Excessive Demand and Boom-Bust Cycles[*]

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Abstract

It has long been argued in the history of economic thought that excessive demand fueled by elastic credit supply may generate boom-bust business cycles (e.g., Tugan-Baranovsky, 1894; and Wicksell, 1906). This paper shows that dynamic interactions between persistent consumption demand (based on catching-up-with-the-Joneses preferences on the borrower side) and elastic credit supply (based on collateralized assets on the lender side) indeed generate a multiplier-accelerator mechanism that transforms small and serially-uncorrelated shocks into large and hump-shaped boom-bust cycles. Such results confirm in dynamic general equilibrium Tugan and Wicksell's ideas regarding the importance of the credit channel.

Keywords: Excess Demand, Over-Investment, Borrowing Constraints, Multiplier-Accelerator, Elastic Credit Supply.

JEL codes: E21, E22, E32, E44, E63.

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1 Introduction

Casual observation indicates that business cycles are characterized by excessive demand during booms followed by insufficient demand during recessions. Typically, periods of persistent consumption growth forecast investment booms and output expansion, and periods of underconsumption are associated with economic downturns. Post-war economic data show that measures of consumer confidence lead the business cycle, and aggregate consumption Granger-causes aggregate investment and output.[1] Among the potential driving forces behind sustained excessive consumption demand,[2] competition for living standards among consumers (– a well-documented social behavior of consumption) appears as a natural candidate. Economic intuition and the history of economic thought suggest that competition-for-consumption could lead to over-investment when fueled by procyclical credit supply, which in turn could generate boom-bust cycles in aggregate output (see, e.g., Tugan-Baranovsky, 1894; Wicksell, 1898 and 1906).[3]

This paper models the interaction between excessive consumption demand due to competition-for-status and procyclical credit supply. We introduce catching-up-with-the-Joneses preferences to capture the social behavior of competition-for-status, and borrowing limits based on the value of collateralized assets to capture the elastic supply of credit. Our main result is that such an interplay indeed gives rise to over-investment and hump-shaped boom-bust cycles in output.

The boom-bust cycles are created by a multiplier-accelerator mechanism, which translates small temporary shocks (say, a one-time shock to total factor productivity or credit demand) into large and highly persistent movements in aggregate investment and output. At the peak of the expansion, the increases in the capital stock and output are larger than their initial responses to the shock and they over-shoot their long-run level from above in the contraction phase. In this process an initial boom also plants the seed for a future recession, and vice versa. This business cycle theory overcomes an important shortcoming of the standard real-business-cycle (RBC) theory, which relies on large and persistent shocks in total factor productivity (TFP) to explain the large and persistent fluctuations in output.

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[1]See, e.g., Cochrane (1994) and Wen (2007), among others.

[2]By "excessive" consumption demand, we mean persistent above-steady-state consumption over time.

[3]In the history of economic thought, the concept of "over-investment" has at least two meanings: (i) investment in excess of savings and (ii) excessive investment volatility. In this paper we use this concept with mainly the second meaning in mind. For a review of Tugan-Baranovsky's economic theory on business cycles, see Barnett (2001).

Competition for living standards leads to persistent rises in consumption demand over time. To fulfill the persistent consumption demand, firms must increase production capacity continuously. Since investment is financed by household saving which curtails consumption, above steady-state consumption is not sustainable in general equilibrium unless there exist idle resources for lending and such credit lending is strongly elastic and procyclical (as suggested by Tugan-Baranovsky, 1894; and Wicksell, 1906). Procyclical credit supply arises naturally from endogenous borrowing limits based on the value of collateralized assets. Under collateral constraints, a positive investment improves firms' credit worthiness, which in turn enables them to obtain more credit and make more investment in the collateralized assets (e.g., capital and land), thereby further increasing firms' net worth and relaxing future borrowing constraints. This generates a dynamic multiplier effect on investment and output. Since consumption demand has strong inertia (due to catching-up-with-the-Joneses), the interplay of the above propagation mechanisms results in a cumulative process of expansion in aggregate demand and output after a shock to the economy. However, a perpetual boom in aggregate demand is not sustainable because of diminishing returns to investment. As the boom continues, diminishing marginal products dictate that the speed of the increase in aggregate output will slow down and that the "natural" rate (in the terminology of Wicksell) will fall below the borrowing interest rate;[4] hence, sooner or later falling profits and rising debt payment will erode investment spending and eventually cause a downturn in income and consumption. Again because of consumption inertia, the initial decline in consumption triggers a persistent process of under-consumption. Hence, expectations turn pessimistic, the incentive for investment reduces, thereby investment falls and firms' net worth declines. In this period the multiplier-accelerator effect reverses itself, generating a cumulative process of contraction. During the contraction phase, insufficient aggregate demand and tightened credit supply reinforce each other, causing the economy to over-shoot its steady state from above and create a recession. This explains why periods of excessive consumption go hand-in-hand with periods of over-investment and credit expansion, and why such an investment boom is ultimately followed by a recession. The larger the initial boom, the deeper the afterward recession.

This mechanism of recurrent booms and slumps is reminiscent of the conventional wisdom about the business cycle. However, here it is obtained in a general-equilibrium model with rational agents. Our formulation of procyclical credit supply borrows from Kiyotaki and Moore (1997), who have shown that endogenous credit limits based on the value of collateralized assets lead to a dynamic multiplier mechanism. When combined with a particular form of lumpy

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[4]According to Wicksell (1906), the "natural" rate is determined by the marginal products of assets.

investment, this mechanism may give rise to credit cycles. However, subsequent investigations have found that such a propagation mechanism is neither robust nor empirically important. For example, in a partial-equilibrium analysis, Kocherlakota (2000) shows that the amplification and persistence of the impact of shocks under endogenous credit constraints depend crucially on the shares of capital and land in production. If the cost shares are small enough to be empirically plausible, then both the amplification and the persistence are small and insignificant. In their general-equilibrium analysis, Cordoba and Ripoll (2004a) argue that the dynamic multiplier mechanism of collateral constraints is extremely difficult to obtain under standard preferences and technologies with realistic parameter values. For example, for the widely-used values of 1/3 for the capital share and 1 for the elasticity of intertemporal substitution, the amplification is close to zero and there is no multiplier-accelerator effect.[5]

An important implication of this paper is that the credit-cycle theory of Kiyotaki-Moore (henceforth, KM) is revived in a real-business-cycle (RBC) framework with conventional preferences and standard neoclassical technologies. A notable difference between our approach and the existing literature is that in our model the financial sector (the lender) does not produce tangible goods. This is consistent with the role of financial institutions in the real world where the major role of banks is to provide loans (credit) rather than directly engaging in goods production. Under collateral constraints, a small transitory shock can generate a large response in aggregate output because of the reallocation of productive resources ("free loanable capital" in the terminology of Tugan-Baranovsky) from unproductive agents (the lenders) to the productive but credit-constrained agents (the borrowers). In our model, since lenders do not produce goods, any extension of credit from lenders to borrowers strictly increases aggregate output. This feature captures the fact emphasized by Tugan-Baranovsky and Wicksell that resources (credit or money) hoarded by banks do not contribute to GDP unless they are lent out. In the model of KM and Cordoba and Ripoll (2004a), both the borrowers and the lenders produce goods and contribute to aggregate output. Thus, extending credit from lenders to borrowers increases the borrowers' output but decreases the lenders' output. As a result, output of the banking sector is countercyclical and the multiplier effect on aggregate output is insignificant. In reality, lending out resources by the banking sector does not reduce the sector's output; if anything, it increases it.[6]

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[5]The same arguments apply to the model used by Cordoba and Ripoll (2004b).

[6]Although financial services are a component of GDP, its share in GDP is trivial. On the other hand, the financial (banking) sector is the single most important asset holder and loanable-funds provider of the economy. The assets and credit resources of this sector are not used for goods production, but for generating loans. In this paper, we model the lenders as providing financial services only, in constrast to KM and Cordoba-Ripoll.

The key distinction between our approach and that of the existing literature is that we focus on the role of excessive aggregate demand arising from competition-for-status in creating boom-bust business cycles.[7] Although its importance in understanding asset returns and long-run growth has been well acknowledged in the literature, the role of consumption inertia under habit formation in generating boom-bust business cycles has not been thoroughly analyzed.[8] The main contribution of this paper is to show that the dynamic interaction between endogenous credit constraints (on the supply side of credit) and competition for living standards (on the demand side of credit) creates over-investment and boom-bust business cycles. This result is obtained despite strongly diminishing returns to investment, in sharp contrast to KM (1997) and Aghion, Banerjee, and Piketty (1999), who assume linear technologies and constant savings rates. As a by-product, the credit-cycle theory of KM is resuscitated in a RBC framework with conventional preferences, standard technologies, and empirically plausible parameter values.[9]

Given that the boom-bust cycles are welfare reducing (as we show in the paper when we derive the first-best allocation), stabilization policies are called for. We discuss optimal tax policies that implement the first-best risk-sharing allocation. The analysis shows that the optimal policy is a time-varying consumption tax levied on the workers (the borrowers), but not on the capitalists (the lenders). This tax policy is reminiscent of the zero-capital tax in the optimal taxation literature. We also discuss the stabilization effects of conventional constant-rate

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[7]Both Tugan-Baranovsky and Wicksell argued that over-investment is the cause, rather than a consequence, of the boom-bust cycles. In particular, Wicksell (1906) emphasized the gap between the natural rate (determined by the marginal products of assets) and the loan rate as an important mechanism for driving investment booms and slumps. However, Wicksell seems to also have acknowledged that the natural rate must be realizable in terms of revenue; otherwise firms will not undertake investment no matter how high the natural rate is above the loan rate (see Boianovsky, 1995). That is, the marginal product of capital is measurable only in terms of marginal utilities of consumption. This is why in our general-equilibrium model persistent consumption demand (or expected consumption growth) is needed, in addition to any deviations of the natural rate from the loan rate, for triggering the multiplier-accelerator mechanism. This is similar to Wicksell's emphasis on persistent commodity-price increases as a trigger of an investment boom and the credit cycle.

[8]See, e.g., Abel (1990), Constantinides (1990), Campbell and Cochrane (1999), Caroll, Overland, and Weil (2000), Fuhrer (2000), Ljungqvist and Uhlig (2000), Boldrin, Christiano, and Fisher (2001), and Alvarez-Cuadrado, Monteiro, and Turnovsky, (2004), among others. In this literature, habit formation takes two forms, external and internal. We use the former, which corresponds to catching-up-with-the-Joneses. However, internal habit formation gives similar results because it acts as competition for living standards with one's own historical self. For the early literature on the relationship between habit-formation and cycles, see Ryder and Heal (1973) and their followers.

