

# The tenuous relationship between effort and performance pay\*

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## Abstract

When a worker is offered performance related pay, the incentive effect is not only determined by the shape of the incentive contract, but also by the probability of contract enforcement. We show that weaker enforcement may reduce the worker's effort, but lead to higher-powered incentive contracts. This creates a seemingly negative relationship between effort and performance pay.

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# 1 Introduction

The last two decades have seen a strong growth in the use of performance related pay. An increasing fraction of jobs explicitly pays workers for their performance, using bonuses, commissions or some other kinds of merit pay (see Lemieux et al, 2009). At the same time there seem to be an increase in complaints and even lawsuits over unpaid bonuses. As a recent example, 104 bankers in London were suing Dresdner Kleinwort and Commerzbank for \$66 worth of unpaid bonuses in the biggest case of its kind in the UK.<sup>1</sup> The increased use of discretionary bonuses, for which employers are expected to exercise their discretion reasonably and fairly, has also resulted in a number of employer-employee disputes and lawsuits.<sup>2</sup>

These two trends - more use of performance related pay and complaints about unpaid bonuses - coincide with what seems to be an increasing skepticism over what performance related pay actually can achieve. Standard economic models that predict a positive relationship between effort and performance pay are challenged by empirical work suggesting that performance pay mitigates motivation and reduces effort (see e.g. surveys by Weibel et al, 2010, Frey and Jegen 2001 and Jenkins et al. 1998).

In this paper we show that these phenomena may be closely related. Uncertainty over bonus payments or weaker enforcement of bonus contracts, may lead to higher bonuses and lower effort, creating a negative equilibrium relationship between performance pay and effort. The relationship that we propose, contrasts with the standard explanation based on motivation crowding out. The common denominator of the crowding out theories is that non-monetary intrinsic motivation is treated as a variable as opposed to a fixed attribute. Higher monetary rewards may reduce intrinsic motivation to such an extent that effort is reduced.<sup>3</sup> We show that variations in enforcement

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<sup>1</sup>See e.g. Financial Times, January 25, 2012

<sup>2</sup>See e.g. Alexandra Carn in Financial World, 2006-07, and Howard Meyers in New York Law Journal, June 27, 2008).

<sup>3</sup>Recent papers show how the structure of monetary rewards may undermine incentives

probability can have similar effects as variations in intrinsic motivation, and that the former can be an alternative explanation for a negative association between performance pay and effort.

With "enforcement probability" we here mean the probability that an employee who is entitled to a bonus actually receives the bonus. There are a number of reasons why the employee may not be paid as promised. If the incentive contract is incomplete, the employer may deliberately choose not to honor the contract hoping that the court will not be able to enforce it. The employer may also provide discretionary bonuses, where the bonus is paid at the employer's discretion and the employee is not protected by a legally enforceable contract. Reputational concerns may then affect decisions whether or not to pay any bonus, although the court can also play a role if the employer clearly has acted unreasonably. Finally, there may be more or less unexpected contingencies that arise during the employment relationship that make it costly, or even impossible, for the employer to pay the bonus as promised.<sup>4</sup>

In this paper, we show how exogenous variations in enforcement probability affect both incentive design and effort. Clearly, weaker probability *cet. par.* reduces the employee's effort, because the expected bonus decreases. But weaker enforcement may also lead to higher-powered incentive contracts. Why is this? At the outset one might expect the opposite. No incentive contract can be implemented in a situation where the firm certainly won't pay. And high-powered incentives can certainly be enforced if the contract is honored for sure. Also, risk aversion on the part of the agent can make it quite costly for the firm to offer incentives where very high bonuses

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for social esteem (Benabou and Tirole, 2006, and Ellingsen and Johannesson, 2008), affect agents' internal rewards from norm adherence (Sliwka, 2007), or affect agents' perception of their tasks or own abilities (Benabou and Tirole, 2003). See Frey and Regel (2001) for a review of previous literature on motivation crowding out.

<sup>4</sup>In the Dresdner Kleinwort case, the loss of 6.5 billion euros made them unwilling to pay the bonuses. In the aftermath of the financial crisis one has also seen examples where CEOs give up their bonuses after pressure from stakeholders or politicians.

are paid with low probability, as the agent must be compensated for the high risk associated with such schemes. However, it turns out that on the margin, the incentive intensity of the contract can be negatively related to the probability of enforcement under quite standard assumptions, creating a negative equilibrium relationship between performance pay and effort.

We show this in a simple moral hazard model where a principal must provide an agent with incentives to exert effort, and where the incentive contract is honored with a probability  $v < 1$ . Our modeling set-up can account for both legal and non-legal, or informal, enforcement mechanisms. With legal enforcement,  $v$  is the probability that the court can verify performance and thus enforce the contract. With informal enforcement,  $v$  is the probability that the principal feels morally or socially committed to honor the contract. It is natural to consider the probability of both legal and informal enforcement as a variable rather than as a fixed parameter. Generally, the complexity of the transactions, the strength of the enforcement institutions and the practice of legal courts are factors that affect legal enforcement. Also, informal contract enforcement, such as the environments for reputational enforcement may vary.

For contracting parties these may constitute exogenous variations. But one can also think of the enforcement probability as an endogenous variable, since the contracting parties' effort in writing a contract that describes a job's tasks and operational performance metrics may also affect this probability (see Kvaløy and Olsen, 2009). In this paper, however, we abstract from endogenous verifiability, and treat enforcement as an exogenous variable. Exogenous variations occur naturally across countries and industries, but can also affect a given contractual relationship via legal reforms, changes in legal practice, standardization of industry contracts, changes in (labor) law or other institutional or organizational changes.

We first adopt the classical model on risk sharing vs. incentives (e.g. Holmström 1979), and show that when enforcement is probabilistic, then

under certain conditions contractual incentive intensity and effort are negatively related. We then show that a similar result can also be obtained under risk neutrality and limited liability. This negative relationship is a "false crowding out effect" since total monetary incentives, which is the product of the enforcement probability and contractual incentives, is positively related to effort. But since the enforcement probability does not show up in the incentive contract, it *appears* that incentives and effort are negatively related.

To see the intuition, note that if the enforcement probability increases, this has a positive effect on effort, but it also increases expected wage costs per unit of effort since the probability that the principal actually has to pay as promised increases. In order to reduce wage costs, the principal can simply reduce expected contractual wage payments. Hence, effort increases, but the contractual incentives are lower-powered. And the other way around: Weaker enforcement induces lower effort since the probability that the agent actually is paid decreases. In order to mitigate the reduction in effort, the principal can thus provide higher-powered incentives.

This result has an important empirical implication: When observing a negative relationship between performance pay and effort, one has to control for the probability that incentive contracts are actually honored. If not, one may wrongfully infer that monetary incentives crowd out non-monetary motivation. Controlling for enforcement probability is quite easy in experimental work.<sup>5</sup> In empirical work, however, this is much more of a challenge. Take the empirical work on New Public Management (NPM) as an example. NPM describes reforms in the public sector that are characterized by an emphasis on output control, performance related pay and introduction of market mechanisms. Scholars argue that NPM undermines - or crowd out - intrinsic motivation and thus the effort of public servants, see e.g. Weibel, Rost,

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<sup>5</sup>There are a few laboratory and field experiments documenting a negative causal relationship between effort and monetary incentives (e.g. Frey and Oberholzer-Gee, 1997; Gneezy and Rustichini, 2000, and Fehr and Gächter, 2002).

