Financial Liberalization: Efficiency Gains and Black-Holes

Romain Ranciere
PSE, IMF and CEPR

Aaron Tornell
UCLA

April 2011 (Preliminary and Incomplete)

JEL Classification No.F34, F36, F43, O41

1 Introduction

Financial liberalization tends to enhance growth, but it also generates greater crisis-volatility, induced by risk-taking and lending booms. Here, we analyze the gains and costs of financial liberalization in a setup that incorporates this growth-crisis trade-off. The paper makes three contributions.

The first contribution is positive. Given the availability of new micro-level data sets, we now know much more about the key empirical regularities associated with financial liberalization, crises and growth. This paper provides a theoretical framework to integrate them. The second contribution is normative. Our framework allows us to decompose the welfare consequences of financial liberalization into gains from higher production efficiency and losses associated with crisis-induced volatility. Third, the paper contributes to the debate on financial regulatory design. It helps understand how, in a world with systemic bailout guarantees, the regulatory environment shapes the outcome of financial liberalization.

We show that, even when taking into account the costs of crisis and the existence of bailout guarantees, there are net gains from liberalization provided there are regulatory limits on the types of liabilities that can be issued. The micro-level risk-taking mechanism by which liberalization spurs growth–and which is motivated by firm-level evidence–generates aggregate boom-bust cycles.

With regulatory limits, these booms fund productive investment, athough they can be punctuated by rare crises. In contrast, in the absence of regulatory limits, the issuance of financing instruments that allows for the concentration of most repayments in crisis states, can undermine and even overturn the gains from financial liberalization. In this case, liberalization turns into an "anything goes" regime in which the breakdown of financial discipline leads large scale funding of unprofitable projects and to a reduction in production efficiency.

In this paper financial liberalization is growth enhancing because it improves allocative efficiency. This channel is important in economies where financial frictions hinder the growth of sectors that are more dependent on external finance. By allowing for new financing instruments, liberalization relaxes the financing constraints faced by these sectors, which in turn increases aggregate growth and consumption possibilities. It does so by allowing agents to take on credit risk and in this way increase leverage. As a consequence, sectors more dependent of external finance invest more and grow faster. The rest of the economy benefits from this relaxation of the bottleneck via input-output linkages. However, because financial liberalization induces credit risk taking, it generates financial fragility and might lead to crises, which although rare, are severe.

Here, we analyze this trade-off between risk-taking, growth and production efficiency in a two-sector model with financial frictions. Our model is designed to capture three prominent empirical regularities associated with financial liberalization. First, although crises have been costly, countries that have liberalized financially, and have experienced booms and busts have been, on average, growing faster than non-liberalized countries.[1] Second, financial liberalization spurs aggregate growth mainly through TFP gains rather than aggregate capital accumulation.[2] Such aggregate TFP gains are associated with a sectoral reallocation of resources. Typically, following liberalization sectors more dependent on external finance grow more, but crash more severely during a crisis and subsequently suffer a greater decline during the credit crunch.[3] Third–implicit and explicit–guarantees to bailout lenders during systemic crises have been widespread the world over.[4]

The argument relies on how, in the presence systemic bailout guarantees, the financial regulatory regime influences financing decisions, and on how financing constraints in one sector affect the performance of the whole economy via input-output linkages. In a financially repressed economy, there is misallocation because the input producing sector depends on external finance to fund its investment and faces borrowing constraints due to contract enforceability problems. These constraints generate a bottleneck that limits the supply of intermediate inputs for the final-goods sector, and thus impacts negatively the growth performance and the production efficiency of the economy as a whole.

Because both sectors compete every period for the available supply of inputs, when contract enforceability problems are severe, the input producing sector attains low leverage and commands only a small share of inputs for investment: there is a misallocation of inputs which results in a socially inefficient low aggregate growth path. A central planner would increase the input sector investment share to attain the Pareto optimal allocation. In a decentralized economy, the first best can be attained by reducing the enforceability problems that generate the financing constraints. However, if such a reform is not feasible, financial liberalization, may be seen as an alternative way to improve the allocation despite the financial fragility it entails.

Financial liberalization allows for new instruments , which leads to a relaxation of the constraints and the bottleneck. However, and this is key, the new instruments generate new states of the world in which insolvencies occur, and so a riskless economy is endogenously transformed into a risky one. Our framework provides an internally consistent mechanism under which such transformation emerges, and helps understand how it can enhance long-run growth and production efficiency even though occasional crises occur during which the input sector suffers the costs associated with widespread bankruptcies.

[1] Bekaert, et.al. (2006) and Ranciere et.al. (2008), Henry (2007)

[2] Bonifliogli (2008). Beck, Loayza, Levine (2000) find a similar result for financial development.

[3] Dell'Arricia, Detragiache, and Rajan (2008), Klingebiel, Kroszner and Laven (2007), Levchenko, Ranciere, and Thoenig (2009)

[4] See Jeanne-Zettlemeyer (2001) and Ranciere, et. al.(2008) for evidence on systemic bailouts.

In order to analyze the link between financial regulation and production efficiency, we consider two classes of one-period securities–standard bonds and catastrophe bonds–and three financial regulatory regimes: repression, liberalization and an anything-goes regime. With standard bonds a debtor must promise to repay the same nominal amount in all states, and if it fails to repay it must default. In contrast, with catastrophe bonds a debtor can promise to repay an arbitrarily large amount in bad states and nothing in good states.[5]

Under financial repression a firm can only issue standard bonds and must denominate repayments in the good which it produces–i.e., cannot take on insolvency risk. In a liberalized regime a firm can only issue standard bonds, but can take on risk through a mismatch between the unit of the good they produce and the unit of the good on which they denominate their liabilities. In the context of emerging markets, this mechanism corresponds to the famous currency mismatch by which firms in nontradable sectors, issue liabilities denominated in foreign currency. When a critical mass of agents undertake currency mismatch, this micro-level risk generates systemic risk. [6] Finally, under the anything-goes regime firms can issue both standard and catastrophe bonds, and can take on insolvency risk.

Under financial repression there exists only one equilibrium where insolvencies and crises never occur. If there are significant contract enforceability problems, along this safe equilibrium the intermediate good sector exhibits low growth because its investment is constrained by its cash flow. Since intermediate goods serve as inputs for both sectors, the intermediate goods sector (N-sector) constrains the long-run growth of the final good sector (T-sector) and that of GDP: there is a bottleneck that constrains aggregate growth. In contrast, under financial liberalization, the economy evolves along a risky equilibrium in which relative price risk arises endogenously and N-firms find it optimal to take on insolvency risk by denominating debt in T-goods (unhedged debt). This risky behavior relaxes borrowing constraints, increases investment, alleviates the bottleneck and allows both sectors to grow faster. However, it also generates financial fragility, as a shift in expectations can cause a sharp fall in the price of N-goods relative to T-goods, bankrupt N-firms and land the economy in a crisis.

In order to address the growth-stability trade-off the model captures two costs typically associated with crises: bankruptcy and financial distress costs. Bankruptcy costs are static and derive from the severe input price decline that leads to firesales and bankrupts input sector firms with mismatch on their balance sheets. Financial distress costs are dynamic and derive from the resultant collapse in internal funds and the reduction in risk taking in the aftermath of crisis, which

[5] The issuance of catastrophe bonds corresponds, for instance, to the sale of options and credit default swaps without collateral.

[6] Ranciere, Tornell and Vamvakidis (2010)

depresses new credit and investment, hampering growth.

Our first result is that if there is a bottleneck, a liberalized–financially fragile–economy will on average grow faster than a repressed–safe–economy even if bankruptcy costs are arbitrarily large, provided that the dynamic crisis costs are not too severe. This result follows from the fact that crises must be rare events in order for credit risk to be profitable for individual borrowers that must risk their own equity, and from the fact that not all the bankruptcy losses experienced by the N-sector during crises are aggregate deadweight losses. The financial distress costs of crises can be far more significant than bankruptcy costs because they spread dynamically: the decline in internal funds and the reduction in risk taking translate into depressed leverage and investment in the input sector that reduces aggregate growth.

Our second result is that when contract enforceability problems are severe, but not too severe, credit risk brings the allocation nearer to the Pareto optimal level. Furthermore, it increases the present value of consumption that the economy can attain, even when we net out the fiscal costs of bailout guarantees, as long as the financial distress costs are not too large.

The efficiency benefits described above rely on the fact that the increase in leverage occurs without loosing financial discipline. In the model this discipline comes about by limiting external finance to standard debt contracts under which agents must repay in all states to avoid bankruptcy. Because of contract enforceability problems, lenders require that borrowers risk their own equity by imposing borrowing constraints. In this way the incentives of borrowers and creditors are aligned in selecting only positive NPV projects. Importantly, systemic bailout guarantees do not undermine this discipline because they are granted only in case of a systemic crisis, not if an idyosincratic default occurs.

This discipline breaks down in an anything-goes regime if bailout guarantees are present. Our third result is that allowing the issuance of catastrophe bonds, that pay zero in good states and promise a huge amount if a–rare–crisis occurs, reduces production efficiency. This is because such bonds allow for the funding of unproductive projects with a negative contribution to national income. These inferior projects are privately profitable because they are a means to exploit the subsidy implicit in the guarantee. A firm with a non-profitable project could issue bonds that promise to repay only in a crisis state. Lenders would be willing to buy them without requiring collateral because they expect the promised repayment to be covered by the bailout. As a result the firm can attain an unreasonable high leverage without risking its own equity, and bet that the project turns out a large profit in good states.

These theoretical results allow us to contrast the experience of emerging markets following financial liberalization, with the recent U.S. experience. Emerging markets'booms have featured mainly standard debt, and while they have experienced crises–the so called '3^{rd} generation' or balance-sheet crises–systemic risk-taking has been, on average, associated with higher long-run growth. In contrast, the recent U.S. boom featured a proliferation of uncollateralized derivatives that supported large-scale funding of negative NPV projects in the housing sector.[7] According to our model, the differences between the two experiences reflect the key role of regulatory limits in a world with systemic bailout guarantees. In absence of limits on the type of liabilities that can be issued, credit market discipline vanishes and the the efficiency gains of financial liberalization are overturned.

The rest of the paper is structured as follows. Section 2 relates our paper to the literature. Section 3 presents the model. Section 4 analyses long-run growth and production efficiency under financial repression and financial liberalization. Section 6 considers the anything-goes regime and characterizes a black-hole equilibrium. Section 7 concludes.

2 Outline and Related Literature.

In a nutshell, the equilibrium path of the economy is captured by three key equations. First, the share of intermediate goods production that is used for investment in the intermediate goods sector.

$$\text{Investment share:} \quad \phi_t = \phi(\text{internal funds, financial regime, enforceability problems}) \tag{1}$$

Second, the equilibrium production of intermediate inputs and final goods.

$$\begin{array}{lll} \text{Intermediate good} & q_{t+1} = q(I_t), & I_t = \phi_t q_t \\ \text{Final good} & y_t = y(d_t), & d_t = [1-\phi_t] \cdot q_t \end{array} \tag{2}$$

The evolution of the investment share ϕ_t is the key determinant of aggregate growth and production efficiency through the input-output linkages in (2).

In equilibrium ϕ_t is determined by the interaction of bailout guarantees with contract enforceability problems. This interaction depends crucially on the regulatory regime. Under financial repression the ϕ_t-sequence is smooth, but it can be inefficiently low and result in slow aggregate growth. Under financial liberalization the ϕ_t-sequence has a higher mean, but it exhibits sharp and sudden contractions–associated with crises. The underlying mechanism is that when agents coordinate on systemic risk-taking–and by doing so exploit systemic bailout guarantees–they attain higher leverage, which increases investment and growth, but it also makes the economy vulnerable to crises.

[7] Ranciere-Tornell (IMFER) , Levitin-Wachter.

We show that, despite bailout costs and bankruptcy costs, a shift from a repressed to a liberalized regime can increase aggregate growth, production efficiency and the present value of consumption. In contrast, a shift from financial repression to an anything-goes regime can reduce production efficiency and create a financial black-hole, in which unproductive projects are funded.

By emphasizing the link between borrowing constraints, sectoral misallocation and input-output linkages, this paper relates to Jones (2010, 2011) who emphasizes the consequence of resource misallocation in terms of intermediate inputs and its consequences on aggregate productivity through input-output linkages. Analogous to Jones steady-state input-ouput multiplier, in our setup higher production efficiency results from a dynamic input-ouput multiplier: an increase in today's investment in the intermediate input sector (ϕ_t) increases tomorrow's production in the final good sector.

There is a vast empirical literature on the growth effects of financial liberalization. Henry (2007) and Bekaert and Harvey (2006) find that it is generally growth enhancing, but earlier literature obtains more mixed results (Edison et. al., 2002). A reason for this is that financial liberalization has typically lead both to higher growth and to more frequent crises. This dual effect is at the core of our theoretical mechanism. Ranciere, Tornell, Westerman (2006) and Bonfiglioli (2008) find robust evidence for this dual effect of financial liberalization. The average growth gains in tranquil times dominate the output costs associated with a higher propensity to crisis.[8] This result is stronger when countries with a low quality of institutions are excluded from the sample (IMF, 2007).This result is consistent with our model in which financial liberalization is growth-enhancing only when the degree of contract enforceability is higher than a threshold so that the leverage effect is strong enough to outweigh the crisis effect.[9] Broner and Ventura (2010) find a similar result but through a different mechanism that focuses on the interaction between the enforcement of domestic vs. international contracts.

Bonfiglioli (2008) finds that the growth gains from financial liberalization come from an increase in aggregate TFP rather than from an increase in aggregate capital accumulation. Our model predicts that financial liberalization promotes a more efficient allocation of intermediate inputs accross sectors and therefore increases aggregate TFP. Abiad, Moday and Ueda (2008) provide evidence for such an allocative efficiency effect by comparing the dispersion of Tobin's Q among

[8] These results are related to Kaminsky and Schmukler (2008) who find that financial liberalization increase stock market volatility in the short run but reduce it in the long run or Loayza and Ranciere (2006) who show that financial development can reduce growth in the short run - through higher volatility and the incidence of crises - but increase it in the long run.

[9] A result consistent with the empirical literature trying to identify the group of countries for which financial liberalization increases growth (Ranciere, Tornell, Westermann; 2008, IMF, 2007)

listed firms in five emerging market countries before and after financial liberalization.

Levchenko, Thoenig and Ranciere (2008) find, using sector-level data, that sectors more depende nt on external finance grow more and become more volatile after financial liberalization.[10] In our model, the N-sector depends on external finance to fund investment but the T-sector does not. In tranquil times, the N-sector grows faster than the T-sector. This implies that N-goods becoming cheaper and more abundant which, in turn, foster growth in the T-sector. This effect is stronger when the economy is financially liberalized and the N-secor less financially constrained. However liberalization also brings crisis risk. During crises, the N-sector, unlike the T-sector, suffers from severe financial distress costs and experiences a credit crunch that sharply reduces investment and output. Dell'Arricia, Detragiache, Rajan (2008) and Klingebiel, Kroszner and Laven (2007) find indeed empirical evidence that sectors more dependent on external finance suffer disproportionatly more during financial crises. Related, but based on a completly different set-up, Buera, Kaboski and Shin (2010) show how a relaxation of financial constraints can result in more efficient allocation of capital and entrepreuneural talent accross sectors.

Other theorerical papers emphasize the welfare gains from financial liberalization coming from intertemporal consumption smoothing (Gourinchas and Jeanne, 2006), better international risk-sharing (Obstfeld, 1994) and better domestic risk-sharing (Ueda and Townsend, 2007). Gourinchas and Jeanne (2006) show that the welfare benefits associated with this mechanism are negligible in comparison to the increase in domestic productivity. The gains from risk-sharing can be much larger: Obstfeld (1994) demonstrates that international risk-sharing, by allowing a shift from safe to risky projects, increases strongly domestic productivity, production efficiency and welfare. In our framework the gains also stem from an increase in *production efficiency* but not from risk-sharing. The gains derive from a reduction of the *contract enforceability* problem not of the incomplete markets problem: efficiency gains are obtained by letting entrepreneurs take on *more risk*, not by having consumers face *less risk.* In Tirole (2000) currency mismatch also results in social welfare gains, but through a discipline effect on government policy, not through a better allocation of resources.

Systemic bailout guarantees play a crucial role in our framework. By affecting collective risk-taking and the set of fundable projects, they shape the growth and production efficiency effects of a regulatory regime. While there is ample evidence of ex-post systemic bailouts (Ranciere, Tornell, Westermann, 2010), evidence on bailout expectations are more difficult to obtain. Ranciere, Tornell

[10] Looking at the finance-growth nexus at the sector-level, Samaniego and Ilyina (2010, 2011) show that what really matters is the interaction between the ability to raise external finance and the need for such financing in order to fund growth-enhancing investment.

and Vamvakidis (2009), using firm-level data on loan pricing for a large sample of firms in Eastern Europe, find that some form of bailout expectations is necessary to rationalize the differences in the pricing of foreign and domestic currency debt accross firms.[11] By comparing the pricing of out-of-the money put options on a financial sector index with option on individual banks forming the index, Kelly, Lustic and Niewerburgh (2011) show that systemic bailouts, but not idyosincratic bailouts are expected.[12] Farhi and Tirole (2011) demonstrate how time-consistent bailout policies, designed by optimizing goverments, generate a collective moral hazard problem that explains the wide-scale maturity mismatch and high leverage observed in the US financial sector before the 2007-2008 crisis.

The cycles our model are very different from Schumpeter's (1934) cycles in which the adoption of new technologies plays a key role. Our credit cycles are more similar to Juglar's credit cycles (Juglar, 1863).[13]

Our model considers a similar credit market game as Schneider and Tornell (2004), ST, and Ranciere, Tornell, Westermann (2008), RTW. However, the questions addressed and the models considered are quite different. Here, we characterize the long-run paths of a two-sector economy, and compare the growth, volatility and productiction efficiency induced by different regulatory regimes. Instead, ST concentrate on how a boom-bust episode can arise from the interaction of contract enforceability problems and bailout guarantees. Here, productive linkages across sectors play a key role: higher investment in the input sector helps the finals goods sector. Neither ST nor RTW consider productive linkages. Finally, RTW is mainly an empirical paper that establishes a positive link between growth and crisis-volatility (negative skewness of credit growth). Such a link is present here, but not in ST. ST is not designed to capture the volatility-growth link.

[11] Bailout expectation are necessary to explain why: (i) firms in the non-tradable sector with currency mismatch on their book borrow at a cheapest rate than similar firms in the tradable sector but with no currency mimatch. (ii) the spread in interest rate between foreign and domestic currency debt is not significantly different for firms in the non-tradables and firms in the tradables sector.

[12] Relatedly, by looking at differences in stock returns between large and small banks, Ghandi and Lustig (2010) provide evidence of an implicit guarantees on large banks in the US economy but not on small banks.

[13] Juglar (1862, 1863) characterize asymmetric credit cycles along with the periodic occurrence of crises in France, England, and United States between 1794 and 1859. He concludes that: "The regular development of wealth does not occur without pain and resistance. In crises everything stops for a while but it is only a temporary halt, prelude to the most beautiful destinies." Juglar (1863), page 13 (our translation).

3 Model

We consider an endogenous OLG growth model of a two-sector small open economy with credit market imperfections. There are two goods: a final consumption good (T) and an intermediate good (N) good, which is used as an input in the production of both goods. We let the T-good be the numeraire and we denote the relative price of N-goods by $p_t = p_t^N / p_t^T$.[14]

Agents. There are competitive risk neutral international investors whose cost of funds equals the world interest rate r. These investors lend any amount as long as they are promised an expected payoff of $1 + r$. They also issue default-free bonds: an N-bond and a T-bond. The T-bond pays $1 + r$ next period, while the N-bond pays $(1 + r_t^n)p_{t+1}$.

There are overlapping generations of consumers that live for two periods and have linear preferences over consumption of T-goods: $c_t + \frac{1}{1+r} c_{t+1}$. Consumers are divided into two groups of measure one: workers and entrepreneurs.

Workers are endowed with one unit of standard labor. In the first period of their life, a worker supplies inelastically his unit of labor ($l_t = 1$) and receives a wage income v_t. At the end of the first period, he retires and invests his wage income in the risk-free bonds.

Entrepreneurs are endowed with one unit of entrepreneurial labor. In the first period of her life, a young entrepreneur supplies inelastically one unit of entrepreneurial labor ($l_t^e = 1$) and receives a wage income v_t^e. At the end of the first period, she starts running an N-firm and makes investment decisions. In the second period of her life, she receives the firm's profits, if any.

