Inflation and welfare in long-run equilibrium with firm dynamics

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Abstract

How do changes in the growth rate of money affect output and welfare in the long-run when the economy's industrial organization is endogenous? To analyse this question we build a model with cash-in-advance constraints and an endogenous distribution of establishments' productivities. Inflation distorts firm entry and exit dynamics. The model is calibrated to the United States economy. We find that increasing the annual inflation rate by 10 percent above the average rate in the U.S. would result in only a modest fall in average productivity (of about 1.3 percent). This result is robust to substantial changes in both parameter values and model specification. However, modest falls in average productivity may be responsible for almost 1/2 of the welfare cost of inflation.

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1 Introduction

To ascertain whether the welfare cost of inflation is important we must ask how does anticipated inflation affect the long-run values of macroeconomic variables. As pointed out by Lucas (1981) the disciplining virtue of applied welfare economics is that it forces one to take a position on all of the issues involved in constructing a quantitatively serious general equilibrium model of the entire economy. If international differences in income per capita are explained by differences in the accumulation of productive factors and by differences in the efficiency in the employment of these factors then the welfare cost of inflation will be high if it discourages the accumulation of factors of production or if it leads to less efficiency in their use. The first possibility has been extensively examined in the literature however the latter has been neglected. In this paper we begin the exploration of this second possibility.

Measures of the welfare cost of inflation are usually derived by comparing steady states levels of aggregate consumption at different rates of money growth within the framework of monetary equilibrium growth models. Money is often introduced by means of cash-in-advance constraints which require agents to hold money balances to facilitate transactions. Cooley and Hansen (1989) show that when the neoclassical growth model is augmented with this structure, the relative price of consumption with respect to leisure increases as the long-run rate of monetary growth increases. Consequently agents substitute away from labor, which induces employment and output to drop. Stockman (1981) shows that, when the cash-in-advance constraint also applies to investment goods, a similar effect operates through lower capital accumulation. At moderate inflation rates, these models produce small welfare costs: in steady state, Cooley and Hansen (1989) report that a 10 percent inflation rate results in a welfare cost of about 0.4 percent of income relative to an optimal monetary policy.

However, in these earlier models average productivity is exogenous and only the accumulation of factors of production matters to determine income. Gomme (1993) and Jones and Manuelli (1995) extend the work on the effects of monetary policy to models of endogenous growth and also find the welfare cost of inflation to be small. However, their work assumes a single representative firm and abstract from heterogeneity in production units. If, however, the industrial organization, i.e. the allocation of aggregate resources across uses, is important in understanding cross-country differences in per capita incomes, then it is not only the level of factor accumulation that matters, but also how these factors are allocated across heterogeneous production units[1]. Since large differences in income per capita cannot be accounted for simply by differences in the accumulation of production factors, to answer the question of whether the welfare cost of inflation is important we should consider a framework where the allocation of factors across establishments with different productivity levels is potentially affected by money. Indeed, the prevailing view in development accounting is that cross-country differences in income per capita are mostly explained by differences in total factor productivity[2]. Thus, to confidently examine whether an economy is wealthier at low levels of inflation in the framework of monetary equilibrium growth models average productivity should be endogenous and potentially affected by the monetary growth rate.

In this paper, we investigate what is the impact of higher rates of monetary growth on the real economy including output, consumption, investment, hours worked and productivity in a model where the productivity distribution of incumbent establishments is endogenous. For this purpose, we build a model characterized with cash-in-advance constraints on consumption and investment goods, and in addition we assume that liquidity constraints also apply to the creation of new establishments. Because efficiency in the use of the factors of production is an important channel influencing output, the model considers establishment heterogeneity along the lines of Hopenhayn (1992), Hopenhayn and Rogerson (1993) and Melitz (2003). In this framework, we are able to analyze the effect of long-run monetary growth on output per worker and we confirm the finding of previous literature that monetary growth has a negative impact on output in a cash-in-advance economy. In addition to discouraging investment and labor supply, an increase in the long-run rate of money growth increases the cost of creating new establishments. As a result, incumbent establishments' profits must increase so as to encourage industry entry. This occurs through a fall in the equilibrium wage-rate. The fall in wages allows new establishments with low

[1] There is substantial evidence of the importance of capital and labor allocation across establishments as a determinant of aggregate productivity. For instance, Baily, Hulten, and Campbell (1992) document that about half of overall productivity growth in U.S. manufacturing in the 1980's can be attributed to factor reallocation from low productivity to high productivity establishments.

[2] See King and Levine (1994), Klenow and Rodriguez-Clare (1997), Prescott (1998), Hall and Jones (1999) and Caselli (2005).

productivity to stay in the industry leading to a reallocation of the factors of production toward less efficient establishments. This adjustment in the size distribution of production plants lowers average productivity in the economy.

We calibrate the model to the U.S. economy and find that increasing the annual inflation rate by 10 percent above the average rate in the U.S. would result in only a modest fall in average productivity (of about 1.3 percent). This result is robust to substantial changes in both parameter values and model specification. Hence, even when the size distribution of productive establishments is endogenous and affected by money growth, the welfare cost of inflation is likely to be modest. We consider several alternative calibrations to the benchmark economy, revealing the importance of the assumptions made regarding the returns to scale and the dispersion of productivities across establishments. We show that heterogeneity is important to understand the impact of inflation on output and welfare. Quantitatively, it may be responsible for almost 1/2 of the effect of inflation on welfare, confirming results by Atkeson et al. (1996) on the importance of heterogeneity and decreasing returns to scale for interpreting cross-country differences in macroeconomic outcomes.

Finally, we show that deflating at the rate of time preference (zero nominal interest rates) yields an equilibrium allocation which is equivalent to the equilibrium allocation obtained without the cash-in-advance constraint and is the optimal policy. Hence, our paper makes a contribution to the literature on optimal policy in monetary economies by showing that the allocation associated with the Friedman Rule is optimal in models with firm entry and exit dynamics and where the size distribution of productive establishments is endogenous, in the context of a cash-in-advance economy of the sort considered by Lucas and Stokey (1987).

The remainder of the paper is organized as follows. In section 2 we lay out the details of our model and describe the stationary competitive equilibrium. In Section 3 we investigate the qualitative effect of changes in the monetary growth rate on the endogenous real aggregates and the size distribution of productive establishments. Section 4 discusses the procedure for calibrating our model and section 5 presents our model-based quantitative findings. Finally, section 6 concludes.

2 The model

We consider a cash-in-advance production economy, which exhibits establishment level heterogeneity as studied by Hopenhayn (1992) and Hopenhayn and Rogerson (2008). Establishments have access to a decreasing returns to scale technology, pay a fixed cost to remain in operation each period and are subject to entry and exit. In what follows we first describe the problem of the household confronted with a cash-in-advance constraint, next we describe the production side in more detail and finally characterize the stationary competitive equilibrium.

2.1 The household

There is an infinitely-lived representative household with preferences over streams of consumption and leisure at each date described by the utility function

$$U = \sum_{t=0}^{\infty} \beta^t \left(\ln C_t + A \ln L_t \right),$$

where C_t is consumption at date t, L_t is leisure and $\beta \in (0, 1)$ is the discount factor. The representative agent is endowed with one unit of productive time each period and has $K_0 > 0$ units of capital at date 0. She owns three types of assets: capital, cash, and production establishments. The mass of (incumbent) establishments at time t is denoted by H_t.

The timing of the household decision problem resembles the one in Stockman (1981). The household enters period t with nominal money balances equal to m_{t-1} that are carried over from the previous period and in addition receives a lump-sum transfer equal to gM_{t-1} (in nominal terms), where M_t is the per capita money supply in period t. Thus, the money stock follows the law of motion

$$M_t = (1 + g) M_{t-1}.$$

Output has three purposes: (i) it can serve as a *consumption good*; (ii) as an *investment good* which increases the stock of capital owned by the household; (iii) as a *marketing good* which has to be purchased in order to create new establishments. Households are required to use their previously acquired money balances to purchase goods. Because we want to compare situations when the constraint applies to some types of good but not to others, we introduce three parameters that we denote by θ_i with $i = c, k, h$. When $\theta_c = 1$ the cash-in-advance constraint applies to the consumption good, when $\theta_k = 1$ purchases of the investment good are constrained and when $\theta_h = 1$ the constraint applies to the marketing good needed to create a new establishment. When $\theta_i = 0$ $(i = c, k, h)$ the constraint does not apply to the specific good and this good is said to be a *credit good* in the Lucas and Stokey (1987) sense. Hence, the constraint reads as

$$\theta_c C_t + \theta_k X_t + \theta_h \kappa E_t \leq \frac{m_{t-1} + gM_{t-1}}{p_t}, \tag{1}$$

where p_t is the price level at time t, X_t is investment, given by

$$X_t = K_{t+1} - (1 - \delta) K_t \tag{2}$$

and κ is the quantity of marketing good that has to be purchased to create each new establishment and constitutes a sunk cost. E_t is the mass of new establishments created.

