Re-examining the Effects of Switching Costs

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Abstract

Consumers often incur costs when switching from one product to another. Recently there has been renewed debate within the literature about whether these switching costs lead to higher prices. We build a theoretical model of dynamic competition and solve it analytically for a wide range of switching costs. We provide a simple condition which determines whether switching costs raise or lower long-run prices. We also show that switching costs are more likely to increase prices in the short-run. Finally switching costs redistribute surplus across time, and as such are shown to sometimes increase consumer welfare.

Keywords: Switching costs, Dynamic competition, Markov perfect equilibrium, Linear-quadratic games

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1 Introduction

In many markets consumers incur costs if they switch from the product they currently purchase, to another product sold by a different company. For example, in the U.S. auto insurance industry Honka (2013) estimates an average switching cost of \$116, while in the U.S. pay-TV market Shcherbakov (2009) estimates switching costs of \$109 for cable and \$186 for satellite. These amount to respectively 20%, 32% and 52% of what a typical consumer would spend annually on these products. Evidence of significant switching costs has also been found in several other industries, including cell phones and bank deposits (Shy 2002) and also domestic gas (Giulietti *et al* 2005).

Switching costs partially lock consumers to their initial supplier, creating the well-known trade-off between 'harvesting' and 'investing'. On the one hand a firm might charge a high price and harvest its existing customers, exploiting their reluctance to switch. On the other hand since consumers tend not to switch, there is a strong relationship between current market share and future profitability. A firm might therefore prefer to invest in market share by charging a low price. The conventional wisdom has been that the harvesting effect dominates, such that switching costs increase prices (Farrell and Klemperer 2007). Such wisdom typically draws on two-period models, in which firms offer 'bargains' to consumers when they are young, and 'rip-offs' when they become old. However one drawback of these models is that they artificially separate out the investment and harvesting motives into the first and second periods respectively. In reality firms often compete over a long time horizon, and at any moment are trying to both attract new consumers and sell to old ones. Therefore the subsequent literature has emphasized models with infinite horizons, and has offered some findings where small switching costs can reduce prices.

In this paper we re-examine the effect of switching costs on prices, profits, and consumer surplus within a more general model of dynamic competition. This approach has several distinctive features. Firstly, in contrast to other papers which typically focus only on very small or very large switching costs, or which use numerical simulations, it permits analytical results for a very wide range of switching costs. Secondly, it also allows both firms and consumers to be forward-looking, whereas many other papers make the restrictive assumption that consumers are myopic. Thirdly, it allows us to study the impact of switching costs in both the short- and long-run, whereas the existing literature has tended to focus only on the latter. Distinguishing between the two is important - we show that switching costs can affect prices differently, depending upon the time horizon considered. Finally, it enables us to consider how switching costs affect not only prices but also welfare. To this end we provide a novel explanation for why switching costs can be beneficial to consumers.

In more detail, the model considers two infinitely-lived firms who sell to overlapping generations of consumers. In each period the market is covered and product differentiation is modelled using a linear Hotelling line. Linearity is important because it enables us to find a closed form solution for equilibrium prices. It also allows us to be very general in other dimensions. In particular all agents in the model - including consumers - are forward-looking, and we are able to state our results for very general levels of the switching cost. We first show that the impact of switching costs on steady state prices is almost always ambiguous, and depends upon how patient consumers are relative to firms. We then derive a necessary and sufficient condition for when switching costs are pro-competitive, and show that it is satisfied *unless* consumers are significantly more patient than firms. The intuition, which we expand upon below, is that a firm's incentive to 'lock in' consumers far outweighs a consumer's incentive to avoid being locked in. We therefore find a general presumption that in the long-run, switching costs make markets more competitive.

In the short-run the relationship between switching costs and prices is generally more complicated, and has rarely been studied within the previous literature. Additional complexities arise because firms may start with unequal market shares, and therefore have different pricing incentives. Compared with what happens in the long-run, the firm with larger market share charges a higher price, whilst the firm with the smaller market share charges a lower price. This implies that the average (i.e. consumption-weighted) price can initially be quite high. However over time firms' market shares become more symmetric, and the average price decreases monotonically. We provide a condition which determines whether average price is higher with switching costs than without. We also demonstrate that switching costs can be anti-competitive in the short-run and yet pro-competitive in the long-run.