Production Technologies. There is a continuum, of measure one, of firms run by entrepreneurs that produce N-goods using entrepreneurial labor (l_t^e), and capital (k_t). Capital consists of N-goods invested during the previous period (I_{t-1}), which fully depreciates after one period. The production function is

$$q_t = \Theta_t k_t^{\beta} l_t^{e 1-\beta}, \qquad \Theta_t =: \theta \overline{k_t}^{1-\beta}, \qquad k_t = I_{t-1}, \qquad \beta \in (0, 1) \tag{3}$$

The technological parameter Θ_t embodies an external effect, where $\overline{k_t}$ is the average N-sector capital, that each firm takes as given. Notice that in a symmetric equilibrium, the output of the N-sector is linear in investment:

$$q_t = \theta I_{t-1}$$

There is a continuum, of measure one, of competitive firms that produce the T-good combining standard labor (l_t) and the N-good (d_t) using a Cobb-Douglas technology: $y_t = a d_t^{\alpha} (l_t)^{1-\alpha}$. The

[14] In an international setup p_t is the inverse of the real exchange rate.

representative T-firm maximizes profits taking as given the price of N-goods (p_t) and standard labor wage (v_t).

$$\max_{d_t, l_t} [y_t - p_t d_t - v_t l_t], \quad y_t = a d_t^{\alpha} (l_t)^{1-\alpha}, \quad \alpha \in (0,1). \tag{4}$$

There is an alternative–inferior–technology to produce T-goods that will only be actived in the financial black-hole equilibrium considered in Section **. This technology uses only T-goods as inputs according to:

$$y_{t+1} = \varepsilon_{t+1} I_t^{\varepsilon}, \quad \varepsilon_{t+1} = \begin{cases} \overline{\varepsilon} & \text{with probability} \quad \lambda, \\ 0 & \text{with probability} \quad 1 - \lambda \end{cases} \qquad \overline{\varepsilon} \leq 1 + r, \tag{5}$$

where I_t^{ε} denotes the input of T-goods.

Firm Financing. The investable funds of a firm consist of its internal funds w_t plus the liabilities B_t it issues. These investable funds can be used to buy default-free bonds (s_t, s_t^n) or invest to produce next period. Since N-firms investment consist in buying N-goods ($p_t I_t$), the time t budget constraint and time $t+1$ profits of an N-firm are, respectively:

$$p_t I_t + s_t + s_t^n = w_t + B_t. \tag{6}$$

$$\pi(p_{t+1}) = p_{t+1} q_{t+1} + (1+r) s_t + p_{t+1}(1 + r_t^n) s_t^n - v_{t+1}^e l_{t+1} - L_{t+1}, \tag{7}$$

where the cash flow of the firm equals the entrepreneur's wage ($w_t = v_t^e$), and L_{t+1} is the next period's promised repayment, which we describe below. Since T-firms produce by combining instantanously labor and intermediate inputs, they do not require financing.

There are two types of one-period bonds: standard bonds and catastrophe bonds. Under standard bonds a firm must promise to repay the same nominal amount in all states. In contrast, with catastrophe bonds a debtor can promise to repay an arbitrarily large ammount in bad states and zero in good states.

Standard bonds can promise to repay in either N-goods or T-goods. That is, if at time t a firm issues b_t T-bonds and b_t^n N-bonds, with respective interest rates ρ_{t+1} and ρ_{t+1}^n, then at $t+1$ it will have to repay in all states

$$L_{t+1} = (1 + \rho_{t+1}) b_t \ + \ p_{t+1}(1 + \rho_{t+1}^n) b_t^n. \tag{8}$$

If at $t+1$ the firm does not repay, then it must default.

Credit market imperfections. Firm financing is subject to three credit market imperfections. First, firms cannot commit to repay their liabilities. This imperfection might give rise to borrowing constraints in equilibrium

Contract Enforceability Problems. Entrepreneurs cannot commit to repay their liabilities: if at time t the entrepreneur incurs a non-pecuniary cost $h[w_t + B_t]$, then at $t+1$ she will be able to divert all the returns provided the firm is solvent (i.e., $\pi(p_{t+1}) \geq 0$).

Second, there are systemic bailout guarantees that cover lenders against systemic crises, but not against idyosincratic default. This imperfection might induce N-firms to undertake insolvency risk by denominating their debt in T-goods rather that in N-goods.

Systemic Bailout Guarantees. If a majority of firms become insolvent, a bailout agency pays lenders the outstanding liabilities of each defaulting firm. The guarantee applies to any type of financial liabilities.

Lastly, there are bankcruptcy costs. When a firm defaults, a share $1 - \mu - \mu_w$ of the insolvent firms' revenues is lost in bankcruptcy procedures. In this case, the bailout agency can recoup only $\mu p_t q_t$, and the workers receive a wage of only $\mu_w p_t q_t$. The parameteres μ and μ_w satisfy

$$\mu \in [0, \beta] \qquad \text{and} \qquad \mu_w \in [0, 1 - \beta]. \tag{9}$$

Fiscal Solvency. We impose the condition that bailout guarantees are domestically financed via taxation. We assume that the the bailout agency is run by the government, that has perfect access to perfect capital markets and can levy lump-sum taxes (T_t) on T-production. It follows that the intertemporal government budget constraint is:

$$E_t \sum_{j=0}^{\infty} \delta^j \{[1 - \xi_{t+j}][L_{t+j} - \mu p_{t+j} q_{t+j}] - T_{t+j}\} = 0, \tag{10}$$

where $\xi_{t+j} = 1$ if no bailout is granted and zero otherwise.

Regulatory Regimes. The regulatory regime determines the set of liabilities that firms can issue. There are three regulatory regimes. First, a *"financially repressed regime"* under which a firm can only issue one-period standard bonds and must denominate debt in the good which it produces (i.e, cannot take on insolvency risk). Second, a *"financially liberalized regime"* under which a firm can only issue one-period standard bonds, but is free to take on insolvency risk. Finally, there is an *"anything-goes regime"* under which firms can issue both standard and catastrophe bonds, and can take on insolvency risk.

Consider an N-firm. Since the only source of uncertainty is relative price risk, from the perspective of an N-firm, N-bonds constitute hedged debt. Meanwhile T-bonds generate insolvency risk because there is a mismatch between the denomination of liabilities and the price that will determine future revenues. Thus, its solvency will depend on the price of N-goods tomorrow. The following table describes the allowable repayment patterns of an N-firm under the three regulatory regimes.

Equilibrium Concept. In this economy there is endogenous price risk: in an equilibrium p_{t+1} may equal $\overline{p}_{t+1}$ with probability u_{t+1} or $\underline{p}_{t+1}$ with probability $1 - u_{t+1}$. The probability u_{t+1} may equal either 1 or u, and this is known at t.

A key feature of the mechanism is the existence of correlated risks across agents: since guarantees are systemic, the decisions of agents are interdependent. They are determined in the following credit market game, which is similar to that considered by Schneider and Tornell (2004). During each period t, taking prices as given, every young entrepreneur proposes a plan $P_t = (I_t, s_t, s_t^n, b_t, b_t^n, L_{t+1})$ that satisfies budget constraint (6). Lenders then decide whether to fund these plans. Finally, funded young entrepreneurs make investment and diversion decisions.

Payoffs are determined at $t+1$. Consider first plans that do not lead to diversion. If the firm is solvent ($\pi(p_{t+1}) \geq 0$), the old entrepreneur pays $v_{t+1}^e = [1-\beta]p_{t+1}q_{t+1}$ to the young entrepreneur and L_{t+1} to lenders. She then collects the profit $\pi(p_{t+1})$. In contrast, if the firm is insolvent ($\pi(p_{t+1}) < 0$), young entrepreneurs receive $\mu_w p_{t+1} q_{t+1}$ ($\mu_w < 1 - \beta$), lenders receive the bailout if any is granted, and old entrepreneurs get nothing. Consider next plans that entail diversion. If the firm is solvent, the old entrepreneur gets $\beta p_{t+1} q_{t+1}$, the young entrepreneur gets $[1-\beta]p_{t+1}q_{t+1}$ and lenders receive the bailout if any is granted. Under insolvency entrepreneurs get nothing and lenders receive the bailout if any is granted. The problem of a young entrepreneur is then to choose an investment plan P_t and diversion strategy η_t that solves:

$$\max_{P_t, \eta_t} \quad E_t \left[\zeta_{t+1}\{p_{t+1}q_{t+1} + (1+r)s_t + p_{t+1}(1+r^n)s_t^n - v_{t+1}^e l_{t+1}^e - (1-\eta_t)L_{t+1}\} - \eta_t h \cdot [w_t + B_t]\right]$$

subject to (6), where $\eta_t = 1$ if the entrepreneur has set up a diversion scheme, and zero otherwise; and $\zeta_{t+1} = 1$ if $\pi(p_{t+1}) \geq 0$, and zero otherwise.

Definition. *A symmetric equilibrium is a collection of stochastic processes* $\{I_t, s_t, s_t^n, b_t, b_t^n, L_{t+1}, d_t, y_t, q_t, u_t, p_t, w_t, v_t^e, v_t\}$ *such that, (i) given current prices and the distribution of future prices, the plan* $(I_t, s_t, s_t^n, b_t, b_t^n, L_{t+1})$ *is determined in a symmetric subgame perfect equilibrium of the credit market game, and* d_t *maximizes T-firms' profits; (ii) factor markets clear; and (iii) the market for non-tradables clears:*

$$d_t(p_t) + I_t(p_t, \underline{p}_{t+1}, \overline{p}_{t+1}, u_{t+1}) = q_t(I_{t-1}) \tag{11}$$

To close the model we assume that date zero young entrepreneurs are endowed with $w_0 = (1-\beta)p_o q_o$ units of T-goods, while old entrepreneurs are endowed with q_o units of N-goods and have no debt in the books.

3.1 Discussion of the Setup

Despite the rich structure of our economy, which enables to reproduce several related empirical facts, our equilibria are fully solved in closed-form. This allows us to isolate the investment share ϕ_t as the key determinant of the growth and efficiency properties of alternative regimes. Because of the empirical importance of sectorial asymmetries, a one-sector framework is not appropriate to analyze the financial liberalization policies that tend to generate boom-bust cycles.

Our framework is similar to a Rebelo-type two-sector AK model. The source of endogenous growth is a production externality in the intermediate goods sector, which is also the investment sector. This N-sector uses its own goods as capital, and as a result, the share of N-output commanded by the N-sector for investment (ϕ) is the key determinant of aggregate growth. Because the N-sector is subject to borrowing constraints, ϕ might be too small in equilibrium and so the economy as a whole might experience a *bottleneck* to growth. Our result about the gains from finnacial liberalization will derive from the fact that the undertaking of credit risk–by increasing the mean value of ϕ–may increase production efficiency and aggregate growth via linkages to the T-sector.[15] This modeling choice is consistent with the evidence provided by Harrison (2003) of robust positive externalities in the investment sector but not in the consumption good sector. As shown by Febelmayr and Licandro (2005), the two-sector AK model is consistent with the time serie evidence of a fall in the price of the equipement sector relative to the final good sector (Whelan (2003)). The fall in the price of investment is the consequence of the production externality in the investment good sector and enables sustained growth in the aggregate economy.

Empirical evidence shows that the higher growth associated with FL comes together with more crisis-volatility. To capture this growth-volatility link, we consider a set-up with no exogenous source of shocks. In equilibrium endogenous insolvency risk arises from a self-reinforcing mechanism: N-firms find it profitable to issue T-debt in the presence of systemic guarantees and sufficient expected price variability. This variability, in turn, arises when N-firms issue enough T-debt: since N-goods are inputs in N-production, enough T-debt in the balance sheet of N-firms gives rise to the possibility of a crisis state characterized by the collapse of the N-good price and generalized

[15] In contrast, the assumptions that N-goods are not consumed and T-goods are not intermediate inputs are convenient but not essential. If N-goods were consumed, there would a deeper fall in the demand of N-goods when N-firms become insolvent, accentuating the self-fulfilling depreciation that generates crisis.

bankruptcies.

To capture the dynamic and the static effects of crises we have allowed for two types of crisis costs: financial distress costs–indexed by $(1-\beta)/\mu_w$–and bankruptcy costs–indexed by β/μ. All the equilibria we characterize exist for any $\mu_w \in (0, 1-\beta)$ and $\mu \in [0, \beta]$.

Financing constraints affect sectors asymmetrically. Contract enforceability problems give rise to financing constraints, which affect mainly the N-sector as it needs external financing to invest. In contrast, T-firms that use N-inputs do not require financing because they transform instantaneously inputs into final output. If we assumed instead that T-firms require capital, but have access to perfect capital markets, the dynamics of the model would be the same. In this case T-firms would use the efficient level of capital and the increased T-sector demand for intermediate inputs–because its fall in price–will make capital more productive and increase the level of capital T-firms use.[16] This feature is consistent with the sectoral findings of Levchenko et al. (2008).

The assumption that bailouts are granted only during a systemic crisis is essential. If instead, guarantees were granted whenever a single borrower defaulted, then the guarantees would neutralize the contract enforceability problems and borrowing constraints would not arise in equilibrium as noted by Schneider and Tornell (2004).

The three regulatory regimes we consider–repression, liberalization, and anything-goes–are meant to capture in a simple way three regulatory environments. One in which there is over-regulation, credit policies are restrictive and so leverage is low. Another situation in which agents are free to take on risk but there is financial discipline that ensures lenders impose strict repayment criteria on their loans. Finally, a situation where agents have the ability to implement scams that exploit bailout guarantees, like the ones that were used by AIG.[17] As we shall see, standard bonds induce more financial discipline than catastrophe bonds because if at $t+1$ the firm does not repay, then it must default.

Markets are complete in our framework. Since during each period the price can take only two values, the menu of securities allows agents to hedge all risk.[18] This will allow us to make the point that growth and efficiency gains arise from the undertaking of credit risk, not from consumption smoothing.

[16] To see this consider the following alternative production function in the T-secor: $y = ak^{\alpha}d^{\beta}l^{1-\beta-\alpha}$. The efficient level of capital is such that $(1+r)k = \beta y$. Substituting in the production function makes clear that increase in the demand for inputs increases the demand for capital as well.

[17] AIG issued large amount of CDS prior to the crisis - and cashed default premia - but did not have the collateral necessary to meet large promised payments during the crisis when the CDS were triggered. AIG liabilities from CDS were ultimatly covered in full through to a goverment bailout.

[18] In particular, N-debt is a perfect hedge for N-sector firms.

The agency problem and the two-period lived entrepreneur set-up is considered by Schneider and Tornell (2004). The advantage of this set-up is that one can analyze financial decisions period-by-period. This will allow us to explicitly characterize the stochastic processes of prices and investment. These closed-form solutions are essential to derive the limit distribution of growth rates and establish our efficiency results.

3.2 Symmetric Equilibria (SE)

We construct SE in two steps. First, we take prices (p_t) and the likelihood of crisis ($1 - u_{t+1}$) as given, and derive the equilibrium at a point in time. We then endogeneize p_t and u_{t+1}.

Notice that the existence of risk neutral deep-pocket investors implies that uncovered interest parity will hold in any equilibrium

$$(1 + r_t^n)p_{t+1}^e = 1 + r, \quad \text{where} \;\; p_{t+1}^e := u_{t+1}\overline{p}_{t+1} + (1 - u_{t+1})\underline{p}_{t+1} \tag{12}$$

The representative T-firm maximizes profits, taking goods and factor prices as given. It thus sets $p_t d_t = \alpha y_t$ and $v_t^T l_t^T = (1-\alpha)y_t$. Since consumers supply inelastically one unit of labor, equilibrium T-output, consumer's income and the T-sector demand for N-goods are, respectively:

$$y_t = d_t^\alpha, \qquad v_t^T = [1 - \alpha]y_t, \qquad d(p_t) = \left[\frac{\alpha}{p_t}\right]^{\frac{1}{1-\alpha}} \tag{13}$$

Since $t + 1$ consumption is discounted using the riskless interest rate $\delta = 1/(1 + r)$, consumers born in period t are indifferent between t and $t + 1$ consumption. Thus, we set:

$$c_{t+1} = [1 - \alpha]y_t \tag{14}$$

In any SE the representative N-firm's capital (k_t) is equal to average N-sector capital ($\bar{k}_t$). Thus, (3) implies that N-output equals: $q_{t+1} = \theta k_{t+1} = \theta I_t$. N-sector financing and investment (I_t) plans are determined by the equilibria of the credit market game, characterized by the next proposition.

Proposition 3.1 (Symmetric Non-diversion Credit Market Equilibria (CME)) *Given prices, there is investment in the production of N-goods if and only if*

$$R_{t+1}^e := \beta\theta\left[u_{t+1}\frac{\bar{p}_{t+1}}{p_t} + [1 - u_{t+1}]\frac{\underline{p}_{t+1}}{p_t}\right] \geq \frac{1}{\delta} > \frac{h}{u_{t+1}} \tag{15}$$

If (15) holds, then

i *In a financially repressed regime credit and investment are:*

$$b_t^n = [m^s - 1]w_t, \quad I_t = m^s\frac{w_t}{p_t}, \quad \text{with } m^s = \frac{1}{1 - h\delta}. \tag{16}$$

ii *In a financially liberalized regime there is a 'safe' CME as in (16). If in addition* $u_{t+1} = u < 1$ *and* $\frac{\beta\theta \underline{p}_{t+1}}{p_t} < \frac{h}{u}$, *there also exists a 'risky' CME in which currency mismatch is optimal* ($b_t^n = 0$). *Credit and investment are:*

$$b_t = [m^r - 1]w_t, \quad I_t = m^r \frac{w_t}{p_t}, \quad \text{with } m^r = \frac{1}{1 - u^{-1}h\delta}. \tag{17}$$

This proposition follows from the results in Schneider and Tornell (2004). To see the intuition notice that, given that all other entrepreneurs choose the safe plan (i), an entrepreneur knows that no bailout will be granted next period. Since lenders must break-even, the entrepreneur must internalize all bankruptcy costs. Thus, she will not set a diversion scheme and will hedge insolvency risk by denominating all debt in N-goods. Since the firm will never go bust and lenders must break even, the interest rate that the entrepreneur has to offer satisfies

$$1 + \rho_t^n = [1 + r]/E_t(p_{t+1}).$$

Since (15) holds, investment yields a return which is higher than the opportunity cost of capital.[19] Thus, the entrepreneur will borrow up to an amount that makes the credit constraint binding: $(1+r)b_t^n \leq h(w_t+b_t^n)$. Substituting this borrowing constraint in the budget constraint $p_t I_t = w_t + b_t^n$ generates the investment equation. Notice that a necessary condition for borrowing constraints to arise is $h < 1 + r$. If h, the index of contract enforceability, were greater than the cost of capital, it would always be cheaper to repay debt rather than to divert.

Given that all other entrepreneurs choose the *risky plan* (ii), a young entrepreneur expects a bailout in the low state, but not in the high state. The proposition shows that, in spite of the guarantees, diversion schemes are not optimal. Thus, borrowing constraints bind. Will the entrepreneur choose T-debt or N-debt? She knows that all other firms will go bust in the bad state (i.e., $\pi(\underline{p}_{t+1}) < 0$) provided there is *insolvency risk* – i.e., $\frac{\beta\theta \underline{p}_{t+1}}{p_t} < \frac{h}{u}$. However, since there are systemic guarantees, lenders will get repaid in full. Thus, the interest rate on T-debt that allows lenders to break-even satisfies

$$1 + \rho_t^{risky} = 1 + r$$

It follows that the benefits of a risky plan derive from the fact that choosing T-debt over N-debt reduces the cost of capital from $1+r$ to $[1+r]u$. Lower expected debt repayments ease the borrowing constraint as lenders will lend up to an amount that equates $u[1+r]b_t$ to $h[w_t+b_t]$. Thus, investment is higher relative to a plan financed with N-debt. The downside of a risky plan is that it entails a probability $1 - u$ of insolvency. Will the two benefits of issuing T-debt –more and cheaper funding–

[19]The marginal return to investment is $E_t(p_{t+1})\Theta_t \beta k_t^{\beta-1} l_t^{1-\beta} - (\delta p_t)^{-1} = E_t(p_{t+1})\theta\beta - (\delta p_t)^{-1}$. This is because in an SE $\Theta_t = \theta \bar{k}_t^{1-\beta}$, $\bar{k}_t = k_t$ and $l_t = 1$.

be large enough to compensate for the cost of bankruptcy in the bad state? If there is sufficient real exchange rate variability and u is not too low, expected profits under a risky plan exceed those under a safe plan: $u\pi^r(\overline{p}_{t+1}) > u\pi^s(\overline{p}_{t+1}) + (1-u)\pi^s(\underline{p}_{t+1})$.