The representative household must choose consumption, investment, leisure, nominal money holdings and the mass of new establishments subject to the cash-in-advance constraint (1) and the budget constraint

$$C_t + X_t + \kappa E_t + \frac{m_t}{p_t} \leq w_t (1 - L_t) + r_t K_t + \bar{z}_t H_t + (m_{t-1} + gM_{t-1}) / p_t, \tag{3}$$

where w_t is the wage rate, r_t the interest rate and $\bar{z}_t$ are average dividends across incumbent establishments.

We assume that the gross growth rate of money, $1 + g$, always exceeds the discount factor, β, which is a sufficient condition for (1) to always bind in equilibrium and existence of a stationary equilibrium[3]. We sometimes denote real money balances by $\mu_t = \frac{m_t}{p_t}$.

2.2 Production establishments

Once a new establishment is created at t, its idiosyncratic productivity $s \in S$ is revealed as drawn from a distribution $F(s)$ and remains constant over time until the establishment exits the industry. At $t + 1$ the establishment starts production. Incumbent establishments produce output by renting labor and capital. The production function of an establishment with idiosyncratic productivity s at time t is

$$y_{s,t} = s n_{s,t}^{\alpha} k_{s,t}^{\nu} - \eta, \tag{4}$$

[3]See Abel (1985).

where $n_{s,t}$ and $k_{s,t}$ are labor and capital employed, η is a fixed operating cost, $\alpha \in (0,1)$, $\nu \in (0,1)$ and $\nu + \alpha < 1$. The flow profits of an incumbent establishment are given by

$$z_{s,t} = \max_{n_{s,t},k_{s,t}} \left\{ s n_{s,t}^{\alpha} k_{s,t}^{\nu} - w_t n_{s,t} - r_t k_{s,t} - \eta \right\}, \tag{5}$$

where w_t is the wage rate and r_t is the interest rate.

Establishments exit both because of exogenous exit shocks and endogenous decisions. In particular, in any given period after production takes place, each establishment faces a constant probability of death equal to λ. Moreover, an establishment decides to leave the industry if its discounted profits are negative. Given that we only analyze the stationary equilibrium of the economy and idiosyncratic productivities are constant over time, it turns out that the only moment when an establishment decides to leave the industry is upon entry. This is because profits are constant over time in the stationary equilibrium. Consequently, establishments choose to exit when

$$z_s < 0.$$

We denote by s^* the idiosyncratic productivity threshold below which establishments choose to exit. Specifically, s^* is such that $z_{s^*} = 0$.

Given the first order conditions which solve the problem of incumbent firms (5) the labor demand by an establishment with productivity s is

$$n_{s,t} = s^{\sigma} \left(\frac{\alpha}{w_t}\right)^{(1-\nu)\sigma} \left(\frac{\nu}{r_t}\right)^{\nu\sigma} \tag{6}$$

and the demand for capital reads

$$k_{s,t} = s^{\sigma} \left(\frac{\alpha}{w_t}\right)^{\alpha\sigma} \left(\frac{\nu}{r_t}\right)^{(1-\alpha)\sigma}, \tag{7}$$

where $\sigma = (1-\alpha-\nu)^{-1}$. Replacing the factor demands into the profit function yields

$$z_{s,t} = \Omega \frac{s^{\sigma}}{w_t^{\alpha\sigma} r_t^{\nu\sigma}} - \eta, \tag{8}$$

where $\Omega = \alpha^{\alpha\sigma}\nu^{\nu\sigma} - \alpha^{(1-\nu)\sigma}\nu^{\nu\sigma} - \alpha^{\alpha\sigma}\nu^{(1-\alpha)\sigma}$.

Let $h(s;t)$ denote the mass of incumbent establishments with productivity level s at time t. The motion equation for $h(s;t)$ is given by

$$h(s;t+1) = (1-\lambda)h(s;t) + E_t dF(s) I[s \geq s^*], \tag{9}$$

where I is an indicator function that takes value one if the expression in brackets is true and zero otherwise. With $H_t = \int_{s \in S} h(s;t)ds$ denoting the mass of incumbent establishments. Consequently, firm entry (the mass of start-ups) reads

$$E_t = \frac{H_{t+1} - (1-\lambda) H_t}{1 - F(s^*)}. \tag{10}$$

Finally, following Melitz (2003), it is useful to define average productivity as

$$\bar{s}_t = \left\{ \int_{s \geq s^*} s^{\sigma} \frac{dF(s)}{1 - F(s^*)} \right\}^{\frac{1}{\sigma}}. \tag{11}$$

Hence, with knowledge of s^* one can identify $\bar{s}_t$. From equation (8), this implies that average dividends write as

$$\bar{z}_t = \int_{s \geq s^*} z_{s,t} \frac{dF(s)}{1 - F(s^*)} ds = \Omega \frac{\bar{s}_t^{\sigma}}{w_t^{\alpha\sigma} r_t^{\nu\sigma}} - \eta. \tag{12}$$

2.3 Household optimal behavior

The Bellman equation characterizing household's optimal behavior writes as

$$V(m_{t-1}, K_t, H_t) = \max_{C_t, L_t, m_t, K_{t+1}, H_{t+1}} \left\{ \ln C_t + A \ln L_t + \beta V(m_t, K_{t+1}, H_{t+1}) \right\}, \tag{13}$$

and is subject to the cash-in-advance constraint (1) and the budget constraint (3).

Let ϕ_t and γ_t be the Kuhn-Tucker multipliers for the constraints (1) and (3), respectively. The first-order conditions which characterize the solution to the problem of the household are

$$\frac{1}{C_t} - \theta_c \phi_t - \gamma_t = 0, \tag{14}$$

$$\frac{A}{L_t} - \gamma_t w_t = 0 \tag{15}$$

$$\beta V_1(m_t, K_{t+1}, H_{t+1}) - \frac{\gamma_t}{p_t} = 0, \tag{16}$$

$$\beta V_2(m_t, K_{t+1}, H_{t+1}) - \theta_k \phi_t - \gamma_t = 0, \tag{17}$$

$$\beta V_3(m_t, K_{t+1}, H_{t+1}) - \frac{\kappa}{1 - F(s_t^*)} (\theta_h \phi_t + \gamma_t) = 0, \tag{18}$$

plus the budget constraint and the complementary slackness condition associated with the budget constraint. Moreover, by the envelope theorem, the shadow values of money, capital and the mass of establishments are respectively

$$V_1\left(m_{t-1}, K_t, H_t\right) = \frac{\phi_t + \gamma_t}{p_t}, \tag{19}$$

$$V_2\left(m_{t-1}, K_t, H_t\right) = (1-\delta)\left(\theta_k \phi_t + \gamma_t\right) + \gamma_t r_t. \tag{20}$$

and

$$V_3\left(m_{t-1}, K_t, H_t\right) = \frac{1-\lambda}{1-F(s_t^*)} \kappa\left(\theta_h \phi_t + \gamma_t\right) + \gamma_t \bar{z}_t. \tag{21}$$

Combining (19), (20) and (21) and the first-order conditions (16), (17) and (18) yields the three Euler equations

$$\beta \frac{\phi_{t+1} + \gamma_{t+1}}{p_{t+1}} - \frac{\gamma_t}{p_t} = 0, \tag{22}$$

$$\beta\left(1-\delta\right)\left(\theta_k \phi_{t+1} + \gamma_{t+1}\right) + \beta \gamma_{t+1} r_{t+1} - \theta_k \phi_t - \gamma_t = 0 \tag{23}$$

and

$$\beta \frac{1-\lambda}{1-F(s_{t+1}^*)} \kappa\left(\theta_h \phi_{t+1} + \gamma_{t+1}\right) + \beta \gamma_{t+1} \bar{z}_{t+1} - \kappa \frac{\theta_k \phi_t + \gamma_t}{1-F(s_t^*)} = 0. \tag{24}$$

Equations (14) and (22)-(24), combined with the intra-temporal first-order condition (15) and the budget constraint (3) characterize the solution to the household problem.

