p Z_v^a}{J^D} = \frac{2(1-a)(p^2+2kt)}{J^D} > 0 \\
\frac{dp^D}{dt} &= -\frac{Z_t^p Z_a^a - Z_a^p Z_t^a}{J^D} = \frac{2p^2[(2-a)v - at]}{J^D t(4-3a)} > 0 \\
\frac{dp^D}{dk} &= -\frac{Z_k^p Z_a^a - Z_a^p Z_k^a}{J^D} = \frac{-4at(2t-v)}{J^D(4-3a)} < 0 \\
\frac{da^D}{dv} &= -\frac{Z_p^p Z_v^a - Z_v^p Z_p^a}{J^D} = \frac{4(1-a)(2-a)p}{J^D} > 0 \\
\frac{da^D}{dt} &= -\frac{Z_p^p Z_t^a - Z_t^p Z_p^a}{J^D} = \frac{(2-a)ap[(2-a)t - 2(1-a)v]}{J^D t} \\
\frac{da^D}{dk} &= -\frac{Z_p^p Z_k^a - Z_k^p Z_p^a}{J^D} = \frac{-2at(4-3a)}{J^D} < 0,
\end{aligned}$$

where $J^D = Z_p^p Z_a^a - Z_a^p Z_p^a$ is the Jacobian of the system (24) and (25), which after some rearrangments simplifies to $J^D = \frac{2p^2(1-a)}{a} > 0$, implying a unique and stable equilbrium.

Note that $\frac{da^D}{dt} \geq 0 \Leftrightarrow v < \frac{(2-a)t}{2(1-a)}$, where it is readily checked that $\frac{t(2-a)}{2(1-a)} > \underline{v}^D$. The ambiguity with regard to the effect of t on a arises for the following reason: On the one hand a higher t increases price and therefore renders advertising more attractive. On the other hand, (for a given price) a higher t depresses demand in the monopolisic segment $\widetilde{x} = \frac{v-p}{t}$ and, thereby, renders advertising less attractive. The loss in revenue due to this latter effect increases in v.

Inserting (26) and (27), it is readily verified that equilibrium profits is given by:

$$\pi^D = \frac{a^D\left(2-a^D\right)}{4t} \cdot \left(p^D\right)^2.$$

We then obtain

$$\begin{aligned}\frac{d\pi^D}{dv} &= \frac{p^D}{2t}\left[\left(1-a^D\right)p^D\frac{da^D}{dv}+a^D\left(2-a^D\right)\frac{dp^D}{dv}\right]>0\\ \frac{d\pi^D}{dt} &= \frac{p^D}{2t}\left[\frac{-a^D\left(2-a^D\right)p^D}{2t}+\left(1-a^D\right)p^D\frac{da^D}{dt}+a^D\left(2-a^D\right)\frac{dp^D}{dt}\right]<0\\ \frac{d\pi^D}{dk} &= \frac{p^D}{2t}\left[\left(1-a^D\right)p^D\frac{da^D}{dk}+a^D\left(2-a^D\right)\frac{dp^D}{dk}\right]<0.\end{aligned}$$

Here, the signs of the first and third derivatives are immediate. A higher gross willingness to pay for the product, v, induces the firms both to charge a higher price and engage in more extensive advertising. Both activities contribute to a higher profit. In contrast, by stifling advertising a higher cost of advertising, k, enhances price competition. Both effects tend to depress profits. This is precisely opposite to the result from the earlier case with perfectly price inelastic demand. The impact of the degree of product differentiation, t, is ambiguous a priori. Greater product differentiation allows to charge a higher price, thus enhancing profits, but at the same time lowers the level of demand in the monopolistic segment, thus lowering profit. The effect on advertising activity is ambiguous in of itself. After tedious calculations one can confirm, however, a negative overall effect on profit. This counterintuitive finding stands again in contrast to the case of a perfectly price-inelastic monopolistic segment. We can summarise as follows