To sum up, Proposition 3.1 makes three key points regarding the financially liberalized regime. First, binding borrowing constraints arise in equilibrium and investment is constrained by cash flow, provided the production of N-goods is a positive NPV undertaking: $R^e_{t+1} \geq 1+r$. Second, agents optimally choose T-denominated debt if there is sufficient real exchange rate variability so that firms go bust in the low price state: $\pi(\underline{p}_{t+1}) < 0$. Third, such a risky currency mismatch eases borrowing constraints and allows firms to invest more than under perfect hedging: $m^r > m^s$.

3.2.1 Equilibrium Dynamics

Here, we endogeneize prices and determine the conditions under which there is a self-validating process $\{p_t,\ \bar{p}_{t+1},\ \underline{p}_{t+1},\ u_{t+1}\}_{t=0}^{\infty}$ that satisfies the return conditions specified in Proposition 3.1. We start by characterizing the transition equations. If a firm is solvent, the young entrepreneur's wage equals the marginal product of her labor, while under insolvency she just obtains a share μ_w of revenues. Thus, in any SE the young entrepreneur's cash flow is

$$w_t = \begin{cases} [1-\beta]p_t q_t & \text{if } \pi(p_t) \geq 0 \\ \mu_w p_t q_t & \text{if } \pi(p_t) < 0, \end{cases} \qquad \mu_w \in (0, 1-\beta) \tag{18}$$

Suppose for a moment that (15) holds, so that it is optimal to invest all funds in the production of N-goods: $p_t I_t = m_t w_t$. It then follows from (18) that N-sector investment is

$$I_t = \phi_t q_t, \qquad \phi_t = \begin{cases} [1-\beta]m_t & \text{if } \pi(p_t) \geq 0 \\ \mu_w m_t & \text{if } \pi(p_t) < 0, \end{cases} \qquad m_t \in \{m^s, m^r\} \tag{19}$$

Since in an SE $q_t = \theta I_{t-1}$, it follows from (13), (19) and the market clearing condition ($d_t + I_t = q_t$) that equilibrium N-output, prices and T-output evolve according to

$$q_t = \theta\phi_{t-1}q_{t-1} \tag{20}$$

$$p_t = \alpha\left[q_t(1-\phi_t)\right]^{\alpha-1} \tag{21}$$

$$y_t = [q_t(1-\phi_t)]^{\alpha} = \frac{1-\phi_t}{\alpha}p_t q_t \tag{22}$$

Clearly, for prices to be positive it is necessary that the share of N-output purchased by the N-sector ϕ_t is less than one:

$$h < u_{t+1}\beta\delta^{-1} \tag{23}$$

Equations (19)-(22) form an SE provided the implied returns validate the agents' expectations (specified in Proposition 3.1). The next two propositions characterize two such SE: a *safe* one in which crises never occur, and a *risky* one where all firms become insolvent in the low price state and are solvent in the high price state.

Proposition 3.2 (Safe Symmetric Equilibria (SSE)) *There exists an SSE if and only if the degree of contract enforceability h is low enough and N-sector productivity θ is large enough. In an SSE there is no currency mismatch ($b_t = 0$) and crises never occur ($u_{t+1} = 1$). Thus, the N-sector investment share is*

$$\phi^s = \frac{1-\beta}{1-h\delta}, \qquad \delta \equiv \frac{1}{1+r}. \tag{24}$$

This proposition states that an SSE exists provided enforceability problems are severe, so that (i) there are borrowing constraints and (ii) $\phi_t < 1$; and productivity is high enough, so that the return on investment is attractive enough.

In an SSE all entrepreneurs select the safe plan of Proposition 3.1 during every period. This implies that there is no currency mismatch in the aggregate, and self-fulfilling crises are not possible ($u_{t+1} = 1$). Therefore, the production of N-goods has a positive net present value (i.e., (15) holds) if and only if $\frac{\beta\theta p_{t+1}}{p_t} = \beta\theta^\alpha(\phi^s)^{\alpha-1} \geq \delta^{-1}$. This condition, as well as (23), hold provided h is low enough and θ is high enough.

Next, we characterize *Risky Symmetric Equilibria (RSE).* We have seen that entrepreneurs will take on T-debt only if there is *enough anticipated real exchange rate variability* to generate high returns in the good state and a critical mass of insolvencies in the bad state. We now reverse the question and ask instead when a risky debt structure implies enough real exchange rate variability. That is: (i) will the low price be low enough so that there will be widespread insolvencies ($\pi(\underline{p}_{t+1}) < 0$)? (ii) will there be a sufficiently high return in the good state to ensure that the ex-ante expected return is high enough ($R^e_{t+1} \geq 1 + r$)?

The following proposition provides answers to these questions, and it establishes that the self-reinforcing mechanism we described above is at work. On the one hand, expected real exchange rate variability makes it optimal for entrepreneurs to denominate debt in T-goods and run the risk of going bust. On the other hand, the resulting currency mismatch at the aggregate level makes the real exchange rate variable, validating agents' expectations.

Proposition 3.3 (Risky Symmetric Equilibrium (RSE)) *There exists an RSE if and only if the probability of crisis $(1-u)$ is small enough, N-sector productivity (θ) is large enough, and the degree of contract enforceability (h) is low, but not too low.*

1. *In any RSE multiple crises can occur during which all N-sector firms default and there is a sharp real depreciation. However, two crises cannot occur in consecutive periods.*

2. *In the RSE where there is a reversion back to a risky path in the period immediately after the crisis, all firms choose risky plans in no-crisis times and safe plans in crisis times. The probability of a crisis and the N-sector's investment share satisfy:*

$$1-u_{t+1} = \begin{cases} 1-u & \textit{if } t \neq \tau_i \\ 0 & \textit{if } t = \tau_i \end{cases} \qquad \phi_t = \begin{cases} \phi^l := \frac{1-\beta}{1-h\delta u^{-1}} & \textit{if } t \neq \tau_i \\ \phi^c := \frac{\mu_w}{1-h\delta} & \textit{if } t = \tau_i \end{cases} \tag{25}$$

where τ_i denotes a crisis time.

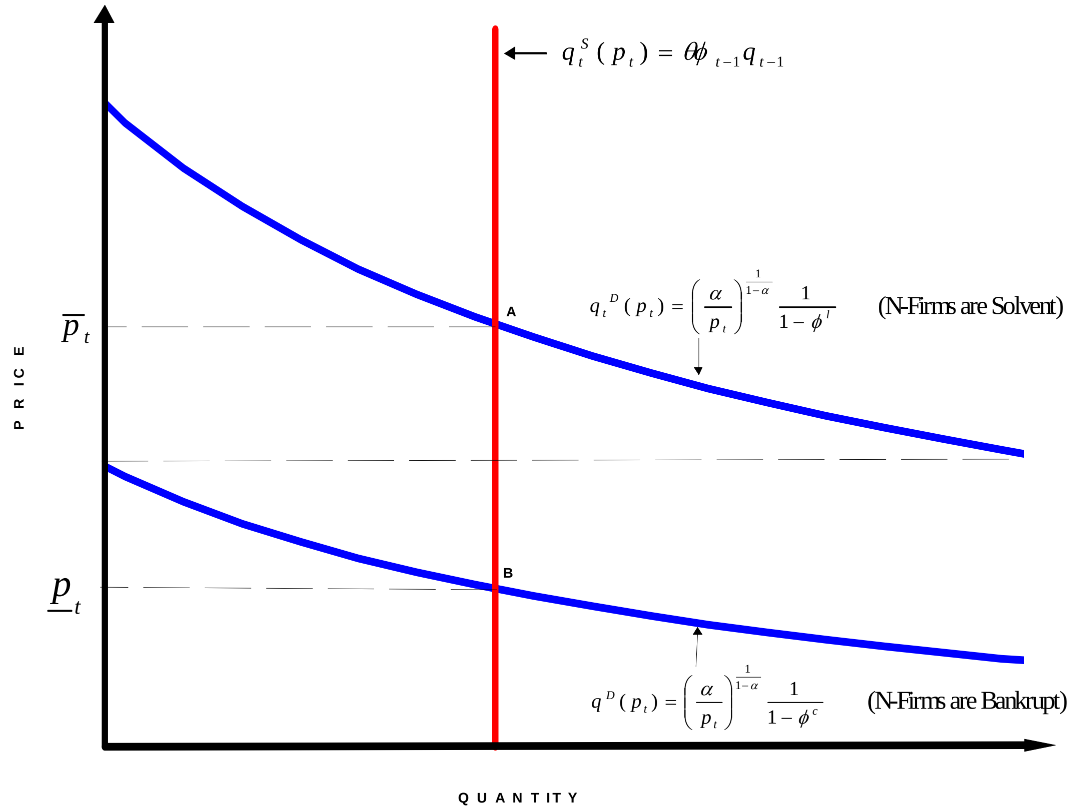
A key property of the RSE characterized in Proposition 3.3 is that a crisis state is not an absorbing state: a crisis can occur every other period independently of the number of previous crises. This property will be essential when we compare financially repressed and liberalized growth paths.

To see the intuition consider a typical period t and suppose that all inherited debt is denominated in T-goods and agents expect a bailout at $t+1$ in case a majority of firms goes bust. Since the debt repayment is independent of prices, there are two market clearing prices as in Figure 1. In the 'solvent' equilibrium (point A in Figure 1), the price is high enough to allow the N-sector to buy a large share of N-output. In contrast, in the 'crisis' equilibrium of point B, the price is so low that N-firms go bust: $\beta \underline{p}_t q_t < L_t$.

The key to having multiple equilibria is that part of the N-sector's demand comes from the N-sector itself. Thus, if the price fell below a cutoff level and N-firms went bust, the investment share of the N-sector would fall (from ϕ^l to ϕ^c). This, in turn, would reduce the demand for N-goods, validating the fall in the prices. Notice that the upper bound on h and the lower bound on θ ensure that when crises are rare events, borrowing constraints arise and investment is profitable (i.e., (15) holds). Meanwhile, the lower bound on h ensures that firms with T-debt go bust in the bad state, and that the fall in cash flow is translated into a large fall in credit and N-investment. This validates the fall in prices.

Three points are worth emphasizing. First, Proposition 3.3 holds for any $\mu_w \in (0, 1-\beta)$ and $\mu \in [0, \beta]$. That is, crisis costs are *not necessary* to trigger a crisis. A shift in expectations is sufficient: a crisis can occur whenever entrepreneurs expect that others will not undertake credit risk, so that there is a reversion to the safe CME characterized in Proposition 3.1. Second, two crises cannot occur consecutively. Since investment in the crisis period falls, the supply of N-goods during the post-crisis period will also fall. This will drive post-crisis prices up, preventing the occurrence of insolvencies even if all debt were T-debt. That is, during the post-crisis period a drop in prices large enough to generate insolvencies is impossible. Third, we focus in the proposition above on a RSE where there is a reversion back to a risky path in the period immediately after the

Figure 1: Non Tradables Market Equilibrium



crisis. In subsection **??**, we will relax this assumption and allow agents to play safe strategies for multiple periods in the aftermath of crisis.

4 Growth

Here, we compare the long-run growth rates along the financially repressed and liberalized regimes–characterized in Propositions 3.2 and 3.3. In Section 6 we consider the anything-goes regime.

Since N-goods are intermediate inputs, while T-goods are final consumption goods, gross domestic product equals the value of N-sector investment plus T-output: $gdp_t = p_t I_t + y_t$. It then follows from (19)-(22) that, in any SE, GDP is given by

$$gdp_t = p_t\phi_t q_t + y_t = q_t^{\alpha} Z(\phi_t) = y_t \frac{Z(\phi_t)}{[1-\phi_t]}, \quad \text{with } Z(\phi_t) = \frac{1-(1-\alpha)\phi_t}{[1-\phi_t]^{1-\alpha}} \tag{26}$$

As we can see, the key determinant of the evolution of GDP is the share of N-output commanded by the N-sector for investment: ϕ_t. This share is determined by the cash flow of young entrepreneurs and by the credit they can obtain.

To relate our findings to the the literature on input misallocation (Jones 2010, 2011) notice that $Z(\phi_t)$ can be interpreted as a contemporaneous measure of TFP {at time t}. Since at time t q_t^α is predetermined by past investment, the contemporeneous effect of investment share changes on aggregate TFP variations at t can be decomposed as follows

$$\frac{\partial gdp_t}{\partial \phi_t} = \frac{\partial Z(\phi_t)}{\partial \phi_t} = p_t q_t - \frac{\alpha y_t}{1-\phi_t} + q_t \phi_t \frac{\partial p_t}{\partial \phi_t} = \; q_t \phi_t \frac{\partial p_t}{\partial \phi_t} > 0$$

The first two terms capture variations in investment and final output, while the third reflects relative price fluctuations. Market clearing in the N-goods market–i.e., $(1-\phi_t)p_t q_t = \alpha y_t$–implies that the induced changes in investment and final output cancel out. Therefore, a change in the investment share ϕ_t affect**s** GDP contemporaneously only through its effect on the relative price of N-output. As an increase in investment raises contemporeanously the price of N-goods, measured aggregate TFP increases.[20]

4.1 Growth in a Financially Repressed Economy

In an SSE the investment share ϕ_t is constant and equal to ϕ^s. Thus, (26) implies that GDP and T-output grow at the same rate.

$$1 + \gamma^s := \frac{gdp_t}{gdp_{t-1}} = \frac{y_t}{y_{t-1}} = \left(\theta \tfrac{1-\beta}{1-h\delta}\right)^\alpha = (\theta\phi^s)^\alpha \tag{27}$$

Absent exogenous technological progress in the T-sector, the endogenous growth of the N-sector is the force driving growth in both sectors. As the N-sector expands, N-goods become more abundant and cheaper allowing the T-sector to expand production. This expansion is possible if and only if N-sector productivity (θ) and the N-investment share (ϕ^s) are high enough, so that credit and N-output can grow over time: $\frac{B_t}{B_{t-1}} = \frac{q_t}{q_{t-1}} = \theta\phi^s > 1$. Notice that for any positive growth rate of N-output, γ^s increases with the intensity of the N-input in the production of T-goods (α).[21]

4.2 Growth in a Financially Liberalized Economy

Proposition 3.3 shows that any RSE is composed of a succession of lucky paths punctuated by crisis episodes. In the RSE characterized by (3.3) the economy is on a lucky path at time t if there has

[20] This result has been derived based on GDP expressed in T-goods. It would be true if we used instead a composite price index since it would put a positive weight on p_t^N.

[21] The mechanism by which higher growth in the N-sector induces higher growth in the T-sector is the decline in the relative price of N-goods that takes place in a growing economy $\frac{p_{t+1}}{p_t} = [\theta\phi^s]^{\alpha-1}$. If there were technological progress in the T-sector, p_t^N/p_t^T would appreciate over time (analogous to the Balassa-Samuelson effect). To see this, we add a technological parameter a_t in the T-production function ($y_t = a_t d_t^\alpha l_t^{1-\alpha}$) and let it grow over time $\frac{a_{t+1}}{a_t} = (1+g)$. Then price dynamics are given by $\frac{p_{t+1}}{p_t} = (1+g)[\theta\phi^s]^{\alpha-1}$.

not been a crisis either at $t-1$ or at t. Since along a lucky path the investment share equals ϕ^l, (26) implies that the common growth rate of GDP and T-output is

$$1+\gamma^l := \frac{gdp_t}{gdp_{t-1}} = \frac{y_t}{y_{t-1}} = \left(\theta \frac{1-\beta}{1-h\delta u^{-1}}\right)^{\alpha} = \left(\theta\phi^l\right)^{\alpha} \tag{28}$$

A comparison of (27) and (28) reveals that as long as a crisis does not occur, growth in a risky economy is higher than in a safe economy. Along the lucky path the N-sector undertakes insolvency risk by issuing T-debt. Since there are systemic guarantees, financing costs fall and borrowing constraints are relaxed, relative to a safe economy. This increases the N-sector's investment share ($\phi^l > \phi^s$). Since there are sectorial linkages ($\alpha > 0$), this increase in the N-sector's investment share benefits both the T- and the N-sectors and fosters faster GDP growth.

However, in a risky economy a self-fulfilling crisis can occur with probability $1-u$, and during a crisis episode growth is lower than along a safe path. We have seen that any crisis episode consists of at least two periods: in the first period the financial position of the N-sector is severely weakened and the investment share falls from ϕ^l to $\phi^c < \phi^s$; then in the second period it jumps back to ϕ^l. Since these transitions occur with certainty, the mean crisis growth rate is given by:

$$1+\gamma^{cr} = \underbrace{\left(\left(\theta\phi^l\right)^{\alpha}\frac{Z(\phi^c)}{Z(\phi^l)}\right)^{1/2}}_{\text{crisis period}} \underbrace{\left((\theta\phi^c)^{\alpha}\frac{Z(\phi^l)}{Z(\phi^c)}\right)^{1/2}}_{\text{post-crisis period}} = \left(\theta(\phi^l\phi^c)^{\frac{1}{2}}\right)^{\alpha} \tag{29}$$

The second equality in (29) shows that the average loss in GDP growth stems only from the fall in the N-sector's average investment share: $(\phi^l\phi^c)^{\frac{1}{2}}$. This reduction comes about through two channels: financial distress (indexed by $\frac{\mu_w}{1-\beta}$) and a reduction in risk taking and leverage (indexed by $\frac{1-h\delta}{1-h\delta u^{-1}}$). Notice that variations in GDP growth generated by real exchange rate changes at τ and $\tau+1$ cancel out. Appendix A analyzes the costs of crises.

A crisis has long-run effects because N-investment is the source of endogenous growth, and so the level of GDP falls permanently. This raises two questions: is mean long-run GDP growth in a risky economy greater than in a safe one? Does an increase in risk taking (i.e., an increase in the probability of crisis) in a risky economy increase mean long-run GDP growth? The answers to these questions are not straightforward because an increase in the probability of crisis $(1-u)$ has opposing effects on long-run growth. One the one hand, a greater $1-u$ increases investment and growth along the lucky path by increasing the subsidy implicit in the guarantee and allowing firms to be more leveraged. On the other hand, a greater $1-u$ makes crises more frequent. Therefore, to give a precise answer to the questions we have raised, we compute the limit distribution of GDP's growth rate.

Growth Limit Distribution. Next, we derive the limit distribution of GDP's compounded growth rate $(\log(gdp_t) - \log(gdp_{t-1}))$ along the RSE characterized in Proposition 3.3. In this RSE firms undertake credit risk the period after the crisis. In subsection **??** we consider alternative RSEs where a crisis is followed by a cool-off phase during which safe plans are undertaken.

Recall that in any RSE two crises cannot occur in consecutive periods. It follows from (25), (28) and (29) that the growth process follows a three-state Markov chain characterized by

$$\Gamma = \begin{pmatrix} \log\left((\theta\phi^l)^\alpha\right) \\ \log\left((\theta\phi^l)^\alpha \frac{Z(\phi^c)}{Z(\phi^l)}\right) \\ \log\left((\theta\phi^c)^\alpha \frac{Z(\phi^l)}{Z(\phi^c)}\right) \end{pmatrix}, \qquad T = \begin{pmatrix} u & 1-u & 0 \\ 0 & 0 & 1 \\ u & 1-u & 0 \end{pmatrix}$$

The three elements of Γ are the growth rates in the lucky, crisis and post-crisis states, respectively. The element T_{ij} of the transition matrix is the transition probability from state i to state j. Since the transition matrix is irreducible, the growth process converges to a unique limit distribution over the three states that solves $T'\Pi = \Pi$. Thus, $\Pi = \left(\frac{u}{2-u}, \frac{1-u}{2-u}, \frac{1-u}{2-u}\right)'$, where the elements of Π are the shares of time that an economy spends in each state over the long-run. It then follows that the mean long run GDP growth rate is $E(1+\gamma^r) = \exp(\Pi'\Gamma)$.[22] That is,

$$E(1+\gamma^r) = (1+\gamma^l)^\omega(1+\gamma^{cr})^{1-\omega} \;=\; \theta^\alpha(\phi^l)^{\alpha\omega}(\phi^l\phi^c)^{\alpha\frac{1-\omega}{2}}, \quad \text{where } \omega = \frac{u}{2-u} \tag{30}$$

A comparison of long run GDP growth rates in (27) and (30) reveals the trade-offs involved in following safe and risky growth paths, and allows us to determine the conditions under which credit risk is growth enhancing. Rearranging (30), we derive in the following proposition.