[9]In the original KM model, the emergence of the credit cycle relies crucially on the interaction between credit constraints and an unconventional form of lumpy investment under linear production technologies. A linear technology implies that output moves one-for-one with capital, which enhances the multiplier effect of credit borrowing and investment on output. Lumpy investment implies an uncoupling of the borrowers' aggregate borrowing from their aggregate asset holdings, which generates an accelerator effect in the setup of KM. In contrast, we assume standard Cobb-Douglas technologies and neoclassical capital accumulation. In fact, the multiplier-accelerator mechanism in our model is much stronger than that under linear technologies and lumpy investment. For example, a one-time increase in productivity can lead to as large as a fivefold increase in aggregate output several periods later in our model with capital and labor, whereas it causes only negligible changes in aggregate output after the impact period in the KM model. In contrast to the existing literature, our results continue to hold even when lenders are risk averse and the share of land in production is very small (e.g., 5 percent or less) despite the fact that land may be the only collateralized asset for the borrower.

tax policies and the business-cycle implications of non-anticipated policy shocks. We find that constant-rate taxes have some stabilization power on the economy; however, unexpected policy shocks intended to stimulate aggregate demand can lead to counter-productive consequences. This tempers the conventional wisdom regarding the necessity and effectiveness of fiscal policy.

The literature on business cycles with credit market frictions has flourished.[10] This literature shows how financial frictions may generate hump-shaped output dynamics. Our paper complements the existing studies, as we show that credit market frictions, when interacted with competition for living standards, create not only hump-shaped dynamics but also highly persistent dampened cycles. Proving the presence of cycles is important because it frees the RBC approach from relying on technological regress (that is, negative TFP shocks) to generate recessions. As we show in the text, certain types of tax-cut policies indeed generate prolonged recessions after a short-lived boom.

The rest of the paper is organized as follows. Section 2 presents a basic general-equilibrium model of credit cycles with reproducible capital. In the model, lenders do not produce goods but own land. Borrowers produce goods by using land and capital as factors of production. There is no labor in the basic model. It is shown that this model can generate boom-bust cycles under standard parameter values. Section 3 discusses optimal tax policies that implement the first-best allocation with perfect risk sharing. Section 4 introduces labor and shows that endogenous labor supply can further amplify the multiplier-accelerator mechanism if the income effect on labor supply is small. Implications of constant-rate tax policies and policy shocks are also analyzed. Section 5 concludes the paper with remarks for future research.

2 The Basic Model

2.1 Structure

There are two types of agents in the economy, lenders and borrowers. Lenders do not produce, but provide loans (credit) to borrowers. In this sense, lenders serve the role of banks or financial institutions in the economy. The type of credit provided by lenders are one-period loans that can be used to finance consumption and investment. Lenders hold assets and derive utilities from consumption and land,[11] do not accumulate fixed capital, and use interest income (profits) from payment on previous loans to finance current consumption and land investment. The budget

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[10]As exemplified by the contributions of Bernanke and Gertler (1989), Carlstrom and Fuerst (1997), Krishnamurthy (2003), Cooley, Marimon, and Quadrini (2004), Iacoviello (2005), Campbell and Hercowitz (2006), Boháček and Rodríguez Mendizábal (2007), Christiano, Motto, and Rostagno (2007), and Iacoviello and Neri (2008) among many others. Iacoviello and Neri (2008) estimate a model that shares many similarities with ours but they do not focus on boom-bust cycles.

[11]Introducing land in the utility function generates a demand for assets.

constraint of a representative lender is given by

$$\tilde{C}_t + Q_t(\tilde{L}_{t+1} - \tilde{L}_t) + B_{t+1} \leq (1 + R_t)B_t, \tag{1}$$

where $\tilde{C}$ denotes consumption, $\tilde{L}_t$ the amount of land owned by the lender in the beginning of period t, Q_t the relative price of land, B_{t+1} the amount of new loans (credit lending) generated in period t, and R_t the real interest rate. The utility function of the lender is given by

$$U(\tilde{C}, \tilde{L}) = \frac{\tilde{C}^{1-\sigma_l}}{1 - \sigma_l} + b\frac{\tilde{L}^{1-\sigma_w}}{1 - \sigma_w}, \qquad \{\sigma_l, \sigma_w, b\} \geq 0; \tag{2}$$

and the time discounting factor is $\tilde{\beta} \in (0, 1)$.

Borrowers can produce goods using land and capital.[12] The production technology is given by

$$Y_t = AK_t^\alpha L_t^\gamma, \quad \alpha, \gamma \in (0, 1), \alpha + \gamma < 1; \tag{3}$$

where A is TFP, L denotes the amount of land owned by the borrower, and K denotes his capital stock.[13] Capital is reproducible and the total amount of land is in fixed supply,

$$L_t + \tilde{L}_t = \bar{L}. \tag{4}$$

A representative borrower in each period needs to finance consumption (C), land investment $(L_{t+1} - L_t)$, capital investment $(K_{t+1} - (1 - \delta)K_t)$, and loan payment that includes both the principal (B) and the interest $(R \times B)$, where $\delta \in (0, 1)$ is the depreciation rate of capital. The budget constraint of the borrower is given by

$$C_t + K_{t+1} - (1 - \delta)K_t + Q_t(L_{t+1} - L_t) + (1 + R_t)B_t \leq B_{t+1} + AK_t^\alpha L_t^\gamma. \tag{5}$$

The momentary utility function of the representative borrower is given by

$$U(C) = \frac{\left[C_t - \rho\bar{C}_{t-1}\right]^{1-\sigma_B}}{1 - \sigma_B}, \qquad \sigma_B \geq 0; \tag{6}$$

where $\rho \in (0, 1)$ measures the degree of habits in consumption and $\bar{C}$ denotes the average consumption of the borrowers.[14] Borrowers are assumed to be less patient than lenders; hence, their time discounting factor satisfies $\beta < \tilde{\beta}$.

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[12] Labor is fixed in the basic model. It will be introduced into the model in section 4.

[13] As in Kiyotaki and Moore (1997), we assume that rental markets for assets do not exist.

[14] The results are similar when habit formation is internalized. We choose to present the external habit model because it is simpler.

The borrowing constraint faced by the borrower is

$$(1 + R_{t+1})B_{t+1} \leq Q_{t+1}L_{t+1} + \theta(1-\delta)K_{t+1}, \tag{7}$$

where $\theta \in (0,1)$ measures the collateral value of the non-depreciated capital stock. KM assume that reproducible capital does not have collateral value, which corresponds to the case with $\theta = 0$.[15] The borrowing constraint imposes that the amount of debt in the beginning of the next period cannot exceed the collateral value of assets owned by the borrower next period. The rationale for this constraint is that, due to lack of contractual enforceability, the lender has incentives to lend only if the loan is secured by the value of the collateral.[16]

2.2   First-Best Allocation

In this subsection, we derive the allocation that obtains in a "first-best" environment with perfect risk sharing, absent the credit constraint (7).[17] We show that there is no credit cycle in the first-best allocation with perfect risk sharing. The allocation is equivalent to the solution to the following representative-agent's program:

$$\max \sum_{t=0}^{\infty} \left\{ \beta^t \frac{\left[C_t - \rho \bar{C}_{t-1}\right]^{1-\sigma_B}}{1-\sigma_B} + \tilde{\beta}^t \left[\frac{\tilde{C}_t^{1-\sigma_l}}{1-\sigma_l} + b \frac{\tilde{L}_t^{1-\sigma_w}}{1-\sigma_w} \right] \right\}$$

subject to

$$C_t + \tilde{C}_t + K_{t+1} - (1-\delta)K_t \leq AK_t^{\alpha}L_t^{\gamma} \tag{8}$$

$$L_t + \tilde{L}_t \leq \bar{L} \tag{9}$$

The first-order conditions are given by

$$\beta^t \left[C_t - \rho C_{t-1}\right]^{-\sigma_B} = \tilde{\beta}^t \tilde{C}_t^{-\sigma_l} \tag{10}$$

$$\tilde{\beta}^t \tilde{C}_t^{-\sigma_l} = \tilde{\beta}^{t+1} \tilde{C}_{t+1}^{-\sigma_l} \left[\alpha \frac{Y_{t+1}}{K_{t+1}} + 1 - \delta \right] \tag{11}$$

$$\tilde{\beta}^t \tilde{C}_t^{-\sigma_l} \gamma \frac{Y_t}{L_t} = \tilde{\beta}^t b \tilde{L}_t^{-\sigma_w} \tag{12}$$

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[15]If capital is firm specific, then it has little collateral value on the market. However, our results are not sensitive to the value of θ.

[16]For more discussions on this, see Kiyotaki and Moore (1997) and Kiyotaki (1998).

[17]By "first-best" allocation we mean allocation with perfect risk sharing without borrowing constraints. The results are derived under external habit formation. But the results are the same under internal habit formation.

In the limit, because $\tilde{\beta} > \beta$, equation (10) implies $\lim_{t \to \infty} [C_t - \rho C_{t-1}]^{-\sigma_B} = 0$ provided that $\lim_{t \to \infty} \tilde{C}_t > 0$; which in turn implies that the borrower's consumption level goes to zero in the limit, $\lim_{t \to \infty} C_t = 0$.[18] Equation (11) gives the modified golden-rule capital-to-output ratio in the steady state, $\frac{K}{Y} = \frac{\alpha \tilde{\beta}}{1 - \tilde{\beta}(1-\delta)}$, where $\tilde{\beta}$ is the inverse of the gross interest rate. The resource constraint (8) implies the lender's consumption-to-output ratio, $\frac{\tilde{C}}{Y} = 1 - \delta \frac{K}{Y} = 1 - \frac{\delta \alpha \tilde{\beta}}{1 - \tilde{\beta}(1-\delta)}$. Equation (12) implies $\gamma \frac{Y}{L} \tilde{C}^{-\sigma_l} = b (\bar{L} - L)^{-\sigma_w}$, which uniquely solves for the steady-state allocation of land between the two agents because the left-hand side is decreasing in the borrower's land holding L, $\lim_{L \to 0} LHS = \infty$, and the right-hand side is increasing in it, $\lim_{L \to \bar{L}} RHS = \infty$.

In the first-best allocation, the dynamics of the model is very similar to that of a standard RBC model. Hence, there is no hump-shaped cyclical propagation mechanism for realistic parameter values. To see this, notice that the above program is a standard RBC model with two consumption goods except the relative price of C is infinity in the steady state. Hence, near the steady state we can ignore the weight of the borrower's consumption in the utility function and set $C_t = 0$. The lender's land $\tilde{L}$ in utility plays the role of leisure and the borrower's land L in the production function plays the role of hours worked. The aggregate land supply $\bar{L}$ is equivalent to time endowment. Therefore, as in a standard RBC model, a one-time shock to productivity will have zero persistence in aggregate output. Replacing external habit formation by internal habit formation does not change this basic feature of standard RBC models.[19]

2.3 Competitive Equilibrium with Borrowing Constraints

Denoting $\tilde{\Lambda}$ as the Lagrangian multiplier of the constraint (1), the first-order conditions of the lender with respect to consumption, land investment, and lending are given, respectively, by

$$\tilde{C}_t^{-\sigma_l} = \tilde{\Lambda}_t \tag{13}$$

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[18]Since the lender is more patient with a lower discounting rate, we must have $\tilde{C} > C$ in the steady state.