Proposition 4 *If the monopoly demand segment is price elastic, then*

(i) a higher advertising cost (k) *lowers advertising, prices, and profits;*

(ii) more product differentiation (t) *increases prices, but has an ambiguous effect on advertising and depresses profits;*

(iii) a greater gross willingness to pay (v)*, increases advertising, prices, and profits.*

3 A generalised model

In this section we will look for more general conditions for when informative advertising can be considered as pro-competitive or not. Let firm i's demand in the monopoly and competitive segments be given by $x_i(p_i)$ and $y_i(p_i, p_j)$, respectively, where $i, j = 1, 2$ and $i \neq j$. Both $x_i(\cdot)$ and $y_i(\cdot)$ are assumed to be continuous and twice differentiable, where $\partial x_i / \partial p_i < 0$, $\partial y_i / \partial p_i < 0$, $\partial y_i / \partial p_j > 0$, and $x_i(p_i) > y(p_i, p_j)$. We can specify firm i's demand as follows:

$$D_i = a_i (1 - a_j) x_i(p_i) + a_i a_j y_i(p_i, p_j). \tag{28}$$

This demand function has the following properties:

$$\frac{\partial D_i}{\partial p_i} = a_i (1 - a_j) \frac{\partial x_i}{\partial p_i} + a_i a_j \frac{\partial y_i}{\partial p_i} < 0,$$

$$\frac{\partial D_i}{\partial p_j} = a_i a_j \frac{\partial y_i}{\partial p_j} > 0,$$

$$\frac{\partial D_i}{\partial a_i} = (1 - a_j) x_i(p_i) + a_j y_i(p_i, p_j) > 0,$$

$$\frac{\partial D_i}{\partial a_j} = -a_i [x_i(p_i) - y_i(p_i, p_j)] < 0.$$

A higher price on own product reduces own demand, with the reduction coming from both the monopoly and the competitive segment. A higher price on rival product increases own the demand, with the increase coming solely from the competitive segment. More consumers informed about own product increases own demand, with the effect coming from both the monopoly and the competitive segment: (i) some uninformed consumers become informed about product i (market expansion); and (ii) some consumers informed about product j become fully informed *and* demand product i (business-stealing). Finally, more consumers informed about rival product reduces demand for product i. The effect comes solely from the competitive segment: consumers partially informed about product i become fully informed and (some of them) demand product j (business stealing).

4 Price equilibrium

Let us now derive the equilibrium when firms set prices simultaneously and independently taking the degree of product information as exogenously given. In this case, each firm i chooses the price that maximises the gross profit function

$$V_i = (p_i - c)\, D_i, \tag{29}$$

where c is a constant marginal cost parameter assumed to be identical across firms. The profit-maximising price of firm i is defined by the following first-order condition[7]

$$\frac{\partial V_i}{\partial p_i} = (1 - a_j) \left(x_i + (p_i - c)\, \frac{\partial x_i}{\partial p_i} \right) + a_j \left(y_i + (p_i - c)\, \frac{\partial y_i}{\partial p_i} \right) = 0. \tag{30}$$

We see that the profit-maximising price is balancing the marginal profitability from the monopolistic (first-term) and the competitive (second-term) segments.[8]

Equation (30) implicitly defines a best-response function $p_i\,(p_j)$. By dif-

[7] The second-order condition requires that

$$\frac{\partial^2 V_i}{\partial p_i^2} = (1 - a_j) \left(2\frac{\partial x_i}{\partial p_i} + (p_i - c)\, \frac{\partial^2 x_i}{\partial p_i^2} \right) + a_j \left(2\frac{\partial y_i}{\partial p_i} + (p_i - c)\, \frac{\partial^2 y_i}{\partial p_i^2} \right) < 0,$$

which is assumed to hold. Note that if $\partial^2 x_i / \partial p_i^2$ and $\partial^2 y_i / \partial p_i^2$ are non-positive, the condition is always fulfilled.

