Distortions in referendum outcomes caused by quorum rules: Evidence from a pivotal-voter model

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March 12, 2009

Abstract

If you were unlucky enough to come accross this paper, please keep in mind that this is a very preliminary version. At this point, we decline any responsability on the consequences of using the implications of our analysis in the real world.

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Keywords:

Jel Codes: D72

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1 Introduction

Direct democracy is becoming increasingly common in Western democracies (Setälä 1999; LeDuc 2003), public support for referendums is high and arguably rising (Bowler *et al.* 2007), and there is some theoretical and empirical work showing that direct democracy outcomes are Pareto efficient most of the times (Noam 1980). Still, poorly designed rules cause voting paradoxes and stimulate the voters to hide or disguise their real preferences. Therefore, the institutional design of referendums is an issue that is likely to raise increasing attention on the part of policy-makers and citizens alike.

One aspect of referendum design is related to quorum rules. Low turnouts are often seen as a threat to referendum legitimacy, because of the fear that an active minority is imposing their will on an apathetic majority. When legitimacy is a concern, it is common to defend the existence of quorum requirements for a referendum to be valid. If, for example, we look at member states of the European Union, we realize that while referenda have a firm tradition in established democracies countries such as Italy, Ireland or Denmark. However, these countries have distinct choices about quorum rules. Typically, quorum rules take two forms: either a participation requirement or majority requirement. The quorum of participation demands a minimum turnout for the results of the referendum to be valid. The majority requirement makes the validity of the results dependent on the approval of a certain percentage of the electorate.

The literature on the effects of quorum requirements on referenda is not extensive, but it is growing. To our knowledge, the first paper dealing with the issue is the one of Côrte-Real and Pereira (2004). Côrte-Real and Pereira use a decision axiomatic approach to show that different types of quorums imply different assumptions about the interpretation of abstention. One corollary of Côrte-Real and Pereira's results was that participation quorums do create conditions under which supporters of the status quo may have incentives to abstain, thus contributing to decrease turnout. The general argument about the relationship between quorum rules and actual turnout was studied by Herrera and Mattozzi (2009), in a group turnout model, and by Aguiar-Conraria and Magalhães (2008). Herrera and Mattozzi show that political parties' and interest groups' behavior is influenced by quorum rules: in their presence, the incentives to mobilize voters are distorted, allowing groups that are in favour of preserving the status quo to use a 'quorum-busting strategy'.The same framework is used by the authors to argue that the effects of 'approval quorums' and 'participation quorums' are equivalent. Aguiar-Conraria and Magalhães (2008) use a rational choice, decision theoretic voting model to demonstrate that certain types of quorum requirements change the incentives some electors face. In particular, participation quorums induce electors who are against changes in the status quo to abstain rather than vote. They do not reach the same conclusion about the approval quorum. In the first empirical study on this subject, Aguiar-Conraria and Magalhães estimate that the existence of participation quorums may increase abstention up to 14 percentage points.

In this paper we go a step further. Our concern is not about the impact of the quorum requirements on turnout, but, instead, its impact on the referendum outcome. We assume that citizens are motivated to vote by the chance that they might swing the election. It is well-known that most of the 'partial equilibrium' rational choice models are incapable of explaining observed levels of turnout in a satisfactory way. This happens because the probability that a single vote is decisive is very close to zero. Therefore, if the costs of voting are positive, these models predict an abstention rate of 100%. But if we take consider a pivotal-voter model, there is a link going from strategies to beliefs about actions. Using a game-theoretic general equilibrium approach, citizens rationally anticipate the probability that their votes will be pivotal and they will vote if the expected benefit outweighs the cost of voting. Assuming away quorum requirements, a strictly positive level of turnout is assured in equilibrium: if no citizen were expected to vote, any deviator would be pivotal with probability one.

The pivotal-voter model, Ledyard (1984) and Palfrey and Rosenthal (1983, 1985), is one of the workhorses of formal political theory. Several implications have been derived from them. For example, Campbell (1999) showed that small minorities, with very strong feelings about the issue to be voted for, can impose their view on an apathetic majority and Borgers (2004) used pivotal model to show that voluntary voting Pareto-dominates compulsory voting. Still, these type of models have some insufficiencies. For example, Coate *et al.* (2008) showed that Pivotal models are incapable of explaining large winning margins.

2 A pivotal voter model

Our base model is a traditional the pivotal voter model of Coate *et al.* (2008), modified to accommodate different quorum rules. We focus on referenda with a binary choice: changing the 'status quo' (the 'Yes' option) or keeping the 'status quo' (the 'No' option). We will call the group of supporters the 'changers', because they want to change the status quo, and the group of opposers the conservatives, because they wish to preserve that status quo.

Each citizen must decide whether to vote or not. It is trivial to show that if they choose to vote, then they will vote according to their preferences: changers will vote 'Yes', conservatives vote 'no'. We assume that there are n electors $(i = 1, \ldots, n)$[1] and that each faces a cost of voting given by c_i. c_i is the realization of a uniformly distributed random variable, $c_i \sim U[0, c]$. If the 'Yes' wins the election, then supporters obtain a benefit b, while opposers suffer a loss x. Each elector knows his/her own type, and knows the probability, μ, that each individual elector favors the proposal. Each voter knows his/her own cost, but only knows the distribution of the other voters.