[19]With internalized habit formation, equation (10) becomes

$$\beta^t [C_t - \rho C_{t-1}]^{-\sigma_B} - \rho \beta^{t+1} [C_{t+1} - \rho C_t]^{-\sigma_B} = \tilde{\beta}^t \tilde{C}_t^{-\sigma_l}.$$

Equations (11) and (12) will remain the same. Hence, if the borrower's consumption level C goes to zero in the steady state, then the model has the same dynamics as that with external habit. In the steady state, the lender's consumption level must be positive because of a lower discounting factor. Hence, the above equation implies that

$$\lim_{t \to \infty} \left\{ [C_t - \rho C_{t-1}]^{-\sigma_B} - \rho \beta [C_{t+1} - \rho C_t]^{-\sigma_B} \right\} = 0,$$

which implies $\lim_{t \to \infty} [C_t - \rho C_{t-1}] = 0$. Hence, $\lim_{t \to \infty} C_t = 0$.

$$Q_t \tilde{\Lambda}_t = \tilde{\beta} Q_{t+1} \tilde{\Lambda}_{t+1} + \tilde{\beta} b \tilde{L}_{t+1}^{-\sigma_w} \tag{14}$$

$$\tilde{\Lambda}_t = \tilde{\beta}(1 + R_{t+1})\tilde{\Lambda}_{t+1}. \tag{15}$$

Denoting $\{\Lambda, \Phi\}$ as the Lagrangian multipliers of constraints (5) and (7), respectively, the first-order conditions of the borrower with respect to consumption, land investment, capital investment, and borrowing are given, respectively, by

$$[C_t - \rho C_{t-1}]^{-\sigma_B} = \Lambda_t \tag{16}$$

$$Q_t \Lambda_t = \beta Q_{t+1}\Lambda_{t+1} + \beta\gamma \frac{Y_{t+1}}{L_{t+1}}\Lambda_{t+1} + Q_{t+1}\Phi_t \tag{17}$$

$$\Lambda_t = \beta\Lambda_{t+1} \left[\alpha \frac{Y_{t+1}}{K_{t+1}} + 1 - \delta \right] + \theta(1-\delta)\Phi_t \tag{18}$$

$$\Lambda_t = \beta(1 + R_{t+1})\Lambda_{t+1} + (1 + R_{t+1})\Phi_t. \tag{19}$$

A competitive equilibrium is a sequence of positive prices $\{Q_t, R_t\}_{t=0}^{\infty}$ and positive allocations $\left\{ C_t, \tilde{C}_t, K_{t+1}, L_{t+1}, \tilde{L}_{t+1}, \right\}$ such that: (i) $\left\{ C_t, \tilde{C}_t, K_{t+1}, L_{t+1}, \tilde{L}_{t+1}, \right\}$ satisfies the first-order conditions (13)-(19), the transversality conditions, $\lim_{t\to\infty} \beta^t \Lambda_t L_{t+1} = 0$, $\lim_{t\to\infty} \beta^t \Lambda_t K_{t+1} = 0$, $\lim_{t\to\infty} \tilde{\beta}^t \tilde{\Lambda}_t \tilde{L}_{t+1} = 0$, and the complementarity condition, $\Phi_t [Q_t L_t + \theta(1-\delta)K_t - (1+R_t)B_t] = 0$ for all $t \geq 0$, given $\{Q_t, R_t\}_{t=0}^{\infty}$ and the initial endowments $L_0 \geq 0, \tilde{L}_0 \geq 0, B_0 \geq 0, K_0 \geq 0$; (ii) The good and asset markets clear for all t, $C_t + \tilde{C}_t + K_{t+1} - (1-\delta)K_t = Y_t$ and $L_t + \tilde{L}_t = \bar{L}$, respectively.

The model has a unique steady-state equilibrium in which the borrower is credit-constrained, i.e., equation (7) binds. In steady state, equation (15) indicates that the interest rate is determined by the lender's time discounting factor, $1 + R = \tilde{\beta}^{-1}$. This interest rate of loanable funds is different from the "natural" rate determined by the firm's marginal product of capital. Equation (19) then implies $\Phi = (\tilde{\beta} - \beta)\Lambda > 0$, suggesting that the borrowing constraint binds around the steady state. Equation (18) implies that the capital-to-output ratio is given by $\frac{K}{Y} = \frac{\beta\alpha}{1-\beta(1-\delta)-\theta(1-\delta)(\tilde{\beta}-\beta)}$, which determines (in conjonction with the marginal product of land) the natural rate of interest in the terminology of Wicksell. The natural rate would equal the loanable funds rate if $\beta = \tilde{\beta}$; or, as in the first-best economy, if there exists perfect risk sharing without borrowing constraints.[20] Notice that the capital-to-output ratio increases with θ, suggesting that borrowing constraints entice consumers to save more than necessary when capital

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[20]The gap between the natural rate and the loan rate in the steady state reflects a premium or wedge created by borrowing constraints.

can serve as a collateralized asset. Equation (17) implies $Q = (1 - \tilde{\beta})^{-1}\beta\gamma\frac{Y}{L} = \sum_{j=0}^{\infty}\tilde{\beta}^j\beta\gamma\frac{Y}{L}$, suggesting that the price of land is determined by the present value of its marginal products. The lender's budget constraint implies $\tilde{C} = \beta\gamma Y + \theta(1-\tilde{\beta})(1-\delta)K$, suggesting that the lender's consumption level is a fraction $\beta\gamma$ of aggregate output plus the average per-period collateral value of capital, $\frac{\theta(1-\delta)K}{\sum_{j=0}^{\infty}\tilde{\beta}^j}$. The borrower's budget constraint implies $C + \left(\delta + \frac{\theta(1-\delta)}{\sum_{j=0}^{\infty}\tilde{\beta}^j}\right)K + \beta\gamma Y = Y$, where the average collateral value of capital reflects the excess savings on capital accumulation besides depreciation. The second part of the savings, $\beta\gamma Y$, finances the loan repayments to the lender. This indicates that investment deviates from savings because of credit lending. All of the great ratios (e.g., capital-to-output ratio, land-to-output ratio, consumption-to-output ratio) are determined as functions of the model's structural parameters only. Once the steady-state distribution of land is determined, the steady-state values of all other variables are determined through the great ratios. Because equation (17) is the demand curve of land and equation (14) gives the supply curve of land, the steady-state distribution of land across agents is determined uniquely by the implicit equation,

$$\beta\gamma\frac{Y(L)}{L} = \tilde{\beta}b\left(\bar{L} - L\right)^{-\sigma_w}\tilde{C}(L)^{\sigma_l}, \tag{20}$$

where the left-hand side decreases in L and the right-hand side increases in L.

2.4 Quantitative Implications

The model's stationary equilibrium path is solved by log-linearizing the model around the steady state. As in KM and others in this literature,[21] we assume that this is a deterministic economy with perfect foresight and the borrowing constraint always binds. And we examine the dynamics of the model near the steady state after a sudden unexpected shock to TFP, which has no persistence.

Calibration. The time period is a quarter. As a benchmark, we set the collateral value of capital $\theta = 0$ (in accord with KM), the lender's discounting factor $\tilde{\beta} = 0.99$ (implying 4% annual interest rate), the borrower's discounting factor $\beta = 0.5$ and risk aversion $\sigma_B = 4$ (suggesting that the borrower has a strong incentive to borrow), the rate of capital depreciation $\delta = 0.025$, the degree of habit persistence $\rho = 0.9$,[22] capital share $\alpha = 0.35$, land share $\gamma = 0.05$, and the

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[21]See, e.g., Kocherlakota (2000), Cordoba and Ripoll (2004a), and Iacoviello (2005).

[22]This value is consistent with the most recent estimates of habit formation in the literature; see, e.g., Chen and Ludvigson (2004).

utility weight parameter b is set so that the steady-state ratio of land allocated between the two types of agents $\frac{\tilde{L}}{L} = 1$. The results are not very sensitive to these particular parameter values (i.e., 1-10% changes in these values give similar results).[23] The risk aversion parameters for the lender, $\{\sigma_l, \sigma_w\}$, determine the volatility of prices in the model and are hence left free for experiments.
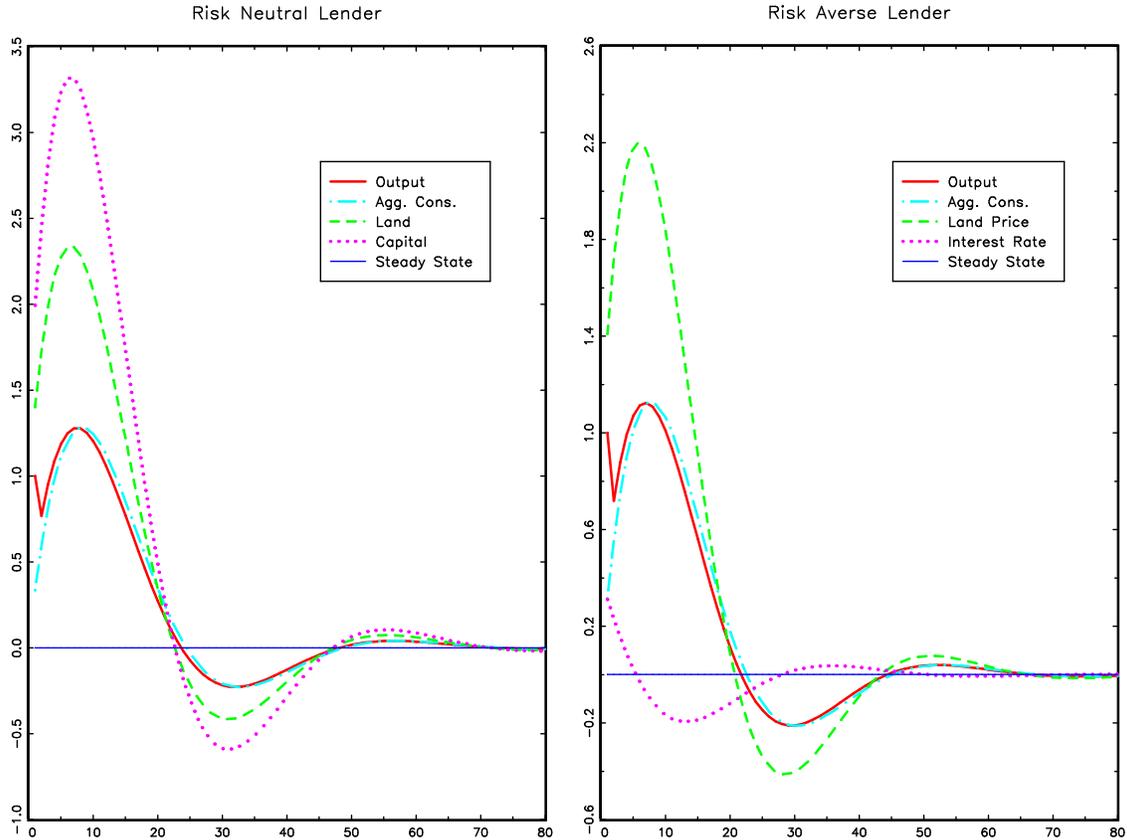

Figure 1. Impulse Responses to a One-Time TFP Shock.