2.4 Market clearing

Market clearing conditions for labor and capital are given, respectively, by

$$N_t = \int_{s \in S} n_{s,t} h(s;t) ds \tag{25}$$

and

$$K_t = \int_{s \in S} k_{s,t} h(s;t) ds. \tag{26}$$

Market clearing in the money market requires

$$m_t = M_t. \tag{27}$$

Finally, the economy's feasibility constraint reads

$$C_t + X_t + \kappa E_t = Y_t, \tag{28}$$

where $Y_t \equiv \int_{s \in S} y_{s,t} h(s;t) ds$.

2.5 Stationary equilibrium

We consider the steady-state competitive equilibrium of the model. In a steady-state equilibrium, all rental rates and real aggregates are constant over time. Moreover, the gross rate of inflation $\Pi \equiv \frac{P_{t+1}}{P_t}$ is also constant, equal to the gross rate of monetary growth $1+g$. Thus, we henceforth ignore all time subscripts to simplify the notation.

We now illustrate three effects of inflation related to the three cash-in-advance constraints of the economy.

Since the shadow values ϕ and γ are each positive and constant in the steady-state[4], from equations (14), (15) and (22), consumption and leisure in the steady-state equilibrium satisfy the condition

$$\frac{L}{C} = \frac{A}{w}\left[1+\theta_c\left(\frac{1+g}{\beta}-1\right)\right]. \tag{29}$$

Equation (29) suggests that, when the cash-in-advance constraint applies to consumption, an increase in inflation raises the cost of consumption relative to leisure. This result corresponds to the effect examined in Cooley and Hansen (1989).

Given equations (22) and (23), the representative household problem yields the stationary equilibrium rental rate of capital, given by

$$r = \left(\frac{1}{\beta}-1+\delta\right)\left[1+\theta_k\left(\frac{1+g}{\beta}-1\right)\right] \tag{30}$$

Equation 30 shows that the rental cost of capital is increasing in the rate of anticipated inflation when the cash-in-advance constraint applies to the capital-good. It also suggests the following mechanism. When the cash-in-advance constraint applies to investment, inflation increases the cost of holding money balances, which reduces capital accumulation. As a result, at higher inflation, the rental cost of capital is higher. This result is due to Stockman (1981).

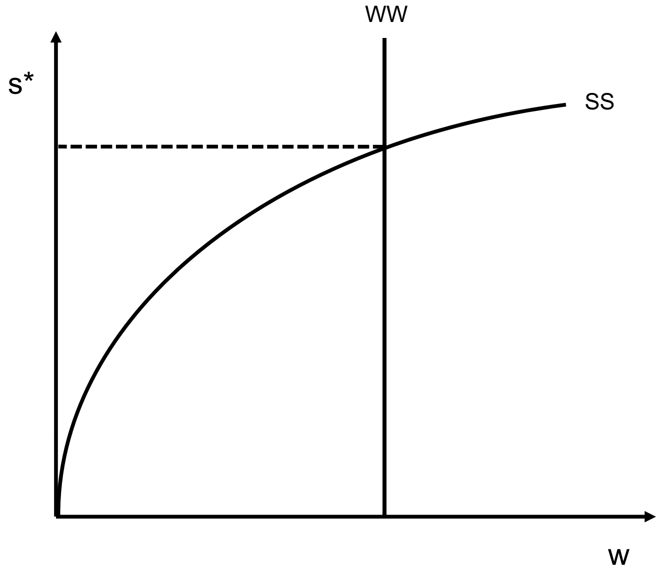
Finally, from equations (22) and (24) a free-entry condition reads

$$\kappa\left[1+\theta_h\left(\frac{1+g}{\beta}-1\right)\right] = \left[1-F(s^*)\right]\frac{\beta\bar{z}}{1-\beta(1-\lambda)}. \tag{31}$$

Equation (31) states that in equilibrium the sunk cost that has to be paid to create a new establishment (the left-hand side of (31)) has to be equal to the expected discounted profits from creating this establishment (the right-hand side of (31)). The rate of discount

[4]See Stockman (1981).

Figure 1: Determination of s^* and w in the stationary economy



of profits depends on the household discount factor β and the probability λ that the new establishment dies in future periods. The probability $[1 - F(s^*)]$ also appears on the right-hand side of (31) because one has to account for the probability of successful entry when evaluating discounted profits.

Equation (31) characterizes the mechanism by which money growth affects firm entry and exit dynamics. When the cash-in-advance constraint applies to the sunk cost, an increase in inflation makes entry more costly. The next Section shows that this has an effect on average productivity too.

Hence, inflation may have three effects, depending on the structure of the cash-in-advance constraint. It may affect labor supply, capital accumulation and the productivity distribution of incumbent establishments. Each effect contributes to lowering the level of output. This allows us in the next Section to state a Proposition on the real effects of inflation. Before doing this, we go through the remaining relations characterizing the equilibrium.

In the stationary competitive equilibrium the optimal exit rule by incumbent establishments requires $z_{s^*} = 0$. This yields a solution for the productivity threshold, given by

$$s^* = w^{\alpha} r^{\nu} \left(\frac{\eta}{\Omega}\right)^{1-\alpha-\nu}. \tag{32}$$

Since the equilibrium interest rate is determined by (30), the exit condition characterizes a relation between the wage rate and the productivity threshold which is represented by the SS locus in Figure 1.

In turn, the expected value of entry, i.e. the right hand side of the free entry condition (31) is locally independent of s^* by the envelope theorem (see Appendix for proof). Consequently, the equilibrium wage rate is independent of s^*, as illustrated by the WW locus in Figure 1. Hence, in an equilibrium with production the free-entry condition determines the wage rate. In particular, consider the comparative statics of moving from a stationary equilibrium with low sunk cost κ to an equilibrium with high sunk cost. For there to be an equilibrium with entry, firms' expected value of entry must increase. Since the rental cost of capital remains unchanged firms are not willing to enter the industry unless the wage rate falls. Accordingly the WW locus has to shift to the left which translates in a movement along the SS curve. This in turn leads to a lower productivity threshold. To examine the impact of an increase in the monetary growth rate g when the marketing good is a *cash good* ($\theta_h = 1$) but investment is a *credit good* ($\theta_k = 0$) requires exactly the same comparative statics[5].

Finally, solving for the fixed point of (9) and integrating over productivity levels yields

$$H = E \int_{s \in S} \frac{I[s \geq s^*]}{\lambda} dF(s), \tag{33}$$

which, combined with the resource constraint (28), gives a solution for the mass of incumbent establishments, completing the characterization of the stationary competitive equilibrium. Specifically, the stationary competitive equilibrium is defined as follows:

Definition 1. *A stationary competitive equilibrium is a wage rate w, a rental rate of capital r, an aggregate distribution of establishments $h(s)$, a mass of entry E, a household value function $V(m, K, H)$, an establishment profit function z_s, a productivity threshold s^*, policy functions for incumbent establishments n_s and k_s and aggregate levels of consumption C, employment N, capital K and real money balances μ, such that:*

i. The household optimizes: equations (29), (30) *and* (31)*;*

ii. Establishments optimize: equations (6), (7) *and* (32)*;*

[5]See Figure 3

iii. Markets clear: equations (25)*,* (26)*,* (27) *and* (28)*;*

iv. $h(s)$ *is an invariant distribution, i.e. a fixed point of* (9)*.*

To summarize, the model is solved as follows. First, the rental cost of capital is pinned down by equation (30). Then, given the value of r, one can solve for the values of the wage rate w and the productivity threshold s^* from (31) and (32). One can consequently characterize fully the stationary distribution of capital, employment, profits and output with equations (4), (6), (7) and (8) across incumbent firms. Finally, the feasibility constraint (28), together with the other market-clearing conditions and the first-order condition for leisure (29), allow to determine the mass of incumbents H and all the aggregates of the economy such as investment, consumption, output, the stock of capital and employment[6].