It is natural to ask whether switching costs can reduce prices by so much that they actually benefit consumers. Young consumers often gain because they pay lower prices. However old consumers always lose out - they bear the brunt of switching costs, and are never fully compensated for this by any price reductions. Therefore switching costs have a tendency to transfer surplus from the old to the young. When consumers have a preference for current over future consumption, this transfer is beneficial. Consequently for a wide range of parameters, a consumer's lifetime expected surplus is larger with switching costs than without.

Our modelling approach is most closely related to papers by Beggs and Klemperer (1992), To (1996), Doganoglu (2010), Somaini and Einav (2012) and Fabra and García (2012). They also use Hotelling-style models and, with the exception of the first and last papers, have overlapping generations of consumers. Beggs and Klemperer (1992) and To (1996) restrict attention to a special case where switching costs are so high that no consumer ever actually switches. They both find that steady state prices are higher compared to a market that has no switching cost. Doganoglu (2010) restricts attention to another special case where switching costs are very low. Using a model of experience goods he shows that starting from zero, a small increase in the switching cost leads to a decrease in the steady state price. Our approach in this paper is very different, because we solve our model for a considerably wider range of switching costs. We show that away from the two extremes which these other papers focus on, the impact of switching costs on prices is ambiguous. We then derive and interpret a condition on parameters which determines whether that impact is positive or negative. Somaini and Einav (2012) solve a model which is even more general than ours, and which allows for cost asymmetries and many (potentially multiproduct) firms. However they are interested in antitrust policy in dynamic markets, rather than in determining analytically how switching costs affect prices and welfare. We also note that by working with a simplified version of their model, we are able to derive all our main results analytically, and are able to go beyond looking at just steady state outcomes. Finally Fabra and García (2012) develop a continuous-time model, and like us they also distinguish between the short- and long-run effects of switching costs. However in their model consumers are myopic. We show that this assumption can be very restrictive, because it makes switching costs seem more pro-competitive than they actually are.

Several other recent papers are also related. Cabral (2013) proves that small switching costs are pro-competitive, whenever firms can price discriminate and consumers' preferences are not too serially correlated across time. He also finds that switching costs can benefit consumers. However the intertemporal transfer effect, which plays an important role in our results, is not present in his model because he does not use an overlapping generations framework. Arie and Grieco (2012) show that firms with low market shares are more likely to be harmed by small switching costs, and to respond by reducing their price. Using a logit model Pearcy (2011) derives a closed form solution for steady state prices, and shows that switching costs are more likely to be pro-competitive in markets that have many firms. Finally Bouckaert *et al* (2012) and Biglaiser *et al* (2013) explore the consequences of heterogeneity in switching costs. They show that an increase in the distribution of switching costs can lead to lower industry profits.

The paper proceeds as follows. Section 2 outlines the model, whilst Section 3 proves existence and uniqueness of an equilibrium in affine strategies. We then examine how switching costs affects prices and profits (in Section 4) and consumer surplus (in Section 5). Section 6 then checks the robustness of our results by allowing for consumer preferences to be correlated across time. Finally we conclude in Section 7 with some directions for future research.

2 Model

Time is discrete and there are infinitely many periods, denoted by $t = 1, 2, \ldots$ There are two firms A and B that are located on a Hotelling line at positions $x = 0$ and $x = 1$ respectively. The marginal cost of production is normalized to zero for both firms. Each period a unit mass of new consumers is born, who then live for two periods before exiting the market. Consequently at any moment there are (equal-sized) overlapping generations of 'young' and 'old' consumers in the economy. At the start of period t each consumer is randomly assigned a location x^t on the Hotelling line, which (for old consumers) is independent of their location in the previous period. A consumer with location x^t values product A at $V - x^t$ and product B at $V - (1 - x^t)$. If an 'old' consumer bought from firm i when young but now wants to buy from firm $j \neq i$, she must incur a switching cost $s \in (0, 7/10]$.[1] As explained more fully below, we assume $s \leq 7/10$ in order to ensure that in equilibrium each firm always has some consumers switching to it and others switching away from it. Consumers and firms are both risk-neutral and have discount factors δ_c and δ_f respectively which lie in $(0, 1)$.