Impulse Responses. The impulse responses of the model to a one-percent increase in TFP are graphed in figure 1. The left window in Figure 1 shows the responses of aggregate output (Y), aggregate consumption ($C + \tilde{C}$), aggregate capital formation (K_{t+1}), and the borrower's land investment (L_{t+1}) when the lender is risk neutral ($\sigma_l = \sigma_w = 0$); and the right window in Figure 1 shows the responses of aggregate output, aggregate consumption, the price of land (Q_t), and the gross interest rate (R_t) when the lender is risk averse: $\sigma_l = \sigma_w = 1$. Since this is a one-period shock with zero persistence, any serial correlation in the impulse responses is generated endogenously within the model. With a risk neutral lender, the land price and

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[23]Under these parameter values, the implied steady-state consumption level of the lender is small, less than 2.5% of aggregate output.

interest rate in the model are constant; hence, credit-resource reallocations or debt fluctuations are driven by the quantities of collateralized assets. Whereas with a risk averse lender, the land possession of both the lender and the borrower becomes constant but the land price fluctuates; hence, credit-resource or debt reallocations are driven by the value of collateralized assets. In either case, changes in the nominal size of collateral can drive the entire economy to fluctuate through credit lending and investment activities.

Figure 1 (left window) shows that a purely transitory shock can generate highly persistent and hump-shaped fluctuations in aggregate activities, due to the presence of stable complex eigenvalues in the linearized system. The dynamic *multiplier-accelerator* effect on aggregate output reaches its maximum after 6 periods of the shock and the increase in output at the peak is about 125% of the shock on TFP.[24] The economy over-shoots its steady state from above as it retreats from the initial boom and enters a recession before settling down on a long-run steady state via dampened cycles. New capital formation and land investment are excessively volatile and procyclical, suggesting that credit resources are rapidly pumped into the production sector from the financial system, resulting in the typical phenomenon of "over-investment" described by Tugan-Baranovsky (1894) and Wicksell (1906).[25] The length of each boom-bust cycle is about $10 - 11$ years long under the current parameterization.[26] Because the lender is risk neutral, the interest rate and land price do not change over time, albeit the marginal product of capital changes dramatically. However, the nature of the credit cycle is not sensitive to the degree of risk aversion of the lender. The right window in figure 1 shows that investment, output, and consumption fluctuate in the same manner with a similar magnitude and cyclical length when the lender's risk aversion parameters are set to $\sigma_l = \sigma_w = 1$. In this case, the quantity of the collateralized asset (land) becomes constant and the land price starts to fluctuate violently, producing cyclical fluctuations in the credit limit and bringing the entire economy to fluctuate along with it.[27] The above results suggest that fluctuations in land price are not crucial for generating the credit cycle and they weaken the criticism raised against the KM model (see, e.g., the discussion regarding the lack of hedging against movements in land price by Krishnamurthy, 2003).

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[24]To see the difference between our model and that of Kiyotaki and Moore (1997), the readers may compare figure 1 with their figure 3 (p.238).

[25]For more discussions on over-investment, see the last paragraph of this subsection.

[26]This cyclical frequency accidently coincides with the length of the business cycles documented by Tugan-Baranovsky (1894) for 19th century England.

[27]The response of aggregate output on impact is one percent because all production factors are predetermined and there is no labor. In the second period and beyond, changes in output are completely driven by land and capital accumulations. There is a downward kink in output in the second period because the accumulated asset stocks are not large enough to completely offset the withdraw of the TFP shock.

As a comparison, the impulse responses of the first-best allocation to a one-time positive shock to TFP are graphed in figure 2, where the parameter values are exactly the same as in the competitive equilibrium with risk averse lenders (i.e., $\sigma_l = \sigma_w = 1$). It shows that the impact of the shock on output is not amplified, and it is short-lived with zero persistence. Although investment is more volatile than output, the capital stock is as smooth as consumption.[28]
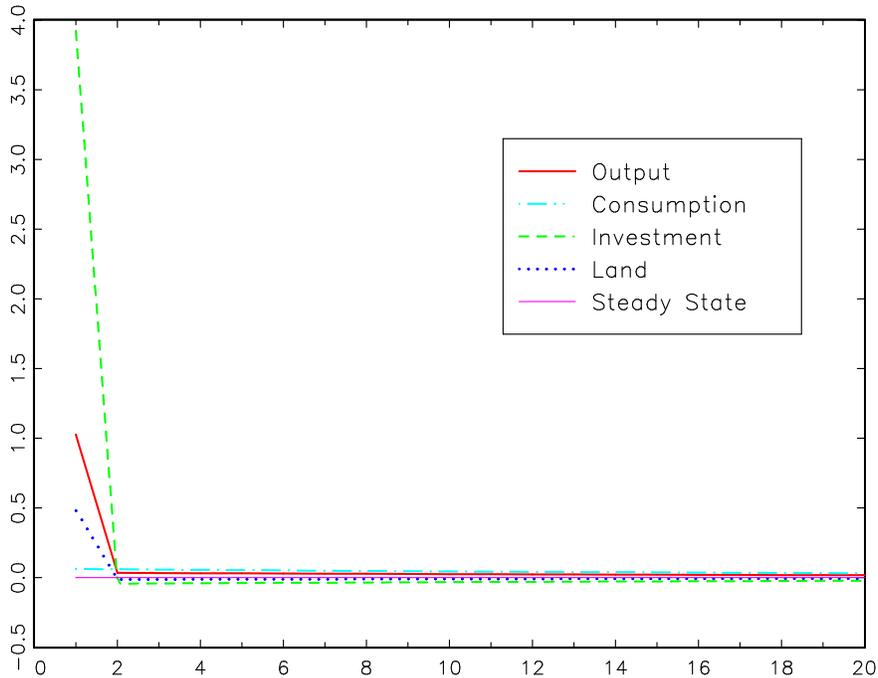

Figure 2. Impulse Responses in a First-Best Allocation.

Over-Investment. Historically, "over-investment" mainly means "investment in excess of savings". Sometimes it also means "excessive investment volatility". Based on the first definition, over-investment is not possible in general equilibrium at the aggregate level in a closed economy. However, it is possible in an open economy, or in a closed economy at the disaggregate level for a subset of the agents, if there exist lending and borrowing among the agents (or countries). In our model, over-investment of the non-banking sector (the borrower) is possible and this takes place when investment of the borrower is partially financed by the her own savings and partially by the lender's savings (loans). The borrower's investment is given by $I_t = K_{t+1} - (1-\delta)K_t + Q_t(L_{t+1} - L_t)$, and her savings given by $S_t = Y_t - C_t = \tilde{C}_t + K_{t+1} - (1-\delta)K_t$.

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[28] As changes of the capital stock, investment is a flow variable and is hence more volatile than capital in percentage terms. The log-linear relationship between investment and capital is given by $i_t = \frac{1}{\delta}(k_{t+1} - (1-\delta)k_t)$. In the competitive equilibrium of our model, the capital stock is far more volatile than output, suggesting an even greater volatility of investment. Because movements in other variables appear to be trivial relative to investment, we plot the capital stock instead of investment series in figure 1.

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Hence, over-investment takes place if $Q_t(L_{t+1} - L_t) > \tilde{C}_t$. Thus, whenever land is reallocated from the lender to the borrower in a sufficiently large amount, there exists over-investment. However, in this paper we focus more on the second aspect of the notion of over-investment, namely, excessive investment volatility.[29]

2.5 Sensitivity Analysis

(i) *Endogenous credit limits are important.* As emphasized by KM and Kocherlakota (2000), credit limits based on the value of collateralized assets are important for generating an endogenous propagation mechanism. This is also true in our model. We have explored a model with constant credit limits, that is, $(1 + R_{t+1})B_{t+1} \leq \bar{B}$, and confirmed that there are no multiplier-accelerator effects, hence no hump-shaped credit cycles under standard and empirically plausible parameter values. This is because the supply of credit is no longer elastic and procyclical with a constant credit limit. Consequently, over-investment will not occur because of the lack of the Tugan-Wicksellian credit channel to finance it. However, albeit necessary, endogenous credit limits are not by themselves sufficient for generating the multiplier-accelerator mechanism (more discussions on this point appear below).

(ii) *Production asymmetry is important.* In our model, the lender provides loans but does not produce goods. This asymmetry between the financial role of the lender and the productive role of the borrower is meant to capture the idea of Tugan and Wicksell and is important for the multiplier effect of credit constraints on aggregate output. If the lender also produces goods, as in the model of KM and Cordoba-Ripoll (2004a), then resource reallocation between the lender and the borrower not only generates counter-cyclical fluctuations in lender's output, but also dampens the magnitude of aggregate output so that the peak response of aggregate output to a one-time aggregate TFP shock takes place only in the impact period and the response is less than one-for-one after the impact period.[30]

(iii) *Habit formation is important.* Without habit formation, the model has no hump-shaped credit cycles. For example, setting $\rho = 0$ in the basic model leads to monotonic impulse responses as shown in figure 3.[31]

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[29]Notice that a larger volatility of investment than that of savings in the log-linear system does not necessarily imply investment in excess of savings, because a log-linear variable measures only percentage deviations relative to its own steady state. Although the excessive volatilities of capital and land investment in our model are due to elastic credit supply, they may or may not indicate over-investment (in excess of savings).

[30]We have experimented with a variant of Cordoba-Ripoll's (2004a) model in which both the lender and the borrower produce goods and there is habit persistence. Our findings are that such a setting still exhibits the accelerator effects (i.e., it can over-shoot the steady state and have cycles), but, not surprisingly, the multiplier effect is significantly weakened.