A voter derives utility from voting only if he/she is pivotal, i.e. if he/she casts the decisive vote . The probability of being pivotal depends on the strategies of the other voters. For example, if an elector believes that all other voters will abstain, then the subjective probability of being pivotal is 1. This is a game of incomplete information in which the preferences and voting costs are exogenously given. The 'Yes' option wins, if it receives at least as many votes as the 'No' and if the quorum requirements are satisfied.

A strategy for elector i is a function that specifies if he/she votes or abstains for each possible realization of c_i. We look for symmetric Bayesian-Nash equilibria: given the strategies of the other citizens and the distribution of supporters and voting costs, each citizen must be happy with his/her strategy. Symmetry implies that all members of a group (supporters or opposers) follow the same strategy. An elector will vote if the voting cost is below some threshold. Let γ_s and γ_o be those cut-off values for supporters and opposers respectively.

[1] For reasons that will become obvious, we assume that n is a multiple of 4. Other cases require obvious modifications.

2.1 No quorum requirements

First we assume that there are no quorum requirements. This means that whoever gets the majority of the votes wins. In case of a tie, victory is for changers.

Consider the choices elector i faces, assuming that the remaining $n-1$ follow their equilibrium strategies. If the $n-1$ remaining electors follow their equilibrium strategies, each changer will vote if his/her voting cost is less than γ_s, while opposers will vote if their voting cost is less than γ_o. Let $\rho(v_s, v_o; \gamma_s, \gamma_o)$ be the probability that among the $n-1$ individuals, v_s vote 'Yes' and v_o vote 'No', given their equilibrium strategies, γ_s and γ_o.

Given our rules, a changer will be pivotal when, among the $n-1$ other electors, the number of 'No' votes is equal to the number of 'Yes' votes plus one. Therefore, the expected benefit of voting is

$$\sum_{v=1}^{n/2} \rho(v-1, v; \gamma_s, \gamma_o)\, b.$$

This change supporter will vote if the expected benefit exceeds the cost of voting. In equilibrium, this means that

$$\sum_{v=1}^{n/2} \rho(v-1, v; \gamma_s, \gamma_o)\, b = \gamma_s. \tag{1}$$

For the opposers, the reasoning is analogous. Just note that an opposer will be pivotal in case of a tie among the other $n-1$ electors. The expected benefit of voting is

$$\sum_{v=1}^{n/2-1} \rho(v, v; \gamma_s, \gamma_o)\, x.$$

while, in equilibrium, we have

$$\sum_{v=1}^{n/2-1} \rho(v, v; \gamma_s, \gamma_o)\, x = \gamma_o. \tag{2}$$

We have two equations (1 and 2) and two unknowns. To compute the equilibrium we need to derive the function $\rho(v_s, v_o; \gamma_s, \gamma_o)$. μ is the probability that each individual is a 'Yes' supporter. Therefore, the probability, $P(s)$, that there are s supporters among the remaining $n-1$ electors is given by

$$P(s)=\binom{n-1}{s}\mu^{s}(1-\mu)^{n-1-s}.$$

Among the s supporters, only the ones whose individual costs are smaller that the expected benefits will vote. Therefore, the probability that v_s of those will vote is

$$V(v_s)=\binom{s}{v_s}\left(\frac{\gamma_s}{c}\right)^{v_s}\left(1-\frac{\gamma_s}{c}\right)^{s-v_s}.$$

Similarly the probability that, among the other $n-1-s$ electors, v_o will vote 'No' is

$$V(v_o)=\binom{n-1-s}{v_o}\left(\frac{\gamma_o}{c}\right)^{v_o}\left(1-\frac{\gamma_o}{c}\right)^{n-1-s-v_o}.$$

Putting these equation together we have that

$$\rho(v_s,v_o;\gamma_s,\gamma_o)=\sum_{s=v_s}^{n-1-v_o}\binom{s}{v_s}\left(\frac{\gamma_s}{c}\right)^{v_s}\left(1-\frac{\gamma_s}{c}\right)^{s-v_s}\binom{n-1-s}{v_o}\left(\frac{\gamma_o}{c}\right)^{v_o}\left(1-\frac{\gamma_o}{c}\right)^{n-1-s-v_o}P(s)$$

Introducing this in equation (1) and (2) we have a nonlinear system of two equations and two unknowns:

$$\begin{cases}\sum_{v=1}^{n/2}\sum_{s=v-1}^{n-1-v}\binom{s}{v-1}\left(\frac{\gamma_s}{c}\right)^{v-1}\left(1-\frac{\gamma_s}{c}\right)^{s-(v-1)}\binom{n-1-s}{v}\left(\frac{\gamma_o}{c}\right)^{v}\left(1-\frac{\gamma_o}{c}\right)^{n-1-s-v}\binom{n-1}{s}\mu^{s}(1-\mu)^{n-1-s}b=\gamma_s\\ \sum_{v=1}^{n/2-1}\sum_{s=v}^{n-1-v}\binom{s}{v}\left(\frac{\gamma_s}{c}\right)^{v}\left(1-\frac{\gamma_s}{c}\right)^{s-v}\binom{n-1-s}{v}\left(\frac{\gamma_o}{c}\right)^{v}\left(1-\frac{\gamma_o}{c}\right)^{n-1-s-v}\binom{n-1}{s}\mu^{s}(1-\mu)^{n-1-s}x=\gamma_o\end{cases}\tag{3}$$

Because there is no closed form solution to this problem, solutions have to be found numerically. Existence of a solution is not a problem (see Ledyard 1984 and Palfrey and Rosenthal 1985), but there are no general uniqueness results. Multiple solutions are possible. To our knowledge the only uniqueness result derived so for is for the case of $\mu=0.5$ and $b=x$.