[8] Obviously, firm i would increase its profits if it could charge different prices to fully and partially informed consumers. However, we do not allow for price discrimination. Uniform pricing can be justified by the fact that firms are not able to observe each consumer's degree of product information.

ferentiation, using the implicit-function rule, we obtain:

$$\frac{dp_i}{dp_j} = -\frac{\frac{\partial^2 V_i}{\partial p_j \partial p_i}}{\frac{\partial^2 V_i}{\partial p_i^2}} = -\frac{a_j \left(\frac{\partial y_i}{\partial p_j} + (p_i - c) \frac{\partial^2 y_i}{\partial p_j \partial p_i} \right)}{\frac{\partial^2 V_i}{\partial p_i^2}}. \qquad (31)$$

Since the denominator is the second-order condition, assumed to be negative, prices are *strategic complements (substitutes)* if the numerator is positive (negative), i.e.,

$$\frac{dp_i}{dp_j} > (<)\, 0 \quad \Leftrightarrow \quad \frac{\partial y_i}{\partial p_j} + (p_i - c) \frac{\partial^2 y_i}{\partial p_j \partial p_i} > (<)\, 0.$$

If $\partial^2 y_i / \partial p_j \partial p_i \geq 0$, then prices are always strategic complements. Otherwise, the strategic relationship is determined by the relative strength of the first-order and the second-order cross-derivatives. Observe from (31) that all strategic interaction is going through the competitive segment. If $a_j = 0$, there is no strategic relationship in prices, and firm i sets prices as a local monopolist.

The set of first-order conditions given by (30) implicitly defines the Nash-equilibrium in prices; $p_1^* (a_1, a_2)$ and $p_2^* (a_1, a_2)$. Using the (own-price) elasticities

$$\varepsilon_{x_i} := \frac{\partial x_i}{\partial p_i} \frac{p_i}{x_i} \quad \text{and} \quad \varepsilon_{y_i} := \frac{\partial y_i}{\partial p_i} \frac{p_i}{y_i},$$

we can write the equilibrium conditions as:

$$(1 - a_j) \frac{\partial x_i}{\partial p_i} \left[\frac{1}{\varepsilon_{x_i}} + \frac{p_i^* - c}{p_i^*} \right] + a_j \frac{\partial y_i}{\partial p_i} \left[\frac{1}{\varepsilon_{y_i}} + \frac{p_i^* - c}{p_i^*} \right] = 0, \qquad (32)$$

where $i, j = 1, 2$ and $i \neq j$. We assume that the price equilibrium defined by (32) is unique and stable.[9]

[9] Uniqueness and stability of the price equilibrium are ensured by a strictly positive Jacobian, i.e.,

$$J := \frac{\partial^2 V_i}{\partial p_i^2} \frac{\partial^2 V_j}{\partial p_j^2} - \frac{\partial^2 V_i}{\partial p_j \partial p_i} \frac{\partial^2 V_j}{\partial p_i \partial p_j} > 0.$$

From (32) we see that the equilibrium prices are determined by both the relative sizes $(a_j, 1-a_j)$ and price elasticities $(\varepsilon_{x_i}, \varepsilon_{y_i})$ of the competitive and the monopolistic segment. To analyse the equilibrium further it is convenient to make the following definitions. Define p_i^m as the price that maximise profits in the monopoly segment[10]

$$x_i\left(p_i^m\right)+\frac{\partial x_i\left(p_i^m\right)}{\partial p_i}=0 \Leftrightarrow -\frac{p_i^m-c}{p_i^m}=\frac{1}{\varepsilon_{x_i}}, \tag{33}$$

and p_i^c as the equilibrium price of the competitive segment

$$y_i\left(p_i^c, p_j^c\right)+\left(p_i^c-c\right)\frac{\partial y_i\left(p_i^c, p_j^c\right)}{\partial p_i}=0 \Leftrightarrow -\frac{p_i^c-c}{p_i^c}=\frac{1}{\varepsilon_{y_i}}. \tag{34}$$