[31]The next section shows that when labor supply is elastic, credit cycles occur for lower values of the habit
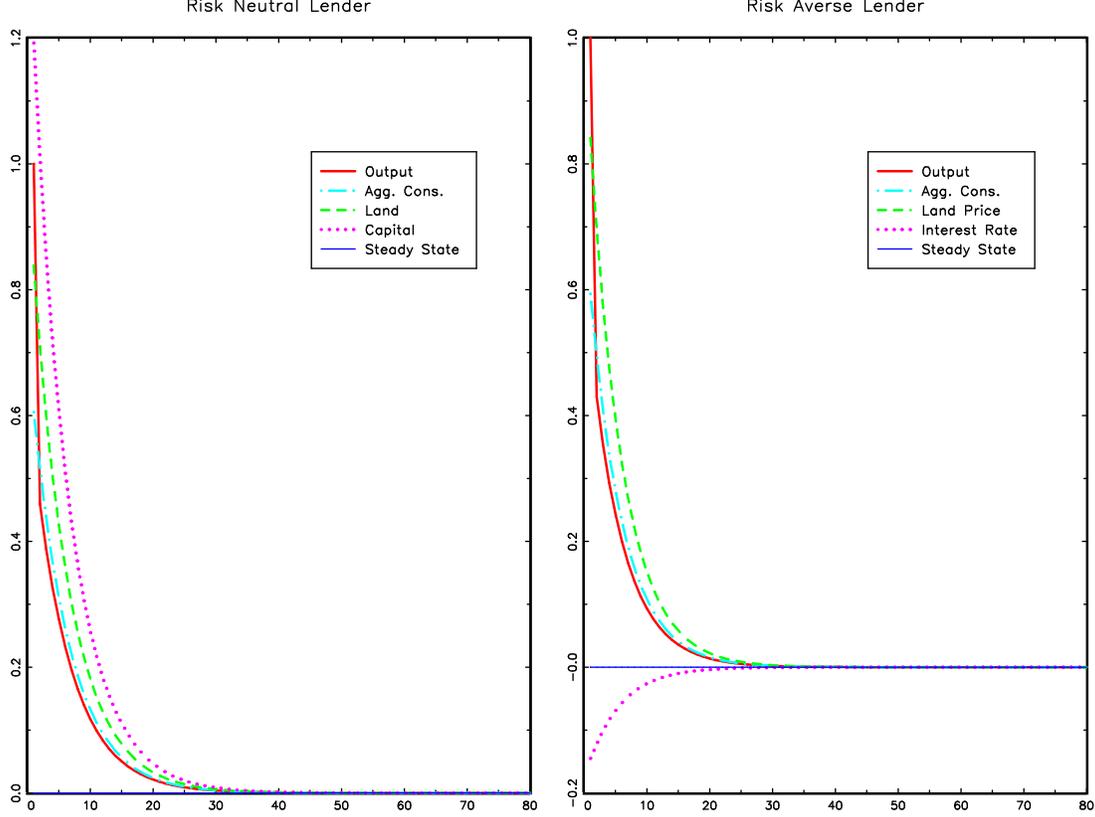
Figure 3. Impulse Responses without Habit Formation ($\rho = 0$).

To understand the intuition behind the above results, consider a simpler version of the basic model where the lender is risk neutral ($\sigma_l = \sigma_w = 0$) and there is no capital. Risk neutrality implies a constant interest rate, $(1 + R) = \tilde{\beta}^{-1}$, and a constant land price Q according to equations (13)-(15). Equation (19) then becomes $\Phi_t = \tilde{\beta}\Lambda_t - \beta\Lambda_{t+1}$. Assume $\sigma_B = 1$, $\rho = 0$ and the borrowing constraint binds, $(1 + R)B_{t+1} = QL_{t+1}$. The leverage effect of collateralized borrowing modifies the borrower's budget constraint in the following way:

$$C_t + QL_{t+1} - (QL_t - (1 + R)B_t) \leq \tilde{\beta}QL_{t+1} + AL_t^\gamma, \tag{21}$$

where the third term on the left-hand side vanishes because the borrower uses the current land value to pay back the last-period loan. In addition, collecting L_{t+1} terms on both sides of the budget constraints gives $Q(1 - \tilde{\beta})L_{t+1}$ as total expenditure on land, which is a fraction $(1 - \tilde{\beta})$ of the land value and equals the user's cost of land. In other words, debt leverage permits investment financing so that the down-payment of future land stock is proportional to the value of land and equals the user's cost. This implies that a one-percent increase in

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persistence parameter ρ. However, $\rho > 0$ is still necessary for generating credit cycles.

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income can translate into a proportional increase in the land stock, which in turn implies a γ-percent increase in output tomorrow. Therefore, the leverage effect creates a powerful dynamic multiplier mechanism on output. To see this more clearly, rewrite the borrower's budget constraint (21) and the first-order condition (17) as

$$C_t + Q(1 - \tilde{\beta})L_{t+1} = AL_t^{\gamma}, \tag{22}$$

$$Q(1 - \tilde{\beta})\frac{1}{C_t} = \beta\gamma\frac{Y_{t+1}}{L_{t+1}}\frac{1}{C_{t+1}}. \tag{23}$$

This model has closed-form solutions, with the decision rules of consumption, debt, and land investment given by the simple relationships,

$$C_t = (1 - \beta\gamma) AL_t^{\gamma}, \tag{24}$$

$$B_{t+1} = \frac{\tilde{\beta}\beta\gamma}{(1 - \tilde{\beta})}AL_t^{\gamma}, \tag{25}$$

$$L_{t+1} = \frac{\beta\gamma}{(1 - \tilde{\beta})Q}AL_t^{\gamma}. \tag{26}$$

Notice that all decision variables are proportional to aggregate output. Log-linearizing the decision rules around the steady state gives $c_t = b_{t+1} = l_{t+1} = \gamma l_t$, where lower-case variables denote percentage deviations from the steady state. In this case, a one-percent increase in current output leads to a one-percent increase in the levels of both consumption and the new loan, which in turn translates into a one-percent increase in land stock (L_{t+1}) and a γ-percent increase in the next period's output. Thus, with the borrower as the single producer in the economy, a one-time shock to TFP can generate serially correlated movements in aggregate output with the degree of persistence determined by γ. The larger the share of land in production, the more persistence is the model. If γ is close to one, for example, then a one-time shock can generate a permanent increase in future output. This roughly explains the result obtained by Kocherlakota (2000) and Cordoba and Ripoll (2004a). In short, endogenous credit constraints, by themselves, generate endogenous persistence but do not give rise to the hump-shaped multiplier-accelerator mechanism, unless, as shown by KM, a particular form of lumpy investment is introduced.

The picture changes dramatically when there is habit formation. A classic predator-prey cyclical mechanism emerges when credit constraints and habit formation interact with each

other. The log-linearized decision rules with habit formation takes the following dynamic form, which exhibits the typical features of the predator-prey theory:

$$\begin{pmatrix} c_t \\ l_{t+1} \end{pmatrix} = \begin{pmatrix} + & + \\ - & + \end{pmatrix} \begin{pmatrix} c_{t-1} \\ l_t \end{pmatrix}, \tag{27}$$

where consumption (c) corresponds to predators and land (l) to prey. More consumption today implies higher consumption tomorrow due to competition for living standards. But a higher consumption level implies a higher debt level, which erodes the available funds for land investment and reduces future output. As output decreases, consumption also declines, leading to boom-bust cycles over time.

3   Optimal Stabilization Policy

Compared to the first-best allocation, credit cycles generated by credit constraints are welfare reducing because of the large and highly persistent cyclical fluctuations induced by such constraints. Therefore, there is an important role for stabilization policies. The goal of optimal stabilization policies is to achieve the first-best allocation with perfect risk sharing.

To find the optimal policies, notice that in the competitive equilibrium the borrower consumes too much because the market interest rate is too low relative to the borrower's time discounting factor. Hence, a stabilization policy that taxes the consumption of the borrower may attain the first-best allocation. The following proposition shows that a time-varying consumption tax on the borrowers does, in the limit, implement the first-best allocation.

Proposition 1 *A sequence of consumption tax policy $\{\tau_t\}_{t=0}^{\infty}$ satisfying $\lim_{t\to\infty} \tau_t = 0$ and the relationship,*

$$\beta^t \left[C_t - \rho C_{t-1} \right]^{-\sigma_B} \frac{1}{1 + \tau_t} = \tilde{\beta}^t \tilde{C}_t^{-\sigma_l}, \tag{28}$$

implements the first-best allocation in the steady state.

Proof. Denote $T_t = \tau_t C_t$ as the lump-sum transfer. The borrower solves

$$\max \sum_{t=0}^{\infty} \beta^t \left\{ \left[C_t - \rho \bar{C}_{t-1} \right]^{1-\sigma_B} \frac{1}{1 - \sigma_B} \right\}$$

subject to

$$(1 + \tau_t)C_t + Q_t(L_{t+1} - L_t) + K_{t+1} - (1 - \delta)K_t + (1 + R_t)B_t \le B_{t+1} + AK_t^\alpha L_t^\gamma + T_t, \tag{29}$$

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$$(1 + R_{t+1})B_{t+1} \leq Q_{t+1}L_{t+1} + \theta(1 - \delta)K_{t+1}. \tag{30}$$

Denoting $\{\Lambda, \Phi\}$ as the Lagrangian multipliers for the two constraints above, the first-order conditions with respect to $\{C_t, L_{t+1}, K_{t+1}, B_{t+1}\}$ are given, respectively, by

$$\beta^t \left[C_t - \rho C_{t-1}\right]^{-\sigma_B} \frac{1}{1 + \tau_t} = \beta^t \Lambda_t, \tag{31}$$

$$\beta^t Q_t \Lambda_t = \beta^{t+1} Q_{t+1} \Lambda_{t+1} + \beta^{t+1} \gamma \frac{Y_{t+1}}{L_{t+1}} \Lambda_{t+1} + \beta^t Q_{t+1} \Phi_t, \tag{32}$$

$$\beta^t \Lambda_t = \beta^{t+1} \Lambda_{t+1} \left[\alpha \frac{Y_{t+1}}{K_{t+1}} + 1 - \delta\right] + \beta^t \theta(1 - \delta)\Phi_t, \tag{33}$$

$$\beta^t \Lambda_t = \beta^{t+1}(1 + R_{t+1})\Lambda_{t+1} + \beta^t(1 + R_{t+1})\Phi_t. \tag{34}$$

Under the tax policy (28), the above first-order conditions becomes

$$\beta^t \left[C_t - \rho C_{t-1}\right]^{-\sigma_B} \frac{1}{1 + \tau_t} = \tilde{\beta}^t \tilde{C}_t^{-\sigma_l} \tag{35}$$

$$\tilde{\beta}^t Q_t \tilde{C}_t^{-\sigma_l} = \tilde{\beta}^{t+1} Q_{t+1} \tilde{C}_{t+1}^{-\sigma_l} + \tilde{\beta}^{t+1} \gamma \frac{Y_{t+1}}{L_{t+1}} \tilde{C}_{t+1}^{-\sigma_l} + \beta^t Q_{t+1} \Phi_t \tag{36}$$

$$\tilde{\beta}^t \tilde{C}_t^{-\sigma_l} = \tilde{\beta}^{t+1} \tilde{C}_{t+1}^{-\sigma_l} \left[\alpha \frac{Y_{t+1}}{K_{t+1}} + 1 - \delta\right] + \beta^t \theta(1 - \delta)\Phi_t \tag{37}$$

$$\tilde{\beta}^t \tilde{C}_t^{-\sigma_l} = \tilde{\beta}^{t+1}(1 + R_{t+1})\tilde{C}_{t+1}^{-\sigma_l} + \beta^t(1 + R_{t+1})\Phi_t \tag{38}$$