Osterloh (2010), and Perry, Engbers and Jun (2009). But if NPM actually undermines effort (which of course is debatable, see Stazyk, 2010), would this necessarily come from crowding out of intrinsic motivation? Important aims of NPM include decentralization of management authority, more discretion and flexibility, less bureaucracy and less rules. These institutional changes may affect both the legal and the informal enforcement environment.

The crux is that enforcement and contractual incentives may be substitutes. In that sense our paper is related to models showing the substitutability between explicit contracts and informal relational contracts (see Baker, Gibbons and Murphy, 1994, and Schmidt and Schnitzer, 1995). In these models, improved explicit contracts may reduce feasible incentive pay under relational contracting, but effort is still positively related to the sum of contractual incentives. In contrast, we find that effort may be negatively related to contractual incentives.

In spirit, our argument also bears similarities to the type of argument Prendergast (2002abc) uses in a series of papers in order to explain a positive relationship between uncertainty and incentives. Prendergast shows that such a relationship may be due to a positive relationship between uncertainty and delegation, which in turn generates a need for incentive pay. Similarly, we point out that a negative relationship between contractual incentives and effort may be due to a negative relationship between incentives and contract enforcement, which in turn generates a negative relationship between incentives and effort.

With respect to the modelling, a contribution of the paper is to consider probabilistic enforcement in an otherwise standard moral hazard model with risk aversion or limited liability. In the classic moral hazard models (e.g. Holmström, 1979), perfect enforcement is assumed, while in models of incomplete contracting, it is commonly assumed that contracting is prohibitively costly so that legal enforcement is impossible (starting with Grossman and Hart, 1986). However, imperfect enforcement is increasingly recognized as

an important ingredient in models of contractual relationships. Some papers focus on the relationship between ex post evidence disclosure and enforceability (Ishiguro, 2002; Bull and Joel Watson, 2004), while others focus on the relationship between ex ante contracting and enforceability (Battigalli and Maggi, 2002, Schwartz and Watson, 2004, Shavell 2006). There is also a growing literature on the interaction between legal imperfect enforcement and informal (relational) enforcement, see Sobel (2006), MacLeod (2007), Battigalli and Maggi (2008) and Kvaløy and Olsen (2009, 2012).

The paper is organized as follows. In Section 2 we present the basic model and study variations in enforcement probability under risk aversion and limited liability, respectively. Section 3 concludes.

## 2 Incentives and enforceability

We consider a relationship between a principal and an agent, where the agent produces output  $x$  for the principal. Output is a random variable ( $x \in X$ ), and the agent's effort  $a$  affects the probability distribution (density)  $f(x, a)$ . Effort costs are given by  $C(a)$ , where  $C'(a) > 0$ ,  $C''(a) > 0$ ,  $C(0) = 0$ . We assume that output is observable to both parties, but that the agent's effort level is unobservable to the principal, so the parties must contract on output: The principal offers a wage  $w(x) = s + \beta(x)$  where  $s$  is a non-contingent fixed salary and  $\beta(x)$  is a contingent bonus. The principal is assumed to be risk neutral, but we allow the agent to be risk averse, with a utility function  $u(w)$ .

We assume that contracts are not perfectly enforceable by the court of law, but that there is a probability  $v \in (0, 1)$  that the principal is committed to honor the full contract, i.e. the fixed salary plus the discretionary bonus. (The fixed salary must always be paid.) Given that the agent accepts the contract, he is thus paid the fixed salary  $s$ , then exerts (hidden) effort  $a$ , after which the output  $x$  is realized and observed by both parties. Finally the agent is paid the discretionary bonus  $\beta(x)$  with probability  $v$ . (So with

probability  $1 - v$  he receives no bonus.)

We abstract here from strategic behavior that may affect the probability of verification. Instead the modeling approach allows for several kinds of exogenous variations. One is that a court decides whether or not the principal has to pay the bonus as promised. The parameter  $v$  is then the ex ante probability (belief shared by the parties) that the contract will be legally enforced. A second interpretation is that the principal learns about the contractual environment ex post, for instance to which extent social or reputational concerns matter for the given contractual relationship. The parameter  $v$  is then the ex ante probability that the principal is committed to pay the discretionary bonus.

We will now deduce the optimal contract and discuss variations in enforcement probability  $v$ . We first assume that the agent is risk averse, and next analyze the case where both parties are risk neutral but subject to limited liability.

## 2.1 Risk aversion

In stage 2 the game  $\Gamma$ , the agent chooses effort to maximize his expected utility, given by

$$U(a, w, v, s) = v \int f(x, a)u(w(x))dx + (1 - v)u(s) - C(a).$$

(Unless otherwise noted, all integrals are over the support  $X$ .) For each outcome  $x$ , the agent gets the payment  $w(x) = s + \beta(x)$  with probability  $v$ , and the payment (fixed salary)  $s$  otherwise, and this gives expected utility as specified. Optimal effort satisfies

$$U_a(a, w, v, s) = v \int f_a(x, a)u(w(x))dx - C'(a) = 0 \quad (\text{IC})$$

(We will invoke assumptions to make the 'first-order approach' valid.)



In stage 1 the principal chooses wages (and effort  $a$ ) to maximize her payoff, subject to the agent's choice, represented by IC, and the agent's participation constraint:

$$U(a, w, v, s) \geq U_o \quad (\text{IR})$$

The principal, assumed risk neutral, has payoff

$$V(a, w, v, s) = \int f(x, a) [x - vw(x)] dx - (1 - v)s$$

Forming the Lagrangian  $L = V + \lambda(U - U_o) + \mu U_a$ , with multipliers  $\lambda$  and  $\mu$  on the IR and IC constraints, respectively, one sees that optimal payments satisfy

$$\frac{1}{u'(w(x))} = \lambda + \mu \frac{f_a(x, a)}{f(x, a)}, \quad \frac{1}{u'(s)} = \lambda \quad (\text{W})$$

These conditions are standard (Holmström 79), and reflect the trade-off between providing insurance and incentives for the agent. This trade off is relevant for the performance dependent bonuses, but not for the fixed payment  $s$ . Given a monotone likelihood ratio  $\frac{f_a(x, a)}{f(x, a)}$  (MLRP), payments  $w(x)$  will be increasing in output  $x$ .

Payments will be chosen to implement the action that is optimal for the principal, and this entails an action that satisfies  $L_a = 0$ . The optimal action and the associated payments (and multipliers) will depend on the parameter  $v$ , i.e. on the level of enforceability.

We now ask, i) will effort increase when the enforcement probability  $v$  increases and ii) may contractual incentives at the same time become weaker? That is: would the new contractual incentives (corresponding to the higher  $v$ ) have induced lower effort under the old  $v$ ? If so, the new contractual incentives are weaker, but the associated effort will be higher.