Obviously, if $a_j \rightarrow 0$, then $p_i^* \rightarrow p_i^m$ and if $a_j \rightarrow 1$, then $p_i \rightarrow p_i^c$. Using the definitions in (33)-(34), we can establish the following result:

Proposition 5 *The price equilibrium defined by (32) implies either of three possibilities: (i) If* $\varepsilon_{x_i}=\varepsilon_{y_i}=const$, *then*

$$-\frac{p_i^*-c}{p_i^*}=\frac{1}{\varepsilon_{x_i}}=\frac{1}{\varepsilon_{y_i}} \quad \textit{and} \quad p_i^*=p_i^m=p_i^c.$$

(ii) If demand in the monopoly segment is less price elastic than in the competitive segment, i.e., $0>\varepsilon_{x_i}|p_i>\varepsilon_{y_i}|p_i$, *then*

$$\frac{1}{\varepsilon_{x_i}}<-\frac{p_i^*-c}{p_i^*}<\frac{1}{\varepsilon_{y_i}} \quad \textit{and} \quad p_i^c<p_i^*<p_i^m.$$

(iii) If demand in the monopoly segment is more price elastic than in the

[10] Note that p_i^m is *not* equivalent to the monopoly price. Monopoly pricing is defined by the set of prices $\left(p_i^M, p_j^M\right):=\arg\max\left\{V_i\left(p_i, p_j\right)+V_j\left(p_i, p_j\right)\right\}$. It is easily verified that monopoly prices always exceed the equilibrium prices defined by (32).

competitive segment, i.e., $0 > \varepsilon_{y_i}|p_i > \varepsilon_{x_i}|p_i$, *then*

$$\frac{1}{\varepsilon_{y_i}} < -\frac{p_i^* - c}{p_i^*} < \frac{1}{\varepsilon_{x_i}} \quad \textit{and} \quad p_i^m < p_i^* < p_i^c.$$

A proof is provided in the Appendix.

From Proposition 1 it follows that the price effect of a larger competitive segment is not as clear-cut as in Grossman and Shapiro (1984). In fact, if demand in the competitive segment is less elastic with respect to own price than demand in the monopoly segment, then a larger fraction of fully informed consumers will result in *higher* prices.

4.1 The impact of information / advertising

The impact of consumer information on price competition is analysed by comparative statics of the effect of a change in a_i and a_j on the equilibrium price p_i^*. The comparative statics are obtained by total differentiation of (30) applying the Cramer's rule.[11]

$$\frac{dp_i^*}{da_i} = \frac{1}{J}\frac{\partial^2 V_j}{\partial a_i \partial p_j}\frac{\partial^2 V_i}{\partial p_j \partial p_i}, \tag{35}$$

$$\frac{dp_i^*}{da_j} = -\frac{1}{J}\frac{\partial^2 V_i}{\partial a_j \partial p_i}\frac{\partial^2 V_j}{\partial p_j^2}. \tag{36}$$

[11] Generally, we have that:

$$\frac{dp_i^*}{da_i} = -\frac{1}{J}\begin{vmatrix} \frac{\partial^2 V_i}{\partial a_i \partial p_i} & \frac{\partial^2 V_i}{\partial p_j \partial p_i} \\ \frac{\partial^2 V_j}{\partial a_i \partial p_j} & \frac{\partial^2 V_j}{\partial p_j^2} \end{vmatrix} \quad \text{and} \quad \frac{dp_i^*}{da_j} = -\frac{1}{J}\begin{vmatrix} \frac{\partial^2 V_i}{\partial a_j \partial p_i} & \frac{\partial^2 V_i}{\partial p_j \partial p_i} \\ \frac{\partial^2 V_j}{\partial a_j \partial p_j} & \frac{\partial^2 V_j}{\partial p_j^2} \end{vmatrix}$$

However, since $\partial^2 V_i/\partial a_i \partial p_i = \partial^2 V_j/\partial a_j \partial p_j = 0$, the comparative statics simplify to those reported in (35)-(36).