Proposition 4.1 (Long-run GDP Growth) *In a RSE the mean long-run GDP growth rate is given by*

$$E(1+\gamma^r) = (1+\gamma^s)^\alpha \left(\frac{\phi^l}{\phi^s}\right)^{\frac{1}{2-u}} \left(\frac{\mu_w}{1-\beta}\right)^{\frac{u}{2-u}} \tag{31}$$

1. *There is risky equilibrium such that mean long-run GDP growth is greater than in a safe equilibrium only if financial distress during crises is not too severe (i.e., $l^d \equiv 1 - \frac{\mu_w}{1-\beta} < \overline{l^d}$).*

2. *If $l^d < \overline{l^d}$, there exists an $h^* < u\beta\delta^{-1}$, such that mean growth is greater in a risky than in a safe equilibrium if and only if the degree of contract enforceability satisfies $h > h^*$:*

$$h^* = \frac{1-\left(1-l^d\right)^{1-u}}{u^{-1}-\left(1-l^d\right)^{1-u}}\frac{1}{\delta} \qquad \overline{l^d} = 1 - \left(\frac{1-\beta}{1-\beta u}\right)^{\frac{1}{1-u}}. \tag{32}$$

[22] $E(1+\gamma^r)$ is the geometric mean of $1+\gamma^l, 1+\gamma^{lc}$ and $1+\gamma^{cl}$.

The proposition establishes two conditions for risk-taking to be growth enhancing. First, financial distress costs, as measured by the fall in a firm's internal funds from $1-\beta$ to μ_w, cannot be excessively large to allow for risk-taking to increase long run growth.[23] Second, the degree of contract enforceability h needs to be high enough so that the leverage effect associated with risk-taking is sufficiently strong .

Rewriting $h > h^*$ as $(1-u)[\log(1-\beta) - \log(\mu_w)] < \log(\phi^l) - \log(\phi^s)$ makes clear what are the costs and benefits associated with a risky path. A risky economy outperforms a safe one if the benefits of higher investment in no-crisis times ($\phi^l > \phi^s$) compensate for the shortfall in internal funds and investment in crisis times ($\mu_w < 1-\beta$) weighted by the frequency of crisis $(1-u)$.

Notice that an increase in distress costs can be compensated by an increase in the degree of contract enforceability. The latter increases leverage and amplifies the benefits of risk-taking ($\partial\phi^l/\partial h > \partial\phi^s/\partial h$). However, as h is bounded above to ensure the existence of an RSE ($\phi^l < 1 \Leftrightarrow h < u\beta\delta^{-1}$), an increase in contract enforceability can compensate for large but not arbitrarily large financial distress costs (i.e., $\mu_w \to 0$).

Figure 2 exhibits one realization of the paths of GDP, credit, T- and N-output associated with a set of parameters satisfying the conditions in Propositions 3.2 and 3.3. This figure makes clear that greater long run growth comes at the cost of (rare) crises. Notice that since N-goods are used as inputs in both sectors, higher N-sector investment leads to a lower initial level of T-ouput in a risky economy ($y_0^l = \left[q_0(1-\phi^l)\right]^\alpha < \left[q_0(1-\phi^s)\right]^\alpha = y_0^s$). Over time, however, T-output along the risky path will overtake that in a safe path.

Figure 3 illustrates the limit distribution of GDP growth rates by plotting different GDP paths corresponding to different realizations of the sunspot process. Most of the risky paths outperform the safe path, except for a few unlucky risky paths. If we increased the number of paths, the cross section distribution would converge to the limit distribution.

Figure 4 exhibits the two effects of an increase in the probability of crisis $(1-u)$. A reduction in u increases the investment multiplier m^r at a point in time, but it also increases the frequency of crises. The figure shows that for high u the first effect dominates and the long-run mean growth rate of GDP goes up. Importantly, u cannot be reduced indefinitely. After a certain point an RSE ceases to exist.

[23] How large can "not too large" be?

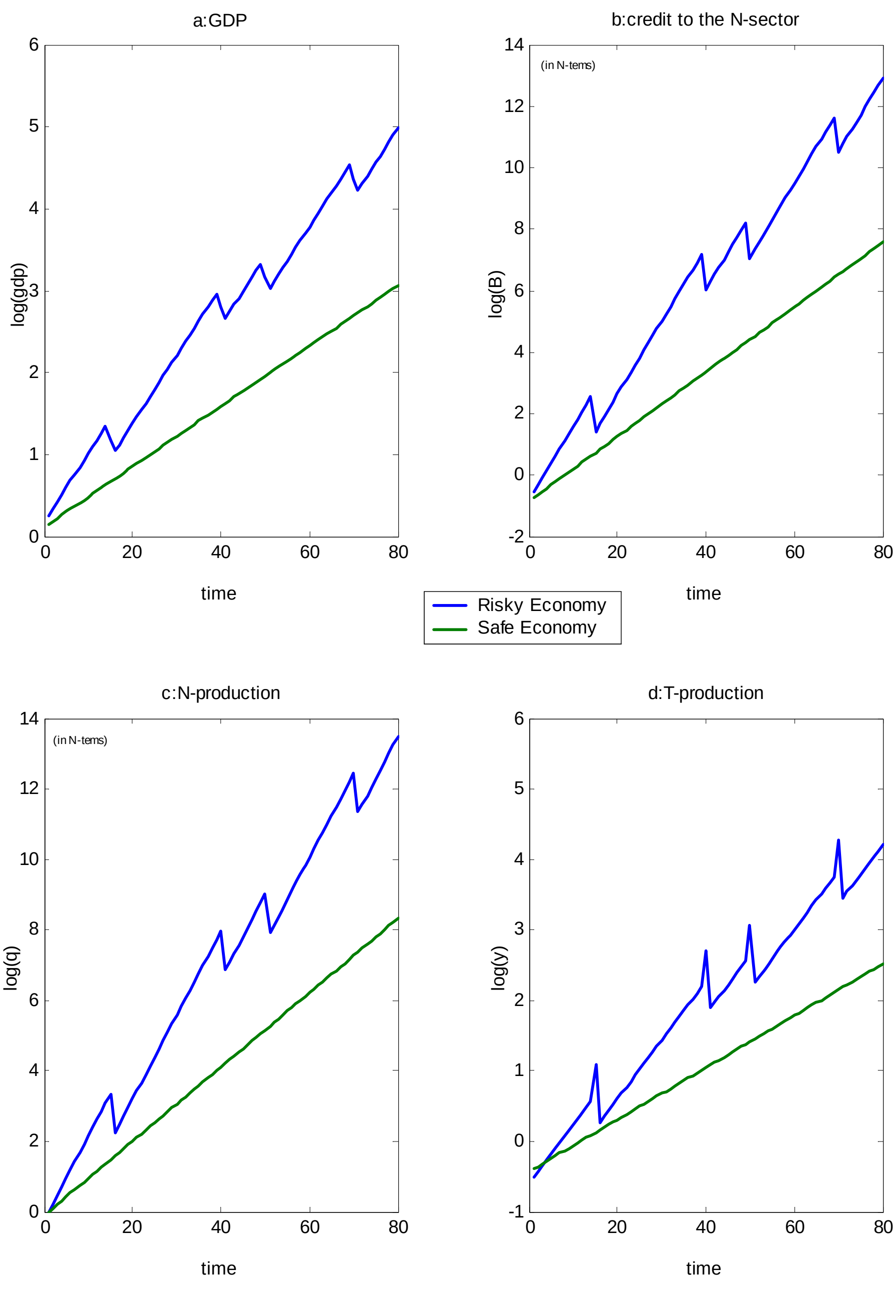
$1-\beta = 0.2$

u	0.85	0.99
$\overline{l^d}$	95.4%	98%

$1-\beta = 0.4$

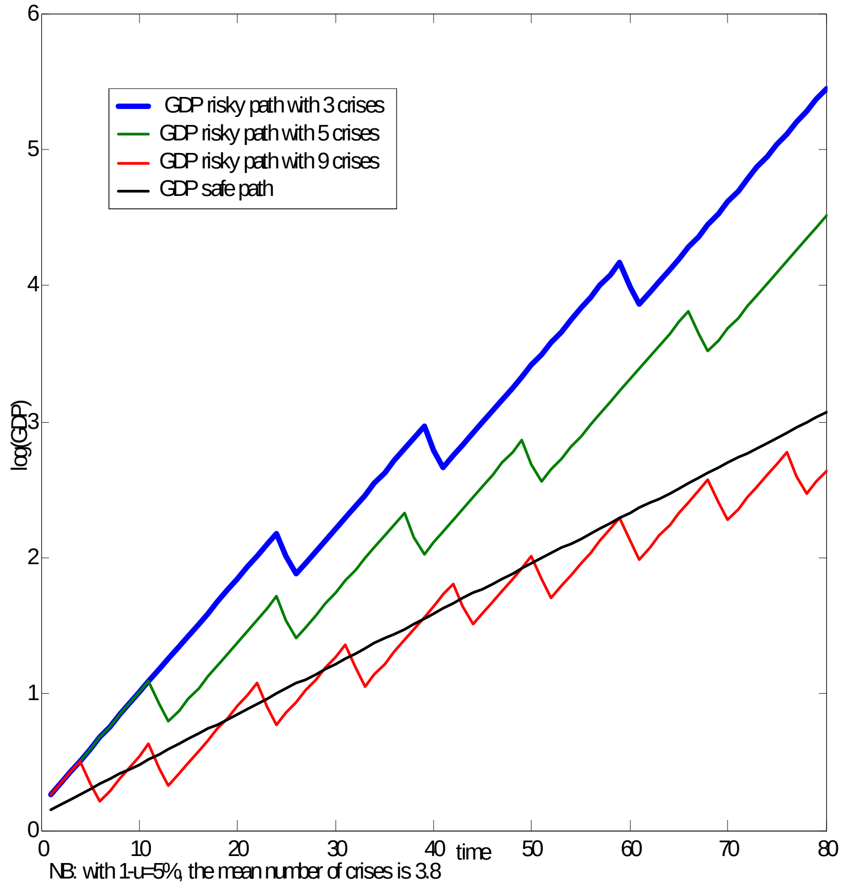
u	0.85	0.99
$\overline{l^d}$	74.2%	77.4%

Figure 2: Risky vs Safe Economy



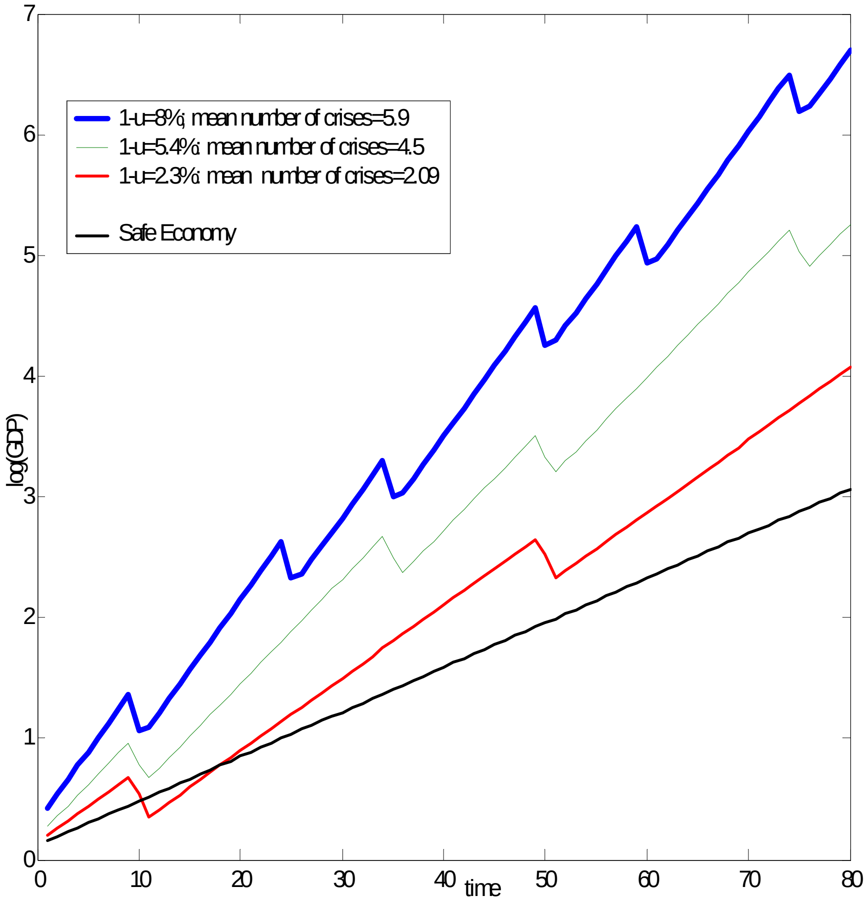
$parameters: \theta = 1.65 \quad \alpha = 0.35 \quad h = 0.76 \quad 1-\beta = 0.2 \quad l^d = 70\% \quad 1-u = 5\%$

Figure 3: Limit Distribution of GDP



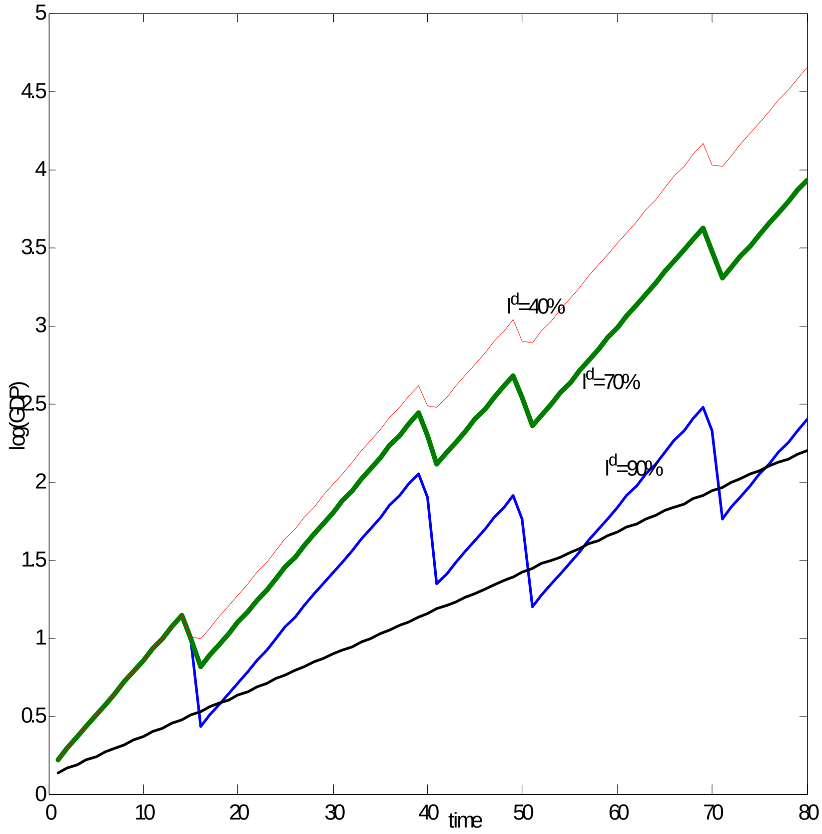
parameters : $\theta = 1.65$ $\alpha = 0.35$ $h = 0.76$ $1 - \beta = 0.2$ $l^d = 70\%$ $1 - u = 5\%$

Figure 4: GDP Growth and Credit Risk



parameters : $\theta = 1.65 \quad \alpha = 0.35 \quad h = 0.76 \quad 1-\beta = 0.2 \quad l^d = 70\ \%$

Figure 5: GDP Growth and Financial Distress Costs ($l^d = 1 - \frac{\mu_w}{1-\beta}$)



parameters : $\theta = 1.65$ $\alpha = 0.35$ $h = 0.76$ $1 - \beta = 0.2$ $1 - u = 5\%$

Finally, Figure 5 shows risky growth paths associated with different degrees of crisis' financial distress. As we can see, even if 90% of N-sector cash flow is lost during a crisis, a risky economy can outperform a safe economy over the long run.

5 Production Efficiency and Consumption Possibilities

We have considered an endogenous growth model where the financially constrained N-sector is the engine of growth because it produces the intermediate input used throughout the economy. Thus, the share of N-output invested in the N-sector, ϕ_t, is the key determinant of economic growth. When ϕ_t is too small T-output is high in the short-run, but long-run growth is slow. In contrast, when ϕ_t is too high, there is inefficient accumulation of N-goods. In this section we ask three questions.

First, what is the Pareto optimal investment share sequence $\{\phi_t\}$? Second, can this Pareto optimal investment sequence be replicated in a financially repressed economy? If not, can the average invesment share be higher in a financially liberalized economy where agents undertake credit risk and crises occur? Third, will the present value of consumption be greater in a liberalized economy after netting out the costs bailouts and of crises? In Section 6 we consider the anything-goes regime.

5.1 Pareto Optimality

Consider a central planner who maximizes the present discounted value of the consumption of workers and entrepreneurs by investing the supply of N-goods in the T-sector ($[1-\phi_t]q_t := d_t$) and in the N-sector ($\phi_t q_t$), as well as by assigning sequences of consumption goods to consumers and entrepreneurs for their consumption.

$$\max_{\{c_t, c_t^e, \phi_t\}_{t=0}^{\infty}} W^{PO} = \sum_{t=0}^{\infty} \delta^t \left[c_t^e + c_t\right], \quad \text{s.t.} \quad \sum_{t=0}^{\infty} \delta^t \left[c_t + c_t^e - y_t\right] \leq 0 \qquad (33)$$
$$y_t = [1-\phi_t]^{\alpha} q_t^{\alpha}, \qquad q_{t+1} = \theta \phi_t q_t$$

Pareto optimality implies efficient accumulation of N-inputs: the planner should choose the investment sequence $\{\phi_t\}$ to maximize the present value of T-production ($\sum_{t=0}^{\infty} \delta^t y_t$). We show in the Appendix that the Pareto optimal N-investment share is *constant* and equal to

$$\phi^{po} = (\theta^{\alpha}\delta)^{\frac{1}{1-\alpha}}, \qquad \text{if } \alpha < \log(\delta^{-1})/\log(\theta) \qquad (34)$$

The Pareto optimal share equalizes the discount rate δ^{-1} to the intertemporal rate of transformation. A marginal increase in the N-sector investment share ($\partial\phi$) reduces today's T-output by $\alpha\left[(1-\phi)q_t\right]^{\alpha-1}\partial\phi$, but increases tomorrow's N-output by $\theta\partial\phi$ and tomorrow's T-output by $\alpha\left[(1-\phi)\theta\phi q_t\right]^{\alpha-1}\theta\partial\phi$. Thus, at an optimum $\theta^{\alpha}\phi^{\alpha-1} = \delta^{-1}$.

Can a decentralized economy replicate the Pareto optimal allocation? The optimal investment share is determined by investment opportunities: $\theta^{\alpha}\delta$. In contrast, in a decentralized safe economy the N-investment share ($\phi^s = \frac{1-\beta}{1-h\delta}$) is determined by the credit market imperfections: the degree of contract enforceability (h) and the constrained sector's cash flow ($1-\beta$). Clearly, if either h or $1-\beta$ are low, the N-sector investment share will be lower than the Pareto optimal share: $\phi^s < \phi^{po}$. That is, when the N-sector is severely credit constrained, low N-sector investment will keep the economy below production efficiency. For future reference we summarize with the following Proposition.

Proposition 5.1 (Bottleneck) *N-sector investment in a safe economy is below the Pareto optimal level (i.e., there is a 'bottleneck') if there is low contract enforceability:*

$$h < (1-(1-\beta)\theta\,(\theta\delta)^{-\frac{1}{1-\alpha}})/\delta.$$

When there is a bottleneck, the share of N-inputs allocated to T-production should be reduced and that allocated to N-production should be increased in order to bring the allocation nearer to the Pareto optimal level. This reallocation reduces the initial level of T-output, but increase its growth rate and the present value of cumulative T-production.

5.1.1 Input-output Linkages and the Dynamic Multiplier Effect

If there is a bottleneck (i.e., $\phi^s < \phi^{po}$), an increase in the investment share ϕ corresponds to a reduction of input misallocation. In the context of our two-sector endogenous growht model this increase in ϕ leads to an increase in future final good production. This dynamic input-multiplier effect is analogous to the steady-state approach proposed by Jones (2010, 2011) in the context of a neo-classical growth model.