Consider the agent's (marginal) incentives for effort; they are given by  $vm(a, w)$ , where

$$m(a, w) \equiv \int f_a(x, a) u(w(x)) dx \quad (\text{M})$$

Thus  $m(a, w)$  is the marginal incentive for effort generated by the contract  $w(x) = s + \beta(x)$ . We call  $m$  the marginal *contractual* incentives.

Consider now  $\tilde{v} > v$ , and suppose the associated optimal efforts satisfy  $\tilde{a} > a$ . A way to interpret question ii) is then to ask whether  $m(a, \tilde{w}) < m(a, w)$ , i.e. whether the monetary payments  $\tilde{w}$  associated with the higher  $\tilde{v}$  yield in isolation lower marginal incentives for the agent.

Now, optimal effort and payments are functions of  $v$ , say  $a(v)$  and (with some abuse of notation)  $w(v)$ , respectively. We thus ask if  $m(a, w(v))$  is decreasing in  $v$ , i.e. if

$$\frac{\partial}{\partial v} m(a, w(v)) = \int f_a(x, a) \frac{\partial}{\partial v} u(w(x; v)) dx < 0$$

Note that in equilibrium the agent's choice of effort will be  $a = a(v)$ , and hence we have from incentive compatibility (IC) that  $vm(a(v), w(v)) = C'(a(v))$ . Differentiating this identity we see that for equilibrium effort  $a = a(v)$  we have

$$v \frac{\partial}{\partial v} m(a, w(v)) = \left[ C''(a) - v \frac{\partial}{\partial a} m(a, w(v)) \right] a'(v) - C'(a)/v \quad (1)$$

From this it follows that if  $a'(v) > 0$  (so effort increases with  $v$ ), and the last term dominates the other terms on the RHS (so  $\frac{\partial}{\partial v} m < 0$ ), then it will be the case that effort and marginal contractual incentives for effort move in opposite directions.<sup>6</sup> We will in the following provide a specification of functional forms where this is precisely the case.

Note from (1) and IC ( $vm = C'$ ) that the sign of  $\frac{\partial}{\partial v} m$  is given by the sign of

$$\left[ \frac{aC''(a)}{C'(a)} - \frac{a}{m(a, w)} \frac{\partial}{\partial a} m(a, w) \right] \frac{va'(v)}{a} - 1 \quad (2)$$

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<sup>6</sup>If on the other hand  $a'(v) < 0$ , then (since the square bracket in (1) is positive by the agent's SOC), we will have  $\partial m / \partial v < 0$ , and thus effort and marginal contractual incentives moving in the same direction.

Hence the sign is determined by the magnitudes of three elasticities; pertaining to marginal costs, marginal contractual incentives and equilibrium effort, respectively. Signing expressions like (1) thus requires properties of equilibrium effort variations in a moral hazard model. To make this tractable we consider specific functional forms. Assume the following specifications for the probability distribution and for the agent's utility:

$$F(x, a) = \Pr(\text{outcome} \leq x|a) = 1 - e^{-x/a}, \quad x \geq 0, \quad u(w) = \sqrt{w} \quad (3)$$

Here the expected output is  $Ex = a$ , so higher effort increases expected output and leads to a more favorable distribution in the sense of first order stochastic dominance. The distribution satisfies MLRP. The utility function implies constant relative risk aversion ( $-wu''/u' = \text{const}$ ).

It turns out that the marginal contractual incentives for effort in this case are constant and independent of effort, i.e.  $\frac{\partial}{\partial a}m(a, w(v)) = 0$ . So from (1) we have here (for  $a = a(v)$ )

$$\frac{v^2}{C'(a)} \frac{\partial}{\partial v} m(a, w(v)) = \frac{vC'''(a)}{C'(a)} a'(v) - 1 \quad (4)$$

Hence we see that if the equilibrium marginal cost  $C'(a(v))$  is inelastic (as a function of  $v$ ) then marginal contractual incentives will be reduced as the level of enforceability  $v$  increases. If at the same time effort increases with higher  $v$ , then clearly effort and contractual incentives will move in opposite directions. It can be shown (see the appendix) that this will indeed be the case if the cost function exhibits inelastic marginal costs ( $aC'''(a)/C'(a) \leq 1$ ) and moreover  $aC''''(a)/C'''(a) > -3$ . (This holds e.g. for quadratic costs;  $C(a) = ca^2$ ). Thus we provide a set of conditions where *effort increases while the incentives for effort generated by the contract decrease*. (A somewhat more general result is given in the appendix; see Lemma 2.)

**Proposition 1** *If functional forms satisfy (3), then effort and contrac-*

*tual incentives are negatively related if marginal effort costs are inelastic ( $aC''(a)/C'(a) \leq 1$ ) and  $aC'''(a)/C''(a) > -3$ .*

The intuition is as follows. Improved enforceability increases the agent's incentives to exert effort (other things equal), but it also increases the principal's wage costs per unit of effort (since the probability that the principal actually has to pay as promised increases). Now, even though the principal finds it optimal to induce higher effort when  $v$  increases, she will make a trade-off between the benefits from higher effort and the expected wage costs from higher  $v$ . She may thus reduce these wage costs by providing lower-powered incentives. In other words, improved enforcement may crowd out contractual incentives.

Note that this type of crowding out appears when effort costs are inelastic, meaning that the agent has a high responsiveness to incentives. The reason is that improved enforcement increases effort and thus wage costs per unit effort to such an extent that the principal finds it optimal to reduce contractual incentives.

## 2.2 Limited liability

We will now show that similar results can be obtained under risk neutrality and limited liability. We assume from now on that the agent is risk neutral in the sense that  $u(w) = w$ , but that he is protected by limited liability so that  $w(x) \geq 0$ . We also assume that the principal has limited means so that  $w(x) \leq x$ . Hence, it is assumed that the principal cannot commit to pay wages above the agent's value added. This constraint resembles Innes (1990) who in a financial contracting setting assumes that the investor's (principal's) liability is limited to her investment in the agent. Finally, it is convenient here to specify that output has support  $X = [\underline{x}, \bar{x}]$

Now, the game proceeds as in the previous section, but under risk neutrality, the agent's expected payoff is simply:  $s + \int_{\underline{x}}^{\bar{x}} v\beta(x)f(x, a)dx - C(a)$ ,

yielding a first order condition for effort as follows:

$$\int_{\underline{x}}^{\bar{x}} v\beta(x)f_a(x,a)dx - C'(a) = 0 \quad (\text{IC}')$$

In stage 1, the principal maximizes her payoff, which is

$$\int_{\underline{x}}^{\bar{x}} (x - v\beta(x))f(x,a)dx - s,$$

subject to incentive (IC'), participation (IR) and limited liability constraints:

$$s + \int_{\underline{x}}^{\bar{x}} v\beta(x)f(x,a)dx - C(a) \geq U_o \quad (\text{IR})$$

$$x \geq w(x) = s + \beta(x) \geq 0$$

Mainly to simplify notation, we will assume  $\underline{x} = 0$  and hence that the fixed salary must be  $s = 0$ . By the same argument as in Innes (1990), it then follows that the optimal wage scheme pays the minimal wage for outcomes below some threshold, and the maximal wage for outcomes above that threshold ( $\beta(x) = 0$  for  $x < x'_0$  and  $\beta(x) = x$  for  $x > x'_0$ ). It is well known that the discontinuity of this scheme is problematic, and for that reason one requires continuity and monotonicity. The optimal such scheme also has a threshold (say  $x_0$ ) and pays  $\beta(x) = 0$  for  $x \leq x_0$  and  $\beta(x) = x - x_0$  for  $x > x_0$ . In the following we will focus on this kind of (constrained optimal) incentive scheme. Since the expected marginal payoff from exerting extra effort is zero as long as output is below  $x_0$ , it is clear that the higher is the threshold  $x_0$ , the lower is the incentive intensity of the contract.