The timing of the model is as follows. In each period t the two firms simultaneously and non-cooperatively choose prices p_A^t and p_B^t, in order to maximize their respective discounted sum of current and future profits. Firms cannot commit to any future prices. Consumers then observe p_A^t and p_B^t as well as their own personal location x^t. Young consumers buy whichever product maximizes their expected lifetime utility. Old consumers either stay with their initial supplier, or pay the switching cost and buy from the competitor.

This is a simplified version of the model in Somaini and Einav (2012), which also allows for arbitrarily many firms whose marginal costs may differ. Their paper is primarily concerned with antitrust policy in markets with switching costs, and does not

[1] We could have written consumer valuations as $V - \tau x^t$ and $V - \tau (1 - x^t)$, and let the switching cost be τs. Equilibrium prices would be scaled up by τ, but otherwise none of the analysis would change. We therefore assume without loss of generality that $\tau = 1$.

analytically determine how these switching costs affect equilibrium prices and welfare. One advantage of our simpler set-up is that we are able to derive all our main results analytically. Our set-up is also isomorphic to Doganoglu (2010) when in our model $\delta_c = \delta_f$, and when in his model $\Delta = 2$. (Δ is a parameter in his model which captures heterogeneity in consumers' product valuations.) The main difference is that he focuses on comparative statics of the steady state price around the point $s = 0$; we study prices both in and out of steady state, as well as consumer surplus, and moreover our analysis is valid for any $s \in (0, 7/10]$.

3 Solving the model

3.1 Consumers

Old consumers Suppose an old consumer bought product A when she was young. When old, she can again buy A and enjoy a surplus $V - x^t - p_A^t$, or she can switch to product B and get $V - (1 - x^t) - p_B^t - s$. Therefore buying A again is optimal if and only if $x^t \leq \dot{x}^t = \left(1 + p_B^t - p_A^t + s\right)/2$. Similarly an old consumer who bought B when she was young, optimally switches to A if and only if $x^t \leq \ddot{x}^t = \left(1 + p_B^t - p_A^t - s\right)/2$. We will show that since the switching cost is not too large, in equilibrium $\dot{x}^t, \ddot{x}^t \in (0, 1)$ i.e. generically each firm has some consumers switching away from it and others switching towards it.

Young consumers born in period t form (rational) expectations about the prices Ep_A^{t+1} and Ep_B^{t+1} they will face when old (we discuss below how these expectations arise). If a young consumer buys A she gets an immediate payoff $V - x^t - p_A^t$; when old, she will stay with A and get $V - x^{t+1} - Ep_A^t$ if x^{t+1} is sufficiently low, otherwise she will switch to B and get $V - (1 - x^{t+1}) - Ep_B^t - s$. Taking an expectation over all possible values of x^{t+1}, the young consumer can calculate her expected lifetime payoff from buying product A. She can similarly calculate the expected utility from product B, and buy whichever product is best.

Lemma 1. *There exists a threshold $\tilde{x}^t$ such that all young consumers with location $x^t \leq \tilde{x}^t$ buy product A, and everyone else buys product B. The threshold satisfies*

$$\tilde{x}^t = \frac{1}{2} + \frac{p_B^t - p_A^t + \delta_c s \left(Ep_B^{t+1} - Ep_A^{t+1} \right)}{2} \tag{1}$$

(Note that all omitted proofs are given in the appendices.) Young consumers located at $\tilde{x}^t$ expect to get the same lifetime utility from both products and are therefore indifferent between them. People located to the left (right) of $\tilde{x}^t$ have a stronger initial preference for product A (B) and therefore buy it.

3.2 Firms

Each firm's strategy specifies a price that should be played, for every time period and for every possible history of the game. This section solves for and characterizes that strategy.

We know from the previous section that in period t product A is bought by $\tilde{x}^t$ young consumers (defined in Lemma 1) and by $\tilde{x}^{t-1}\dot{x}^t + \left(1 - \tilde{x}^{t-1}\right)\ddot{x}^t$ old consumers. Total demand for product A therefore depends not just on current prices, but also on consumers' price expectations and past market share. To reflect this write A's demand as $D_A^t \left(p_A^t, p_B^t, Ep_A^{t+1}, Ep_B^{t+1}, \tilde{x}^{t-1}\right)$. Given an initial history, firm A has the following optimization problem:

$$\max_{\left\{p_A^\tau\right\}_{\tau=t}^\infty} \sum_{\tau=t}^{\infty} \delta_f^{\tau-t} p_A^\tau D_A^\tau \left(p_A^\tau, p_B^\tau, Ep_A^{\tau+1}, Ep_B^{\tau+1}, \tilde{x}^{\tau-1}\right) \tag{2}$$

subject to i). B's strategy, ii). the process by which expectations $\left\{Ep_A^{\tau+1}, Ep_B^{\tau+1}\right\}_{t=\tau}^\infty$ are formed, and iii). subject to equation (1) which specifies how $\tilde{x}^\tau$ evolves over time.