3 The real effects of inflation

We now investigate the relation between inflation, the equilibrium aggregates K and N, and the size distribution of productive establishments, characterized by s^*. Proposition 1 summarizes our main result

PROPOSITION 1. *Consider the stationary competitive equilibrium as defined earlier.*

i. If $\theta_c = \theta_k = \theta_h = 0$*, an increase in the inflation rate* Π *has no effect on the economy.*

ii. If there exists at least one θ_i*, with* $i = c, k$*, that takes value one and* $\theta_h = 0$*, an increase in the inflation rate* Π *is associated with a fall in the equilibrium capital stock* K *and a fall in the employment rate* N*. However, the productivity threshold,* s^**, does not change.*

iii. If $\theta_h = 1$*, an increase in the inflation rate* Π *is associated with a fall in the equilibrium capital stock* K*, a fall in the employment rate* N *and a fall in the productivity threshold,* s^**.*

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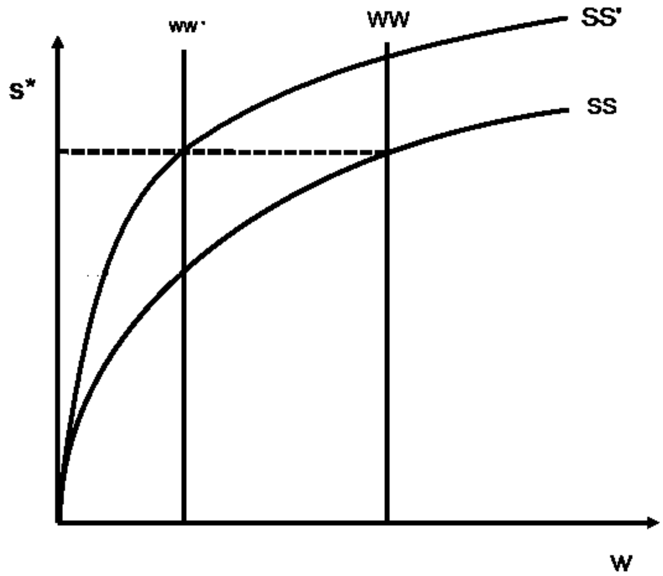
[6]In the Appendix, we present all the equations that characterize the stationary equilibrium for the particular restriction that we impose on the distribution F. See also Section 4, where we describe the calibration procedure.

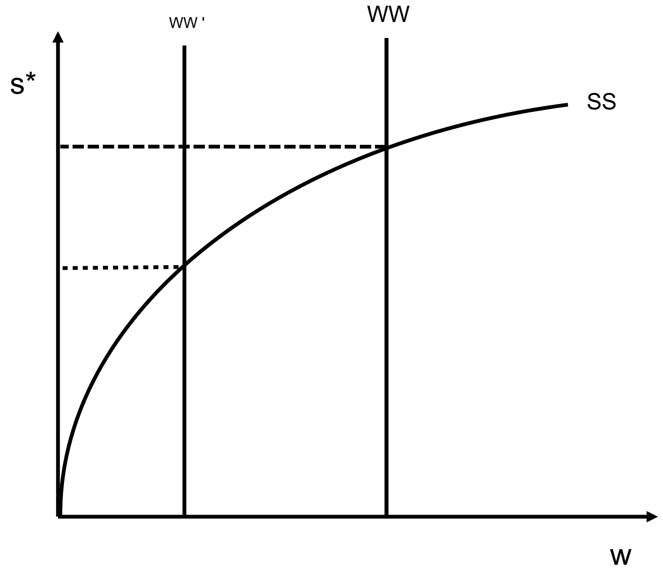
Figure 2: Effect of an increase in the monetary growth rate g on s^* and w when $\theta_h = 1$



In what follows we discuss some aspect related to Proposition 1, however, the detailed proof is developed in the Appendix. When $\theta_i = 0$ for all i, all goods are *credit goods* and therefore money growth has no real effects. When consumption is a *cash good* condition (29) is affected by money growth. At high rates of inflation, the marginal utility of leisure must fall with respect to the product of the wage rate and the marginal utility of consumption, leading the household to supply less labor. Lower hours worked leads to lower output and therefore lower consumption and capital stock. The rental cost of capital, determined by (30), remains the same and, therefore both the SS relation and the WW relation are unaffected. Thus the wage rate and average productivity are unaffected.

When $\theta_k = 1$, i.e. investment is a *cash good*, condition (30) is affected. At high rates of inflation the return on capital must increase as individuals are less willing to invest. The increase in the rental cost of capital lowers profits for the same wage rate and therefore the probability of a successful entry decreases at each wage rate (i.e. the SS locus in Figure 2 shifts upward). However the probability of successful entry must remain unchanged in equilibrium since the cost of creating a new establishment (the left-hand side of equation (31)) has not changed. Thus, for there to be an equilibrium with entry, the wage rate must fall sufficiently for the free entry condition to be satisfied. The WW locus in Figure 1 shifts left. At high rates of inflation the wage rate is lower and the

Figure 3: Effect of an increase in the monetary growth rate g on s^* and w when $\theta_h = 1$



average productivity and the probability of successful entry are unaffected, as illustrated by Figure 2.

When the marketing good is a *cash good*, $\theta_h = 1$, the liquidity constraint increases the cost of creating new establishments and the comparative static is the same as the one corresponding to an increase in the sunk cost, illustrated in Figure 3. At high rates of inflation, the wage rate must fall so that there is firm entry in equilibrium. Thus, the WW locus shifts to the left which translates in a movement along the SS curve. This in turn leads to a lower productivity threshold.

4 Calibration

In this section we describe the model calibration procedure. In order to solve our model we need to specify a distribution for the establishments' productivity draws $F(s)$. Following Helpman et al. (2004), we assume a Pareto distribution for F with lower bound s_0 and shape parameter $\varepsilon > \sigma$, i.e. $F(s) = 1 - \left(\frac{s_0}{s}\right)^{\varepsilon}$. The shape parameter is an index of the dispersion of productivity draws: dispersion decreases as ε increases, and the productivity draws are increasingly concentrated toward the lower bound s_0. This assumption has two advantages: it generates a distribution of idiosyncratic productivities among incumbent

Table 1: Parameters: summary

Notation	Value	Parameter
g	0.0243	Monetary growth rate
α	0.5667	Labor income share
ν	0.2833	Capital income share
δ	0.0956	Depreciation rate of capital
β	0.9775	Household's discount factor
ε	0.2338	Pareto distribution shape parameter
λ	0.0696	Failure rate of incumbent establishments
s_0	1	Pareto distribution lower bound
κ	1	Sunk entry cost
A	2.4199	Disutility of labor

establishments that fits microeconomic data quite well[7] and delivers close-form solutions for the endogenous aggregates[8]. Specifically, the distribution of productivities among incumbent establishments, which is the distribution F truncated at $s = s^*$, is also Pareto with lower bound s^* and shape parameter ε.

We calibrate the model to data for the United States. The length of each period is one year. The growth rate of the money supply g is chosen to be 2.43 percent which matches the average annual rate of inflation in the U.S. between 1988 and 2007, reported in the World Economic Indicators database. For labor and capital income shares, α and ν repectivelly, empirical evidence concerning establishment level returns to scale, reported by Atkenson, Khan and Ohanian (1996) and Atkeson and Kehoe (2005) suggests the relation $\alpha + \nu = 0.85$. The separate identification of α and ν is done according to the income shares of labor and capital. Based on Gomme and Rupert (2007) we assign 28.3 percent to capital and the remainder to labor, yielding $\alpha = 0.5667$ and $\nu = 0.2833$.

The annual depreciation rate δ is chosen to be 9.56 percent based on evidence from the BEA as reported in Gomme and Rupert (2007). In particular, Gomme and Rupert (2007)

[7] See Axtell (2001) and Cabral and Mata (2003).

[8] See the Appendix for the complete description of the model solution.

Table 2: Calibration: targets

Target	Value
U.S.average annual inflation rate (1988-2007)	0.0243
Production function returns to scale	0.85
$\frac{\nu}{\nu+\alpha}$	0.283
Investment/GDP (net of government expenditure)	0.1851
Standard deviation of log U.S.plant sales	1.67
Manufacturing establishments (1-5 years old) failure rates	0.397
Manufacturing establishments (6-10 years old) failure rates	0.303
Hours-work (rate)	0.255

distinguish between capital depreciation of market structures and capital depreciation of equipment and software. The 9.56 percent correspond to the weighted average of the depreciation rate of each component according to their share in GDP. Given the depreciation rate, the rental cost of capital r is chosen to match the investment-output ratio, given by $\frac{X}{Y} = \frac{\delta}{\nu} r$. The implied rental cost of capital return is 12.42 percent, which requires the discount factor $\beta = 0.9775$. Notice that the investment-output ratio is calculated with output net of government expenditure.