Given that the principal cannot extract rent from the agent through the fixed salary component, the IR constraint will not bind unless the agent's reservation utility  $U_o$  is 'large'. Mainly to simplify notation we will assume here that  $U_o = 0$  and hence that this constraint is not binding.

Given the form of the incentive scheme, the expected payment for the

agent is now

$$v \int_{\underline{x}}^{\bar{x}} \beta(x) f(x, a) dx = v \int_{x_0}^{\bar{x}} (x - x_0) f(x, a) dx = v \int_{x_0}^{\bar{x}} G(x, a) dx,$$

where the expression in the last integral follows from integration by parts, and where  $G(x, a) = \Pr(\text{outcome} > x | a) = 1 - F(x, a)$ . By a similar calculation the principal's expected payoff can be written as

$$\int_{\underline{x}}^{\bar{x}} x f(x, a) dx - v \int_{\underline{x}}^{\bar{x}} \beta(x) f(x, a) dx = \int_{\underline{x}}^{\bar{x}} G(x, a) dx - v \int_{x_0}^{\bar{x}} G(x, a) dx \quad (5)$$

The principal's problem is now (for a given  $v$ ) to choose  $x_0, a$  to maximize this payoff subject to the agent's incentive constraint.

We will focus on cases where higher  $v$  is valuable for the principal.<sup>7</sup> Note that a higher  $v$  is beneficial for the principal because it strengthens the agent's incentives, but is on the other hand costly because it increases the total expected payments (and therefore the rent) to the agent. It turns out that a higher  $v$  is valuable if  $G_a(x, a) > 0$ , meaning that more effort yields a shift to a distribution that is more favorable in the sense of first order stochastic dominance. As is well known, this is implied by MLRP.

Again, we analyze the following question: what happens to the optimal effort ( $a$ ) and incentive scheme (represented by  $x_0$ ) when  $v$  varies? Comparative statics yields the following

**Lemma 1** *If (in addition to MLRP) we have*

$$\frac{\partial}{\partial a} \int_{x_0}^{\bar{x}} \frac{G_a(x, a)}{G_a(x_0, a)} dx > 0 \quad (6)$$

*then  $a'(v) > 0$ .*

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<sup>7</sup>If the principal can influence the verification probability  $v$ , e.g. by making costly investments (say  $K(v)$ ) in better contract specifications or performance metrics, we will have  $\partial L / \partial v = K'(v)$  in optimum and thus  $\partial L / \partial v > 0$  for the relevant level  $v$ .

As noted before, a improved enforcement increases the agent's incentives to exert effort (other things equal), but it also increases the principal's wage costs per unit of effort. The proposition gives conditions under which the first effect dominates in the sense that the principal finds it optimal to induce higher effort when enforceability increases. But the principal may still want to mitigate the latter effect, that is to reduce wage costs by providing lower-powered incentives. The next result shows that this is indeed what will occur, under some conditions. The following conditions turn out to be sufficient:

$$G_{aa}(x, a) < 0, \quad \frac{\partial G_{aa}(x, a)}{\partial a} \leq 0 \quad \text{and} \quad \frac{\partial G_{aa}(x, a)}{\partial x} > 0 \quad (7)$$

**Proposition 2** *Suppose that  $C'''(a) \geq 0$  and that  $G(x, a)$  in addition to the assumptions in Lemma 1 satisfies (7). Then both effort and the threshold for the incentive scheme increase with higher enforceability ( $a'(v) > 0$  and  $x'_0(v) > 0$ ), hence higher effort is then associated with lower-powered contractual incentives.*

An example that satisfies all assumptions is  $G(x, a) = \Pr(\text{outcome} > x) = 1 - x^a$ ,  $0 \leq x \leq 1$ , (see the appendix).

The proposition demonstrates that higher effort may be associated with lower-powered contractual incentives (higher  $x_0$ ), and the other way around, even if there is no *motivation-crowding-out*.

### 3 Concluding remarks

We offer a simple model where contractual monetary incentives and effort are negatively related even if there is no crowding out of non-monetary motivation. The idea is simple: Improved enforcement induces higher effort, but increases the principal's expected wage costs, which can be mitigated

by lower-powered incentives. Or: Weaker enforcement induces lower effort, which can be mitigated by higher-powered incentives.

Our model is not an alternative to the behavioral models on crowding out, but a complement. In contrast to (parts of) the crowding out literature, we do not offer a negative *causal* relationship between incentives and effort. Instead we identify a spurious relationship where improved contract enforcement increases effort but "crowd out" contractual incentives. Total monetary incentives, which is the product of the enforcement probability and contractual incentives, are positively related to effort, but since the enforcement probability does not show up in the incentive contract, it *appears* that incentives and effort are negatively related. The empirical implication is clear: When observing a negative relationship between performance pay and effort, one has to control for the probability that the relevant incentive contracts are actually enforced. If not, one may wrongfully infer that monetary incentives crowd out non-monetary motivation.

## Appendix

### Proof of Proposition 1.

From the Lagrangian  $L = V + \lambda(U - U_o) + \mu U_a$ , we obtain the following conditions for optimal bonuses  $\beta(x)$ , or equivalently payments  $w(x) = s + \beta(x)$ :

$$0 = -vf(x, a) + \lambda vf(x, a)u'(w(x)) + \mu f_a(x, a)u'(w(x)),$$

and for the optimal fixed payment  $s$ :

$$0 = -1 + \lambda \left( v \int f(x, a)u'(w(x))dx + (1 - v)u'(s) \right) + \mu v \int f_a(x, a)u'(w(x))dx.$$

The first is equivalent to  $\frac{1}{u'} = \lambda + \mu \frac{f_a}{f}$ , and substituting from the first into the second we get  $\lambda u' = 1$ . This proves (W).

For utility  $u(w) = \sqrt{w}$  we have  $1/u' = 2u$ , hence the conditions for



optimal payments are

$$2u(w(x)) = \lambda + \mu \frac{f_a(x, a)}{f(x, a)} \equiv \lambda + \mu h(x, a), \quad 2u(w(s)) = \lambda \quad (8)$$

where  $h(x, a) = \frac{f_a(x, a)}{f(x, a)}$  denotes the likelihood ratio.