2.2 The pivotal-voter model with an approval quorum

If there is an approval quorum, a change in the 'status quo', i.e. for a proposal to pass, requires the support of at least 50% of the voters and also the support of a certain percentage of the total electorate. In our calculations, we will consider that certain percentage to be 25%.

For a person who is against the proposal, the only modification one has to make to equation 2 is to consider the summation from $v = n/4$ instead of $v = 1$:

$$\sum_{v=n/4}^{n/2-1} \rho_{aq}\left(v, v; \gamma_s, \gamma_o\right) x = \gamma_o \tag{4}$$

This modification is obvious and it happens because if the number of 'Yes' votes is smaller than $\frac{n}{4}$ the 'status quo' wins, independently of who receives the majority of the votes.

For a person who favors the proposal, there are two possibilities of being pivotal. If the quorum is satisfied, then a changer is pivotal if, among the other $n-1$ voter, $v-1$ vote 'Yes' and v vote 'No'. On the other hand, the elector can also be pivotal if his vote is decisive to guarantee that the quorum is satisfied. In equilibrium, we have:

$$\sum_{v=n/4}^{n/2} \rho_{aq}\left(v-1, v; \gamma_s, \gamma_o\right) b + \sum_{v=1}^{n/4-1} \rho_{ap}\left(n/4-1, v; \gamma_s, \gamma_o\right) b = \gamma_s \tag{5}$$

The system of equations to solve is

$$\left\{\begin{array}{l} \sum_{v=n/4}^{n/2} \sum_{s=v-1}^{n-1-v} \binom{s}{v-1} \left(\frac{\gamma_s}{c}\right)^{(v-1)} \left(1-\frac{\gamma_s}{c}\right)^{s-(v-1)} \binom{n-1-s}{v} \left(\frac{\gamma_o}{c}\right)^{v} \left(1-\frac{\gamma_o}{c}\right)^{n-1-s-v} P\left(s\right) b + \\ + \sum_{v=1}^{n/4-1} \left(\sum_{s=n/4-1}^{n-1-v} \binom{s}{(n/4-1)} \left(\frac{\gamma_s}{c}\right)^{n/4-1} \left(1-\frac{\gamma_s}{c}\right)^{s-(n/4-1)} \binom{n-1-s}{v} \left(\frac{\gamma_o}{c}\right)^{v} \left(1-\frac{\gamma_o}{c}\right)^{n-1-s-v} P\left(s\right)\right) b = \gamma_s \\ \\ \sum_{v=n/4}^{n/2-1} \sum_{s=v}^{n-1-v} \binom{s}{v} \left(\frac{\gamma_s}{c}\right)^{v} \left(1-\frac{\gamma_s}{c}\right)^{s-v} \binom{n-1-s}{v} \left(\frac{\gamma_o}{c}\right)^{v} \left(1-\frac{\gamma_o}{c}\right)^{n-1-s-v} P\left(s\right) x = \gamma_o \end{array}\right. \tag{6}$$

Again, existence of a solution is not a problem, as not voting is always an equilibrium strategy. To realize this, just note that if one believes that nobody else is going to vote, his/her incentives to vote are zero. Whether he/she is a supporter (the vote is not enough to meet the quorum) or an opposer (the 'status quo' will win any way) is irrelevant. Uniqueness is not guaranteed either:

Proposition 1 *For some parameter values, it is possible to have more than one equilibrium strategy.*

Proof. To prove this proposition it is enough to create one example with two solutions.

Consider the case of $\mu = 0.5$ and $b = x$, chosen in a way that the solution to system of equations 3 implies a very high turnout rate, close to 100%. Let γ_s^* and γ_o^* be that solution. This means that $\frac{\gamma_s^*}{c}$ and $\frac{\gamma_o^*}{c}$ are very close to 1.

This in turn implies that $\sum_{v=1}^{n/4-1} \rho_{ap}\left(n/4-1, v; \gamma_s^*, \gamma_o^*\right)$ in system (6) is very close to zero.

On the other hand, looking again at system (3), with very high (γ_s^*, γ_o^*), $\rho\left(v-1, v; \gamma_s^*, \gamma_o^*\right)$ and $\rho\left(v, v; \gamma_s^*, \gamma_o^*\right)$ will be close to zero for $v \leq n/4$. Therefore $\sum_{v=1}^{n/4} \rho\left(v-1, v; \gamma_s^*, \gamma_o^*\right)$ and $\sum_{v=1}^{n/4} \rho\left(v, v; \gamma_s^*, \gamma_o^*\right)$ will be close to zero.

Deleting the terms close to zero, one has that systems 6 and 3 are identical.

Using a continuity argument, one concludes that one of the solution to system 6 will be in the neighborhood of (γ_s^*, γ_o^*) . Given that $(0, 0)$ is also a solution, the proposition is proved. ■

2.3 The pivotal-voter with a participation quorum

With a participation quorum, a change in the status quo requires the support of the majority of 50% of the voters and that a given percentage of registered voters take part in the vote. We will consider that certain percentage to be 50%.

For a person that supports the proposal, the modifications to introduce to equation 1 are straightforward:

$$\sum_{v=n/4}^{n/2} \rho_{pq}\left(v-1, v; \gamma_s, \gamma_o\right) b + \sum_{v=n/4+1}^{n/2} \rho_{pq}\left(v-1, n/2-v; \gamma_s, \gamma_o\right) b = \gamma_s \tag{7}$$

The first term applies when the quorum is met, meaning that an elector is pivotal if, among the other $n-1$ voter, $v-1$ vote 'Yes' and v vote 'No'. The second term captures the possibility of being pivotal when his/her vote is decisive to guarantee that the quorum is satisfied (note that $v - 1 + n/2 - v = \frac{1}{2}n - 1$).