A marginal increase in the N-sector investment share ($\partial\phi$) reduces today's T-output by $\alpha\left[(1-\phi)q_t\right]^{\alpha-1}\partial\phi$, but increases tomorrow's N-output by $\theta\partial\phi$ and tomorrow's T-output by $\alpha\left[(1-\phi)\theta\phi q_t\right]^{\alpha-1}\theta\partial\phi$. The intertemporal multiplier effect is therefore:

$$M = \frac{\alpha\left[(1-\phi)\theta\phi q_t\right]^{\alpha-1}\theta}{\alpha\left[(1-\phi)q_t\right]^{\alpha-1}} = \theta^{\alpha}\phi^{\alpha-1}.$$

It follows that the long run dynamic gains in T-output resulting from a marginal increase in the investment rate in the N-sector are given by

$$M + M^2 + ...M^j + ... = \sum_{j=1}^{\infty} M^j = \frac{1}{1-M} - 1.$$

These dynamic gains are maximized if M tends to 1, or equivalently if the investment share ϕ^g tends to $(\theta)^{\alpha/1-\alpha}$. Notice that the value of ϕ^g is increasing in α, the *strength of the input-ouput linkage.*

To see the link between ϕ^g and ϕ^{po} note that ϕ^g maximizes the total expected sum of final good production. If the objective were to maximize the expected discounted sum of final good production, we would obtain instead the Pareto optimal investment share: $\phi^{po} = \delta^{1/1-\alpha}(\theta)^{\alpha/1-\alpha}$.

5.2 Present Value of Consumption in a Descentralized Economy

Consider a decentralized economy with a bottleneck ($\phi^s < \phi^{po}$). Can financial liberalization bring the economy nearer to the Pareto optimum? Recall that liberalization induces the adoption of insolvency risk that makes the economy vulnerable to crises, which entail deadweight losses for the economy. Is the present value of consumption in a finacially liberalized economy be greater than in a repressed economy, after netting out the bailout costs?

The expected discounted value of workers'consumption and entrepreneurs' consumption in our decentralized economy is equal to:

$$W^d = E_0 \left(\sum_{t=0}^{\infty} \delta^t (c_t + c_t^e)\right) = E_0 \left(\sum_{t=0}^{\infty} \delta^t [[1-\alpha] y_t + \pi_t - T_t]\right) \tag{35}$$

To derive the second equation in (35) notice that in equilibrium workers' income at t is $[1-\alpha]y_t$, entrepreneurs' income is equal to their profits π_t, and the fiscal cost of bailouts is financed with lump-sum taxes T_t.

In order to obtain a closed-form solution notice that at any $t \geq 1$ profits equal the old entrepreneurs share in revenues minus debt repayments: $\pi_t = \beta p_t q_t - L_t = \frac{\alpha}{1-\phi^s}\beta y_t - \frac{\alpha}{1-\phi^s}\frac{h}{u}\phi^s y_{t-1}$. Meanwhile, since at $t=0$ there is no debt burden, $\pi_0 = \frac{\alpha}{1-\phi^s}\beta y_0$. In a safe economy firms are always solvent and crises never occur. Thus, there are no bailouts and no taxes. It then follows from (35) that the present value of consumption equals the present value of T-output

$$W^s = \sum_{t=0}^{\infty} \delta^t y_t^s = \frac{1}{1-\delta(\theta\phi^s)^\alpha} y_o^s = \frac{(1-\phi^s)^\alpha}{1-\delta\left(\theta\phi^s\right)^\alpha} q_o^\alpha \qquad \text{if } \delta(\theta\phi^s)^\alpha < 1 \tag{36}$$

Consider a liberalized economy. Along the lucky path, the investment share is greater than in a safe economy. Thus, if there is a bottleneck and crises are rare events, the present value of T-output along the lucky path is greater than in a safe path. However, along a lucky path a crisis can occur with probability $1-u$. The question then arises as to whether it is worthwhile to incur the crisis costs in order to attain higher T-output growth.

A crisis involves three costs. First, there is a fiscal cost. Lenders receive a bailout payment equal to the debt repayment they were promised: $L_\tau = u^{-1} h \phi^l p_{\tau-1} q_{\tau-1}$. Since the bailout agency recuperates only a share $\mu \leq \beta$ of firms revenues $p_\tau q_\tau$, while the rest is dissipated in bankruptcy procedures, the fiscal cost of a crisis is $T(\tau) = L_\tau - \mu p_\tau q_\tau$. Second, investment falls: in a crisis the investment share is $\phi^c = \frac{\mu_w}{1-h\delta}$ instead of ϕ^s in a safe economy. During crisis borrowing constraints are tighter than in a safe economy because an N-firm's net worth is $\mu_w p_\tau q_\tau$ instead of $[1-\beta]p_\tau q_\tau$ and risk taking is curtailed: only safe plans are financed. Finally, since during a crisis all N-firms go bust, old entrepreneurs' profits are zero.

The deadweight loss of a crisis for the economy as a whole is lower than the sum of these three costs. During a crisis there is a sharp *redistribution* from the N- to the T-sector generated by a large fall in the relative price of N-goods (a firesale). Thus, some of the costs incurred in the N-sector show up as greater T-output and consumers' income. We show in the Appendix that after netting out the costs and redistributions, a crisis involves two deadweight losses: (i) the revenues dissipated in bankruptcy procedures: $[\beta - \mu]p_\tau q_\tau$; and (ii) the fall in N-sector investment due to its weakened financial position: $[(1-\beta) - \mu_w]p_\tau q_\tau$. Using the market clearing condition $\alpha y_t = [1-\phi_t]p_t q_t$, we have that the sum of these two deadweight losses equals $\frac{\alpha}{1-\phi^c}[1-\mu-\mu_w]y_\tau$ in terms of T-goods. Thus, in an RSE the present value of consumption is given by

$$W^r = E_0 \sum_{t=0}^{\infty} \delta^t k_t y_t, \qquad k_t = \begin{cases} k^c := 1 - \frac{\alpha[1-\mu-\mu_w]}{1-\phi^c} & \text{if } t = \tau_i \\ 1 & \text{otherwise,} \end{cases} \tag{37}$$

where τ_i is a crisis time. In order to compute this expectation we need to calculate the limit distribution of $k_t y_t$. We do this in the Appendix and show that it is equal to

$$W^r = \frac{1+\delta(1-u)\left[\theta\phi^l \frac{1-\phi^c}{1-\phi^l}\right]^\alpha k^c}{1-\left[\theta\phi^l\right]^\alpha \delta u - \left[\theta^2\phi^l\phi^c\right]^\alpha \delta^2(1-u)}[(1-\phi^l)q_0]^\alpha \tag{38}$$

By comparing (36) and (38) we can determine the conditions under which the ex-ante present value of consumption is greater in a risky economy.

Proposition 5.2 *In an economy where crisis are rare events:*

1. *Financial liberalization increases the present value of consumption only if the investment share in a repressed regime (ϕ) is less than the Pareto investment share (ϕ^{po}).*
2. *When $\phi < \phi^{po}$, financial liberalization increases the present value of consumption for any level of bankrupcy costs μ, if financial distress in the wake of crisis is not too high ($\mu_w > \mu_w^*$) and the discount rate δ is not too low.*

Proposition 5.2 is proved by taking the derivative of W^r with respect to u and letting $u \to 1$. Since $W^r|_{u=1} = W_s$, financial liberalization, which allows for systemic risk-taking, increases the present value of consumption if and only if $\frac{\partial W^r}{\delta u}\big|_{u=1}$ is negative. We have:

$$\frac{\partial W^r}{\delta u}\bigg|_{u=1} = \left\{ \underbrace{\alpha\phi'(\frac{D}{\phi}-1)}_{\text{Efficiency gains}} + \underbrace{(1-D)(1-k_c(\frac{1-\phi^c}{1-\phi^l})(1-\phi))}_{\text{Bankrupcy costs}} + \underbrace{(1-\phi)^\alpha D\delta(\theta)^\alpha((\phi)^\alpha-(\phi^c)^\alpha)}_{\text{Financial distress costs}} \right\} K \tag{39}$$

where $D = \delta(\theta\phi)^\alpha = (\phi^{po})^{1-\alpha}\phi^\alpha$ and K a stricly positive number.[24] Since the derivative is evaluated at $u = 1$, we have $\phi \equiv \phi^l = \phi^s$.

The first term in (39) captures the efficiency gains from financial liberalization. It can be rewritten as $\alpha\phi'((\frac{\phi^{po}}{\phi})^{1-\alpha}-1)$, which is negative if and only if $\phi < \phi^{po}$. The second term captures

[24] $K = \left[\frac{q_0^\alpha(1-\phi)^{\alpha-1}}{1-[\theta\phi]^\alpha\delta u-\left[\theta^2\phi\phi^c\right]^\alpha\delta^2(1-u)}\right]^2 > 0.$

the bankrucpcy costs associated with crises. The third term reflects the financial distress crisis costs, which are increasing in the difference between the tranquil times investment share (ϕ) and the crisis investment share (ϕ^c). When $\phi^c < \phi < \phi^{po}$, financial distress costs correspond to production efficiency losses since they bring the allocation of intermediate inputs farther away from the pareto optimal level.

Since the second and third term in (39) are positive, a necessary condition for $W^r > W^s$ is that the first term be negative, which occurs only if $\phi < \phi^{po}$. In other words, there are efficiency gains associated with financial liberalization only if there is a bottleneck.

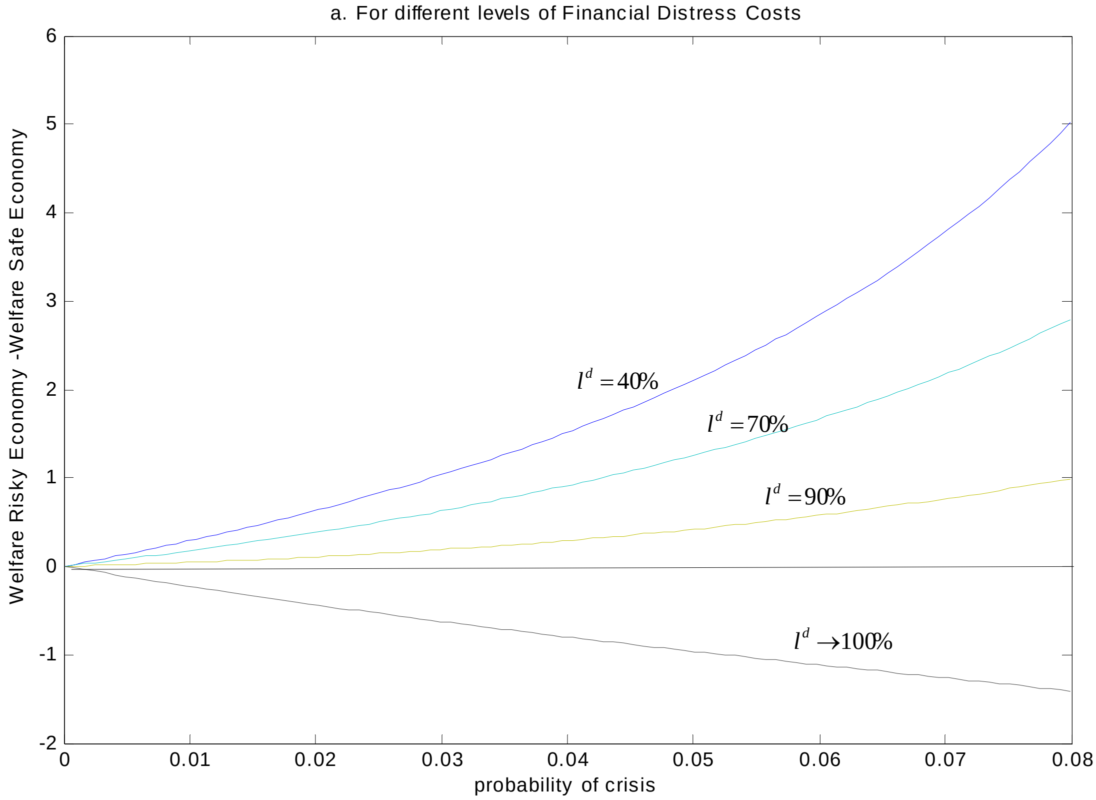
The rest of the proof discusses conditions under which the crises costs are outweighed by the efficiency gains. If the discount rate is high enough, the bankrucpcy costs, which are static in nature, become vanishingly small. In this case, the gains (losses) from financial liberalization depend on the comparison between the efficiency gains–which depend on how much risk-taking reduces the distorsion in the allocation of intermediate input in tranquil times–and financial distress costs, which measure how the allocation of intermediate inputs become more distorted in crises times. Both efficiency gains and financial distress costs are dynamic, which means that they propagate to future periods through the investment chanel and affect future levels of T-production through input-output linkages. We show that when financial distress costs are below a threshold ($\mu_w > \mu_w^*$), there are positive production efficiency gains from financial liberalization.

The gain associated with undertaking credit risk is increasing in the probability of crisis $(1-u)$. This does not mean that this probability can be arbitrarily large. As we have discussed earlier, an RSE exists only if crises are rare events. In panel (a) of Figure 6, we show how $W^r - W^s$ varies over a range of crisis probabilities between 0 and 8%. Except when the financial distress cost of crises is very high, the risky economy dominates the safe economy. This difference is amplified by a limited increase in credit risk. In contrast, if crisis costs are very large, $W^r - W^s < 0$ and any increase in risk reduces W^r further. Finally, panel (b) of Figure 6 shows that the gains, in terms of the present value of consumption, are increasing in the intensity of N-inputs in T-production (α). A greater α strengthens the sectorial linkage and thus increases the benefits of relaxing the borrowing constraint in the N-sector.

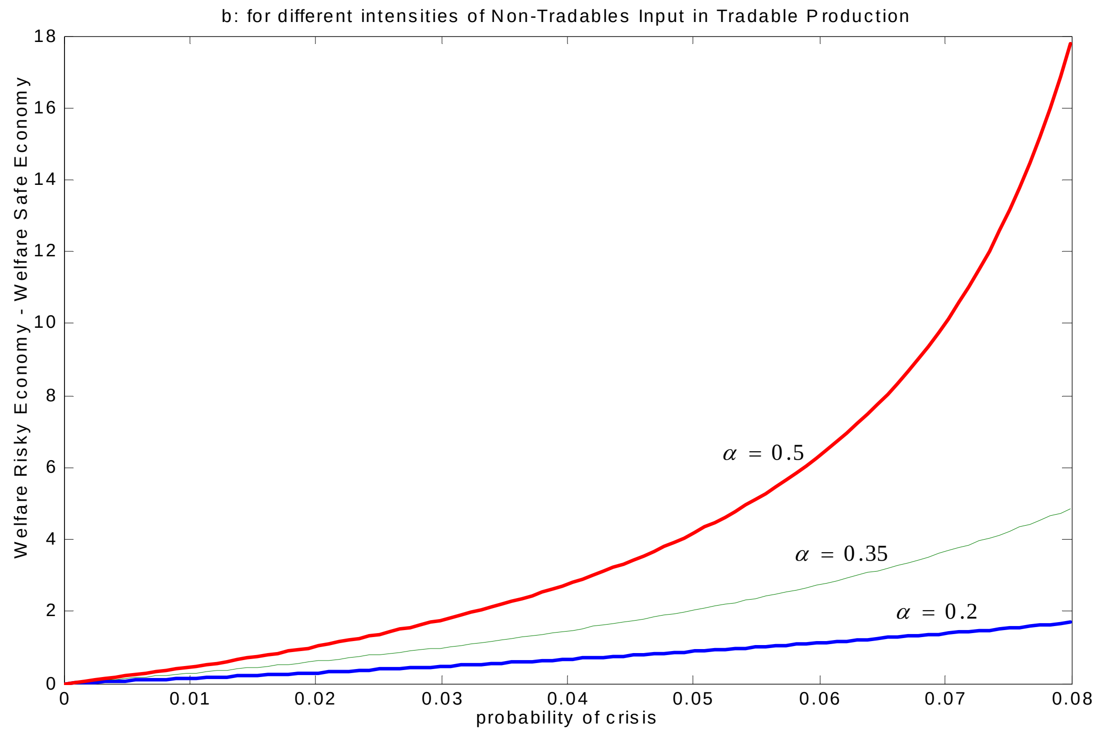
6 Anything-Goes Regulatory Regime

Here we analyze the consequences of relaxing the constraint on the issuance of catastrophe bonds that promise a very big payoff in bad states and nothing otherwise. Such an anything-goes regime per-se does not eliminate financial discipline.The toxic cocktail is the combination of this anything-

Figure 6: Production Efficiency and Credit Risk



parameters $: \theta = 1.65 \quad \alpha = 0.35 \quad h = 0.76 \quad 1 - \beta = 0.2 \quad l^b = 70\%$



parameters $: \theta = 1.65 \quad h = 0.76 \quad 1 - \beta = 0.2 \quad l^d = l^b = 70\%$

goes regime with bailout guarantees. This combination breaks down financial discipline and activates the negative NPV production ε-technology (5).

Notice that in the repressed and liberalized regimes, the $\varepsilon-$technology is not used in equilibrium, and all production of T-goods is carried out with the positive NPV technology (4) that uses intermediate N-inputs. Here, we characterize a 'financial black-hole' equilibrium where such a negative NPV technology is active. To do so we add two elements to the setup of Section 3. First, we introduce a new set, of measure one, of entrepreneurs that have access to the ε-technology and that live for two periods. When young an ε-entrepreneur (who has zero internal funds) issues debt, and uses the proceeds to buy T-goods (I_t^ε), which he invests to produce T-goods using production function (5): $y_{t+1} = \varepsilon_{t+1} I_t^\varepsilon$.

Second, we add an upper bound on the bailout rule to ensure fiscal solvency.

Bailout Guarantees A bailout up to and ammount Γ_t is granted to lenders of defaulting borrowers if half of borrowers defaults. The bailout Γ_t is prorated among all defaulting bonds.

We parametrize the bailout upper bound Γ_t as a share γ of T-output produced by the non-diverting part of the economy

$$\Gamma_{t+1} = \gamma[y_{t+1}^{\theta,nd} + y_{t+1}^{\varepsilon,nd}], \tag{40}$$

where $y_{t+1}^{\theta,nd}$ is the T-output produced using N-inputs from non-diverting N-firms, and $y_{t+1}^{\varepsilon,nd}$ is the T-output from non-diverting ε-firms. Bailouts are financed via lump-sum taxes on the T-output produced by the non-diverting part of the economy.[25]

The rest of the setup is the same as in Section 3. In particular, all entrepreneurs can issue both standard and catastrophe bonds with the following repayment schedule:

$$L_{t+1}^c = \begin{cases} 0 & \text{if } \varepsilon_{t+1} = \overline{\varepsilon} \\ 1 + \rho_t^c & \text{if } \varepsilon_{t+1} = 0 \end{cases} \tag{41}$$

Each lender oberves whether the borrower is an ε- or θ-entrepreneur, and decides whether to buy the bonds. At time $t+1$, lenders receive the promised repayment from non-defaulting borrowers, or a bailout if one is granted.

[25] That is, the government cannot tax the diverting part of the economy–i.e., the black market. This is a realistic assumption, and it is also important for the working of the model. If output of the diverting sector were taxable, then one could construct equilibria where diversion is desirable because it would relax borrowing constraints as lenders would not impose the no-diversion condition.

6.1 Equilibrium

First, notice that in the absence of bailout guarantees, ε-agents are not funded in any equilibrium. Since the $\varepsilon-$technology yields less than the riskless return in all states–because $\overline{\varepsilon} < 1 + r$–any profitable strategy for an ε-entrepreneur involves the issuance of catastrophe bonds. Lenders, however, are unwilling to buy catastrophe bonds as they will never be repaid. Only $\theta-$agents will be funded and, as in Section 3, they will choose a safe plan with no insolvency risk. Thus, in the abscence of bailout guarantees, neither the catastrophe bonds nor the inferior ε-technology are used in equilibrium. They are irrelevant for the allocation of resources.

In the presence of bailout guarantees, however, there are equilibria where the inferior ε-technology is active. We will characterize one of these equilibria where $\varepsilon-$agents are funded, and $\theta-$agents choose the no-diversion safe plans that we characterized in Section 3. It is necessary that in equilibrium θ-entrepreneurs do not choose diversion schemes so that there is a source of taxation to fund the bailouts. We will refer to this equilibrium as a "black-hole equilibrium."[26]

We characterize first the behaviour of θ-entrepreneurs. In a black-hole equilibrium each θ-entrepreneur beleives that all other θ-entrepreneurs will not default next period, and that ε-entrepreneurs will default if $\varepsilon_{t+1} = 0$. Thus, she expects a unique price p_{t+1} and that a bailout will be granted if $\varepsilon_{t+1} = 0$. Given these expectations, the representative $\theta-$agent chooses whether to issue standard bonds or catastrophe bonds, and whether to implement a diversion scheme or not. The following Lemma shows that if the bailout is not 'too generous,' θ-entrepreneurs will behave as in Section 3. However, if bailouts are too generous, financial discipline in the N-sector breaks down.