Proposition 1 now follows from the lemma below. To state the lemma define

$$M(a) = \int f_a(x, a)h(x, a)dx \quad (9)$$

$$M_1(a) = \int f_{aa}(x, a)h(x, a)dx \quad (10)$$

$$N(a) = \int f_a(x, a)h^2(x, a)dx \quad (11)$$

Define also

$$\begin{aligned} p(a) &= 2(U_o + C(a))C'(a) - \int x f_a(x, a)dx \\ q(a) &= 2 \left[ \frac{N(a) - 2M_1(a)}{2M(a)} + \frac{C''(a)}{C'(a)} \right] \frac{C'(a)^2}{M(a)} \end{aligned}$$

Then we have

**Lemma 2** *Assume  $u(w) = w^{-1/2}$ . Then optimal effort satisfies  $p(a) + q(a)/v = 0$ . If  $q(a) > 0$  then  $a'(v) > 0$ . If in addition condition (12) below holds, then  $\frac{\partial m}{\partial v} < 0$ .*

$$\left[ \frac{C''(a)}{C'(a)} - \frac{M_1(a)}{M(a)} \right] \frac{q(a)/v}{p'(a) + q'(a)/v} - 1 < 0 \quad (12)$$

As we will show below, the LHS of (12) coincides with (2). Consider now Proposition 1. For  $F(x, a) = 1 - e^{-x/a}$  it is straightforward to verify (see below) that we have  $M(a) = 1/a^2$ ,  $M_1(a) = 0$ ,  $N(a) = 2/a^3$  and

$\int x f_a(x, a) dx = 1$ , and hence that

$$q(a) = 2 \left[ \frac{1}{a} + \frac{C''(a)}{C'(a)} \right] C'(a)^2 a^2$$

For this distribution, condition (12) in the lemma is thus

$$\left[ \frac{aC''(a)}{C'(a)} \right] \frac{q(a)/v}{ap'(a) + aq'(a)/v} < 1$$

Since  $p'(a) > 0$ , we see that for inelastic marginal costs this condition holds if  $q(a) \leq aq'(a)$ . This holds if  $\frac{aC'''(a)}{C''(a)} \geq -3$  (see below), proving Proposition 1.

**Proof of the lemma.** Consider first the agent's marginal contractual incentive  $m(a, w)$ , where payments  $w()$  are optimal, and thus given by (8) for the optimal action  $a = a^*$ , say. We then have

$$m(a, w) = \int f_a(x, a) u(w(x)) dx = \int f_a(x, a) \frac{\lambda + \mu h(x, a^*)}{2} dx \quad (13)$$

$$= \int f_a(x, a) h(x, a^*) dx \frac{\mu}{2} \equiv M(a, a^*) \frac{\mu}{2} \quad (14)$$

where  $M(a, a^*)$  is (with a slight abuse of notation) defined by the identity, and the third equality follows from  $\int f_a = 0$  (since  $\int f = 1$ ).

Note that the agent's choice problem is concave if  $vm_a(a, w) - C''(a) \leq 0$ , which holds if  $M_a(a, a^*) \leq 0$  and  $C'' \geq 0$ , and that the optimal choice of effort is then given by the FOC  $vm(a, w) = C'(a)$ . In equilibrium we have  $a = a^*$  and thus

$$C'(a) = v \int f_a(x, a) u(w(x)) dx = v \frac{\mu}{2} M(a, a) \equiv v \frac{\mu}{2} M(a) \quad (15)$$

Note also from IR (which will be binding) and (8) that we have

$$\begin{aligned} U_o + C(a) &= v \int f(x, a)u(w(x))dx + (1 - v)u(s) \\ &= v \int f(x, a) [\lambda + \mu h(x, a)] / 2 dx + (1 - v)\lambda/2 = \lambda/2 \quad (16) \end{aligned}$$

where the last equality follows from the fact that  $\int fh = \int f \frac{f_a}{f} = \int f_a = 0$ . Hence we see that  $\lambda = 2(U_o + C(a))$ .

To characterize the optimal effort for the principal, consider

$$\begin{aligned} L_a &= V_a + \lambda U_a + \mu U_{aa} \\ &= \int f_a(x, a) [x - vw(x)] dx + 0 + \mu(v \int f_{aa}(x, a)u(w(x))dx - C''(a)) \\ &= e(a) - v \int f_a(x, a)w(x)dx + \mu(vm_a(a, w) - C''(a)) \quad (17) \end{aligned}$$

where we have defined  $e(a) = \int x f_a(x, a)dx$  as the marginal value of effort on output.

Consider the second term in (17). Since  $u = \sqrt{w}$  we have  $w = u^2$ , and substituting from (8) we can write

$$\begin{aligned} \int f_a(x, a)w(x)dx &= \int f_a(x, a) ([\lambda + \mu h(x, a)] / 2)^2 dx \\ &= \int f_a(x, a) [\lambda^2 + 2\lambda\mu h(x, a) + \mu^2 h^2(x, a)] dx / 4 \\ &= \lambda \frac{\mu}{2} M(a) + \frac{\mu^2}{4} N(a) \quad (18) \end{aligned}$$

where the last equality follows from  $\int f_a = 0$  and the definitions of  $M(a)$  and  $N(a)$ , see (9) and (11).

We see from (13) and (10) that we (in equilibrium) have  $m_a(a, w) =$

$M_1(a)\mu/2$  and hence that (17) can be written as

$$L_a = e(a) - v \left( \lambda \frac{\mu}{2} M(a) + \frac{\mu^2}{4} N(a) \right) + \mu \left( v \frac{\mu}{2} M_1(a) - C''(a) \right)$$

Substituting for  $\mu$  from (15) and for  $\lambda$  from (16) we obtain the following condition for optimal effort

$$\begin{aligned} 0 &= L_a = e(a) - \left( \lambda C'(a) + \frac{C'(a)\mu}{M(a)} N(a) \right) + \mu \left( \frac{C'(a)}{M(a)} M_1(a) - C''(a) \right) \\ &= e(a) - 2(U_o + C(a))C'(a) - \left[ \frac{C'(a)}{M(a)} N(a) - 2 \left( \frac{C'(a)}{M(a)} M_1(a) - C''(a) \right) \right] \frac{C'(a)}{M(a)} \frac{1}{v} \\ &= -p(a) - q(a) \frac{1}{v} \end{aligned}$$

where the last equality follows from the definitions of  $p(a)$ ,  $q(a)$  and  $e(a) = \int x f_a$ .

This shows that optimal effort is given by  $p(a) + q(a)\frac{1}{v} = 0$ , as stated in the lemma, and that  $a'(v) = \frac{q(a)/v^2}{p'(a)+q'(a)/v}$ . Concavity of the principal's optimization w.r.t. effort requires  $p'(a) + q'(a)/v > 0$ , and hence we have  $a'(v) > 0$  when  $q(a) > 0$ . Substituting for  $a'(v)$  in the condition (2) for  $\frac{\partial m}{\partial v} < 0$  and noting (from (14) and (9)-(10)) that  $m_a/m = M_1(a)/M(a)$ , we see that condition (2) is equivalent to (12) in the lemma. This completes the proof.