Following Ghironi and Melitz (2005), we choose the shape parameter of the F distribution in order to match the standard deviation of log U. S. plant sales, which in our case is also output and is reported to be 1.67 in Bernard et al. (2003). Since in our model, this standard deviation is $\frac{1}{\varepsilon-\sigma}$, this implies that the value for ε is 7.27.

The establishments death rate λ is chosen based on empirical evidence reported in Dunne et al. (1989). These authors perform an empirical investigation of establishment turnover using data on plants that first began operating in the 1967, 1972, or 1977 Census of Manufacturers, a rich source of information concerning the U.S. manufacturing sector. They report five-year exit rates among plants 1-5 year old (39.7 percent), 6-11 year old (30.3 percent) and older (25.5 percent). As expected, plant failure rates decline with age. We assume entering establishments do not produce in the first year but simply discover their productivity level. Thereafter, establishments choosing not to exit the industry only exit when hit by the exogenous exit-shock. Thus, we decompose the five-year failure rate of young firms (1-5 years) into two components,

$$0.397 = F(s^*) + [1 - F(s^*)]\, \mathcal{B}_{4,1-\lambda}(3)\,, \tag{34}$$

where $\mathcal{B}_{4,1-\lambda}(3)$ the cumulative probability of 3 successes associated with the binomial distribution with 4 draws and success probability $1 - \lambda$. The first term on the right-hand side of (34) is the probability of an establishment drawing a low productivity level and decide to exit. The second term is the probability of an incumbent establishment dieing over the four following years. This yields an equation in s^* and λ. The value for λ is set to match the failure rate of older incumbent firms (6-11 year old), by solving

$$0.303 = \mathcal{B}_{5,1-\lambda}(4)\,. \tag{35}$$

This yields $\lambda = 0.0696$. Equipped with λ we use equation (34) to find a relation between s^* and s_0. However, s_0 can be normalized to 1 without loss of generality because it has no impact on the endogenous exit-decision of new establishments. This yields a solution for s^*.

Finally, A, the parameter measuring the disutility of labor, is chosen so that the household spends 25.5 percent of its endowment of time working, based on Gomme and Rupert (2007), who account for retired people and interpret evidence from the American Time-use Survey accordingly.

This completes the calibration description. Table 1 summarizes the parameter values and Table 2 the targets informing our choices.

5 Results

We use the model economy just described to examine the interaction between money and the real sector of the economy. We first compare alternative steady states, describing how the macroeconomic aggregates, including output, consumption, investment and aggregate hours, and average productivity vary with respect to a benchmark level at various rates of money growth. We then use data from OECD countries on output and capital per worker to determine the model ability to explain cross-country evidence. Finally, we use the model to measure the welfare costs of anticipated inflation under alternative model specifications.

5.1 Steady State Properties

The model baseline parametrization is characterized by a monetary growth rate of 4.87 percent. This value corresponds to the average inflation rate in the U.S. between 1970 and 1996[9]. Accordingly, Tables 3 and 4 report the log deviation of each macroeconomic aggregate of interest and of average productivity with respect to the levels corresponding to the benchmark steady state. We will begin by interpreting the results in each table.

Table 3 corresponds to model specifications where $\theta_h = 1$ and hence the marketing good is a *cash good*. The Table includes four Panels, each corresponding to an alternative configuration of the cash-in-advance constraints. When the liquidity constraint applies to the creation of new productive establishments , the size distribution of productive of incumbents moves toward lower productivity levels at higher monetary growth rates. Hence, the average productivity of incumbent establishments is lower at high rates of inflation. The bottom row of each Panel of Table 3 reports the level of average productivity at various rates of money growth. When all goods are cash goods (Panel A) productivity falls by 1.3 percent when the rate of money growth is 15 percent, which is roughly 10 percentage points above the average U.S. rate of inflation. Instead, by moving from the benchmark monetary rule to the optimal money growth rule ($g = 1 - \beta$) productivity would increase in steady state by 1 percent. Moreover, inspecting each panel reveals that the money growth rule affects productivity in the same way for each possible configuration of the cash-in-advance constraint as long as $\theta_h = 1$. Thus, the monetary growth rate has a clear, although modest, impact on average productivity.

The results regarding the other macroeconomic aggregates are of course more sensitive to the model specification. Examining Panel A of Table 3 again reveals that, when all goods are *cash goods*, the change in the steady state levels of investment and output associated with the optimal money growth rule with respect to the benchmark money rule are 18 percent and 11 percent respectively. These adjustments are less substantial when capital is a *credit good* but consumption is a *cash good* as shown in Panel B. The results in Panel C are roughly the same as the results in Panel A which suggests a prominent role for the liquidity

[9]The price inflation data is collected from the World Bank Development Indicators (WBDI). This sample period was chosen because the cross-section data on capital and output, which corresponds to the data used in Caselli (2005), is from 1996.

Table 3: Steady States Associated with Various Annual Monetary Growth Rates in Log-deviation From Benchmark when the Marketing Good is a *Cash Good*, i.e.: $\theta_h = 1$

	Panel A: $\theta_c = 1$ and $\theta_k = 1$					Panel B: $\theta_c = 1$ and $\theta_k = 0$				
100×										
g	$\beta - 1$	0.00	4.87*	10	15	$\beta - 1$	0	4.87*	10	15
Output	0.110	0.074	0.000	-0.075	-0.144	0.073	0.046	0.000	-0.047	-0.091
Consumption	0.091	0.062	0.000	-0.063	-0.122	0.073	0.046	0.000	-0.047	-0.091
Investment	0.180	0.122	0.000	-0.122	-0.237	0.073	0.046	0.000	-0.047	-0.091
Hours	0.066	0.045	0.000	-0.045	-0.087	0.056	0.035	0.000	-0.036	-0.070
Productivity	0.010	0.007	0.000	-0.007	-0.013	0.010	0.007	0.000	-0.007	-0.013
	Panel C: $\theta_c = 0$ and $\theta_k = 1$					Panel D: $\theta_c = 0$ and $\theta_k = 0$				
100×										
g	$\beta - 1$	0.00	4.87*	10	15	$\beta - 1$	0	4.87*	10	15
Output	0.058	0.039	0.000	-0.039	-0.074	0.017	0.011	0.000	-0.011	-0.021
Consumption	0.039	0.026	0.000	-0.027	-0.052	0.017	0.011	0.000	-0.011	-0.021
Investment	0.128	0.086	0.000	-0.086	-0.166	0.017	0.011	0.000	-0.011	-0.021
Hours	0.014	0.009	0.000	-0.009	-0.017	0.000	0.000	0.000	0.000	0.000
Productivity	0.010	0.007	0.000	-0.007	-0.013	0.010	0.007	0.000	-0.007	-0.013

Notes: * average U.S. inflation rate over the 1970-1996 period. The steady states are shown in log-deviation from the benchmark model which corresponds to the economy where the monetary growth rate is given by the U.S. average inflation rate.

constraint on the investment good. Finally, Panel D is of interest because it illustrates that when the liquidity constraint only applies to the *marketing good* changes in the monetary growth rate have qualitatively the same effects although these are quantitatively small. This suggests that the cash-in-advance constraint may amplify the burden of inflation when it distorts firm entry and exit dynamics.