The most interesting case is for a person who opposes a proposal. A conservative can be pivotal in two contradicting ways. He/She can be pivotal because his/her vote is decisive to guarantee a 'No' majority. But, on the other hand, he/she can also be decisive to guarantee that the quorum is met. In such case, even if the person votes no, his/her vote is decisive to guarantee that the 'Yes' wins. So his/her utility decreases. Accordingly, we have:

$$\sum_{v=n/4}^{n/2} \rho_{pq}(v, v; \gamma_s, \gamma_o)\, x + \sum_{v=n/4+1}^{n/2} \rho_{pq}(v-1, n/2 - v; \gamma_s, \gamma_o)(-x) = \gamma_o \tag{8}$$

Once more, existence of a solution is not a problem, as not voting is always an equilibrium strategy. Uniqueness is not guaranteed either:

Proposition 2 *For some parameter values, it is possible to have more than one equilibrium strategy.*

Proof. As before, to prove this proposition it is enough to construct one example with two solutions. It is a simple exercise to create an example analogous to the one use in the proof of the previous proposition. ■

2.4 Expected outcomes

To estimate the participation rate, note that $\frac{\gamma_s}{c}$ and $\frac{\gamma_o}{c}$ provide the expected value of the percentage of supporters and opposers, respectively, that will cast their vote. So the expected turnout rate is given by

$$E(turnout) = \mu\frac{\gamma_s}{c} + (1-\mu)\frac{\gamma_o}{c}. \tag{9}$$

It is also easy to compute the expected percentages of 'Yes' and 'No' votes and the margin of victory:

$$E\left(\%Yes\right) = \frac{\mu\gamma_s}{\mu\gamma_s + (1-\mu)\gamma_o} \tag{10}$$

$$E\left(\%No\right) = \frac{(1-\mu)\gamma_o}{\mu\gamma_s + (1-\mu)\gamma_o} \tag{11}$$

$$E\left(Margin\right) = \frac{|\mu\gamma_s - (1-\mu)\gamma_o|}{\mu\gamma_s + (1-\mu)\gamma_o} \tag{12}$$

2.5 Graphical illustration

The ideas formalized so far can be illustrated with the aid of a simple picture. In Figure 1, let the vertical axis represent the percentage of the population that favors the proposal submitted to referendum. In the horizontal axis, we have the percentage of people that oppose the proposal. If there are no quorum requirements (left picture), there is a change in the status quo if the outcome of the referendum places the results on or above the 45 degree line (meaning that at least 50% of the voters vote "Yes"). Therefore, a supporter who believes that the referendum outcome will be placed on the red line, believes that he/she will be pivotal. On the other hand, a conservative believes that he/she will be pivotal if he/she believes that the outcome will be on the green line.

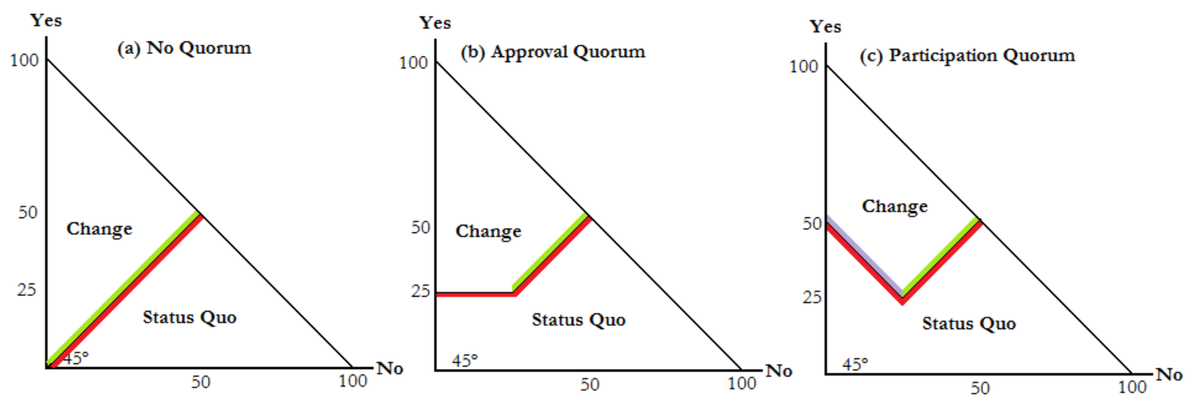

Figure 1: Pivotal lines

The picture in the middle describes a situation where there is an approval quorum of 25 percent. Therefore, a change the status quo requires the results to be above the 45 degree line and above the 25 percent-Yes line. In this case, the 'Status Quo' region increases. There is also qualitative change on the probability of being pivotal. A supporter may be pivotal if his vote is decisive for reaching 50% of the votes (given that the quorum is met) or to reach the quorum (given that at least 50% of the voters choose 'Yes'). These possibilities are represented by the red line. For the opponent, the situation is different. A 'No' vote that guarantees a 'No' majority is decisive only if the quorum is met. Therefore the pivotal green line is reduced, when compared to the first case.