For completeness we finally verify the assertions stated above regarding the distribution  $F(x, a) = 1 - e^{-x/a}$ . We have here density  $f(x, a) = \frac{1}{a} e^{-x/a}$  and likelihood ratio  $h(x, a) \equiv \frac{f_a(x, a)}{f(x, a)} = \frac{1}{a} \left( \frac{x}{a} - 1 \right)$ . Hence

$$\begin{aligned} M(a, a^*) &= \int f_a(x, a) h(x, a^*) dx = \int_0^\infty \frac{1}{a^2} e^{-x/a} \left( \frac{x}{a} - 1 \right) \frac{1}{a^*} \left( \frac{x}{a^*} - 1 \right) dx \\ &= \frac{1}{aa^*} \int_0^\infty e^{-y} (y - 1) \left( y \frac{a}{a^*} - 1 \right) dy = \frac{1}{(a^*)^2} \end{aligned}$$

This shows that  $M(a) = M(a, a) = 1/a^2$  and  $M_1(a) = M_a(a, a^* = a) = 0$ .

We further have

$$N(a) = \int f_a(x, a)h^2(x, a)dx = \int_0^\infty \frac{1}{a^4}e^{-x/a}\left(\frac{x}{a}-1\right)^3dx = \frac{1}{a^3} \int_0^\infty e^{-y}(y-1)^3dy = \frac{2}{a^3}$$

Finally note that  $q(a) = 2a\psi(a)$ , with  $\psi(a) = (C')^2 + aC''C'$  and hence that  $aq'(a) = 2a\psi(a) + 2a^2\psi'(a) \geq q(a)$  if  $\psi'(a) \geq 0$ . We have  $\psi'(a) = 3C'C'' + aC'''C' + aC''C''' > 0$  certainly if  $aC'''/C'' \geq -3$ . This verifies the stated assertions.

**Remark.** As another application of the Lemma, one can show that effort and contractual incentives move in opposite directions ( $a'(v) > 0$ ,  $\frac{\partial m}{\partial v} < 0$ ) for the distribution  $F(x, a) = x^a$ ,  $x \in [0, 1]$  if  $C'(a)$  is sufficiently inelastic and  $v$  is sufficiently large (close to 1). For this distribution one finds  $M(a) = \frac{1}{a^2}$ ,  $N(a) = -\frac{2}{a^3}$ ,  $M_1(a) = \frac{-2}{a^3}$  (and hence marginal incentives are decreasing in effort, since  $m_a = M_1\mu/2 < 0$ ). Assuming  $C'(a) = k = \text{const}$ , we then find  $q(a) = \frac{N(a)-2M_1(a)}{M(a)} \frac{k^2}{M(a)} = 2ak^2$  and  $p(a) = 2(U_o + ka)k - \frac{1}{(a+1)^2}$ , and thus

$$\begin{aligned} \left[ \frac{C''(a)}{C'(a)} - \frac{M_1(a)}{M(a)} \right] \frac{q(a)/v}{p'(a)+q'(a)/v} &= \left[ \frac{2}{a} \right] \frac{2ak^2/v}{2k^2 + \frac{2}{(a+1)^3} + 2k^2/v} \\ &= 2 \frac{1}{v + \frac{v/k^2}{(a+1)^3} + 1} \rightarrow 2 \frac{1}{2 + \frac{1/k^2}{(a+1)^3}} < 1 \quad \text{as } v \rightarrow 1 \end{aligned}$$

This shows that the condition in the Lemma is fulfilled for  $v$  close to 1.

### Proof of Lemma 1

The principal chooses  $x_0, a$  to maximize her payoff (5) subject to the agent's incentive constraint, which here takes the form

$$v \int_{x_0}^{\bar{x}} G_a(x, a)dx - C'(a) = 0 \quad (19)$$

The Lagrangian for this problem is

$$L = \int_{\underline{x}}^{\bar{x}} G(x, a)dx - v \int_{x_0}^{\bar{x}} G(x, a)dx + \mu \left[ v \int_{x_0}^{\bar{x}} G_a(x, a)dx - C'(a) \right] \quad (20)$$

As noted we focus on cases where higher  $v$  is valuable for the principal, i.e.

where  $\frac{\partial L}{\partial v} > 0$ . Since optimization with respect to the threshold parameter  $x_0$  yields  $vG(x_0, a) - v\mu G_a(x_0, a) = 0$  and hence  $\mu = \frac{G(x_0, a)}{G_a(x_0, a)}$ , we have

$$\frac{\partial L}{\partial v} = - \int_{x_0}^{\bar{x}} G(x, a) dx + \mu \int_{x_0}^{\bar{x}} G_a(x, a) dx = \int_{x_0}^{\bar{x}} \left[ \frac{G(x_0, a)}{G_a(x_0, a)} - \frac{G(x, a)}{G_a(x, a)} \right] G_a(x, a) dx \quad (21)$$

We see that we will have  $\frac{\partial L}{\partial v} > 0$  if  $G_a(x, a) > 0$  and the ratio  $\frac{G(x, a)}{G_a(x, a)}$  is decreasing in  $x$ . Both properties follow from MLRP; we demonstrate the latter below (at the end of this proof).

Consider now the Lagrangian (20) and write the constraint (19) as

$$H(x_0, a, v) \equiv v \int_{x_0}^{\bar{x}} G_a(x, a) dx - C'(a) = 0 \quad (22)$$

The FOCs for optimal choices are  $L_{x_0} = L_a = H = 0$ . (Subscripts denote partials.) Differentiation of these conditions yields

$$\begin{bmatrix} L_{x_0 x_0} & L_{x_0 a} & H_{x_0} \\ L_{a x_0} & L_{aa} & H_a \\ H_{x_0} & H_a & 0 \end{bmatrix} \begin{bmatrix} x'_0(v) \\ a'(v) \\ \mu'(v) \end{bmatrix} = \begin{bmatrix} -L_{x_0 v} \\ -L_{av} \\ -H_v \end{bmatrix}$$

and hence the standard comparative statics formulae

$$x'_0(v) = \frac{1}{D} \begin{vmatrix} -L_{x_0 v} & L_{x_0 a} & H_{x_0} \\ -L_{av} & L_{aa} & H_a \\ -H_v & H_a & 0 \end{vmatrix} = \frac{1}{D} [H_a^2 L_{v x_0} - H_a H_{x_0} L_{av} - H_a H_v L_{a x_0} + H_v H_{x_0} L_{aa}]$$

$$a'(v) = \frac{1}{D} \begin{vmatrix} L_{x_0 x_0} & -L_{x_0 v} & H_{x_0} \\ L_{a x_0} & -L_{av} & H_a \\ H_{x_0} & -H_v & 0 \end{vmatrix} = \frac{1}{D} [H_{x_0}^2 L_{av} - H_v H_{x_0} L_{a x_0} - H_a H_{x_0} L_{v x_0} + H_a H_v L_{x_0 x_0}]$$

where

$$D = \begin{vmatrix} L_{x_0x_0} & L_{x_0a} & H_{x_0} \\ L_{ax_0} & L_{aa} & H_a \\ H_{x_0} & H_a & 0 \end{vmatrix} = -L_{x_0x_0}H_a^2 + 2L_{ax_0}H_aH_{x_0} - L_{aa}H_{x_0}^2 > 0 \quad (\text{SOC})$$

From FOC we have  $0 = L_{x_0} = vG(x_0, a) - \mu vG_a(x_0, a)$  and hence

$$L_{vx_0} = G(x_0, a) - \mu G_a(x_0, a) = 0 \quad (23)$$

Hence we can write

$$x'_0(v)D = -H_aH_{x_0}L_{av} - H_aH_vL_{ax_0} + H_vH_{x_0}L_{aa} \quad (24)$$

$$a'(v)D = H_{x_0}^2L_{av} - H_vH_{x_0}L_{ax_0} + H_aH_vL_{x_0x_0} \quad (25)$$

Writing  $g(x, a) = G_x(x, a)$  and using (23) we have

$$L_{x_0x_0}/v = g(x_0, a) - \mu g_a(x_0, a) = g(x_0, a) - \frac{G(x_0, a)}{G_a(x_0, a)}g_a(x_0, a) < 0$$

where the inequality holds because we have assumed  $G_a > 0$  and it follows from MLRP (as shown below) that  $\frac{d}{dx} \frac{G_a}{G} = \frac{1}{G^2}(g_aG - G_ag) > 0$ .