Table 4 corresponds to model specifications where $\theta_h = 0$ and hence the marketing good is a *cash good*. Examining each Panel and comparing it to the corresponding Panel in Table 3 indicates that, although the variations across money growth rates are of the same order of magnitude, they are considerably smaller when the sunk cost is not subject to the liquidity constraint. This confirms the amplification role played by the distortion on the firms' entry and exit dynamics. In particular, Panel B in Table 4 shows that when only consumption is a cash good, moving from the benchmark money rule to the optimal money rule increases consumption by just 5.6 percent. When instead the liquidity constraint applies to the sunk entry cost (Table 3) the impact is 30 percent greater. Comparing Panels C from each Table reveals that the increase in consumption associated with the adoption of the optimal money growth rule when investment is a *cash good* and consumption is a *credit good* is 70 percent higher if the cash-in-advance constraint applies to the sunk entry

Table 4: Steady States Associated with Various Annual Monetary Growth Rates in Log-deviation From Benchmark when the Marketing Good is a *Credit Good*, i.e.: $\theta_h = 0$

	Panel A: $\theta_c = 1$ and $\theta_k = 1$					Panel B: $\theta_c = 1$ and $\theta_k = 0$				
100×										
g	$\beta - 1$	0.00	4.87*	10	15	$\beta - 1$	0	4.87*	10	15
Output	0.094	0.064	0.000	-0.064	-0.124	0.056	0.035	0.000	-0.036	-0.070
Consumption	0.075	0.051	0.000	-0.052	-0.101	0.056	0.035	0.000	-0.036	-0.070
Investment	0.164	0.111	0.000	-0.112	-0.216	0.056	0.035	0.000	-0.036	-0.070
Hours	0.066	0.045	0.000	-0.045	-0.087	0.056	0.035	0.000	-0.036	-0.070
Productivity	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	Panel C: $\theta_c = 0$ and $\theta_k = 1$					Panel D: $\theta_c = 0$ and $\theta_k = 0$				
100×										
g	$\beta - 1$	0.00	4.87*	10	15	$\beta - 1$	0	4.87*	10	15
Output	0.042	0.028	0.000	-0.028	-0.053	0.000	0.000	0.000	0.000	0.000
Consumption	0.023	0.016	0.000	-0.016	-0.031	0.000	0.000	0.000	0.000	0.000
Investment	0.112	0.076	0.000	-0.076	-0.145	0.000	0.000	0.000	0.000	0.000
Hours	0.014	0.009	0.000	-0.009	-0.017	0.000	0.000	0.000	0.000	0.000
Productivity	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

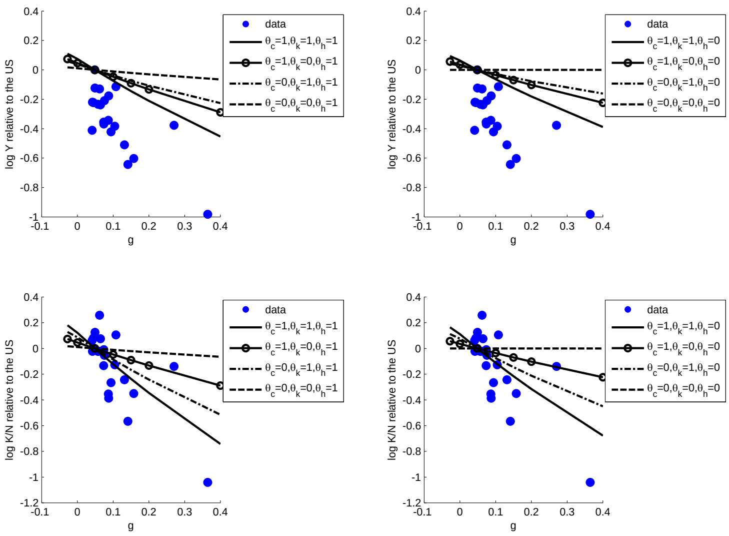
Notes: * average U.S. inflation rate over the 1970-1996 period. The steady states are shown in log-deviation from the benchmark model which corresponds to the economy where the monetary growth rate is given by the U.S. average inflation rate.

cost. This illustrates clearly the gains from improvements in the allocation of the factors of production. Changes in the size distribution of incumbent establishments may therefore amplify the welfare cost of inflation because of the lost efficiency in the allocation of productive factors. Panel D in Table 4 simply illustrates that the cash-in-advance constraints are the single source of money non-neutrality.

5.2 The Model Empirical Fit

As just shown, our model predicts that anticipated inflation has a significant influence on the economy's steady-state. In particular, steady-state output and the capital stock fall as the growth rate of the money supply rises above the optimal level ($g = \beta - 1$). Moreover, when the cash-in-advance constraint applies to the sunk entry cost, average productivity is also predicted to fall as firms entry and exit dynamics are distorted. Here we consider cross-section data on both output and capital per worker, and inflation rates for a sample of OECD countries. The data on output per worker and capital per worker are as reported in Caselli (2005). The purpose is to compare the empirical relations to the predictions of the model as a *first pass* test to the goodness of fit of the model. Figure 4 illustrates the relations between output per worker and inflation and capital per worker and inflation.

Figure 4: Output and Capital vs Inflation Rate across OECD Countries and Model Fit



Note: The upper panels represent the relation between output per worker and inflation and the lower panels the relation between capital per worker and inflation in the semi-log scale. The left-hand side panels compare it to model specifications where $\theta_h = 1$ and the right-hand side panels compare it to model specifications where the $\theta_h = 0$.

The upper Panels both illustrate the relation between output per worker and inflation and compare it to the predictions of the model, for different specifications of the borrowing constraint. Inspection of Figure 4 suggests the model performs well when all cash-in-advance constraints apply. A more formal examination requires that we consider first the empirical relation between output and inflation and between the capital stock and inflation. The 95 % confindence interval for the regression coefficient on inflation corresponding to the linear projection of the logarithm of output on inflation and an intercept is $(-2.94, -1.26)$ with an associated R^2 of 0.58. Similarly, the 95 % confindence interval for the regression coefficient on inflation corresponding to the linear projection of the logarithm of capital per worker on inflation and an intercept is $(-3.85, -1.64)$ with an associated R^2 of 0.57. In turn, the coefficients on inflation associated to the best linear fit of the model predicted relation between output and inflation and capital and inflation are -1.32 and -2.17, respectivelly. Both estimates are inside the respective confidence interval characterizing the empirical relation. These findings represent a good *first pass* for testing the model goodness of fit and therefore support the view that inflation causes output.

Table 5: Welfare Costs Associated with Various Annual Growth Rates of Money

	$\theta_h = 1$				$\theta_h = 0$			
	$\theta_c = 1$	$\theta_c = 1$	$\theta_c = 0$	$\theta_c = 0$	$\theta_c = 1$	$\theta_c = 1$	$\theta_c = 0$	$\theta_c = 0$
100× g	$\theta_k = 1$	$\theta_k = 0$	$\theta_k = 1$	$\theta_k = 0$	$\theta_k = 1$	$\theta_k = 0$	$\theta_k = 1$	$\theta_k = 0$
100× $(\beta - 1)$	3.49	2.52	2.72	1.73	1.81	0.73	1.09	0.00
0	2.39	1.59	1.84	1.08	1.28	0.50	0.75	0.00
4.87*	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
10	-2.46	-1.63	-1.86	-1.07	-1.46	-0.61	-0.81	0.00
15	-4.71	-3.11	-3.56	-2.04	-2.91	-1.25	-1.59	0.00
20	-6.97	-4.62	-5.27	-3.00	-4.45	-1.97	-2.42	0.00
40	-14.65	-9.89	-11.24	-6.32	-10.21	-4.94	-5.54	0.00

Note: * average U.S. inflation rate over the 1970-1996 period. The measure of the welfare cost of inflation is $\Delta C/C \times 100$ where ΔC is the consumption compensation needed for the representative agent to achieve the same steady state utility associated to the U.S. average rate of inflation.

5.3 Welfare Costs of Inflation

To obtain a measure of the welfare cost associated with inflation we proceed in the same way as in Cooley and Hansen (1989) with the single difference that we consider as a benchmark for the monetary growth rate the average rate of inflation for the U.S. instead of considering the optimal money rule. We do so, because it allows us to characterize a more immediate way what would be the benefit from adopting optimal policy and it also allows us to consider the welfare loss if inflation rates increased by 10 percent compared to the average for our sample period.

To compute the welfare cost associated with variations of money growth around its benchmark value, we solve for ΔC in the equation

$$\bar{U} = \ln(C^* + \Delta C) + A \ln(1 - N^*), \tag{36}$$

where $\bar{U}$ is the level of utility attained under the benchmark rate of growth of money, $g = 4.87$, and C^* and N^* are the steady-state consumption and hours associated with the alternative money growth rule. The results of the welfare calculations are expressed as a percent of steady-state consumption ($\Delta C/C^*$), as in Cooley and Hansen. Table 5 shows our findings. The left-hand side Panel corresponds to the specifications where the cash-in-advance constraint applies to the entry sunk cost and the right-hand side Panel consider the other cases. The welfare costs of inflation are of the same order of magnitude as in Cooley and Hansen although they are uniformly larger. We consider first the specification where consumption is the single *cash good* because this corresponds more closely to the Cooley and Hansen model. In this specification, the welfare cost of a 10 percent rate of inflation, relative to the benchmark of $g = 4.87$, is 0.6 percent of steady state consumption. The welfare gain associated with moving from the benchmark money growth rule to the optimal rule is 0.73 percent of steady state consumption. These numbers are roughly three times as large as the ones reported in Cooley and Hansen, even if average productivity is not distorted by monetary policy. When both consumption and investment are *cash goods* but the marketing good is a *credit good*, the welfare cost estimates roughly double. For example, the welfare gain associated with adopting the optimal policy becomes 1.81 percent of consumption. If only investment is a cash good the welfare gain is just over 1 percent.