Applying the assumptions on the Jacobian ($J > 0$) and the second-order conditions $\left(\partial^2 V_j / \partial p_j^2 < 0\right)$, it follows that

$$sign\left(\frac{dp_i^*}{da_i}\right) = sign\left(\frac{\partial^2 V_j}{\partial a_i \partial p_j} \frac{\partial^2 V_i}{\partial p_j \partial p_i}\right),$$

$$sign\left(\frac{dp_i^*}{da_j}\right) = sign\left(\frac{\partial^2 V_i}{\partial a_j \partial p_i}\right).$$

Using (30), we get

$$\frac{\partial^2 V_i}{\partial a_j \partial p_i} = -\frac{1}{a_j}\left(x_i + (p_i - c)\frac{\partial x_i}{\partial p_i}\right)$$

Using the elasticity, it follows that

$$\frac{\partial^2 V_i}{\partial a_j \partial p_i} > (<)\, 0 \Leftrightarrow \frac{1}{\varepsilon_{x_i}} > (<) - \frac{p_i - c}{p_i},$$

which corresponds to the above *case 3 (case 2)*, where the monopolistic segment is *more (less) elastic* than the competitive segment. Equivalently, $\frac{\partial^2 V_j}{\partial a_i \partial p_j} > (<)\, 0 \Leftrightarrow \frac{1}{\varepsilon_{x_j}} > (<) - \frac{p_j - c}{p_j}$. We can thus conclude the following.

Proposition 6 *(i) Own information reduces (increases) own price – i.e.,* $\frac{dp_i^*}{da_i} < (>)\, 0$ *– (through a strategic effect via competitior's price) if and only if either of the following is true:*

(a) prices are strategic complements and the monopolistic segment is less (more) price elastic than the competitive segment; or

(b) prices are strategic substitutes and monopolistic segment is more (less) price elastic than competitive segment.

(ii) Rival's information reduces (increases) own price – i.e., $\frac{dp_i^*}{da_j} < (>)\, 0$ *– (through a direct effect) if and only if the monopolistic segment is less (more) price elastic than the competitive segment.*

5 Price and advertising equilibrium

Let us now endogenise the degree of information in the market by allowing firms to advertise their products. As in Grossman and Shapiro (1984), we let the cost of reaching a fraction a of the population be given by $C(a;k)$, where k is a shift parameter, where $C_k > 0, C_{ak} > 0$. We assume that $C_a > 0$ and $C_{aa} > 0$.[12] The net profit of firm i is then given by

$$\pi_i = V_i - C(a_i;k). \tag{37}$$

Following Grossman and Shapiro (1984), we assume that the firms choose prices and advertising simultaneously and independently. Thus, each firm i chooses a_i and p_i in order to maximise profits taking the rival's decision as given. Since $\partial\pi_i/\partial p_i = \partial V_i/\partial p_i$, the profit-maximising price is defined by the first-order condition in (30), whereas the profit-maximising advertising level is given by the following first-order condition[13]

$$\frac{\partial \pi_i}{\partial a_i} = (p_i - c)\left[(1-a_j)\cdot x_i + a_j \cdot y_i\right] - C_{a_i} = 0. \tag{38}$$

where $i,j = 1,2$ and $i \neq j$. Assuming symmetry, the equilibrium is defined by the following two implicit functions:

$$Z^p = (1-a)\left[x + (p-c)\frac{\partial x}{\partial p}\right] + a\left[y + (p-c)\frac{\partial y}{\partial p}\right] = 0, \tag{39}$$

$$Z^a = (p-c)\left[(1-a)x + ay\right] - C_a(a,k) = 0, \tag{40}$$

where a and p are endogenous variables and k is an exogenous cost parameter. By differentiation, using the implicit-function rule, we obtain the following

[12] For details about the underlying advertising technology, see Grossman and Shapiro (1984). Here, we simply adopt their cost function.