It is natural to simplify the optimization problem (2) by restricting attention to linear Markovian pricing strategies - meaning that each firm's price is a linear function of its stock of old consumers, but does not otherwise depend upon the history of the game.[2] In particular recall that $\tilde{x}^{t-1}$ is a threshold such that in period $t-1$, all young

[2]This is natural because in the finite-horizon version of our model, there is a unique equilibrium and

consumers located to the left of $\tilde{x}^{t-1}$ bought from firm A, and all other young consumers bought from firm B. We therefore suppose that firms use the following pricing strategies

$$p_A^t\left(\tilde{x}^{t-1}\right) = J + K\left(\tilde{x}^{t-1} - 1/2\right) \tag{3}$$

$$p_B^t\left(\tilde{x}^{t-1}\right) = J - K\left(\tilde{x}^{t-1} - 1/2\right) \tag{4}$$

The interpretation is that when $\tilde{x}^{t-1} = 1/2$, each firm sold to half of the young consumers born in period $t-1$, so come period t the two firms are symmetric and both charge the same price J. We will prove later that $K > 0$ so if instead $\tilde{x}^{t-1} > 1/2$, firm A sold to more than half of young consumers in period $t-1$, and therefore charges more than B does in period t.

Forward-looking young consumers can use equations (3) and (4) and predict that $Ep_B^{t+1} - Ep_A^{t+1} = -2K\left(\tilde{x}^t - 1/2\right)$. Substituting this into equation (1) the marginal young consumer in period t has location

$$\tilde{x}_t = \frac{1}{2} + \frac{p_B^t - p_A^t}{2\left(1 + K\delta_c s\right)} \tag{5}$$

It follows that if firms use the pricing strategies (3) and (4), their demands in period t will be linear in $\tilde{x}^{t-1}$ and their profits quadratic in $\tilde{x}^{t-1}$. This suggests that in period t the net present value to a firm of its current and future profits, will also be quadratic in $\tilde{x}^{t-1}$. Therefore we look for value functions $V_A^t\left(\tilde{x}^{t-1}\right)$ and $V_B^t\left(\tilde{x}^{t-1}\right)$ of the form

$$V_A^t\left(\tilde{x}^{t-1}\right) = M + N\left(\tilde{x}^{t-1} - 1/2\right) + R\left(\tilde{x}^{t-1} - 1/2\right)^2 \tag{6}$$

$$V_B^t\left(\tilde{x}^{t-1}\right) = M - N\left(\tilde{x}^{t-1} - 1/2\right) + R\left(\tilde{x}^{t-1} - 1/2\right)^2 \tag{7}$$

A linear Markovian strategy is characterized by the parameters J, K, M, N and R.

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it involves firms using symmetric linear pricing strategies. This can be proved using a simple inductive argument. In particular let $V_i^j\left(\tilde{x}^{j-1}\right)$ be firm i's value function in period j, and let T denote the final period. It is straightforward to show that if $V_A^{t+1}(\cdot)$ and $V_B^{t+1}(\cdot)$ are symmetric and quadratic, then (1). $V_A^t(\cdot)$ and $V_B^t(\cdot)$ are also symmetric and quadratic, and (2). $p_A^t(\cdot)$ and $p_B^t(\cdot)$ are symmetric and linear. Moreover it is clear that $p_A^T(\cdot)$ and $p_B^T(\cdot)$ are symmetric and linear, whilst $V_A^T(\cdot)$ and $V_B^T(\cdot)$ are symmetric and quadratic. Therefore by induction the firms must use symmetric linear pricing policies in each period.