Finally, on the right, we describe a situation where there is a participation quorum of 50%. The 'Change' region is now reduced to the area above the 50 percent participation rate and above the 45 degree line. A supporter is pivotal if his/her vote is necessary either to reach 50% of the votes (given that the quorum is met) or to meet that the quorum (given the 50% is met). This is represented by the red line. A vote of an opposer may be decisive in two different ways. Given that the quorum is satisfied, then an opposer is pivotal, in a good way, if his/her vote is decisive to guarantee the majority (green line). On the other hand, given that 'Yes' will receive majority, an opposer is pivotal, in a bad way, if his/her vote is decisive to meet the quorum requirement (violet line). It is pivotal in a bad way, because the vote is decisive to guarantee an undesired outcome.

Looking at these pictures, it is clear what is the main objective of imposing a quorum requirement. The idea is to create a bias for the status quo, by enlarging the 'Status Quo' region. This way, it is more difficult for very active minorities to change the status quo. We see next if this objective is achieved.

3 Can a small noisy minority be decisive?

It is known that a pivotal voter model is a bad model to explain big margins of victory (Coate *et al.* 2008). This is so because electors only have an incentive to vote if there is a reasonable chance that their vote is pivotal. An equilibrium with a large margin of victory is, therefore, rather difficult to obtain, precisely because the high margin implies that the probability of casting the decisive vote is close to zero. This reasoning is no longer valid, as we will see, if there are quorum requirements, which do change the structure of the decision.

In the context of a pivotal voter model, with no type of quorum, Campbell (1999) showed that it is possible that small minorities, with very strong feelings about the issue to be voted for, impose their view on an apathetic majority. Campbell considers two different normative criteria to check if this outcome is socially desirable. (1) According to a 'democratic criterion' an outcome is desirable only if the majority of the population prefers that same outcome. Obviously, under this criterion, a minority winning the referendum is undesirable. (2) According to what Campbell calls an 'economic criterion', one should sum all the individuals utility and choose the one that provides the greater total utility. Campbell shows that this criterion will also be frequently violated. The possibility of this type of outcome is one a the most common arguments for the existence of quorum requirements.

We will analyze whether the existence of quorum rules changes the argument. With that purpose, we will construct four scenarios. In one of them, under no quorum, a very active small minority group of changers achieves a close result, which is not enough to win the election. In the second scenario, this active minority is able to win the election. The last two scenarios are similar, but the active minority is for the 'status quo' instead.

To simulate our model, we fix the number of eligible voters (for computational reasons we consider only $n = 200$ voters and approximate the Binomial distribution by a Normal distribution). We have to calibrate the parameter values for b, x, μ, c. We choice of c is irrelevant and can be normalized to $c = 1$. To look for the equilibria, we start with a thin grid search. After that, using a standard algorithm, we check the most promising equilibria.

3.1 Active minority for 'Change'

3.1.1 Almost decisive minority for change

We consider $\mu = 0.30$, meaning that only an expected 30% of the population is for change, while an expected overwhelming majority of 70% is against. Following Campbell (1999), we introduce a twist. We assume that the minority is strongly in favor of the proposal while the majority doesn't have such strong feelings for the subject. This is captured by assuming that $b = 30$, while $x = 15$, so that the utility of winning the referendum is twice as much for the supporters than for the opposers.

The scenarios in which 'Change' is more likely are highlighted in yellow. Scenarios in which the 'Status Quo' is expected to prevail are shaded in grey and, when the results are too close to call the background is in white.

		Opposers turnout rate	Supporters turnout rate	General turnout rate	Yes Votes Percentage
No quorum		45.4%	95.4%	60.4%	47.4%
Participation Quorum	eq1	45.5%	96.2%	60.7%	47.5%
	eq2	0.0%	0.0%	0.0%	NA
Approval Quorum	eq1	45.5%	95.6%	60.5%	47.4%
	eq2	35.3%	95.6%	53.7%	53.4%
	eq3	0.0%	94.6%	28.4%	100.0%
	eq4	0.0%	67.9%	20.4%	100.0%
	eq5	0.0%	0.0%	0.0%	NA

Table 1. Equilibrium outcomes for $n = 200, \frac{b}{c} = 30, \frac{x}{c} = 15, \mu = 0.30$

In this scenario, we see that in spite of the huge majority that favors the 'No', under the no quorum equilibrium, the 'status quo' wins by a relatively small margin. The same happens in equilibrium 1, under the participation or under the approval quorum. The most interesting result is that there are two equilibria under the approval quorum under which 'Change' is expected to win. Both of them involve desertion from the status quo supporters. Therefore, ironically, in this scenario of a very active minority, the approval quorum, instead of biasing the results towards the 'status quo', causes the reverse.

3.1.2 Decisive minority for change

In this scenario, we consider an even more apathetic majority. The calibration is the same except for x, which now decreases to 10. As we can see, the no quorum equilibrium implies an expected victory for 'Change'. Participation quorum guarantees the 'status quo': under the participation quorum, there is no equilibrium in the neighborhood of the no quorum equilibrium. This is to be expected, after all, if conservatives can almost guarantee the victory by not showing up, why bother risking a probable defeat by showing up.

Quorum		Opposers turnout rate	Supporters turnout rate	General turnout rate	Yes Votes Percentage
None		30.4%	83.61%	46.38%	54.1%
Participation	eq1	0.0%	0.0%	0.0%	NA
Approval	eq1	0.0%	94.57%	28.37%	100.0%
	eq2	0.0%	67.9%	20.4%	100.0%
	eq3	0.0%	0.0%	0.0%	NA

Table 2. Equilibrium outcomes for $n = 200, \frac{b}{c} = 30, \frac{x}{c} = 10, \mu = 0.30$

With the approval quorum, desertion from conservatives is not enough to defeat 'Change'. This happens, of course, because we have assumed the percentage of people for change to be above the 25% of the majority quorum requirement. These simulations suggest that, in these circumstances, the best way to guarantee that a small minority will not impose its view is by imposing a participation quorum. Next we analyze the case of a small minority of conservatives.