Clearly, equation (35) implies that, for any finite-valued sequences $\{\tau_t\}$ and $\{\tilde{C}_t > 0\}$, we have $\lim_{t \to \infty} \left[C_t - \rho C_{t-1}\right] = 0$. This implies $\lim_{t \to \infty} C_t = 0$. It is also identical to its counterpart (equation (10)) in the first-best allocation in the limit, as $\lim_{t \to \infty} \tau_t = 0$. The other three first-order conditions are the same as those of the first-best allocation if $\Phi_t = 0$. The lender's first-order conditions are the same as before:

$$\tilde{C}_t^{-\sigma_l} = \tilde{\Lambda}_t \tag{39}$$

$$Q_t \tilde{\beta}^t \tilde{C}_t^{-\sigma_l} = \tilde{\beta}^{t+1} Q_{t+1} \tilde{C}_{t+1}^{-\sigma_l} + \tilde{\beta}^{t+1} b \tilde{L}_{t+1}^{-\sigma_w} \tag{40}$$

$$\tilde{\beta}^t \tilde{C}_t^{-\sigma_l} = \tilde{\beta}^{t+1}(1 + R_{t+1})\tilde{C}_{t+1}^{-\sigma_l}. \tag{41}$$

Comparing the borrower's first-order conditions with the lender's gives $\Phi_t = 0$. Hence, the borrowing constraint (30) does not bind and the tax policy (28) implements the first-best allocation in the steady state. ∎

The optimal tax rule suggests that only the borrowers, not the lenders, should be taxed. This is reminiscent of the zero-taxation on capital because lenders are the capital (loan) providers in the economy. However, although the tax policy can achieve the first-best allocation, it does so only in the limit and it is difficult to implement in reality because it requires information about both the borrowers' and the lenders' marginal utilities. Hence, it is worthwhile to investigate whether simple constant-tax rules often observed in real life can at least stabilize the economy. Such an investigation is conducted in the next section after labor has been introduced.

4   Model with Labor

This section introduces endogenous labor by allowing borrowers to supply hours worked elastically. Because habit formation induces a strong negative income effect, with standard separable preferences, labor supply decreases after a positive TFP shock. This is inconsistent with the data. Therefore, the absence of income effects is needed to ensure that labor is procyclical, in accord with the US data. For this reason, we follow Greenwood, Hercowitz, and Huffman (1988) by adopting the following utility function with no income effect:

$$\frac{1}{1 - \sigma_B} \left[C_t - \rho \bar{C}_{t-1} - P_t \frac{N_t^{1+\eta}}{1 + \eta} \right]^{1 - \sigma_B}, \qquad \eta \geq 0; \tag{42}$$

where P is the population size of the representative family of borrowers, N hours worked for each member of the family, and C the family's total consumption.[32] This type of utility function without income effect on labor supply is widely used in the RBC literature.[33] Taking the normalization $P = 1$, the aggregate production function is given by

$$Y_t = A K_t^\alpha L_t^\gamma N_t^{1-\alpha-\gamma}. \tag{43}$$

The first-order condition with respect to hours worked is given by

$$N_t^\eta = (1 - \alpha - \gamma) \frac{Y_t}{N_t}, \tag{44}$$

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[32]We assume perfect risk sharing among family members.

[33]See, e.g.,Greenwood et al. (1988), Mendoza (1991), Correia et al. (1995), Perri and Neumeyer (2005), and Jaimovich and Rebelo (2008).

which shows that labor supply depends only on the real wage (the marginal product of labor) and not on consumption. The elasticity of labor supply is $\frac{1}{\eta}$. Based on this first-order relationship, the steady state utility level is strictly positive only if the inequality, $(1+\eta)(1-\rho) > (1-\alpha-\gamma)\frac{Y}{C}$, holds; which imposes constraints on the values of ρ and η. For example, if ρ is close to one, then η must be very large. This model reduces back to the basic model with fixed labor if $\eta = \infty$.

4.1 Competitive Equilibrium

To facilitate comparison, all common parameters are set at exactly the same values as those in the basic model of section 2, which imply labor's share $(1-\alpha-\gamma) = 0.6$. The elasticity parameter of labor supply is set at $\eta = 6$, implying a labor supply elasticity of 0.17, which is consistent with the microeconomic literature's finding of a relatively small labor supply elasticity. Reducing η (increasing the elasticity of labor supply) further will make the multiplier-accelerator effect even stronger and more dramatic, and the system may converge to a Hopf limit cycle as η tends to zero. On the other hand, increasing η (reducing the labor supply elasticity) reduces the multiplier-accelerator effect and in the limit as η tends to infinity, the system converges back to the basic model presented above.

With endogenous labor, the multiplier-accelerator mechanism of collateral constraints is amplified, as shown in figure 4. The peak response of output is larger and the length of the cycle is longer. For example, with $\eta = 6$, the peak response of output is 5 times the size of the shock and it is reached 28 periods (7 years) after the impact period. The length of the cycle is around 120 periods (30 years). Also, the recession following the initial boom period is longer and more pronounced.
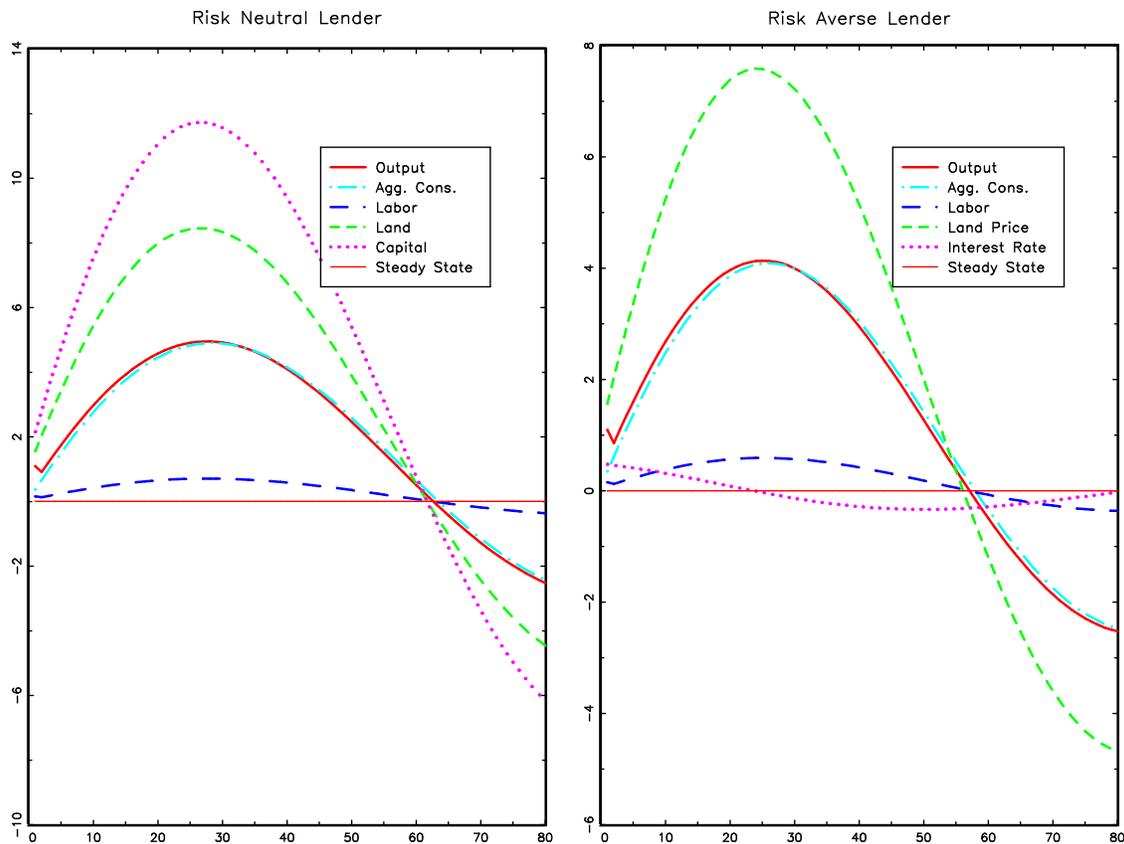
Figure 4. Impulse Responses with Flexible Labor.

The nature of the multiplier-accelerator can be adjusted by varying the borrower's incentives to borrow. For example, if the borrower is less risk averse, less habit forming in consumption, or more patient, then the length of the cycle and the magnitude of the hump are altered. As an example, figure 5 graphs the impulse responses of the model when the borrower's risk aversion parameter is set to $\sigma_B = 1$. As the left window and the right window of figure 5 show, regardless of the lender's degree of risk aversion, the model now has a smaller multiplier effect and a shorter credit cycle. The peak response of output is reached after around 12 periods with a magnitude of 2.5 at the peak, and the length of the cycle is about 60 periods long.