[13] The second-order conditions require that $\frac{\partial^2\pi_i}{\partial p_i^2} < 0$, $\frac{\partial^2\pi_i}{\partial a_i^2} = -\frac{\partial^2 C}{\partial a_i^2} < 0$, and $\frac{\partial^2\pi_i}{\partial p_i^2}\frac{\partial^2\pi_i}{\partial a_i^2} - \frac{\partial^2\pi_i}{\partial a_i \partial p_i}\frac{\partial^2\pi_i}{\partial p_i \partial a_i} > 0$. Since $\frac{\partial^2\pi_i}{\partial a_i \partial p_i} = 0$, the third condition is fulfilled by the two first ones.

(second-order) partial derivatives

$$\begin{aligned}
Z_p^p &= (1-a)\left[2\frac{\partial x}{\partial p}+(p-c)\frac{\partial^2 x}{\partial p^2}\right]+a\left[2\frac{\partial y}{\partial p}+(p-c)\frac{\partial^2 y}{\partial p^2}\right]<0 \\
Z_a^p &= \left[y+(p-c)\frac{\partial y}{\partial p}\right]-\left[x+(p-c)\frac{\partial x}{\partial p}\right] \\
Z_k^p &= 0 \\
Z_p^a &= (1-a)\left[x+(p-c)\frac{\partial x}{\partial p}\right]+a\left[y+(p-c)\frac{\partial y}{\partial p}\right]=0 \\
Z_a^a &= (p-c)(y-x)-C_{aa}<0 \\
Z_k^a &= -C_{ak}<0
\end{aligned}$$

It is instructive to consider relationship between equilibrium price and advertising levels

$$\begin{aligned}
\frac{\partial a}{\partial p} &= -\frac{Z_p^a}{Z_a^a}>0 \\
\frac{\partial p}{\partial a} &= -\frac{Z_a^p}{Z_p^p}>0 \Leftrightarrow Z_a^p>0.
\end{aligned}$$

Whereas higher prices unambigously fuel advertising competition, the impact of advertising on price competition is ambiguous. Noting that $Z_a^p > 0$ holds under the same condition as in (**??**) it follows that $Z_a^p > (<)\, 0$ if the monopolistic segment is more (less) price elastic than the competitive segment. Hence, for a symmetric equilibrium it is true that advertising stifles price competition if and only if the monopolistic market segment exhibits a higher price elasticity of demand. Note that this results holds irrespective of whether prices are strategic complements or strategic substitutes.

5.1 The impact of more costly advertising

Denoting the Jacobian for the system (**??**) and (**??**) by $J = Z_p^p Z_a^a - Z_a^p Z_p^a > 0$, we can now examine the impact of the costliness of advertising on the equilibrium[14]

$$\begin{aligned}\frac{da}{dk} &= \frac{-Z_p^P Z_k^a}{J} < 0 \\ \frac{dp}{dk} &= \frac{Z_a^p Z_k^a}{J} = \frac{\partial p}{\partial a}\frac{da}{dk} < 0 \Leftrightarrow Z_a^p < 0.\end{aligned}$$

While a higher advertising cost always stifles advertising competition, the effect on price competition is now ambiguous. In contrast to GS, more costly advertising may well lead to a stifling of price competition if the monopolistic segment being *more* price elastic than the competitive segment. Note that this result is in line with our previous findings. If the monopolistic segment is relatively price elastic, then higher levels of both *own and rival's* informative advertising tend to boost prices. In this case, increases in advertising cost lower both equilibrium advertising and equilibrium price.