From (22),  $G_a > 0$  and the SOC for the agent we have

$$H_{x_0} = -vG_a(x_0, a) < 0, \quad H_v = \int_{x_0}^{\bar{x}} G_a(x, a)dx > 0, \quad H_a = v \int_{x_0}^{\bar{x}} G_{aa}(x, a)dx - C'''(a) < 0 \quad (26)$$

These inequalities imply  $H_aH_vL_{x_0x_0} > 0$ , and we thus have from (25):  $a'(v)D > [H_{x_0}L_{av} - H_vL_{ax_0}]H_{x_0}$ .

Since  $H_{x_0} = -vG_a < 0$  we then have  $a'(v) > 0$  if  $H_{x_0}L_{av} - H_vL_{ax_0} < 0$ .

To show that this condition implying  $a'(v) > 0$  is satisfied, consider

$$\begin{aligned}
H_{x_0}L_{av} - H_vL_{ax_0} &= -vG_a(x_0, a) \left[ -\int_{x_0}^{\bar{x}} G_a(x, a)dx + \mu \int_{x_0}^{\bar{x}} G_{aa}(x, a)dx \right] \\
&\quad - \left( \int_{x_0}^{\bar{x}} G_a(x, a)dx \right) [vG_a(x_0, a) - \mu vG_{aa}(x_0, a)] \\
&= \mu v \left[ -G_a(x_0, a) \int_{x_0}^{\bar{x}} G_{aa}(x, a)dx + G_{aa}(x_0, a) \int_{x_0}^{\bar{x}} G_a(x, a)dx \right] \\
&= -\mu v G_a^2(x_0, a) \left[ \frac{\partial}{\partial a} \int_{x_0}^{\bar{x}} \frac{G_a(x, a)}{G_a(x_0, a)} dx \right] < 0
\end{aligned}$$

The last inequality follows from the assumption (6) and proves that  $a'(v) > 0$ .

It remains to verify the assertion – stated after (21) – that MLRP implies that the ratio  $\frac{G(x,a)}{G_a(x,a)}$  is decreasing in  $x$ . To this end consider

$$\frac{\partial}{\partial x} \frac{G_a}{G} = \frac{1}{G^2} (g_a G - G_a g) = \frac{g}{G} \left( \frac{g_a}{g} - \frac{G_a}{G} \right) \quad (27)$$

The derivative is positive, and the proof is thus complete, if the last parenthesis is positive. Note that

$$\frac{G_a(x, a)}{G(x, a)} = \frac{\frac{\partial}{\partial a} \int_{\underline{x}}^x g(x', a) dx'}{G(x, a)} = \int_{\underline{x}}^x \frac{g_a(x', a)}{g(x', a)} \frac{g(x', a)}{G(x, a)} dx' \leq \frac{g_a(x, a)}{g(x, a)} \cdot 1$$

where the inequality follows by MLRP ( $\frac{g_a(x,a)}{g(x,a)} = \frac{f_a(x,a)}{f(x,a)}$  increasing). Hence the derivative in (27) is positive, and this completes the proof.

### Proof of Proposition 2

First note that  $G_{aa} < 0$  implies  $L_{av} = -\int_{x_0}^{\bar{x}} G_a(x, a)dx + \mu \int_{x_0}^{\bar{x}} G_{aa}(x, a)dx < 0$ , and hence from (26) that  $H_a H_{x_0} L_{av} < 0$ . We then have from (24):

$$x'_0(v)D = -H_a H_{x_0} L_{av} - H_a H_v L_{ax_0} + H_v H_{x_0} L_{aa} > [-H_a L_{ax_0} + H_{x_0} L_{aa}] H_v \quad (28)$$



Consider  $[-H_a L_{ax_0} + H_{x_0} L_{aa}]$ . Since  $H_a < v \int_{x_0}^{\bar{x}} G_{aa}(x, a) dx$  by (26), and since  $G_{aa} < 0$  implies  $L_{ax_0} = v G_a(x_0, a) - \mu v G_{aa}(x_0, a) > 0$ , we have

$$\begin{aligned} -H_a L_{ax_0} + H_{x_0} L_{aa} &> - \left( v \int_{x_0}^{\bar{x}} G_{aa}(x, a) dx \right) [G_a(x_0, a) - \mu G_{aa}(x_0, a)] v + (-v G_a(x_0, a)) L_{aa} \\ &= v G_a(x_0, a) \left( - \int_{x_0}^{\bar{x}} G_{aa}(x, a) dx \left[ 1 - \mu \frac{G_{aa}(x_0, a)}{G_a(x_0, a)} \right] v - L_{aa} \right) \end{aligned} \quad (29)$$

Consider  $L_{aa}$ . Since  $G_{aa} < 0$  and  $C'''(a) \geq 0$  we have

$$\begin{aligned} L_{aa} &= \int_{\underline{x}}^{\bar{x}} G_{aa}(x, a) dx - v \int_{x_0}^{\bar{x}} G_{aa}(x, a) dx + \mu \left[ v \int_{x_0}^{\bar{x}} G_{aaa}(x, a) dx - C'''(a) \right] \\ &< \int_{x_0}^{\bar{x}} G_{aa}(x, a) \left[ 1 - v + \mu v \frac{G_{aaa}(x, a)}{G_{aa}(x, a)} \right] dx \end{aligned}$$

Hence from (29) we now have

$$\frac{-H_a L_{ax_0} + H_{x_0} L_{aa}}{v G_a(x_0, a)} > - \int_{x_0}^{\bar{x}} G_{aa}(x, a) \left[ 1 + \mu v \left( \frac{G_{aaa}(x, a)}{G_{aa}(x, a)} - \frac{G_{aa}(x_0, a)}{G_a(x_0, a)} \right) \right] dx > 0 \quad (30)$$

where the last inequality will be shown to follow from (7). From (28) and the fact that  $H_v > 0$  we then see that  $x'_0(v) > 0$ .