3.2 Active minority for the 'Status Quo'

3.2.1 Almost decisive minority of conservatives

We now consider the reverse scenario. There is a noisy minority for the status quo and an apathetic majority for change:$\mu = 0.7, b = 15, x = 30$. In this scenario, although the vast majority of citizens support 'Change', without a quorum the percentage of 'Yes' votes is barely enough to win.

		Opposers turnout rate	Supporters turnout rate	General turnout rate	Yes Votes Percentage
No quorum		94.1%	44.60%	59.46%	52.5%
Participation Quorum	eq1	0.0%	73.9%	51.7%	100.0%
	eq2	0.0%	67.8%	47.4%	100.0%
	eq3	0.0%	0.0%	0.0%	NA
Approval Quorum	eq1	92.36%	44.17%	58.6%	52.7%
	eq2	0.00%	41.11%	28.8%	100.0%
	eq3	0.00%	0.00%	0.0%	NA

Table 3. Equilibrium outcomes for $n = 200, \frac{b}{c} = 15, \frac{x}{c} = 30, \mu = 0.70$

In this case, either of the quorums seems to be a bad idea. By introducing quorum requirements, on is also introducing new possible equilibria in which the preferences of the minority are expected to prevail.

3.2.2 Enough to win

		Opposers turnout rate	Supporters turnout rate	General turnout rate	Yes Votes Percentage
No quorum	eq1	86.5%	31.24%	47.80%	45.7%
Participation	eq1	0.0%	0.0%	0.0%	NA
Approval Quorum	eq1	94.95%	37.62%	54.8%	48.0%
	eq2	84.11%	39.72%	53.0%	52.4%
	eq3	0.00%	29.86%	20.9%	100.0%
	eq4	0	0	0	NA

Table 4. Equilibrium outcomes for $n = 200, \frac{b}{c} = 10, \frac{x}{c} = 30, \mu = 0.70$

In this scenario, several equilibria emerge under the approval quorum. However, only in one of them, the strong minority fails to achieve its objectives. Overall, one has to conclude that if the purpose is to avoid decisive minorities quorum requirements are not very efficient at it.

4 A competitive scenario

We now assume away overwhelming majorities. Actually, one may even wonder what is the utility of a referendum if, a priori, it is known that there is a major majority that favors one of the options.

For our benchmark we consider that supporters and opposers have equally strong feelings about the issue, $b = x = 22.5$, and that, ex ante, it is not clear who is in majority, $\mu = 0.5$. The value for b and x was chosen in such way that, with no quorum requirements, the participation rate is close to 75%.

Table 5 tells us the possible equilibria in our benchmark scenario. In the case of no quorum, 74% of the opposer cast their vote. The percentage among the supporters is slightly less because we introduced a small asymmetry in our model when we assumed that for supporters a tie was enough to win the election.

		Opposers turnout rate	Supporters turnout rate	General turnout rate	Yes Votes Percentage
No quorum		74.0%	73.8%	73.9%	49.9%
Participation Quorum	eq1	74.0%	73.8%	73.9%	49.9%
	eq2	53.7%	62.3%	58.0%	53.7%
	eq3	0.0%	94.1%	47.0%	100%
	eq4	0.0%	0.0%	0.0%	NA
Approval Quorum	eq1	74.0%	73.8%	73.9%	49.9%
	eq2	49.3%	59.2%	54.3%	54.5%
	eq3	0.0%	57.9%	29.0%	100%
	eq4	0.0%	39.8%	19.9%	100%
	eq5	0.0%	0.0%	0.0%	NA

Table 5. Equilibrium outcomes for $n = 200$, $\frac{b}{c} = 22.5$, $\frac{x}{c} = 22.5$, $\mu = 0.5$

Under a participation quorum , several equilibria emerge. Two of them were to be expected, given the proofs of proposition 1 and 2. One equilibrium, the high participation one, is basically the same is the equilibrium with no quorum. Another possible equilibrium is simply the no show equilibrium. But there are also some intermediate cases.

In equilibrium 2, conservatives vote less than changers. The turnout rate is smaller than in the case of no quorum, but it is still enough to reach the participation requirement. Sometimes, the argument for the participation quorum relies on the idea that there should be a bias for the 'status quo', which in a way represents an equilibrium that the society achieved. As we saw in Figure 1.c, this is accomplished by increasing the 'status quo' region. What this exercise shows is that, in some cases this system may be a blessing in disguise to changers. Equilibrium 3 shows a more radical result. The abstention rate among opposers is 100%. This way, abstention is almost a functional equivalent of a "No" vote and, as a result, although the 'change' option receives 100% of the votes, the 'status quo' wins. This equilibrium, which is very common in practice, creates an awkward political situations where 'the majority will feel that they have been deprived of victory without an adequate reason' (Venice Commission 2007: 22-23).

The approval quorum has similar, although not identical, effects. Approval quorum equilibria 1,2 and 5 are similar to the participation quorum equilibria 1,2 and 4. The main difference is that there are two different reactions when opposers decide not to participate. In one of them, turnout among supporters is almost 58%, while in the other it is less than 40%. In one case the quorum is met. In the other case the 'status quo' prevails, generating the same awkward situation described before.