Lemma 6.1 (Investment plans of the input sector entrepereneurs) *Consider the following threshold for the generosity of the bailout*

$$\overline{\gamma}' = \frac{2[m^s - 1]}{\delta[1-\lambda]} \frac{1}{[\theta\phi]^\alpha} \frac{[1-\beta]\alpha}{1-\phi} \tag{42}$$

1. *If $\gamma \leq \overline{\gamma}'$ and returns satisfy condition (15), the $\theta-$agent chooses to issue standard debt and diversion schemes are not optimal. N-sector output is $q_t = \theta m^s w_t$, and the production of T-output using N-inputs is : $y_t^\theta = \left(\frac{\alpha}{p_t}\right)^{\frac{\alpha}{1-\alpha}}$.*

2. *If $\gamma > \overline{\gamma}'$, $\theta-$agents have incentives to issue catastrophe bonds and to implement diversion schemes.*

To see the intuition notice first that under a no-diversion plan, the $\theta-$agent is indifferent between both types of debt. Under standard debt, the best plan of a $\theta-$agent is the same as that in the safe

[26] One could construct other black-hole equilibria where $\theta-$agents choose risky plans.

equilibrium characterized in Proposition 3.2: there is no diversion, all debt is indexed to p_{t+1}, and borrowing constraints bind with $b_t^s = [m^s - 1]w_t$. With catastrophe bonds the lender will require an interest rate no smaller than

$$1 + \rho^c = \frac{1+r}{1-\lambda}. \tag{43}$$

To satisfy the no-diversion constraint, lenders lend the $\theta-$agent up to an ammount that satifies the no-diversion condition

$$[1-\lambda][1+\rho^c]b_t \le h[w_t + b_t] \tag{44}$$

Since $[1-\lambda][1+\rho^c] = 1+r$ (by (43)), condition (44) implies that the borrowing constraint with catastrophe bonds is $b_t \le \frac{h\delta}{1-h\delta}w_t = [m^s - 1]w_t$, which is the same as with standard debt. Since under no-diversion the expected debt repayments are the same, the borrower is indifferent between both types of debt, a result similar to the Modigiliani-Miller theorem.

Second, consider plans where the $\theta-$agent chooses a diversion plan and issues catastrophe bonds. Lenders lend her up to the present value of the bailout

$$b_t^{c,\theta} = \frac{1}{2}\delta[1-\lambda]\Gamma_{t+1} = \frac{1}{2}\delta[1-\lambda]\gamma y_{t+1}^{\theta} \tag{45a}$$

Next, notice that no-diversion is preferred to diversion if and only if borrowing under no-diversion ($[m^s - 1]w_t$) is greater than under diversion ($b_t^{c,\theta}$). The proof of the Lemma shows that this condition is equivalent to $\gamma \le \overline{\gamma}'$.

Consider now the $\varepsilon-$agents. Since the ε-technology has negative NPV, ε-agents find it profitable only to issue catastrophe bonds. In the presence of bailout guarantees, lenders are willing to buy these catastrophe bonds. Given the expected bailout Γ_{it+1}, at time t lenders are willing to lend to each ε-agent up to an ammount b_{it}^c (in (45a)) at a rate ρ_t^c (in (43)). At $t+1$, if the good state realizes ($\varepsilon_{t+1} = \overline{\varepsilon}$), lenders will get zero–as promised–while if $\varepsilon_{t+1} = 0$ lenders will get the bailout $\Gamma_{it} = b_{it}^c[1+\rho_t^c]$. It follows that an ε-agent will de-facto repay zero in all states of the world, and so he does not gain anything by implementing a diversion scheme. His expected payoff is $E\pi_{t+1}^{\varepsilon} = \lambda\overline{\varepsilon}b_{it}^c = \lambda\overline{\varepsilon}\frac{1}{2}\delta[1-\lambda]\gamma y_{t+1}^{\theta}$.

We next verify the fiscal solvency of the bailout agency. Recall that the bailout sequence $\{\Gamma_t\}_{t=1}^{\infty}$ is financed via lump-sum taxes on the production of T-goods in the non-diverting part of the economy. Since a bailout occurs with probability $1-\lambda$, assuming that starting at $t=1$ the entire T-output of the non-diverting part of the economy can be taxed in a lump-sum way, fiscal solvency requires

$$E\left(\sum_{t=0}^{\infty}\delta^{t+1}[y_{t+1}^{\theta,nd} + y_{t+1}^{\varepsilon,nd}]\right) \ge E\left(\sum_{t=0}^{\infty}\delta^{t+1}\Gamma_{t+1}\right)$$

We show in the appendix that this inequality holds if and only if

$$\gamma \leq \overline{\gamma}'' := \frac{1}{[1-\lambda][1-\lambda\overline{\varepsilon}\delta]}.$$

Combining the results above, we have

Proposition 6.1 *A black-hole equilibrium exists if and only if the generosity of the bailout guarantee (γ) is below a threshold given by*

$$\overline{\gamma} = \min\left\{\overline{\gamma}', \overline{\gamma}''\right\}$$

In this equilibrium θ-agents issue standard bonds, hedge price risk and never default; ε-agents issue captastrophe bonds and default in the $\varepsilon = 0$ state with probability $1-\lambda$.

If $\gamma > \overline{\gamma}''$, there is not enough fiscal revenue to fund bailouts, even if 100% of Y-output were taxable.

If $\gamma > \overline{\gamma}'$, θ-agents issue catastrophe bonds and divert, so bailout are not fundable.

Consider now production efficiency. The GNP generated by the ε-agents follows

$$GNP_t^{\varepsilon} = \begin{cases} \overline{\varepsilon} b_{t-1}^c & \text{with probability} \quad \lambda \\ -\frac{1+r}{1-\lambda} b_{t-1}^c & \text{with probability} \quad 1-\lambda \end{cases}$$

Therefore, the average contribution of ε-agents to GNP is negative

$$E(GNP_{t+1}^c) = b_t^c \underbrace{[\lambda\overline{\varepsilon} - 1 + r]}_{\text{social cost}} < 0$$

That is, although during good times the ε-sector is seemingly profitable–from the ε-agent individual perspective– the losses it incurs during crisis times more than offset those private profits. Thus, a financial black hole arises in equilibrium.

Why are stocks different from catastrophe bonds? Although stocks are liabilities that might promise very little in some states of the world, the issuance of stocks does not bring with it the political pressure for systemic bailout guarantees.

7 Conclusions

TBA

Appendix

A. The Costs of Crises

During a crisis there are widespread bankruptcies, which generate deadweight losses as well as sectorial redistributions. Here, we net out these crises costs and show that the growth costs of crisis reduce to the fall in the N-sector's investment share, as expressed in (29).[27]

If a crisis occurs at some date, say τ, there is a firesale: there is a steep real exchange rate depreciation, and since there is currency mismatch, all N-firms default. As a result, the investment share falls from ϕ^l to ϕ^c.[28] The price of N-goods must fall to allow the T-sector to absorb a greater share of N-output, which is predetermined by $\tau - 1$ investment. At $\tau + 1$, N-output contracts due to the fall in investment at the time of the crisis. However, entrepreneurs adopt risky plans again, so the investment share increases from ϕ^s back to ϕ^l. Thus, there is a real appreciation. At $\tau + 2$, the economy is back on a lucky path, but the level of cash flow and N-output are below their pre-crisis trend.

Although GDP fluctuations are affected by changes in the real exchange rate, T-output and N-investment, GDP growth during a crisis episode is solely determined by the mean investment share $[\phi^l \phi^c]^{\frac{1}{2}}$ (by (29)). To understand why this is so note that GDP growth has two components: (i) real exchange rate fluctuations (captured by $\frac{Z(\phi_t)}{Z(\phi_{t-1})}$) and (ii) output fluctuations (captured by $(\theta\phi_t)^\alpha$).[29] In the crisis period, GDP growth falls below trend because there is a real exchange rate depreciation ($\frac{Z(\phi^l)}{Z(\phi^c)} < 1$). In the post crisis period, there are two effects: (i) since investment contracted during the previous period, N-output falls below trend and depresses growth; but (ii) there is a rebound of the real exchange rate as the investment share jumps from its crisis level $\left(\frac{Z(\phi^c)}{Z(\phi^l)} > 1\right)$. As we can see, variations in GDP growth generated by real exchange rate changes

[27] Although the main objective of the model is to address long-run issues, it is reassuring that it can account for **these** key stylized facts of recent financial crises, a sharp real depreciation that coincides with a fall in credit growth, as well as the *asymmetric sectorial response* of N- and T-sectors.

[28] This is because young entrepreneurs income is only $\mu_w p_\tau q_\tau$ instead of $[1-\beta]p_\tau q_\tau$, and at τ entrepreneurs can only choose safe plans in which there is no currency mismatch (by Proposition 3.3).

[29] To interpret (29) note that variations in the investment share ϕ_t have lagged and contemporaneous effects on GDP. The lagged effect comes about because a change in ϕ_t affects next period's GDP via its effect on N-output: $q_{t+1} = \theta I_t = \theta\phi_t q_t$. Using (26) and $y_t = ([1-\phi_t]q_t)^\alpha$, the contemporeneous effect can be decomposed as:

$$\frac{\partial gdp_t}{\partial \phi_t} = -\frac{\alpha y_t}{1-\phi_t} + p_t q_t + q_t \phi_t \frac{\partial p_t}{\partial \phi_t} = q_t \phi_t \frac{\partial p_t}{\partial \phi_t}$$

The first two terms capture variations in T-output and N-investment, while the third reflects real exchange rate fluctuations. Market clearing in the N–goods market –i.e., $(1-\phi_t)p_t q_t = \alpha y_t$– implies that the induced changes in N-sector investment and T-output cancel out. Therefore, the contemporeneous changes in the investment share affect GDP contemporaneously only through its effect on the real exchange rate. Since $GDP_t = Z(\phi_t)q_t^\alpha$, we can express $q_t \phi_t \frac{\partial p_t}{\partial \phi_t}$ as $q_t^\alpha \frac{\partial Z_t}{\partial \phi_t}$. Thus, we can interpret $\frac{Z(\phi_t)}{Z(\phi_{t-1})}$ as the effect of real exchange rate fluctuations on GDP.

at τ and $\tau + 1$ cancel out. Thus, the average loss in GDP growth stems only from the fall in the N-sector's average investment share.

In sum, a crisis has two distinct effects: sectorial redistribution and deadweight losses. At the time of the crisis the T-sector benefits from the financial collapse of the N-sector because it can buy N-output at firesale prices and expand production. This leads to a sharp fall in the N-to-T output ratio in the wake of crisis. The deadweight losses derive from the financial distress and the bankruptcy costs generated by crises. The former leads to a contraction in N-investment and thus has a long-run effect on output. In contrast, bankruptcy costs have only a static fiscal impact, which is the cost of the bailout.

B. Post-Crisis Cool-Off Phase and Growth

Here, we show that form the perspective of long-run growth, nothing is gained by delaying the onset of the new risky phase.

In Proposition 3.3, we characterized a RSE where there is a reversion back to a risky path in the period immediately after the crisis. We then compared growth in such a risky economy–where risk-taking occurs whenever it is possible–to growth in a safe economy where risk-taking never occurs. The comparison of these polar cases makes the argument transparent, but opens the question of whether the growth results presented in Proposition 4.1 are applicable to recent experiences in which systemic crises have been followed by protracted periods of low leverage, low investment and low growth.[30] In order to address this issue, we construct an alternative RSE under which a crisis is followed by a cool-off phase during which all agents choose safe plans. The cool-off phase can be interpreted either as a period in which agents believe that others are following safe strategies or as a period during which agents are prevented from taking on risk.[31]

To keep the model tractable, we assume that in the aftermath of a crisis, all agents follow safe plans with probability ζ. Hence, a crisis is followed by a cool-off phase of average length $1/(1-\zeta)$ before there is reversion to a risky path.[32] We show in the appendix that in this case, the mean long-run GDP growth rate is

$$E(1+\gamma^r) = (\theta\phi^s)^\alpha \left(\frac{\phi^l}{\phi^s}\right)^{\frac{1-\zeta}{(1-\zeta)+(1-u)}} \left(\frac{\mu_w}{1-\beta}\right)^{\frac{u(1-\zeta)}{(1-\zeta)+(1-u)}}, \tag{46}$$

[30] Figure 1 is suggestive of such reversion to a safe path in Thailand after the 1997 crisis.

[31] Or alternatively as a period where agents revise downwards their bailout expectations because they perceive that the surge in public debt associated with prior bailouts makes future bailout less likely.

[32] The average lenght of the cooling off period is computed as:

$$\lambda = (1-\xi)\sum_{k=0}^{\infty} \xi^{k-1} k = \frac{1}{1-\xi}$$

which generalizes the growth rate of proposition 4.1. Comparing (46) with (27) we can prove the following Lemma.

Lemma .1 *Consider an RSE where a crisis is followed by a cool-off period of average length* $1/(1-\zeta)$. *Then:*

1. *The conditions under which mean long-run GDP growth is greater in a risky than in a safe equilibrium are independent of* ζ, *and are the same as those in Proposition 4.1.*
2. *The shorter the average cool-off period* $1/(1-\zeta)$, *the higher the mean long-run GDP growth in a RSE.*

The reason why the growth-enhancing properties of risk taking–stated in Proposition 4.1–are independent of ζ is that during the cool-off phase the economy grows at the same rate as in a safe equilibrium. Part 2 makes the important point that the faster risk-taking resumes in the wake of crisis, the higher will be mean long-run growth.

C. Proofs and Derivations

Proof of Proposition 3.2. In an SSE, during every period, all entrepreneurs choose the safe plan characterized in Proposition 3.1. Each entrepreneur will find it optimal to do so provided a majority of entrepreneurs chooses a safe plan and the marginal return to investment in the production of N-goods is no lower than $1+r$: $R^e_{t+1} := \frac{\beta\theta p^e_{t+1}}{p_t} \geq \delta^{-1}$. Since in an SSE crises never occur, prices are deterministic: $u_{t+1} = 1$ and $p^e_{t+1} = p_{t+1}$. Using (20) and (21) it follows that $R^e_{t+1} = \beta\theta^\alpha(\phi^s)^{\alpha-1}$. Thus, an SSE exists if and only if $\beta\theta^\alpha(\phi^s)^{\alpha-1} > \delta^{-1}$ and (23) holds. These two conditions are equivalent to

$$h < \bar{h} = \beta\delta^{-1}, \qquad \theta > \underline{\theta} = [\delta\beta(\phi^s)^{\alpha-1}]^{-1/\alpha} \tag{47}$$

Proof of Proposition 3.3. The proof is in two parts. In part A we consider the case in which two crises do not occur in consecutive periods. Then, in part B we show that two crises cannot occur in consecutive periods.

Part A. Consider an RSE in which all entrepreneurs choose the risky plan characterized in Proposition 3.1 during every period, except when a crisis erupts, in which case they choose safe plans. In a no-crisis period, given that all other entrepreneurs choose a risky plan, an entrepreneur will find it optimal to do so if and only if $R^e_{t+1} := u\beta\theta\frac{\bar{p}_{t+1}}{p_t} \geq 1+r$, and $\pi(\underline{p}_{t+1}) < 0$. To determine whether these conditions hold note that in an RSE the investment share ϕ_{t+1} equals ϕ^l if N-firms are solvent, while $\phi_{t+1} = \phi^c$ if they are insolvent. Replacing these expressions in the equations for cash flow (18), N-output (20) and prices (21), it follows that

$$R^e_{t+1} \geq \tfrac{1}{\delta} \;\Leftrightarrow\; u\overline{R}(u) + [1-u]\underline{R}(u) \geq \tfrac{1}{\delta}, \quad \overline{R}(u) := \beta\theta^\alpha \left[\frac{1}{\phi^l}\right]^{1-\alpha} \tag{48}$$

$$\pi(\underline{p}_{t+1}) < 0 \;\Leftrightarrow\; \underline{R}(u) < \tfrac{h}{u}, \quad \underline{R}(u) := \beta\theta^\alpha \left[\frac{1}{\phi^l}\right]^{1-\alpha} \left[\frac{1-\phi^l}{1-\phi^c}\right]^{1-\alpha} \tag{49}$$

To derive (49) we have used $\pi(\underline{p}_{t+1}) = \beta \underline{p}_{t+1} q_{t+1} - L_{t+1} = \beta\alpha[1-\phi^c]^{\alpha-1}[\theta\phi^c q_t]^\alpha - u^{-1}h\alpha[1-\phi^l]^{\alpha-1}q_t^\alpha$. Consider next a crisis period. Given that all other entrepreneurs choose a safe plan, an entrepreneur will find it optimal to do so if and only if $R^e_{t+1} := \beta\theta p^e_{t+1}/p_t \geq \delta^{-1}$. Since in the post-crisis period there can be no crisis, it follows from the proof of Proposition 3.2 that this condition is equivalent to $\beta\theta^\alpha(\phi^s)^{\alpha-1} \geq \delta^{-1}$. Clearly, this condition is implied by (48). It follows that there exists an RSE where two crises do no occur in consecutive periods if and only if (48) and (49) hold and parameters satisfy (23), which is given by

$$h\delta < u\beta \tag{50}$$

"Only if." We prove that an RSE exists only if $u > \underline{u}$, $\theta > \overline{\theta}$, and $\underline{h} < h < \overline{h}$ in three steps.
Step 1. For any $\theta \in \Re^+$ and any $h \in \Re^+$ there exists no RSE if $u \to 0$. To prove this, let $u \to 0$. Since θ is bounded and $1-\beta < \phi^l < 1$, it follows that $\lim_{u\to 0^+} u\overline{R}(u) = 0$. Therefore, (48)-(50) imply that when $u \to 0$ an RSE exists if and only if $\frac{h}{u} < \frac{\beta}{\delta}$ and $\frac{1}{\delta} < \underline{R}(u) < \frac{h}{u}$, which is a contradiction.
Step 2. For any $u \in (0,1)$ and for any $\theta \in \Re^+$ there exists no RSE if $h > \underline{h}$ or $h < \overline{h}$, where

$$\overline{h} = \frac{\beta u}{\delta}, \quad \underline{h} = \frac{1}{\delta}\left(\left(\frac{1-\phi^c}{1-\phi^l}\right)^{1-\alpha} + \left(\frac{1}{u} - 1\right)\right)^{-1}, \qquad 0 < \underline{h} < \overline{h} \tag{51}$$

Notice that $h < \overline{h}$ is equivalent to (50), and that (48) and (49) hold if and only if $\delta^{-1}\left(u + (1-u)\left[\frac{1-\phi^l}{1-\phi^c}\right]^{1-\alpha}\right)^{-1}$ $\overline{R}(u) < \frac{h}{u}\left[\frac{1-\phi^l}{1-\phi^c}\right]^{\alpha-1}$, which holds only if $h > \underline{h}$.
Step 3. For any $u \in (0,1)$ and for any $h \in \Re^+$ there exists no RSE if $\theta < \underline{\theta}$, where

$$\underline{\theta} = \left(\frac{h}{u\beta}\left[\phi^l\right]^{1-\alpha}\left[\frac{1-\phi^c}{1-\phi^l}\right]^{1-\alpha}\right)^{1/\alpha} \tag{52}$$

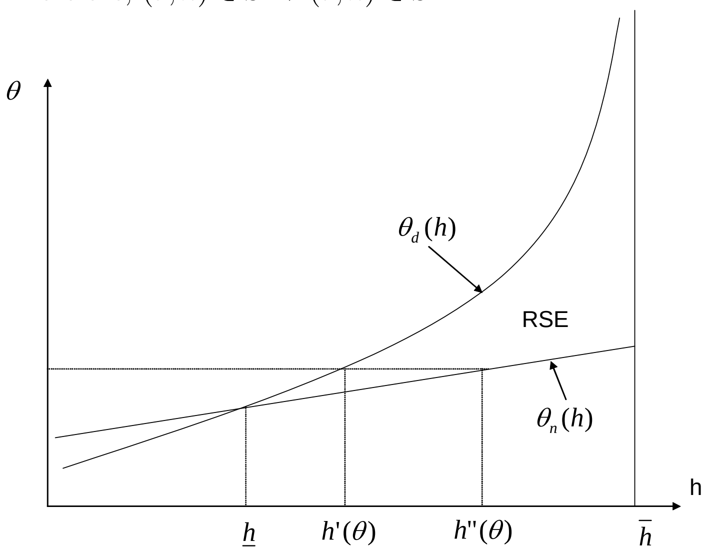
Notice that $u\overline{R}(u) + (1-u)\underline{R}(u)$ is decreasing in h and an RSE exist only if $h > \underline{h}$. Thus, a necessary condition for an RSE to exist is $u\overline{R}(u) + (1-u)\underline{R}(u)\rfloor_{h=\underline{h}} > \delta^{-1}$, which is equivalent to (52).
"If." To establish the existence of an RSE we show that when $u \to 1$ parameter restrictions (48), (49) and (50) are mutually consistent if $(\theta, h) \in S = \{(\theta, h) \in R^2_+ |\ \theta > \underline{\theta},\ h' < h < h''\}$, with $\underline{h} \leq h' < h'' \leq \overline{h}$. We do this in two steps. First, we allow for an upper bound $\theta < \theta_d(h)$. Then, we replace $\theta < \theta_d(h)$ by tighter bounds on h.