In the basic model, we have set $\beta = 0.5$ and $\sigma_B = 4$, which imply extremely high degrees of impatience and risk aversion. With endogenous labor, these parameter values can be significantly relaxed while retaining the boom-bust cycles. For example, with $\beta = 0.8$, $\sigma_B = 0.2$, the model generates a very similar picture to that in figure 1, although the implied degree of risk aversion is 2, much lower than that of the basic model without labor. Also, if $\sigma_B = 1$ and $\rho = 0.8$, then the peak of the first hump in the responses of aggregate output is reached only after 3 periods and the length of the cycle is about $20 - 24$ periods, much shorter than before. This scenario is very similar to the high-frequency business cycle observed in the U.S. data.
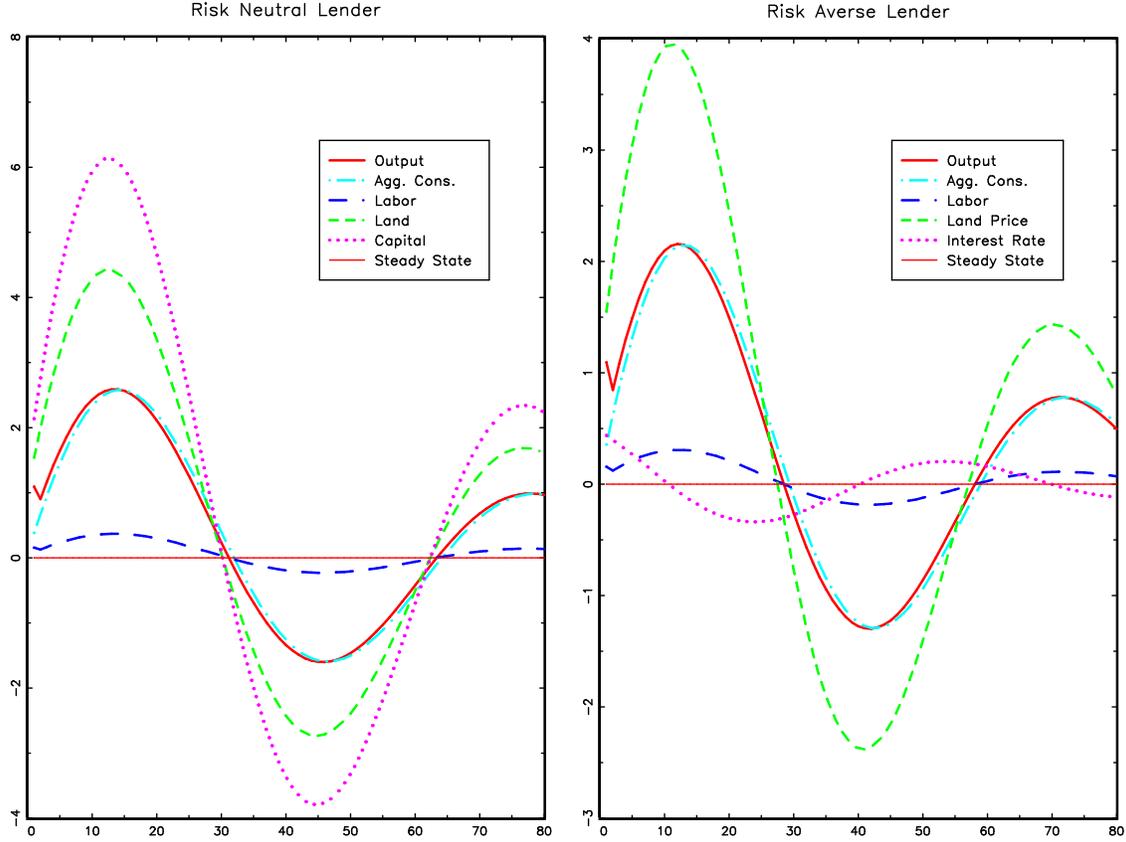
Figure 5. Impulse Responses with $\sigma_B = 1$.

4.2 Policy Implications

As in the model without labor, a time-varying consumption-tax rule implements the first-best allocation in the limit. This is summarized in the following proposition.

Proposition 2 *A sequence of consumption tax policy $\{\tau_t\}_{t=0}^{\infty}$ satisfying $\lim_{t\to\infty} \tau_t = 0$ and the relationship,*

$$\beta^t X_t^{-\sigma_B} \frac{1}{1 + \tau_t} = \tilde{\beta}^t \tilde{C}_t^{-\sigma_l}, \tag{45}$$

where $X_t \equiv C_t - \rho C_{t-1} - \frac{N_t^{1+\eta}}{1+\eta}$, implements the first-best allocation in the steady state.
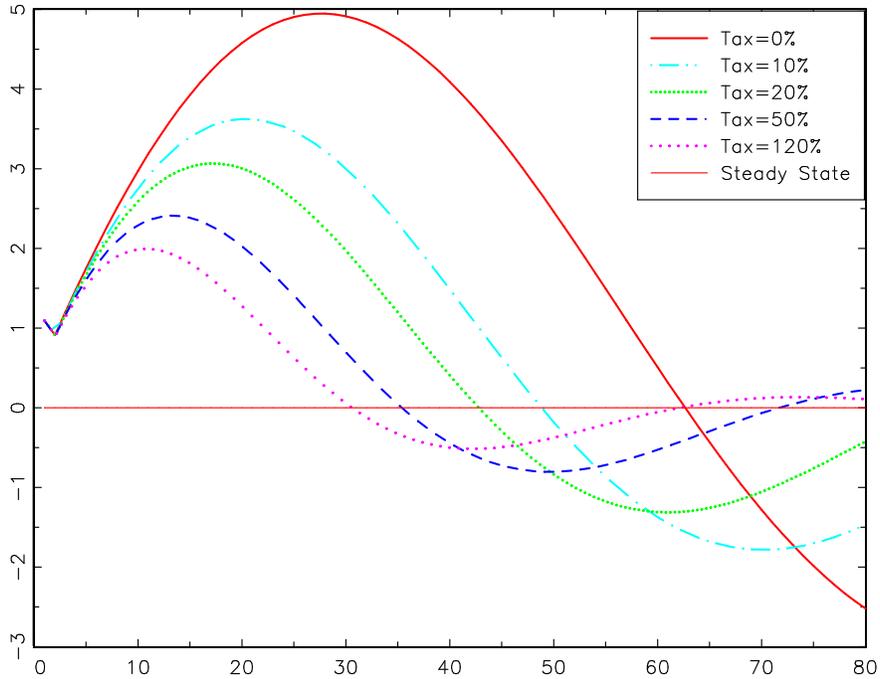
Proof. See the Appendix. ∎

Figure 6. Stabilization Effects of a Consumption Tax.

In real life, such a complex optimal tax rule is difficult to bring into play. What we observe are most often simple tax policies, such as constant-rate sales tax. What are the effects of such simple policies? Figure 6 shows the impulse responses of aggregate output (in the economy with labor and risk neutral lender) to a one-time TFP shock under different steady-state consumption tax rates. The results show that as the tax rate increases, aggregate output is gradually stabilized with smaller amplification and reduced persistence. Therefore, a constant-rate consumption tax does have stabilization effects when the tax rate is high enough. The intuition for the stabilization effect is that consumption tax discourages consumption demand, which reduces the incentive for borrowing, hence mitigating the multiplier-accelerator effects of the credit constrains on investment. Similar results can also be obtained under income tax policies.

However, simple tax policies cannot achieve the first-best allocation, more often they also introduce further distortions into the economy. As an example, we examine the business cycle effects of a sudden, unexpected, (one-period) 1% income-tax cut on the competitive economy with labor. Such a tax reduction is meant to boost the economy by increasing the after-tax marginal rates of return to work and investment. However, we show that such policies intended to stimulate the economy can be counter-productive and generate a long-period of recession instead of a boom.

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Consider a standard income tax τ on aggregate output Y. The borrower's resource constraint becomes

$$C_t + Q_t(L_{t+1} - L_t) + K_{t+1} - (1-\delta)K_t + (1+R_t)B_t \leq B_{t+1} + (1-\tau_t)AK_t^\alpha L_t^\gamma N_t^{1-\alpha-\gamma} + T_t, \quad (46)$$

where $T = \tau Y$ is a lump-sum transfer payment. Suppose the steady-state income tax rate is 20%; then a one-percent sudden decrease in the income tax rate has the following dynamic effects shown in figure 7:
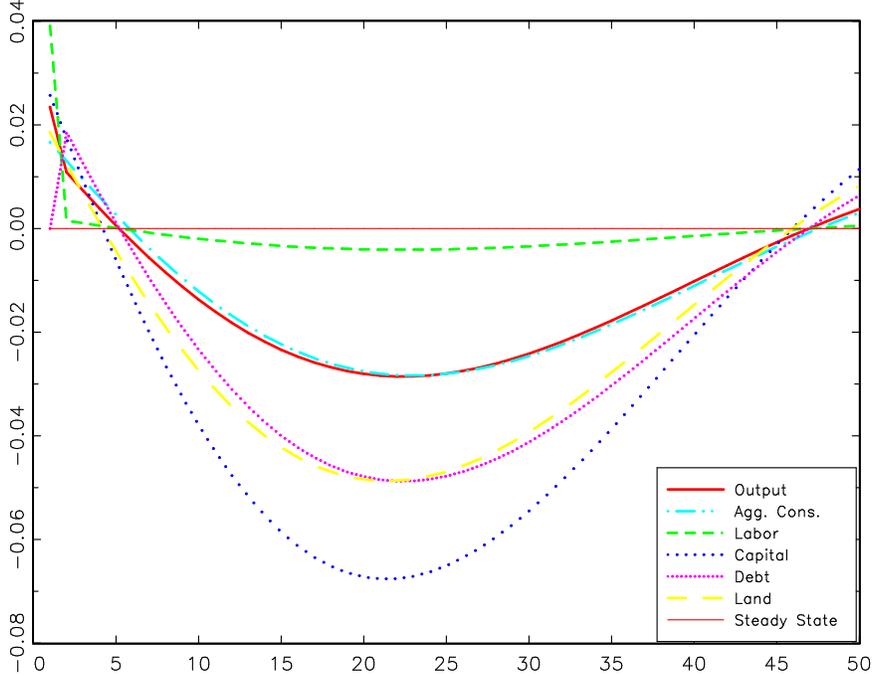

Figure 7. Impulse Responses to an Income-Tax Cut.

The intuition for the prolonged recession caused by a tax cut is as follows. Initially, a tax cut increases the incentives for working and investing. Hence, there is a short boom in the initial period in aggregate consumption, investment, labor, and output. However, since TFP has not changed, the increase in output is fully due to higher labor supply. Also, because the tax cut is financed by an equal decrease in the lump-sum transfer, the initial increase in aggregate demand is supported heavily by borrowing. Therefore, the debt level increases sharply in the second period and it chokes off investment because the natural rate is below the loan rate. As investment decreases in the second period, the multiplier-accelerator mechanism kicks in and generates a cumulative process of contraction. Therefore, the stimulative package of a tax cut is counter-productive.

5   Conclusion

The history of economic thought has long suggested that boom-bust business cycles may be driven by excessive consumption demand and over-investment fueled by credit expansion. Tugan-Baranovsky (1894) argued that industrial cycles were driven by an independent investment function and that, ultimately, over-investment was the cause of recessions. Similarly, Wicksell (1906) proposed making investment independent of savings so aggregate demand is free to rise above or fall below a given level of aggregate supply. A common theme of this line of economic reasoning is to emphasize the important role of credit. In general equilibrium, consumption reduces savings, yet investment requires savings to finance. Hence, boom-bust cycles featuring simultaneous increases in consumption and investment (i.e., co-movements) are difficult to generate in standard general-equilibrium models without persistent shocks to the TFP. Using a two-agent RBC model featuring a productive borrower who is credit-constrained and an unproductive lender who hoards idle resources, this paper shows that dynamic interactions between excessive consumption demand due to competition-for-status on the borrower side and elastic credit supply on the lender side creates a boom-bust cyclical mechanism that embodies some of the ideas and insights of Tugan-Baranovsky and Wicksell.