It is also interesting to highlight what is known in the literature as the "No-Show paradox": it is possible that the quorum is not reached precisely because of its existence or, in other words, turnout is guaranteed to exceed the quorum only if this requirement does not exist. As Aguiar-Conraria and Magalhães (2008) showed, this possibility is empirically very plausible.

4.1 What if the majority of the population is for the status quo?

In this example we consider the same parameter values of our benchmark. The only difference is that we now assume that $\mu = 0.45$. This means that the percentage of conservative potential voters will be close to 55%. Therefore, a referendum outcome that mirrors the majority must be one in which the 'No' is expected to win.

		Opposers turnout rate	Supporters turnout rate	General turnout rate	Yes Votes Percentage
No quorum		60.5%	64.5%	62.3%	46.6%
Participation Quorum	eq1	60.6%	64.7%	62.4%	46.6%
	eq2	48.0%	68.0%	57.0%	53.7%
	eq3	0.0%	0.0%	0.0%	NA
Approval Quorum	eq1	60.5%	64.6%	62.3%	46.6%
	eq2	0.0%	63.8%	28.7%	100.0%
	eq3	0.0%	44.7%	20.1%	100%
	eq4	0.0%	0.0%	0.0%	NA

Table 6. Equilibrium outcomes for $n = 200, \frac{b}{c} = 22.5, \frac{x}{c} = 22.5, \mu = 0.45$

Table 6 gives us quite striking results that can only be labelled as ironic. If there is no quorum, the 'No' is expected to win with a comfortable margin of 6.8 percentage points. If there is a quorum requirement, no matter which, the first equilibrium is in the neighborhood of the 'no quorum' equilibrium, but the expected outcome is reversed in the second equilibrium.

With a participation quorum, the second possible equilibrium implies a smaller percentage of opposers voting. This happens a conservative vote can have two contradicting consequences: (1) it contributes to the status quo majority but (2) it helps 'Change' to reach the quorum. The implication is that if a conservative is afraid that the proposal is supported by a majority of votes, his/her best option is to abstain, rather than voting. Under this equilibrium, 68% of the supporters will vote, which is enough to give them a solid majority (7.4 percentage points ahead).

Under an approval quorum requirement, the second and third equilibria involve the total abstention of opposers. Among the supporters, if they are able to coordinate to show up to the ballots in big numbers (equilibrium 2), they will win the referendum. Note that the approval quorum requirement is that 25% of the electorate votes 'Yes' and equilibrium 2 implies that almost 29% of the electorate votes 'Yes'.

Therefore, this means that one of the arguments for the quorum requirements looses its strength. The quorum requirement, instead of promoting the 'status quo' may actually be working the other way around, creating a bias for 'Change'. To be harmless, the equilibrium under the quorum requirement should be in the neighborhood of the 'no quorum' equilibrium, but, of course, in this case it is just simpler not to have quorum.

4.2 What if the majority of the population is for change?

We consider now the reverse scenario: $\mu = 0.55$. There is a majority of people who is for change. Which of the three systems is the best to reflect these choices? Looking at Table 3, once again, we confirm that the design whose outcome is closer to the preferences of the electors is the no quorum requirement.

		Opposers turnout rate	Supporters turnout rate	General turnout rate	Yes Votes Percentage
No quorum		63.1%	58.8%	60.8%	53.3%
Participation Quorum	eq1	0.0%	95.4%	52.5%	100.0%
	eq2	0.0%	84.7%	46.6%	100%
	eq3	0.0%	0.0%	0.0%	NA
Approval Quorum	eq1	64.0%	59.4%	61.5%	53.2%
	eq2	0.0%	53.0%	29.2%	100.0%
	eq3	0.0%	35.9%	19.7%	100%
	eq4	0.0%	0.0%	0.0%	NA

Table 7. Equilibrium outcomes for $n = 200, \frac{b}{c} = 22.5, \frac{x}{c} = 22.5, \mu = 0.55$

Both the participation and approval quorum equilibria show two possibilities in which the status quo wins. Once again, we have the interesting result that any of the quorum requirements is met when there is no quorum. On the other hand, when there is a quorum there are equilibria in which the quorum is not satisfied. Particularly striking are the equilibria associated with the participation rule. All the equilibria that we found have one common characteristic: the desertion of people who favor the status quo. This makes sense, it is very unlikely that conservatives will have the majority of votes; therefore by showing up they would helping the changers to meet the quorum requirement.

5 Is it possible that quorum rules enhance turnout?

In table 1, we can see that the first equilibria with quorum are slightly higher than with no quorum. But this effect is very marginal and does not change the general impression that quorum requirements promote abstention. This is particularly true in the case of the participation quorum, in which, for several times, we have the extreme case that the only equilibrium is the no show equilibrium. The effects of the majority quorum are not that linear. For example, in table 2, every equilibria associated with the approval quorum imply a significant decrease in turnout; while in table 4, we studied a scenario in which the existence of a majority quorum promotes turnout in a very sensible way. Both equilibrium 1 and 2 of the approval quorum in the latter scenario describe situations in which turnout raises by 5 to 7 percentage points. Still, we may consider that scenario to be uninteresting, given that it describes a situation in which 70% of the people share the same opinion, and looking at the several tables, one may still have the general impression that approval quorums promote abstention. But then, how can one explain the results of Aguiar-Conraria and Magalhães (2008), according to whom, approval quorums have no statistically significant effects of turnout rates? We now provide a realistic example, in which the approval quorum may, significantly increase turnout.