Step 1. We show that for any $\delta \in (0,1)$, $\alpha \in (0,1)$, and $\mu_w \in (0, 1-\beta)$ an RSE exists if $(\theta, h) \in S' = \{(\theta,h) \in R^2_+ |\ \underline{h} < h < \overline{h},\ \theta_n(h) < \theta < \theta_d(h)\}$. Let $u = 1$, for any $\delta \in (0,1)$ and $\alpha \in (0,1)$, (50) holds iff $h < \overline{h} = \beta\delta^{-1}$ and (48) holds iff $\theta \geq \theta_d(h) = [\delta\beta(\phi^s)^{\alpha-1}]^{-1/\alpha}$. Next, if $u = 1$, (49) becomes $\left[\frac{1-\phi^s}{1-\phi^c}\right]^{1-\alpha} < h\frac{(\phi^s)^{1-\alpha}}{\beta\theta^\alpha}$. This condition holds for any $\mu_w \in (0, 1-\beta)$, $h < \bar{h}$ and $\theta > \theta_n(h)$ iff

$$\theta < \theta_d(h) = \left(\left[\frac{1-\phi^c}{1-\phi^s}\frac{1}{\phi^s}\right]^{1-\alpha}\frac{h}{\beta}\right)^{1/\alpha} \quad \text{and} \quad h > \underline{h} = \frac{1}{\delta}\left[\frac{1-\phi^s}{1-\phi^c}\right]^{1-\alpha} \tag{53}$$

Notice that $h > \underline{h}$ is necessary for $\theta_n(h) < \theta_d(h)$ and that $\underline{h}$ is unique. Furthermore, $\theta_n(h) < \theta_d(h) \Leftrightarrow h - \frac{1}{\delta}\left[\frac{1-\phi^s}{1-\phi^c}\right]^{1-\alpha} > 0$. This expression is strictly increasing in h, it is satisfied if $h \to \overline{h}$ and violated if $h = 0$. This ensures existence and unicity of a lower bound $\underline{h}$.

Step 2. We show that the sets S' and S are equivalent. Consider the following three properties of $\theta_n(h)$ and $\theta_d(h)$ over $(\underline{h}, \overline{h})$, which are illustrated in the figure below: (i) $\theta_n(h) < \theta_d(h)$; (ii) $\theta_n(h)$ and $\theta_d(h)$ are continuous and strictly increasing in h; and (iii) $\theta_n(\underline{h}) = \theta_d(\underline{h}) = \underline{\theta}$; $\lim_{h->\overline{h}} \theta_n(h) = \infty$ and $\lim_{h->\overline{h}} \theta_d(h) = (\beta\delta^{-1})^{1/\alpha}$. It follows that for any $(\theta, h) \in S'$, $\theta > \underline{\theta}$ and $h \in (h', h'')$, where $h' = \theta_n^{-1}(\theta)$ and $h'' = \min(\theta_n^{-1}(\theta), \overline{h})$ where $\theta^{-1}()$ denotes the inverse function. Since $\underline{h} \leq h' < h'' \leq \overline{h}$, we have that $(\theta, h) \in S' \Rightarrow (\theta, h) \in S$. Similarly, for any $(\theta, h) \in S$, $\underline{h} < h < \overline{h}$ and $\theta_n(h) < \theta < \theta_d(\theta)$. Therefore, $(\theta, h) \in S \Rightarrow (\theta, h) \in S'$.



Part B. We prove by contradiction that two crises cannot occur in consecutive periods. Suppose that if a crisis occurs at τ, firms choose risky plans at τ. We will show that it is not possible, under any circumstances, for firms to become insolvent in the low price state at $\tau+1$ (i.e., $\pi(\underline{p}_{\tau+1}) < 0$). It suffices to consider the case in which firms undertake safe plans at $\tau+1$, as $\underline{p}_{\tau+1}$ is the lowest in this case. Along this path the N-investment share equals $\phi_\tau = \tilde{\phi}^c := \mu_w m^r$ and $\phi_{\tau+1} = \phi^c := \mu_w m^s$.

Thus, $\pi(\underline{p}_{\tau+1}) = \beta\alpha[1-\phi^c]^{\alpha-1}[\theta\tilde{\phi}^c q_\tau]^\alpha - u^{-1}h\alpha[1-\phi^c]^{\alpha-1}q_\tau^\alpha$, and

$$\tilde{\pi}(\underline{p}_{\tau+1}) < 0 \quad \Leftrightarrow \quad \beta\theta^\alpha \left[\frac{1-\tilde{\phi}^c}{1-\phi^c}\frac{1}{\tilde{\phi}^c}\right]^{1-\alpha} < \frac{h}{u} \tag{54}$$

Notice that the LHS of (48) is strictly lower than the LHS of (54) because: (i) $\mu_w < 1-\beta$, so $\frac{1-\tilde{\phi}^c}{\tilde{\phi}^c} > \frac{1-\phi^c}{\phi^c}$; and (ii)$\phi^l > \phi^c$. However, the RHS of (48) is strictly higher than the RHS of (54) because $u > h\delta$ is necessary for an RSE to exist. This is a contradiction.□

Proof of Proposition 4.1. *Growth Limit Distribution.* In any RSE two crises cannot occur in consecutive periods. Here, we will derive the limit distribution of GDP's compounded growth rate ($\log(gdp_t) - \log(gdp_{t-1})$) along the RSE characterized in Proposition 3.3. In this RSE firms undertake credit risk the period after the crisis. It follows from (25), (28) and (29) that the growth process follows a three-state Markov chain characterized by

$$\Gamma = \begin{pmatrix} \log\left((\theta\phi^l)^\alpha\right) \\ \log\left((\theta\phi^l)^\alpha \frac{Z(\phi^c)}{Z(\phi^l)}\right) \\ \log\left((\theta\phi^c)^\alpha \frac{Z(\phi^l)}{Z(\phi^c)}\right) \end{pmatrix}, \qquad T = \begin{pmatrix} u & 1-u & 0 \\ 0 & 0 & 1 \\ u & 1-u & 0 \end{pmatrix} \tag{55}$$

The three elements of Γ are the growth rates in the lucky, crisis and post-crisis states, respectively. The element T_{ij} of the transition matrix is the transition probability from state i to state j. Since the transition matrix is irreducible, the growth process converges to a unique limit distribution over the three states that solves $T'\Pi = \Pi$. Thus, $\Pi = \left(\frac{u}{2-u}, \frac{1-u}{2-u}, \frac{1-u}{2-u}\right)'$, where the elements of Π are the shares of time that an economy spends in each state over the long-run. It then follows that the mean long run GDP growth rate is $E(1+\gamma^r) = \exp(\Pi'\Gamma)$.[33]

We derive first the limit distribution of the growth rate process $\Delta\log(gdp_t) := \log(gdp_t) - \log(gdp_{t-1})$. Since in an RSE crises cannot occur in two consecutive periods, $\Delta\log(gdp_t)$ follows a three-state Markov chain characterized by the following growth vector and transition matrix

$$\Gamma = \begin{pmatrix} \log((\theta\phi^l)^\alpha) \\ \log((\theta\phi^l)^\alpha \frac{Z(\phi^c)}{Z(\phi^l)}) \\ \log((\theta\phi^c)^\alpha \frac{Z(\phi^l)}{Z(\phi^c)}) \end{pmatrix} \quad T = \begin{pmatrix} u & 1-u & 0 \\ 0 & 0 & 1 \\ u & 1-u & 0 \end{pmatrix}$$

Since the transition matrix is irreducible, the growth process converges to a unique limit distribution over the three states that solves $T'\Pi = \Pi$. Thus, $\Pi' = \left(\frac{u}{2-u}, \frac{1-u}{2-u}, \frac{1-u}{2-u}\right)$ and the geometric mean long run GDP growth rate –equation (30) in the text– is $E(1+\gamma^r) = \exp(\Pi'\Gamma)$. It then follows from (27) and (30) that

[33] $E(1+\gamma^r)$ is the geometric mean of $1+\gamma^l, 1+\gamma^{lc}$ and $1+\gamma^{cl}$.

$$\gamma^r > \gamma^s \Leftrightarrow \left(\frac{\mu_w}{1-\beta}\right)^{1-u} > \frac{1-h\delta u^{-1}}{1-h\delta} \Leftrightarrow h > \bar{h}' := \frac{1}{\delta}\frac{1-\left(\frac{\mu_w}{1-\beta}\right)^{1-u}}{\frac{1}{u}-\left(\frac{\mu_w}{1-\beta}\right)^{1-u}}$$

Notice that an RSE exists only if $h < \bar{h} = u\beta/\delta$. Thus, $\bar{h}' < \bar{h}$ if and only if $\frac{\mu_w}{1-\beta} > \left(\frac{1-\beta}{1-\beta u}\right)^{\frac{1}{1-u}}$.□

Derivation of (33). Any solution to the Pareto problem is characterized by the optimal accumulation of N-goods that maximizes the discounted sum of T-production

$$\max_{\{d_t\}\in\mathcal{C}^1} \sum_{t=0}^{\infty} \delta^t d_t^{\alpha}, \quad \text{s.t.} \quad k_{t+1} = \begin{cases} \theta k_t - d_t & \text{if } t \geq 1 \\ q_0 - d_0 & \text{if } t = 0 \end{cases}, \quad d_t \geq 0,\ q_o \text{ given}$$

The Hamiltonian associated with this problem is $H_t = \delta^t[d_t]^\alpha + \lambda_t[\theta k_t - d_t]$. Since $\alpha \in (0,1)$, the necessary and sufficient conditions for an optimum are

$$0 = H_d = \delta^t\alpha[d_t]^{\alpha-1} - \lambda_t, \qquad \lambda_{t-1} = H_k = \theta\lambda_t, \qquad \lim_{t\to\infty}\lambda_t k_t = 0 \tag{56}$$

Thus, the Euler equation is

$$\hat{d}_{t+1} = [\delta\theta]^{\frac{1}{1-\alpha}} d_t = \theta\hat{\phi} d_t, \quad \hat{\phi} := [\delta\theta^\alpha]^{\frac{1}{1-\alpha}} \qquad t \geq 1 \tag{57}$$

To get a closed form solution for d_t we replace (57) in the accumulation equation:

$$k_t = \theta^{t-1}k_1 - d_0\sum\nolimits_{s=0}^{t-2}\theta^{t-s-2}[\delta\theta]^{\frac{s+1}{1-\alpha}} = \theta^{t-1}\left[k_1 - d_0\hat{\phi}\frac{1-\hat{\phi}^{t-1}}{1-\hat{\phi}}\right] = \theta^{t-1}\left[k_1 - \frac{d_1}{\theta}\frac{1-\hat{\phi}^{t-1}}{1-\hat{\phi}}\right] \tag{58}$$

Replacing (57) and (58) in the transversality condition we get

$$\begin{aligned} 0 &= \lim_{t\to\infty}\delta^t\alpha[d_t]^{\alpha-1}k_t = \lim_{t\to\infty}\delta^t\alpha\left[[\delta\theta]^{\frac{t}{1-\alpha}}d_0\right]^{\alpha-1}\left[\theta^{t-1}k_1 - d_0\hat{\phi}\frac{1-\hat{\phi}^{t-1}}{1-\hat{\phi}}\right] \\ &= \frac{\alpha d_0^{\alpha-1}}{\theta}\left[k_1 - d_0\hat{\phi}\frac{1}{1-\hat{\phi}}\right] \qquad \text{iff} \quad \hat{\phi} < 1 \end{aligned}$$

Since $k_1 = q_0 - d_0$, the bracketed term equals zero if and only if $\hat{d}_0 = [1-\hat{\phi}]q_0$. The accumulation equation then implies that the unique optimal solution is $\hat{d}_t = [1-\hat{\phi}]q_t$.□

Derivation of (37). To simplify notation we assume temporarily that there is only one crisis (at time τ). It follows that profits and the bailout cost are:

$$\begin{aligned} &\pi_t = \tfrac{\alpha}{1-\phi^l}\beta y_t - \tfrac{\alpha\phi^l}{1-\phi^l}\tfrac{h}{u}y_{t-1}, \qquad t \neq \{0,\tau,\tau+1\} \\ &\pi_0 = \tfrac{\alpha}{1-\phi^l}\beta y_0, \quad \pi_\tau = 0, \quad \pi_{\tau+1} = \tfrac{\alpha}{1-\phi^l}\beta y_{\tau+1} - \tfrac{\alpha\phi^c}{1-\phi^c}hy_\tau \end{aligned} \tag{59}$$

$$T(\tau) = L_{\tau-1} - \mu p_\tau q_\tau = \frac{\alpha}{1-\phi^l}\frac{h}{u}\phi^l y_{\tau-1} - \mu p_\tau q_\tau = \frac{\alpha}{1-\phi^l}\frac{h}{u}\phi^l y_{\tau-1} - \mu\frac{\alpha}{1-\phi^c}y_\tau \tag{60}$$

Replacing these expressions in welfare function (35) and using the market clearing condition $p_t q_t[1-\phi_t] = \alpha y_t$, we get

$$\begin{aligned}
W(\tau) &= (1-\alpha)y_o + \tfrac{\alpha\beta y_o}{1-\phi^l} + \sum_{t=1}^{\tau-1}\delta^t\left[[(1-\alpha)y_t + \tfrac{\alpha\beta y_t}{1-\phi^l} - \tfrac{\alpha\phi^l y_{t-1}}{1-\phi^l}\frac{h}{u}\right] + \delta^\tau\left[(1-\alpha)y_\tau + \tfrac{\mu\alpha y_\tau}{1-\phi^c} - \tfrac{\alpha\phi^l y_{\tau-1}}{1-\phi^l}\frac{h}{u}\right] \\
&\quad + \delta^{\tau+1}\left[(1-\alpha)y_{\tau+1} + \tfrac{\alpha}{1-\phi^l}\beta y_{\tau+1} - \frac{\alpha h\phi^c}{1-\phi^c}y_\tau\right] + \sum_{t=\tau+2}^{\infty}\delta^t\left[(1-\alpha)y_t + \frac{\alpha\beta}{1-\phi^l}y_t - \frac{\alpha\phi^l}{1-\phi^l}\frac{h}{u}y_{t-1}\right] \\
&= \sum_{t\neq\tau}\delta^t\left[(1-\alpha)y_t + \frac{\alpha}{1-\phi^l}\beta y_t - \frac{\alpha}{1-\phi^l}\frac{\delta h}{u}\phi^l y_t\right] + \delta^\tau\left[(1-\alpha)y_\tau + \mu\frac{\alpha}{1-\phi^c}y_\tau - \frac{\alpha\phi^c}{1-\phi^c}\delta h y_\tau\right] \\
&= \sum_{t\neq\tau}\delta^t y_t + K^c y_\tau, \qquad K^c := 1-\alpha+\mu\frac{\alpha}{1-\phi^c} - \frac{\alpha}{1-\phi^c}\delta h\phi^c = 1 - \frac{\alpha\left[1-(\mu+\mu_w)\right]}{1-\phi^c}
\end{aligned}$$

Notice that K_c can be simplified as follows

$$K_c = \alpha + \frac{\alpha}{1-\phi^c}(\mu - (1-\mu_w) + (1-\mu_w) - \delta h\phi^c) = \alpha + \frac{\alpha}{1-\phi^c}((1-\mu_w) - \delta h\phi^c) - \frac{\alpha\left[1-(\mu+\mu_w)\right]}{1-\phi^c}$$

Notice that $\frac{1}{1-\phi^c}((1-\mu_w) - \delta h\phi^c) = \frac{(1-\mu_w)(1-h\delta)-h\delta\mu_w}{1-h\delta-\mu_w} = \frac{1-h\delta-\mu_w}{1-h\delta-\mu_w} = 1$. Thus, $K_c = 1 - \frac{\alpha[1-(\mu+\mu_w)]}{1-\phi^c}$. The expression for expected welfare in (37) follows by allowing multiple crises to take place.

Derivation of (38). Consider T-output net of bankruptcy costs: $\tilde{y}_t = K_t y_t$, where K_t is defined in (37). Notice that $W^r = E_0\sum_{t=0}^{\infty}\delta^t K_t y_t = E_0\sum_{t=0}^{\infty}\delta^t\tilde{y}_t$, and $\frac{\tilde{y}_t}{\tilde{y}_{t-1}}$ follows a three-state Markov chain defined by:

$$\widetilde{T} = \begin{pmatrix} u & 1-u & 0 \\ 0 & 0 & 1 \\ u & 1-u & 0 \end{pmatrix}, \qquad \widetilde{G} = \begin{pmatrix} g_1 \\ g_2 \\ g_3 \end{pmatrix} = \begin{pmatrix} (\theta\phi^l)^\alpha \\ \left[\theta\phi^l\frac{1-\phi^c}{1-\phi^l}\right]^\alpha K_c \\ \left[\theta\phi^c\frac{1-\phi^l}{1-\phi^c}\right]^\alpha \frac{1}{K_c} \end{pmatrix} \tag{61}$$

To derive W^r in closed form consider the following recursion

$$\begin{aligned}
V(\widetilde{y}_0, g_0) &= E_0\sum\nolimits_{t=0}^{\infty}\delta^t\widetilde{y}_t = \widetilde{y}_0 + \delta E_0 V(\widetilde{y}_1, g_1) \\
V(\widetilde{y}_t, g_t) &= y_t + \beta E_t V(\widetilde{y}_{t+1}, g_{t+1})
\end{aligned} \tag{62}$$

Suppose that the function V is linear: $V(\widetilde{y}_t, g_t) = \widetilde{y}_t w(g_t)$, with $w(g_t)$ an undetermined coefficient. Substituting this guess into (62), we get $w(g_t) = 1 + \delta E_t g_{t+1} w(g_{t+1})$. Combining this condition with (61), it follows that $w(g_{t+1})$ satisfies

$$\begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + \delta\begin{pmatrix} u & 1-u & 0 \\ 0 & 0 & 1 \\ u & 1-u & 0 \end{pmatrix}\begin{pmatrix} g_1 w_1 \\ g_2 w_2 \\ g_3 w_3 \end{pmatrix} \Rightarrow \begin{aligned} w_1 &= \tfrac{1+(1-u)\delta g_2}{1-(1-u)\delta^2 g_2 g_3 - u\delta g_1} \\ w_2 &= \tfrac{1+\delta g_3 - u\delta g_1}{1-(1-u)\delta^2 g_2 g_3 - u\delta g_1} \\ w_3 &= \tfrac{1+(1-u)\delta g_2}{1-(1-u)\delta^2 g_2 g_3 - u\delta g_1} \end{aligned}$$

This solution exists and is unique provided $g_1\delta u + g_2 g_3\delta^2(1-u) < 1$. Equation (38) follows by noting that at time 0 the economy is in the lucky state: $V(y_0, g_0) = w_1 y_0^l$, and by making the substitution $g_2 g_3 = \left(\theta\phi^l\right)^\alpha(\theta\phi^c)^\alpha$.

Proof of Proposition ??.

A Proof of Proposition 5.2.