Finally, when the cash-in-advance constraint applies to the sunk entry cost, the welfare cost of inflation increases substantially. For example, the welfare gain associated with adopting the optimal policy corresponds to 3.49 percent of consumption. This number is already about an order of magnitude greater than the findings in Cooley and Hansen. Moreover, it seems that between 1/3 and 1/2 of the welfare cost of inflation is driven by the distortions to firm entry and exit dynamics. Thus, a substantial part of the welfare losses at high rates of inflation are explained by less efficiency in the allocation of resources across incumbent establishments and not just by less accumulation of factors of production.

The wage rate is often a convenient measure of welfare. Figure 5 illustrates how small movements in productivity are associated with strong movements in the wage rate. Thus, even if high rates of inflation are associated with only modest falls in average productivity, the welfare loss is important because the fall in the wage rate is strong. The movements in the wage rate are largely driven by the firm entry and exit dynamics. Thus, having an endogenous distribution of productive establishments is important to characterize fully the welfare cost of inflation.

6 Conclusion

In this paper we set out to investigate whether it is important to model heterogeneity across productive establishments when estimating the welfare cost of inflation. For this purpose, we studied a model characterized with cash-in-advance constraints on consumption and investment goods, and in addition we assume that liquidity constraints also apply to the creation of new establishments. In addition to discouraging investment and labor supply, an increase in the long-run rate of money growth increases the cost of creating new establishments and distorts firm entry and exit dynamics. As a result, incumbent establishments' profits must increase so as to encourage industry entry. This occurs through a fall in the equilibrium wage-rate. These adjustments are responsible for a substantial part of the welfare cost of inflation although they are associated only with modest decreases in average productivity.

As was mentioned earlier, Baily, Hulten, and Campbell (1992) document that about half of overall productivity growth in U.S. manufacturing in the 1980's can be attributed to factor reallocation from low productivity to high productivity establishments. It is tempting to imagine that the monetary policy tightening and resulting disinflation which occurred over the same period may have contributed to the reallocation of factors and improvements in efficiency.

A Proof of Proposition 1

We organize the proof as follows. First, we consider the cases where only one of the three cash-in-advance constraints applies, i.e. there exists a unique $i \in \{c, k, h\}$ such that $\theta_i = 1$. Then, based on the effects of inflation under those three frameworks, it is easy to complete the proof of Proposition 1.

A.1 Case where $\theta_c = 1$, $\theta_k = 0$ and $\theta_h = 0$

We consider first the case where $\theta_c = 1$, $\theta_k = 0$ and $\theta_h = 0$. Notice that in this context inflation does not affect the rental cost of capital in (30), nor the productivity threshold and the wage rate in (31) and (32). From (4), (6), (7) and (8), this implies that average output, employment, capital use and profits are affected by inflation either.

To determine the effect of inflation on the other aggregates, notice that in the stationary equilibrium $X = \delta K = \delta \bar{k} H$, $\kappa E = \kappa \frac{\lambda}{1-F(s^*)} H$ and $Y = \bar{y} H$. Replace those equations and (29) in (28) to get:

$$\frac{Lw}{A\left[1 + \theta_c\left(\frac{1+g}{\beta} - 1\right)\right]} + \delta \bar{k} H + \kappa \frac{\lambda}{1 - F(s^*)} H = \bar{y} H \tag{37}$$

Given the labor-market clearing condition, we can write $L = 1 - N = 1 - \bar{n} H$. Replacing this relation in the above equation and rearranging terms leads:

$$H = \frac{w}{A\left[1 + \theta_c\left(\frac{1+g}{\beta} - 1\right)\right]} \left(\bar{y} - \delta \bar{k} - \kappa \frac{\lambda}{1 - F(s^*)} + \frac{w\bar{n}}{A\left[1 + \theta_c\left(\frac{1+g}{\beta} - 1\right)\right]}\right)^{-1} \tag{38}$$

Equation(38) shows that when $\theta_c = 1$, an increase in the anticiapted rate of inflation g decreases the mass of incumbent firms H. Given that average employment, capital and output are not affected, this implies that an increase in the anticiapted rate of inflation g also decreases the aggregate level of capital, employment and output.

A.2 Case where $\theta_c = 0$, $\theta_k = 1$ and $\theta_h = 0$

When $\theta_k = 1$, equation (30) shows that an increase in g increases the rental cost of capital r.

To determine the effect of inflation on the productivity threshold and the wage rate in this context, first replace (32) in (8) to get

$$\bar{z} = \eta \left[\left(\frac{\bar{s}}{s^*} \right)^{\sigma} - 1 \right]. \tag{39}$$

When replacing this equation in the free-entry condition (31), we then have

$$\kappa \left[1 + \theta_h \left(\frac{1+g}{\beta} - 1 \right) \right] = [1 - F(s^*)] \frac{\beta}{1 - \beta(1-\lambda)} \eta \left[\left(\frac{\bar{s}}{s^*} \right)^{\sigma} - 1 \right]. \tag{40}$$

Hence, the productivity threshold does not depend on the rental cost of capital. Following an increase in g, the negative effect of the increase in r on profits cancels out with the positive effect of a decrease in wages. This latter can be seen from equations (30), (32) and (40).

Regarding the effect of inflation on average output per establishment, remark that, from equations (4), (6) and (7), average output can be written as

$$\bar{y} = \bar{s}^{\sigma} \left(\frac{\alpha}{w} \right)^{\alpha\sigma} \left(\frac{\nu}{r} \right)^{\nu\sigma}. \tag{41}$$

By replacing (32) in the above equation, one gets

$$\bar{y} = \frac{\eta}{\Omega} \left(\frac{\bar{s}}{s^*} \right)^{\sigma} \alpha^{\alpha\sigma} \nu^{\nu\sigma}. \tag{42}$$

Hence inflation does not affect average output.

To determine the impact on average capital and employment, notice from (6) and (7) and the fact that the productivity threshold is not affected by inflation that

$$d \ln \bar{n} = -(1-\nu)\sigma d \ln w - \nu\sigma d \ln r \tag{43}$$

$$d \ln \bar{k} = -\alpha\sigma d \ln w - (1-\alpha)\sigma d \ln r \tag{44}$$

Given that

$$\alpha d \ln w = -\nu d \ln r \tag{45}$$

from equation (32) and the fact that s^* is not affected by inflation, this set of equations can be rewritten as

$$d \ln \bar{n} = \frac{\nu}{\alpha} d \ln r \tag{46}$$

$$d \ln \bar{k} = -d \ln r \tag{47}$$

Thus an increase in inflation increases the average level of employment per establishment, while it decreases average capital use.

Equation (38) is still valid if the cash-in-advance constraint only applies to investment. Consequently, if inflation increases average employment, decreases the wage rate and average capital and does not affect average output and the productivity threshold, then it decreases the mass of incumbent establishments from equation (38). Hence, aggregate output and stock of capital decrease too. But, the effect on aggregate employment is a priori ambiguous given that H decreases and $\bar{n}$ increases. To show that the effect on aggregate employment is actually negative, first notice that

$$d\ln N = d\ln\bar{n} + d\ln H. \tag{48}$$

Next, from equation (38), observe that

$$d\ln H = d\ln w - Nd\ln w - Nd\ln\bar{n} + \frac{\delta KA\left(1+\theta_c\left(\frac{1+g}{\beta}-1\right)\right)}{w}d\ln\bar{k}. \tag{49}$$

Replacing the above equation and (45) and (46) in (49)

$$d\ln N = \frac{\delta KA\left(1+\theta_c\left(\frac{1+g}{\beta}-1\right)\right)}{w}d\ln\bar{k}. \tag{50}$$

Thus, aggregate employment decreases following an increase in inflation.

A.3 Case where $\theta_c = 0$, $\theta_k = 0$ and $\theta_h = 1$

Here the rental cost of capital is not affected by inflation (see equation (30)).

To understand the effect on the productivity threshold and the wage rate, combine (8) and (32) with (31) to get

$$\kappa\left[1+\theta_h\left(\frac{1+g}{\beta}-1\right)\right] = [1-F(s^*)]\,\frac{\beta}{1-\beta(1-\lambda)}\eta\left[\left(\frac{\bar{s}}{s^*}\right)^{\sigma}-1\right]. \tag{51}$$

Hence an increase in inflation decreases the productivity threshold s^*.