To show the last inequality in (30), note that the assumptions in (7) imply

$$\frac{\partial}{\partial a} \frac{G_{aa}(x, a)}{G_a(x, a)} = \frac{G_{aa}(x, a)}{G_a(x, a)} \left( \frac{G_{aaa}(x, a)}{G_{aa}(x, a)} - \frac{G_{aa}(x, a)}{G_a(x, a)} \right) \leq 0$$

and  $\frac{G_{aa}(x, a)}{G_a(x, a)} > \frac{G_{aa}(x_0, a)}{G_a(x_0, a)}$  when  $x > x_0$ . These inequalities in turn imply

$$\frac{G_{aaa}(x, a)}{G_{aa}(x, a)} \geq \frac{G_{aa}(x, a)}{G_a(x, a)} > \frac{G_{aa}(x_0, a)}{G_a(x_0, a)} \quad \text{when } x > x_0$$

This implies that the expression in (30) is positive, and hence completes the proof that  $x'_0(v) > 0$ .

To illustrate the assumptions stated in Proposition 3, we finally show that

they are all satisfied by  $G(x, a) = 1 - x^a$ ,  $0 \leq x \leq 1$ . For this distribution we have

$$G_a(x, a) = -x^a \ln x > 0$$

$$G_x(x, a) = -ax^{a-1} = -f(x, a)$$

$$G_{xa}(x, a) = -f_a(x, a) = -x^{a-1}(a \ln x + 1)$$

Hence  $\frac{f_a(x, a)}{f(x, a)} = \ln x + 1/a$  is increasing in  $x$ , so MLRP holds. Moreover, we also have

$$\frac{\partial}{\partial a} \int_{x_0}^{\bar{x}} \frac{G_a(x, a)}{G_a(x_0, a)} dx = \frac{\partial}{\partial a} \int_{x_0}^1 \frac{x^a \ln x}{x_0^a \ln x_0} dx = \int_{x_0}^1 \left(\frac{x}{x_0}\right)^a \ln\left(\frac{x}{x_0}\right) \frac{\ln x}{\ln x_0} dx > 0$$

hence the condition stated in Lemma 1 holds.

Next note that

$$G_{aa}(x, a) = -\frac{d}{da} x^a \ln x = -x^a (\ln x)^2 = G_a(x, a) \ln x < 0$$

and hence that  $\frac{G_{aa}(x, a)}{G_a(x, a)} = \ln x$ . The additional assumptions (7) in Proposition 2 are therefore also satisfied.

## References

- [1] Bajari, Patrick and Steve Tadelis. 2001. "Incentives versus Transaction Costs: A theory of procurement contracts." *RAND Journal of Economics* 32: 387–407.
- [2] Baker, George, Gibbons, Robert and Murphy, Kevin J. 1994. "Subjective Performance Measures in Optimal Incentive Contracts." *Quarterly Journal of Economics* 109: 1125–56.
- [3] Battigalli, P. and G. Maggi. 2002. "Rigidity, Discretion, and the Costs of Writing Contracts." *American Economic Review* 92: 798–817.
- [4] Battigalli, P. and G. Maggi. 2008. "Costly Contracting in Long-term Relationship." *RAND Journal of Economics* 39: 359–377.

- [5] Bénabou, Roland, and Jean Tirole. 2003. "Intrinsic and Extrinsic Motivation." *Review of Economic Studies*, 70(3): 489–520.
- [6] Bénabou, Roland, and Jean Tirole. 2006. "Incentives and Prosocial Behavior." *American Economic Review*, 96(5): 1652–78.
- [7] Bull, Jesse and Joel Watson. 2004. "Evidence Disclosure and Verifiability". *Journal of Economic Theory*, 118: 1-31.
- [8] Charness, G., and Dufwenberg, M. 2006. "Promises and partnership", *Econometrica* 74, 1579-1601.
- [9] Ellingsen, Tore and Magnus Johannesson. 2004. "Promises, Threats and Fairness," *Economic Journal*, 114: 397–420.
- [10] Ellingsen, Tore and Magnus Johannesson. 2008. "Pride and Prejudice: The Human Side of Incentive Theory." *American Economic Review*, 98: 990-1008.
- [11] Fehr, Ernst, and Simon Gächter. 2002. "Do Incentive Contracts Undermine Voluntary Cooperation?" Institute for Empirical Research in Economics Working Paper 34.
- [12] Frey, Bruno S., and Felix Oberholzer-Gee. 1997. "The Cost of Price Incentives: An Empirical Analysis of Motivation Crowding-Out." *American Economic Review*, 87: 746–55.
- [13] Frey, Bruno S., and Reto Jegen. 2001. Motivation crowding out. *Journal of Economic Surveys* 15: 589-611.
- [14] Greif, Avner. 1994. "Cultural Beliefs and the Organization of Society: A Historical and Theoretical Reflection on Collectivist and Individualist Societies." *The Journal of Political Economy*, 102: 912-50.

- [15] Grossman, Sanford J. and Hart, Oliver. 1986. "The Costs and Benefits of Ownership: A theory of Lateral and Vertical Integration." *Journal of Political Economy*, 94: 691-719.
- [16] Gneezy, Uri, and Aldo Rustichini. 2000a. "A Fine Is a Price." *Journal of Legal Studies*, 29(1): 1–17.
- [17] Innes, Robert D. 1990. "Limited liability and Incentive Contracting with Ex Ante Action Choices." *Journal of Economic Theory*, 52: 45-67.
- [18] Holmström, Bengt. 1979 "Moral Hazard and Observability." *Bell Journal of Economics*,10: 74–91.
- [19] Ishiguro, Shingo. 2002. "Endogenous Verifiability and Optimality in Agency." *Journal of Economic Theory* 105: 518-530.
- [20] Kvaløy, Ola and Trond E. Olsen. 2009. "Endogenous Verifiability and Relational Contracting." *American Economic Review*, 99: 2193-2208.
- [21] Kvaløy, Ola and Trond E. Olsen. 2012. "Incentive Provision when Contracting is Costly, working paper, University Of Stavanger.
- [22] MacLeod, W. Bentley. 2007. "Reputations, Relationships and Contract Enforcement." *Journal of Economic Literature*, XLV 595-628.
- [23] Perry, James L, Trent Engbers, and So Yun Jun. 2009. Back to the future? Performance related pay, empirical research, and the perils of persistence. *Public Administration Review* 69: 39-51.
- [24] Schmidt, Klaus M. and Schnitzer, Monika . "The interaction of Explicit and Implicit contracts." *Economics Letters*, 1995, 48(2): 193-99.
- [25] Schwartz Alan and Joel Watson. 2004. "The Law and Economics of Costly Contracting." *Journal of Law, Economics and Organization*, 20: 2-31.

- [26] Shavell, Steven. 2006. "On the Writing and Interpretation of Contracts." *Journal of Law, Economics and Organization*, 22: 289-314.
- [27] Sliwka, Dirk. 2007. "Trust as a Signal of a Social Norm and the Hidden Costs of Incentive Schemes." *American Economic Review*, 97: 999–1012.
- [28] Sobel, Joel. 2006. "For Better or Forever: Formal versus Informal Enforcement." *Journal of Labor Economics* 24: 217-298.
- [29] Stazyk, Edmund C. 2010. "Crowding out intrinsic motivation? The role of performance related pay". Working paper, American University.
- [30] Weibel, A., Rost, K., & Osterloh, M. 2010. Pay for performance in the public sector: Benefits and (hidden) costs. *Journal of Public Administration Research and Theory*, 20: 387-412.