Consider the value functions W^s and W^r given by (36) and (37), respectively, and notice that if $u=1$, both are equal. Since W^s does not depend on u, we will prove the proposition by determining conditions under which $W^r_u := \partial W^r/\partial u|_{u=1}$ is negative. That is, an increase in crisis-risk improves the present value of consumption along a risky path. Let's denote

$$L = 1 - \left[\theta\phi^l\right]^\alpha \delta u - \left[\theta^2\phi^l\phi^s\right]^\alpha \delta^2(1-u), \quad T = \left(1 + \delta(1-u)\left[\theta\phi^l \frac{1-\phi^c}{1-\phi^l}\right]^\alpha k_c\right)(1-\phi^l)^\alpha,$$

so that

$$W^r = \frac{T}{L} q_0^\alpha, \qquad \text{and} \quad W^r_u := \frac{\partial W^r}{\partial u}|_{u=1} = \frac{LT_u - L_u T}{L^2} q_0^\alpha. \tag{63}$$

The derivatives $L_u := \partial L/\partial u|_{u=1}$ and $T_u := \partial T/\partial u|_{u=1}$ are

$$\begin{aligned} L_u &= -\delta(\theta\phi)^\alpha - \alpha\phi'\delta(\theta\phi)^{\alpha-1} + \left[\theta\phi^l\phi^c\right]^\alpha \delta^2 \\ T_u &= -\alpha\phi'[(1-\phi)]^{\alpha-1} - \delta\left[\theta\phi\right]^\alpha (1-\phi)^\alpha = (1-\phi)^{\alpha-1}\left[-\alpha\phi' - \delta\left[\theta\phi\right]^\alpha k_c(1-\phi)\left(\frac{1-\phi^c}{1-\phi}\right)^\alpha\right], \end{aligned}$$

where $\phi = \phi^s = \phi^l|_{u=1}$ and $\phi' = \partial\phi^l/\partial u|_{u=1}$. It then follows from (63) that:

$$\begin{aligned} \tfrac{L^2}{q_0^\alpha} W^r_u &= (D-1)(1-\phi)^{\alpha-1}(\alpha\phi' + D(1-\phi)k_c\left(\frac{1-\phi^c}{1-\phi}\right)^\alpha) + (1-\phi)^\alpha(D + \alpha\phi'\frac{D}{\phi} - D\delta(\theta\phi^c)^\alpha) \\ \tfrac{L^2}{q_0^\alpha(1-\phi)^{\alpha-1}} W^r_u &= (D-1)(1-\phi)^{\alpha-1}(\alpha\phi' + D(1-\phi)k_c\left(\frac{1-\phi^c}{1-\phi}\right)^\alpha) + (1-\phi)^\alpha(D + \alpha\phi'\frac{D}{\phi} - D\delta(\theta\phi^c)^\alpha), \end{aligned}$$

with $D = \delta\left(\theta\phi\right)^\alpha$. Note that $D < 1$ because $\delta < \delta_{\max} := (\theta\phi)^{-\alpha}$ is necessary for W^s in (36) to be well defined. After some algebraic manipulations, the expression above can be expressed as follows:

$$\tfrac{L^2 W^r_u}{q_0^\alpha(1-\phi)^{\alpha-1}} = \underbrace{\alpha\phi'(\frac{D}{\phi} - 1)}_{\text{Pareto gains}} + \underbrace{(1-D)(1-k_c(\frac{1-\phi^c}{1-\phi^l})(1-\phi))}_{\text{Bankupcy costs}} + \underbrace{(1-\phi)^\alpha D\delta(\theta)^\alpha((\phi)^\alpha - (\phi^c)^\alpha)}_{\text{Financial distress costs}} \tag{64}$$

Since $D = \delta\left(\theta\phi\right)^\alpha = \left(\phi^{po}\right)^{1-\alpha}\phi^\alpha$, the first term can be rewritten as $\alpha\phi'((\frac{\phi^{po}}{\phi})^{1-\alpha} - 1)$, which is negative if and only if $\phi < \phi^{po}$ because ϕ' is negative (a reduction in u increases leverage). Since the two other terms are positive, a necessary condition for $W^r > W^s$ is:

$$\phi < \phi^{po},$$

where ϕ^{po} is the Pareto optimal investment share. This establish the part (i) of the proposition. To prove part (ii), consider first the case in which financial distress costs are small ($\mu_w \to 1-\beta$).

In this case the last term in (64) is zero as $\phi^c = \phi$. Thus, W_u^r is negative if and only if:

$$\begin{aligned} \tfrac{T^2 W_u^r}{q_0^\alpha (1-\phi)^{\alpha-1}} &= \alpha\phi' \left(\left[\frac{\phi^{po}}{\phi} \right]^{1-\alpha} - 1 \right) + (1-D) \left(\frac{\alpha[\beta-\mu]}{1-\phi} \right) (1-\phi) < 0 \\ \Leftrightarrow \quad & \frac{\mu}{\beta} > 1 + \beta^{-1}\phi' \left(\left[\frac{\phi^{po}}{\phi} \right]^{1-\alpha} - 1 \right) (1-D)^{-1} \end{aligned} \tag{65}$$

Therefore, a necessary and sufficient condition for (65) is:

$$\phi < \phi^{po} \text{ and } \mu > \mu^* := \max \left\{ 0,\ 1 + \frac{\phi'}{\beta(1-D)} \left(\left[\frac{\phi^{po}}{\phi} \right]^{1-\alpha} - 1 \right) \right\} \cdot \beta.$$

If in addition δ is large enough, this condition holds for any $\mu \geq 0$. To see this observe that $\lim_{\delta \to \delta_{\max}} D = 1$, and D is continuous and inceasing in δ.

Second, consider the more general case where $\mu_w < 1 - \beta$, but let the discount factor $\delta \to \delta_{\max}$, so that $D \to 1$. In this case the second term in (64) converges to zero. Therefore, $W_u^r < 0$ is equivalent to

$$(1-\phi)^\alpha (1 - (\frac{\phi^c}{\phi})^\alpha) < -\alpha\phi' \left(\left[\frac{1}{\phi} \right]^{1-\alpha} - 1 \right). \tag{66}$$

A necessary and sufficient condition for (66) is:

$$\phi < \phi^{po} \text{ and } \mu_w < \mu_w^* := \mu_w^* = (1-\beta) \left(1 + \alpha\phi'((\frac{1}{\phi})^{1-\alpha} - 1)(1-\phi)^{-\alpha} \right)^{1/\alpha}.$$

To see this, develop (66):

$$\begin{aligned} (1 - (\frac{\phi^c}{\phi})^\alpha) &< -\alpha\phi'((\frac{1}{\phi})^{1-\alpha} - 1)(1-\phi)^{-\alpha} \\ (\frac{\phi^c}{\phi})^\alpha &< \alpha\phi'((\frac{1}{\phi})^{1-\alpha} - 1)(1-\phi)^{-\alpha} \\ \left(\frac{\mu_w}{1-\beta} \right)^\alpha &< 1 + \alpha\phi'((\frac{1}{\phi})^{1-\alpha} - 1)(1-\phi)^{-\alpha} \\ \mu_w &< \mu_w^* = (1-\beta) \left(1 + \alpha\phi'((\frac{1}{\phi})^{1-\alpha} - 1)(1-\phi)^{-\alpha} \right)^{1/\alpha} \end{aligned}$$

old prooF The welfare of a risky and a safe economy are given by (36) and (37), respectively. Clearly, if $u = 1$, both are equal. Since W^s does not depend on u, we will prove the proposition by showing that when crises costs are small (i.e., $\mu \to \beta$ and $\mu_w \to 1 - \beta$, so that $k_c \to 1$) the derivative $W_u^r := \partial W^r / \partial u|_{u=1}$ is negative if and only if $\phi^s < \phi^{po}$. Let us denote:

$$L = 1 - \left[\theta\phi^l \right]^\alpha \delta u - \left[\theta^2 \phi^l \phi^s \right]^\alpha \delta^2 (1-u), \quad T = \left(1 + \delta(1-u) \left[\theta\phi^l \frac{1-\phi^s}{1-\phi^l} \right]^\alpha \right) (1-\phi^l)^\alpha$$

The derivatives of L and T evaluated at $u=1$ are:

$$\begin{aligned} L_u &= -\delta(\theta\phi)^\alpha - \alpha\phi'\delta(\theta\phi)^{\alpha-1} + [\theta\phi]^{2\alpha}\,\delta^2 \\ T_u &= -\alpha\phi^{'}[(1-\phi)]^{\alpha-1} - \delta\,[\theta\phi]^\alpha\,(1-\phi)^\alpha = (1-\phi)^{\alpha-1}(-\alpha\phi^{'} - \delta\,[\theta\phi]^\alpha\,(1-\phi)), \end{aligned}$$

where $\phi = \phi^s$ and $\phi' = \partial\phi^l/\partial u|_{u=1}$. Since $W^r(u) = T/L$, it follows that

$$\begin{aligned} \tfrac{T^2 W^r_u}{q_0^\alpha} &= (D-1)(1-\phi)^{\alpha-1}(\alpha\phi' + D(1-\phi)) + (1-\phi)^\alpha(D + \alpha\phi'\frac{D}{\phi} - D^2) \\ \tfrac{T^2 W^r_u}{(1-\phi)^{\alpha-1}q_0^\alpha} &= (D-1)(\alpha\phi' + D(1-\phi)) + (1-\phi)D(1 + \tfrac{\alpha\phi'}{\phi} - D) = \alpha\phi'(\tfrac{D}{\phi} - 1) = \alpha\phi'(\delta(\theta)^\alpha\phi^{\alpha-1} - 1) \end{aligned}$$

where $D = \delta\,(\theta\phi)^\alpha$. Since $\phi < 1$ and $\phi' < 0$, we have that $W^r_u < 0$ if and only if $\delta(\theta)^\alpha(\phi^s)^{\alpha-1} > 1$. Recall from (34) that the Pareto optimal share is $\phi^{po} = (\theta^\alpha\delta)^{\frac{1}{1-\alpha}}$. Hence, we can rewrite this condition as $W^r_u < 0$ if and only if $\phi^s < (\delta\theta^\alpha)^{\frac{1}{1-\alpha}} = \phi^{po}$. Since the system is continuous in u, μ and μ_w, the result in the Proposition follows.□

Derivation of (??). Suppose for a moment that there is only one crisis (at τ). Then consumers welfare is

$$C(\tau) = (1-\alpha)y_o + \sum_{t\neq\tau}\delta^t(1-\alpha)y_t + \delta^\tau\left[(1-\alpha)y_\tau - T(\tau)\right]$$

Using $T(\tau) = \frac{\alpha}{1-\phi}\frac{h}{u}\phi^l y_{\tau-1} - \mu\frac{\alpha}{1-\phi^c}y_\tau$ and $y_\tau = (\theta\phi^l)^\alpha\left[\frac{1-\phi^c}{1-\phi^l}\right]^\alpha y_{t-1}$, it follows that

$$\begin{aligned} (1-\alpha)y_\tau - T(\tau) &= y_\tau\left(1 - \alpha - \frac{\alpha}{1-\phi^c}\left[\frac{h}{u\theta^\alpha}\left[\frac{1-\phi^c}{\frac{1}{\phi^l}-1}\right]^{1-\alpha} - \mu\right]\right) \qquad (67) \\ &= (1-\alpha)y_\tau\left(1 - \tfrac{\alpha}{(1-\phi^c)(1-\alpha)}\left[\tfrac{h}{u\theta^\alpha}\left[\tfrac{1-\phi^c}{\frac{1}{\phi^l}-1}\right]^{1-\alpha} - \mu\right]\right) \equiv (1-\alpha)y_\tau K^T_c \end{aligned}$$

If we allow multiple crises to occur, consumer's welfare is

$$C^r = (1-\alpha)E_0\sum_{t=0}^\infty \delta^t K_t y_t, \qquad K_t = \begin{cases} 1 & \text{if } t \neq \tau_i \\ K^T_c & \text{if } t = \tau_i \end{cases}$$

Following the same steps as in the derivation of (38) we get (**??**).

Proof of Lemma 6.1. To see that diversion is preferred to no-diversion if and only if borrowing is higher under diversion, note that under diversion $[1-\lambda][1+\rho^c]b^{c,\theta}_t = h[w_t + b^{c,\theta}_t]$, and consider a unilateral deviation from an equilibrium where there is no diversion by θ-agent's

$$\begin{aligned} \pi^{c,d}_{t+1} - \pi^{c,nd}_{t+1} &= \beta p_{t+1}\Theta_{t+1}l^{1-\beta}_{t+1}[\hat{k}^\beta_{t+1} - k^\beta_{t+1}] - [h[w_t + \hat{b}_t] - h[w_t + b_t]] \\ &= \beta p_{t+1}\Theta_{t+1}l^{1-\beta}_{t+1}[\hat{k}^\beta_{t+1} - k^\beta_{t+1}] - hp_t[\hat{k}_t - k_t], \end{aligned}$$

where we have used the budget constraint $p_t k_t = w_t + b_t$. It follows that

$$\left.\frac{\partial\left(\pi^{c,d}_{t+1} - \pi^{c,nd}_{t+1}\right)}{\partial \hat{k}_t}\right|_{\hat{k}_t = k_t} = \beta p_{t+1}\Theta_{t+1} l_{t+1}^{1-\beta}\beta\theta k_{t+1}^{\beta-1} - hp_t$$
$$= p_{t+1}\theta - hp_t > 0 \quad \text{by (RoR)}$$

Since the condition on returns (RoR) holds (i.e., $\theta p_{t+1}/p_t > h$) and $\pi^{c,d}_{t+1} - \pi^{c,nd}_{t+1}$ is concave in $\hat{k}_t$ (as $\beta < 1$), the θ-agent will find it profitable to unilaterally deviate and divert if and only if she can attain a higher investment level (i.e., issue more debt). Thus, comparing the debt ceilings under diversion and no-diversion, it follows that there is no-diversion iff

$$b_t^{c,\theta} = \frac{1}{2}\delta[1-\lambda]\gamma y^{\theta}_{t+1} < [m^s - 1]w_t = b^s_t$$
$$\Leftrightarrow \quad \gamma < \overline{\gamma}' = \frac{2[m^s-1]}{\delta[1-\lambda]}\frac{w_t}{y^{\theta}_{t+1}} = \frac{2[m^s-1]}{\delta[1-\lambda]}\frac{1}{[\theta\phi]^{\alpha}}\frac{[1-\beta]\alpha}{1-\phi}$$

This bound is time-invariant because along the equilibrium path $\frac{w_t}{y^{\theta}_{t+1}}$ is constant

$$\frac{w_t}{y^{\theta}_{t+1}} = \frac{w_t}{w_{t+1}}\frac{w_{t+1}}{y^{\theta}_{t+1}} = \frac{p_t q_t}{p_{t+1}q_{t+1}}\frac{[1-\beta]p_{t+1}q_{t+1}}{\frac{1-\phi}{\alpha}p_{t+1}q_{t+1}} = \frac{p_t q_t}{p_{t+1}q_{t+1}}\frac{[1-\beta]\alpha}{1-\phi}$$
$$= \frac{q_{t-1}q_t^{\alpha}}{q_{t+1}^{\alpha-1}}\frac{[1-\beta]\alpha}{1-\phi} = \frac{[\theta\phi]^{-1}q_t q_t^{\alpha}}{[\theta\phi q_t]^{\alpha-1}}\frac{[1-\beta]\alpha}{1-\phi} = \frac{1}{[\theta\phi]^{\alpha}}\frac{[1-\beta]\alpha}{1-\phi}$$

Proof of Proposition 6.1. To prove this proposition we show that condition (**??**) holds iff $\gamma \leq \overline{\gamma}'' := \frac{1}{[1-\lambda][1-\lambda\bar{\varepsilon}\delta]}$.

$$E\left(\textstyle\sum_{t=0}^{\infty}\delta^{t+1}[y^{\theta,nd}_{t+1} + y^{\varepsilon,nd}_{t+1}]\right) \geq E\left(\textstyle\sum_{t=0}^{\infty}\delta^{t+1}\Gamma_{t+1}\right)$$
$$\textstyle\sum_{t=0}^{\infty}\delta^{t+1}\left[y^{\theta,nd}_{t+1} + \lambda\bar{\varepsilon}b^c_t\right] \geq \textstyle\sum_{t=0}^{\infty}\delta^{t+1}\gamma[1-\lambda]y^{\theta,nd}_{t+1}$$
$$\textstyle\sum_{t=0}^{\infty}\delta^{t+1}y^{\theta,nd}_{t+1}\left[1 + \lambda\bar{\varepsilon}\delta[1-\lambda]\gamma - \gamma[1-\lambda]\right] \geq 0$$
$$\textstyle\sum_{t=0}^{\infty}\delta^{t+1}y^{\theta,nd}_{t+1}\left[1 + \gamma[1-\lambda][\lambda\bar{\varepsilon}\delta - 1]\right] \geq 0$$
$$\frac{y^s_o}{1-\delta(\theta\phi^s)^{\alpha}}\left[1 + \gamma[1-\lambda][\lambda\bar{\varepsilon}\delta - 1]\right] \geq 0 \quad \text{if } \delta(\theta\phi^s)^{\alpha} < 1$$
$$\frac{(1-\phi^s)^{\alpha}}{1-\delta\left(\theta\phi^s\right)^{\alpha}}q_o^{\alpha}\cdot\left[1 + \gamma[1-\lambda][\lambda\bar{\varepsilon}\delta - 1]\right] \geq 0 \quad \text{if } \delta(\theta\phi^s)^{\alpha} < 1$$

Since $\phi^s < 1$ and $\delta(\theta\phi^s)^{\alpha} < 1$, the LHS is non-negative iff $\gamma \leq \overline{\gamma}'' := \frac{1}{[1-\lambda][1-\lambda\bar{\varepsilon}\delta]}$.

C. Model Simulations

The behavior of the model economy is determined by eight parameters: $u, r, \alpha, \theta, h, \beta$, μ_w and μ. We will set the probability of crisis $1-u$, the world interest rate r and the share of N-inputs in T-production α equal to some empirical estimates. Then, given the values of u, r and α, we determine the feasible set for the degree of contract enforceability h and the index of total factor productivity in the N-sector θ such that both an RSE and an SSE exist. The values of β, μ_w and μ are irrelevant for the existence of equilibria.

In a panel of 39 MECs studied in Tornell and Westermann (2002), the probability of a crisis in a given period ranges from 5% to 9%. The interest rate r, is set to the average US interest rate from 1980:1 to 1999:4, which equals 0.075. A survey of Mexican manufacturing firms suggests a conservative value for α equal to 35%. We then choose β, θ and h so that: (i) both an RSE and an SSE exist for the range $u \in [0.91, 1]$, and (ii) we obtain plausible values for the growth rates along a safe economy and along a lucky path. In the baseline case: $h = 0.76$, $\theta = 1.65$, $\beta = 0.8$ and $u = 0.95$. These parameters imply a safe GDP growth rate of $(1+\gamma^s) = (1-\beta)^\alpha \frac{\theta}{1-h\delta} = 3.8\%$ and a lucky GDP growth rate of $(1+\gamma^l) = (1-\beta)^\alpha \left(\frac{\theta}{1-h\delta u^{-1}}\right)^\alpha = 8.7\%$. By comparison, the average growth rate of India over the period is 5.14% and that of Thailand is 8.14%.

We choose the financial distress costs of crises $l^d = 1 - \frac{\mu_w}{1-\beta}$ so that the cumulative decrease of GDP during a crisis episode is 13%, which is the mean value in the sample considered by Tornell and Westermann (2002). In the model, the cumulative decrease in GDP growth during a crisis episode is $(1+\gamma^{cr})^2 = \left[\frac{\mu_w}{1-\beta}\right]^\alpha (\theta^2\phi^l\phi^s)^\alpha$. Using the baseline case $h = 0.76$, $\theta = 1.65$, and $\alpha = 0.35$ we get that $(1+\gamma^{cr})^2 = (1-0.13)$ if $\left[\frac{\mu_w}{1-\beta}\right] = 0.45$. Thus, we set conservatively $l^d = 0.7$. In the baseline case, the level of bankruptcy costs is free.

Finally, in order for the welfare measures to be bounded, the expected discounted sum of tradable production has to be finite. In the safe economy this requires $\delta(\theta\phi^s)^\alpha < 1$. In the risky economy: $\left[\theta\phi^l\right]^\alpha \delta u + \left[\theta^2\phi^l\phi^c\right]^\alpha \delta^2(1-u) < 1$. These two conditions impose an upper bound on α.[34] In particular, they hold if $\alpha < 0.6$. Summing up:

[34] Notice that the interior condition for the pareto optimal share, $\phi^{po} = [\theta^\alpha\delta]^{\frac{1}{1-\alpha}} < 1$ is sufficient for all boundness conditions if $\phi^l < \phi^{po}$. This condition is equivalent to an upper bound on $\alpha : \overline{\alpha} = \frac{\log(1+r)}{\log(\theta)}$.

Parameters	*baseline value*	*range of variation*	*sources*
Probability of crisis	$1 - u = 0.05$	$[0, 0.9]$	Average probability of crisis in the MEC sample of 39 countries in Tornell-Westerman (2002)
Intensity of N-inputs in T-production	$\alpha = 0.35$	$[0.2, 0.6]$	Annual Industrial Survey of Mexico (1994-1999)
Risk-free interest rate Discount factor	$r = 0.075$ $\delta = 0.925$		Average U.S. Interest Rate (1980:1-1999:4)
Financial distress costs	$l^d = 70\%$	$[30\%, 99\%]$	Based on Tornell-Westerman (2002)
Bankruptcy costs	$l^b = 100\%$	$[30\%, 100\%]$	
N-sector Productivity	$\theta = 1.6$		
Contract enforceability	$h = 0.76$		
Cash Flows/Sales in N-Sector	$1 - \beta = 20\%$		