Consider now the effect of k on firm i's profit. The equilibrium profits can be written as:

$$\pi_i^*(k) = (p_i^* - c)\, D_i\left(a_i^*, a_j^*, p_i^*, p_j^*\right) - C\left(a_i^*, k\right)$$

Total differentiation, observing the envelope theorem ($\frac{d\pi_i}{da_i} = \frac{d\pi_i}{dp_i} = 0$) and symmetry ($\frac{\partial a_i^*}{\partial k} = \frac{\partial a_j^*}{\partial k}$ and $\frac{\partial p_i^*}{\partial k} = \frac{\partial p_j^*}{\partial k}$), yields

$$\frac{d\pi_i^*}{dk} = (p_i^* - c)\left(\frac{\partial D_i}{\partial a_j}\frac{\partial a_i^*}{\partial k} + \frac{\partial D_i}{\partial p_j}\frac{\partial p_i^*}{\partial k}\right) - \frac{\partial C}{\partial k}.$$

Hence: Direct effect + strategic effect

[14]It is easy to find conditions such that $J > 0$. For instance, $J > 0$ if price elasticities between the monopolistic and competitive segment do not vary much such that Z_a^p is small. Another case is a situation, where the advertising intensity a is low, implying that Z_p^a is small. We show for an example in section 5.2 that $J > 0$ is satisfied.

$$\frac{d\pi_i}{dk} = -C_k(a,k) + (p-c)\left[\frac{\overset{+}{\partial D_i}}{\partial p_j}\frac{dp}{dk} + \frac{\overset{-}{\partial D_i}}{\partial a_j}\frac{\overset{-}{da}}{dk}\right]$$
$$= -C_k(a,k) + (p-c)\left[\frac{\partial p}{\partial a}\frac{\overset{+}{\partial D_i}}{\partial p_j} + \frac{\overset{-}{\partial D_i}}{\partial a_j}\right]\frac{\overset{-}{da}}{dk}$$

Hence, there are three effects: A higher k

- raises cost directly (provided $C_k(a,k) > 0$, which is true in general);
- increases own demand as the rival engages in less advertising[15] ;
- increases own demand if and only if it raises the rival's price. This is true in GS but not necessarily in our model. Here, if $\frac{dp}{dk} < 0$ the increase in competition tends to depress profits.

The effect of advertising cost on operating profit is thus ambiguous for $\frac{dp}{dk} < 0$. For instance, if the level of demand in the competitive segment is close to the level of demand in the monopolistic segment, i.e. if $y \to x$, then $\frac{\overset{-}{\partial D_i}}{\partial a_j} = y - x \to 0$. In this case, the effect through price dominates, leading to an unambiguous reduction in operating profit and, for the corresponding increase in advertising cost, to a reduction in overall profit. In section 5.2.2 we show for a Hotelling-model with price elastic demand in the monopolistic segment how the effect of advertising cost on profit is negative.[16] We can summarise our general results as follows.

Proposition 7 *Consider a symmetric price-advertising equilibrium. (i) A higher advertising cost (k) always induces lower levels of advertising and*

[15]Recall that for a given price $x \geq y$ is always true.

[16]In the duopoly version of G&S $C_k(a,k) = (p-c)\frac{\overset{+}{\partial D_i}}{\partial p_j}\frac{\overset{+}{dp}}{dk}$ implying $\frac{d\pi_i}{dk} = (p-c)\frac{\overset{-}{\partial D_i}}{\partial a_j}\frac{\overset{-}{da}}{dk} > 0$. Obviously, this is not neccessarily true in a more general model.

induces a higher (lower) price if and only if the monopolistic segment is less (more) price elastic than the competitive segment. (ii) A higher advertising cost (k) leads to a higher profit if the monopolistic segment is less price elastic than the competitive segment. Otherwise the effect on operating profit as well as on total profit is indeterminate.

6 Concluding Remarks

To appear.

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