Competition for living standards under catching-up-with-the-Joneses preferences induces persistent consumption demand. With the interest rate of loanable funds below the inverse of the household's time discount rate, this leads to strong incentives for continuous borrowing so as to win the competition by increasing not only current consumption but, more importantly, future consumption. This also generates excessive demand for greater production capacity. Because of endogenous credit constraints, firms have incentives to over-invest in productive assets (including the collateral), which enhances firms' credit worthiness, enabling them to borrow even more both in the current and in the future periods. This procyclical supply of credit reallocates resources from unproductive agents to productive agents; hence, it enhances aggregate productivity and fulfills the excessive consumption demand, leading to a cumulative process of expansion and investment boom. However, as the expansion continues, the debt level rises persistently while the marginal product of assets diminishes quickly. The rising debt level and the growing costs of borrowing erode the available funds for investment, eventually putting an end on the boom and setting off a contraction. The contraction process accelerates itself towards the steady state because less investment implies less collateral, which implies less credit-worthiness and less loans; in addition, competition-for-status implies that less aggregate consumption today leads to less incentive for individual consumption tomorrow, which further reduces investment demand. Thus, an important feature of the contraction process is the lack of

sufficient demand and sharp drop in investment. This process will continue until a point where the marginal product of assets is significantly higher than its steady-state level and the interest rate so that borrowing and investment become profitable again.[34] Thus, under the interaction between competition-for-status and procyclical credit supply, a small shock can trigger a process of boom-bust cycles in credit lending and aggregate activities. In this process, credit resources are unleashed out from the banking sector to the public during an expansion, and sucked back to the banking sector during a contraction.[35]

Our analysis of the business cycle is consistent with the conventional Keynesian idea about the role of aggregate demand in economic fluctuations. Because boom-bust cycles are known to be associated with over-spending in an expansion and under-spending in a recession, they provided the basis for Keynesian policy that calls for government intervention by using demand-stimulus packages. However, our analysis of stabilization policies suggests that stabilization is indeed a delicate job in an economy with endogenous borrowing constraints. Optimal policies, even if they exist, are difficult to implement. Policy shocks have unintended consequences. Although rule-of-thumb policies, such as constant-tax rate policies, may have some stabilization effects, quantitatively these effects may be weak for realistic tax rates. Such mixed results and, especially, the counter-productive outcomes of policy shocks, call for further research.

Our results confirm some of Tugan and Wicksell's views on the trade cycle and reinforce the findings of Kiyotaki and Moore (1997) that highly elastic credit supply has devastating consequences. This may help explaining not only why developing countries (where the supply of credit is severely constrained yet at the same time highly elastic because of endogenous credit limits, insider dealing, corruption, weak corporate governance, and speculative international capital flows) are more volatile and susceptible to economic crises than developed countries, but also why lowered credit standards in the subprime mortgage market designed to meet persistent housing demand for low-income households could have been responsible for the recent financial turmoil in the U.S.

Our model may be viewed as a prototype for many possible extensions. For example, asset pricing, the housing market, oil shocks, sticky prices, monetary policies, imperfect competition, international trade, small-open economy, and so on, can be embedded into our model to study their implications for boom-bust cycles.

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[34]Neither Tugan-Baranovsky (1894) nor Wicksell (1906) had a formal theory of the turning points of the business cycle, although Wicksell conjectured that the fluctuations in the marginal product of capital were important for understanding the turning points.

[35]This process of credit cycles was visualized by Tugan-Baranovsky (1894) as the motion of a steam engine. When the pressure of the steam attained a certain level, the resistance of the piston was overcome and it was set in motion, before returning again to its original position when the steam was exhausted (Barnett, 2001).

Appendix: Proof of Proposition 2

The first-best allocation with labor is equivalent to the solution to the following program:

$$\max \sum_{t=0}^{\infty} \left\{ \beta^t \frac{1}{1-\sigma_B} \left[C_t - \rho \bar{C}_{t-1} - \frac{N_t^{1+\eta}}{1+\eta} \right]^{1-\sigma_B} + \tilde{\beta}^t \left[\frac{\tilde{C}_t^{1-\sigma_l}}{1-\sigma_l} + b \frac{\tilde{L}_t^{1-\sigma_w}}{1-\sigma_w} \right] \right\}$$

subject to

$$C_t + \tilde{C}_t + K_{t+1} - (1-\delta)K_t \le A K_t^{\alpha} L_t^{\gamma} N_t^{1-\alpha-\gamma} \tag{47}$$

$$L_t + \tilde{L}_t \le \bar{L}. \tag{48}$$

Denoting $X_t \equiv \left[C_t - \rho C_{t-1} - \frac{N_t^{1+\eta}}{1+\eta} \right]$, the first-order conditions are given by

$$\beta^t X_t^{-\sigma_B} = \tilde{\beta}^t \tilde{C}_t^{-\sigma_l} \tag{49}$$

$$(1-\alpha-\gamma)\frac{Y_t}{N_t} = N_t^{\eta} \tag{50}$$

$$\tilde{\beta}^t \tilde{C}_t^{-\sigma_l} = \tilde{\beta}^{t+1} \tilde{C}_{t+1}^{-\sigma_l} \left[\alpha \frac{Y_{t+1}}{K_{t+1}} + 1 - \delta \right] \tag{51}$$

$$\tilde{\beta}^t \tilde{C}_t^{-\sigma_l} \gamma \frac{Y_t}{L_t} = \tilde{\beta}^t b \tilde{L}_t^{-\sigma_w} \tag{52}$$

Equation (49) implies $X_t^{\sigma_B} = \left(\frac{\beta}{\tilde{\beta}} \right)^t \tilde{C}_t^{\sigma_l}$, the right-hand side of which goes to zero as $t \to \infty$ for any finite positive value of $\tilde{C}_t$. Hence, if $\sigma_B > 0$, we have $\lim_{t\to\infty} X_t = 0$. This implies $(1-\rho)C = \frac{N^{1+\eta}}{1+\eta} = \frac{1-\alpha-\gamma}{1+\eta}Y$ in the steady state. Equation (51) implies $\frac{K}{Y} = \frac{\tilde{\beta}\alpha}{1-\tilde{\beta}(1-\delta)}$. The aggregate resource constraint then implies $\frac{\tilde{C}}{Y} = 1 - \delta\frac{K}{Y} - \frac{C}{Y} = 1 - \frac{\delta\tilde{\beta}\alpha}{1-\tilde{\beta}(1-\delta)} - \frac{1-\alpha-\gamma}{(1-\rho)(1+\eta)}$, which confirms that $\tilde{C}$ is positive and finite in the steady state provided that ρ is small enough or η is large enough. Notice that the same results will obtain if the consumption habit is internal.

In a competitive equilibrium with consumption tax, the borrower solves

$$\max \sum_{t=0}^{\infty} \beta^t \left\{ \left[C_t - \rho \bar{C}_{t-1} - \frac{N_t^{1+\eta}}{1+\eta} \right]^{1-\sigma_B} \frac{1}{1-\sigma_B} \right\}$$

subject to

$$(1+\tau_t)C_t + Q_t(L_{t+1} - L_t) + K_{t+1} - (1-\delta)K_t + (1+R_t)B_t \le B_{t+1} + A K_t^{\alpha} L_t^{\gamma} N_t^{1-\alpha-\gamma} + T_t \tag{53}$$

$$(1 + R_{t+1})B_{t+1} \le Q_{t+1}L_{t+1} + \theta(1 - \delta)K_{t+1} \tag{54}$$

The first-order conditions are given by

$$\beta^t X_t^{-\sigma_B} \frac{1}{1 + \tau_t} = \beta^t \Lambda_t \tag{55}$$

$$N_t^\eta = (1 - \alpha - \gamma)\frac{Y_t}{N_t} \tag{56}$$

$$\beta^t Q_t \Lambda_t = \beta^{t+1} Q_{t+1}\Lambda_{t+1} + \beta^{t+1}\gamma\frac{Y_{t+1}}{L_{t+1}}\Lambda_{t+1} + \beta^t Q_{t+1}\Phi_t \tag{57}$$

$$\beta^t \Lambda_t = \beta^{t+1}\Lambda_{t+1}\left[\alpha\frac{Y_{t+1}}{K_{t+1}} + 1 - \delta\right] + \beta^t\theta(1 - \delta)\Phi_t \tag{58}$$

$$\beta^t \Lambda_t = \beta^{t+1}(1 + R_{t+1})\Lambda_{t+1} + \beta^t(1 + R_{t+1})\Phi_t \tag{59}$$

Under the tax policy (45), equations (55), and (57)-(59) become

$$\beta^t X_t^{-\sigma_B} \frac{1}{1 + \tau_t} = \tilde{\beta}^t \tilde{C}_t^{-\sigma_l} \tag{60}$$

$$\tilde{\beta}^t Q_t \tilde{C}_t^{-\sigma_l} = \tilde{\beta}^{t+1} Q_{t+1}\tilde{C}_{t+1}^{-\sigma_l} + \tilde{\beta}^{t+1}\gamma\frac{Y_{t+1}}{L_{t+1}}\tilde{C}_{t+1}^{-\sigma_l} + \beta^t Q_{t+1}\Phi_t \tag{61}$$

$$\tilde{\beta}^t \tilde{C}_t^{-\sigma_l} = \tilde{\beta}^{t+1}\tilde{C}_{t+1}^{-\sigma_l}\left[\alpha\frac{Y_{t+1}}{K_{t+1}} + 1 - \delta\right] + \beta^t\theta(1 - \delta)\Phi_t \tag{62}$$

$$\tilde{\beta}^t \tilde{C}_t^{-\sigma_l} = \tilde{\beta}^{t+1}(1 + R_{t+1})\tilde{C}_{t+1}^{-\sigma_l} + \beta^t(1 + R_{t+1})\Phi_t \tag{63}$$

Clearly, equation (60) implies that, for any finite-valued sequences $\{\tau_t\}$ and $\{\tilde{C}_t > 0\}$, we have $\lim_{t\to\infty} X_t = 0$. It is also identical to its counterpart in the first-best allocation in the limit as $\lim_{t\to\infty} \tau_t = 0$. Notice that equation (56) is identical to its counterpart in the first-best allocation. Hence, if N_t is a solution of labor in the first-best, it is also a solution under the optimal consumption tax. The other three first-order conditions (61)-(63) are the same as those of the first-best allocation if $\Phi_t = 0$. The lender's first-order conditions are the same as before:

$$\tilde{C}_t^{-\sigma_l} = \tilde{\Lambda}_t \tag{64}$$

$$Q_t\tilde{\beta}^t \tilde{C}_t^{-\sigma_l} = \tilde{\beta}^{t+1} Q_{t+1}\tilde{C}_{t+1}^{-\sigma_l} + \tilde{\beta}^{t+1}b\tilde{L}_{t+1}^{-\sigma_w} \tag{65}$$

$$\tilde{\beta}^t \tilde{C}_t^{-\sigma_l} = \tilde{\beta}^{t+1}(1 + R_{t+1})\tilde{C}_{t+1}^{-\sigma_l}. \tag{66}$$

Comparing the borrower's first-order conditions with the lender's gives $\Phi_t = 0$. Hence, the borrowing constraint does not bind and, the tax policy (45) implements the first-best allocation in the limit. ∎

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