From equation (32) it follows that the wage rate decreases too.

From (42), average output decreases given s^* decreases.

To determine the effect on average employment and capital, notice from (32) that

$$d\ln s^* = \alpha d\ln w. \tag{52}$$

By replacing the above equation in (6) and (7), we have

$$d\ln\bar{n} = \sigma\left[d\ln\bar{s} - \frac{1-\nu}{\alpha}d\ln s^*\right] \tag{53}$$

$$d\ln\bar{k} = \sigma\left[d\ln\bar{s} - d\ln s^*\right] \tag{54}$$

Hence, average capital decreases following an increase in the rate of money growth and the impact of inflation on average employment is ambiguous. ... incomplete.

B Solutions

$$r = \left(\frac{1}{\beta} - 1 + \delta\right)\left[1 + \theta_k\left(\frac{1+g}{\beta} - 1\right)\right]$$

$$w = \left(\frac{\beta\sigma/(\varepsilon-\sigma)}{\kappa\left[1+\theta_h\left(\frac{1+g}{\beta}-1\right)\right]\left[1-\beta(1-\lambda)\right]}\right)^{\frac{1}{\alpha\varepsilon}}\left(\frac{s_0\Omega^{\frac{1}{\sigma}}\eta^{\frac{\sigma-\varepsilon}{\sigma\varepsilon}}}{r^{\nu}}\right)^{\frac{1}{\alpha}}$$

$$s^* = \left(\frac{\beta}{1-\beta(1-\lambda)}\frac{\sigma}{\varepsilon-\sigma}\frac{\eta}{\kappa}\frac{1}{1+\theta_h\left(\frac{1+g}{\beta}-1\right)}\right)^{\frac{1}{\varepsilon}} s_0$$

$$\bar{s} = \left(\frac{\varepsilon}{\varepsilon-\sigma}\right)^{1/\sigma} s^*$$

$$\bar{k} = \frac{\varepsilon}{\varepsilon-\sigma}\left(\frac{\alpha}{w}\right)^{\alpha\sigma}\left(\frac{\nu}{r}\right)^{(1-\alpha)\sigma} s^{*\sigma}$$

$$\bar{n} = \frac{\varepsilon}{\varepsilon-\sigma}\left(\frac{\alpha}{w}\right)^{(1-\nu)\sigma}\left(\frac{\nu}{r}\right)^{\nu\sigma} s^{*\sigma}$$

$$\bar{y} = \frac{\varepsilon}{\varepsilon-\sigma}\left(\frac{\alpha}{w}\right)^{\alpha\sigma}\left(\frac{\nu}{r}\right)^{\nu\sigma} s^{*\sigma} - \eta$$

$$\bar{z} = \Omega\frac{\varepsilon}{\varepsilon-\sigma}\frac{s^{*\sigma}}{w^{\alpha\sigma}r^{\nu\sigma}} - \eta$$

$$H = \frac{w}{A\left[1+\theta_c(\frac{1+g}{\beta}-1)\right]}\left(\bar{y} - \delta\bar{k} - \kappa\lambda\left(\frac{s^*}{s_0}\right)^{\varepsilon} + \frac{w\bar{n}}{A\left[1+\theta_c(\frac{1+g}{\beta}-1)\right]}\right)^{-1}$$

$$E = \frac{\lambda}{(s_0/s^*)^{\varepsilon}} H$$

$$K = H\bar{k}$$

$$X = \delta K$$

$$N = H\bar{n}$$

$$C = \frac{(1-N)w}{A\left[1+\theta_c(\frac{1+g}{\beta}-1)\right]}$$

$$Y = H\bar{y}$$

References

[1] A. B. Abel, *Dynamic behavior of capital accumulation in a cash-in-advance model*, Journal of Monetary Economics, 16 (1985), pp. 55–71.

[2] A. Atkenson, A. Khan, and L. Ohanian, *Are data on industry evolution and gross job turnover relevant for macroeconomics?*, Carnegie-Rochester Conference Series on Public Policy, 44 (1996), pp. 215–239.

[3] A. Atkeson and P. J. Kehoe, *Modeling and measuring organization capital*, Journal of Political Economy, 113 (2005), pp. 1026–1053.

[4] R. L. Axtell, *Zipf distribution of u.s. firm sizes*, Science, 293 (2001), pp. 1818–1820.

[5] M. N. Baily, C. Hulten, D. Campbell, T. Bresnahan, and R. E. Caves, *Productivity dynamics in manufacturing plants*, Brookings Papers on Economic Activity. Microeconomics, 1992 (1992), pp. 187–267.

[6] A. B. Bernard, J. Eaton, J. B. Jensen, and S. Kortum, *Plants and productivity in international trade*, American Economic Review, 93 (2003), pp. 1268–1290.

[7] L. M. B. Cabral and J. Mata, *On the evolution of the firm size distribution: Facts and theory*, American Economic Review, 93 (2003), pp. 1075–1090.

[8] F. Caselli, *Accounting for cross-country income differences.*

[9] T. F. Cooley and G. D. Hansen, *The inflation tax in a real business cycle model*, American Economic Review, 79 (1989), pp. 733–48.

[10] ——, *The welfare costs of moderate inflations*, Journal of Money, Credit and Banking, 23 (1991), pp. 483–503.

[11] T. Dunne, M. J. Roberts, and L. Samuelson, *The growth and failure of u.s. manufacturing plants*, The Quarterly Journal of Economics, 104 (1989), pp. 671–98.

[12] F. Ghironi and M. J. Melitz, *International trade and macroeconomic dynamics with heterogeneous firms*, The Quarterly Journal of Economics, 120 (2005), pp. 865–915.

[13] P. Gomme, *Money and growth revisited : Measuring the costs of inflation in an endogenous growth model*, Journal of Monetary Economics, 32 (1993), pp. 51–77.

[14] P. Gomme and P. Rupert, *Theory, measurement and calibration of macroeconomic models*, Journal of Monetary Economics, 54 (2007), pp. 460–497.

[15] R. E. Hall and C. I. Jones, *Why do some countries produce so much more output per worker than others?*, The Quarterly Journal of Economics, 114 (1999), pp. 83–116.

[16] E. Helpman, M. J. Melitz, and S. R. Yeaple, *Export versus fdi with heterogeneous firms*, American Economic Review, 94 (2004), pp. 300–316.

[17] H. Hopenhayn and R. Rogerson, *Job turnover and policy evaluation: A general equilibrium analysis*, Journal of Political Economy, 101 (1993), pp. 915–38.

[18] H. A. Hopenhayn, *Entry, exit, and firm dynamics in long run equilibrium*, Econometrica, 60 (1992), pp. 1127–50.

[19] A. Janiak, *Welfare in models of trade with heterogeneous firms.* Oct. 2008.

[20] L. E. Jones and R. E. Manuelli, *Growth and the effects of inflation*, Journal of Economic Dynamics and Control, 19 (1995), pp. 1405–1428.

[21] R. G. King and R. Levine, *Capital fundamentalism, economic development, and economic growth*, Carnegie-Rochester Conference Series on Public Policy, 40 (1994), pp. 259–292.

[22] J. Lucas, Robert E and N. L. Stokey, *Money and interest in a cash-in-advance economy*, Econometrica, 55 (1987), pp. 491–513.

[23] R. E. Lucas, *Discussion of : Stanley fischer, "towards an understanding of the costs of inflation: Ii"*, Carnegie-Rochester Conference Series on Public Policy, 15 (1981), pp. 43–52.

[24] E. C. Prescott, *Needed: A theory of total factor productivity*, International Economic Review, 39 (1998), pp. 525–51.

[25] D. Restuccia and R. Rogerson, *Policy distortions and aggregate productivity with heterogeneous plants*, Review of Economic Dynamics, 11 (2008), pp. 707–720.

[26] J. Robert E. Lucas, *Inflation and welfare*, Econometrica, 68 (2000), pp. 247–274.

[27] A. C. Stockman, *Anticipated inflation and the capital stock in a cash in-advance economy*, Journal of Monetary Economics, 8 (1981), pp. 387–393.

[28] M. Veracierto, *Employment flows, capital mobility, and policy analysis*, International Economic Review, 42 (2001), pp. 571–95.

Figure 5: Wage Rate Associated with Various Annual Growth Rates of Money