We consider a scenario in which the public opinion is completed divided ($\mu = 0.5$); there are no strong feelings about the issue, meaning that turnout rate will be, under no quorum rules, below 50%; and, finally, we assume that conservatives feel stronger about the issue than changers, $b = 12.5$ and $x = 17.5$.

Quorum		Opposers turnout rate	Supporters turnout rate	General turnout rate	Yes Votes Percentage
None	eq1	45.7%	36.35%	41.91%	44.6%
Participation	eq1	0.0%	0.0%	0.0%	NA
Approval	eq1	55.19%	52.03%	53.6%	48.5%
	eq2	0	0	0	NA

Table 8: Equilibrium outcomes for $n = 200, \frac{b}{c} = 12.5, \frac{x}{c} = 17.5, \mu = 0.5$

As we can see in Table 8, the existence of a majority quorum may fight abstention. The strategic reasoning is clear. Under the no quorum benchmark, conservatives will win the referendum. This happens because conservatives care more deeply about the issue, not because they were in majority. Now, if we introduce a participation quorum, given that the quorum is not met, it makes sense not to show up in the election day, as the status quo victory is guaranteed. On the other hand, if a majority quorum is introduced, the expected benefit of voting for a changer increases, because it increases the probability of being pivotal (the elector have two chances of being pivotal: his vote may be decisive to reach the quorum or to reach majority). Therefore, there is an increase in the participation of the changers. This, in turn, implies a reaction from the conservatives. If they want to win the election they have to increase their participation too. Overall, participation of members of both groups increase significantly, but it increases more on the side of the changers.

Can this increase be strong enough for the 'Yes' to win? In table 9, after slightly changing the parameters, we show that this may indeed be the case.

Quorum		Opposers turnout rate	Supporters turnout rate	General turnout rate	Yes Votes Percentage
None	eq1	51.4%	44.62%	48.02%	46.5%
Participation	eq1	0.0%	0.0%	0.0%	NA
Approval	eq1	53.07%	54.63%	53.8%	50.7%
	eq2	0	0	0	NA

Table 9: Equilibrium outcomes for $n = 200, \frac{b}{c} = 13, \frac{x}{c} = 16, \mu = 0.5$

6 Conclusions

Direct democracy is becoming increasingly common in Western democracies and public support for referendums is high and rising. The institutional design of referendums is, therefore, an issue that is likely to raise increasing attention on the part of policy-makers and citizens alike. The existence of quorum rules is one crucial aspect of that design, common to many European democracies, and typically seen as a way of preventing active minorities from imposing their will or even as a way of lending resistance to the status quo. However, although these may be acceptable goals from a normative point of view, there are good reasons to reject quorums as a way to achieve them.

To study the effects of quorum requirements on referendum outcomes, we used a standard pivotal-voter model and computed the equilibria for different scenarios. Our analysis shows that the main argument for the existence of quorum rule is not valid: it is not clear that quorum rules acts against small decisive minorities. Actually, it may even help them to achieve their objectives. Therefore, the problem is much more serious than the ones pointed out by Aguiar-Conraria and Magalhães (2008) and Herrera and Mattozzi (2009), who claimed that quorum requirements increased abstention. Our results show that, even when this is not the case, distortions caused by quorum rules are such that it is rather difficult, based on the results of a particular referendum, to make inferences about the true preferences of the electorate.

References

[1] Aguiar-Conraria, Luís and Pedro Magalhães. 2008. **Referendum Design, Quorum Rules and Turnout**. NIPE working-paper

[2] Borgers, Tilman. 2004. **Costly Voting**. American Economic Review 94 (1), 57–66.

[3] Bowler, Shawn, Todd Donovan, and Jeffrey A. Karp. 2007. **Preferences for Direct Citizen Participation in Affluent Democracies**. Political Research Quarterly 60: 351–362.

[4] Campbell, Colin, 1999. **Large electorates and decisive minorities**. Journal of Political Economy 107 (6), 1199–1217.

[5] Coate, Stephen, Michael Conlin, Andrea Moro. 2008. **The performance of pivotal-voter models in small-scale elections: Evidence from Texas liquor referenda**. Journal of Public Economics 92, 582–596

[6] Côrte-Real, Paulo and Paulo Trigo Pereira. 2004. **The Voter who Wasn't There: Referenda, Representation and Abstention**. Social Choice and Welfare 22: 349–369.

[7] Herrera, Helios and Andrea Mattozzi. 2009. **Quorum and Turnout in Referenda**. Journal of the European Economic Association, forthcoming.

[8] LeDuc, Lawrence. 2003. **The Politics of Direct Democracy: Referendums in Global Perspective**. Toronto: Broadview Press.

[9] Ledyard, John. 1984. **The pure theory of large two-candidate elections**. Public Choice 44, 7–41.

[10] Noam, Eli. 1980. **The Efficiency of Direct Democracy**. The Journal of Political Economy 88: 803–810.

[11] Palfrey, Thomas and Howard Rosenthal. 1983. **A Strategic Calculus of Voting**. Public Choice 41, 7–53.

[12] Palfrey, Thomas and Howard Howard. 1985. **Voter participation and strategic uncertainty**. American Political Science Review 79, 62–78.

[13] Setälä, Maija. 1999. **Referendums in Western Europe – A wave of direct democracy?** Scandinavian Political Studies 22: 327–340.

[14] Venice Commission. 2007. "Code of good practice on referendums." Document CDL-AD(2007)008. Available at http://www.venice.coe.int/docs/2007/CDL-AD(2007)008-e.asp.