A linear Markov perfect equilibrium (MPE) exists whenever the two firms' strategies are subgame perfect (see Fudenberg and Tirole 1991). Proposition 2 below shows that there is a unique such equilibrium. The proof (in the appendix) proceeds by showing that dynamic optimality imposes just enough conditions to uniquely pin down the five behavioral parameters. A sketch of the proof is as follows. Since product A is bought by $\tilde{x}^t + \tilde{x}^{t-1}\dot{x}^t + (1-\tilde{x}^{t-1})\,\ddot{x}^t$ consumers, its demand can be written as

$$D_A^t\left(p_A^t, p_B^t, \tilde{x}^{t-1}\right) = \frac{1}{2} + \frac{p_B^t - p_A^t}{2\left(1+K\delta_c s\right)} + \frac{1+p_B^t - p_A^t}{2} + s\left(\tilde{x}^{t-1} - 1/2\right) \tag{8}$$

Firstly A's strategy must be subgame perfect. The principle of optimality says that A will

$$\max_{p_A^t} p_A^t D_A^t\left(p_A^t, p_B^t, \tilde{x}^{t-1}\right) + \delta_f V_A^{t+1}\left(\tilde{x}^t\right) \tag{9}$$

subject to i). B playing the linear strategy in equation (4) and ii). subject to $\tilde{x}^t$ satisfying equation (5). Subgame perfection requires that the solution to this maximization problem, is precisely the linear strategy given by equation (3). In the appendix we show that this imposes two restrictions on the behavioral parameters. Secondly A's value function must be consistent with the hypothesized pricing strategies. In particular take the righthand side of (9), then use (6) to substitute out for $V_A^{t+1}\left(\tilde{x}^t\right)$, (5) to substitute out for $\tilde{x}_t$, and finally (3) and (4) to substitute out for p_A^t and p_B^t. This leaves an expression for A's period-t value, which is only a function of $\tilde{x}^{t-1}$. Consistency then requires that this should equal the value function given in equation (6); in the appendix we show that imposing this gives three more restrictions on the behavioral parameters. Combining these various restrictions, we can prove:

Proposition 2. *For any $s \in (0, 7/10]$ there is a unique MPE in linear strategies. The behavioral parameter J satisfies*

$$J = \frac{2 + 2K\delta_c s + \delta_f K}{2 + K\delta_c s + \delta_f s} \tag{10}$$

whilst K lies in $[s/3, 3s/8)$ and satisfies the following equation

$$\delta_f K^3\left(2+K\delta_c s\right) - 3K\left(2+K\delta_c s\right)\left(1+K\delta_c s\right)^2 + 2s\left(1+K\delta_c s\right)^3 = 0 \tag{11}$$

Proposition 2 imposes the restriction $s \leq 7/10$. This is because when setting up demand in equation (8), we assumed that generically each firm has both some consumers switching to it and others switching from it. However switching in both directions can only happen if the difference in prices $|p_B^t - p_A^t| = K\,|2\tilde{x}^{t-1} - 1|$ is not too large. Since K is related to s this means that s cannot be too large either. We show in the appendix that it is sufficient to restrict attention to switching costs that are less than $7/10$.[3] To put this into perspective, we show in the next section that although a wide range of prices may be charged by the two firms, in equilibrium no firm will ever charge more than about 1.2. Consequently the equilibrium in Proposition 2 is (at a bare minimum) valid for any switching cost between about 0 and 60% of market prices. Most real-world estimates of switching costs, some of which were summarized in the introduction, comfortably lie in this interval.

Proposition 3. *The market converges to a steady state in which firms split demand equally and charge a price J. In period t the location of the marginal young consumer satisfies*

$$\tilde{x}^t - 1/2 = -\frac{K}{1 + K\delta_c s}\left(\tilde{x}^{t-1} - 1/2\right) \tag{12}$$

Proof. To derive equation (12) simply substitute (3) and (4) into (5). Proposition 2 says that $K \in [s/3, 3s/8)$ and therefore $K < 1 + K\delta_c s$. This implies that $\lim_{t\to\infty} \tilde{x}^t = 1/2$ and, using equations 3 and 4, also implies that $\lim_{t\to\infty} p_A^t = \lim_{t\to\infty} p_B^t = J$. □

If the firms start off with unequal market shares, over time they will converge to a steady state in which they both sell to exactly half of the consumers. During this convergence process, the position of the marginal young consumer $\tilde{x}^t$ oscillates

[3] For example when $\delta_c = \delta_f = 0$ equation (11) has a unique solution $K = s/3$, and switching occurs in both directions if and only if $s < 3/4$. More generally the relevant solution to equation (11) lies in $(s/3, 3s/8)$, and therefore the critical switching cost will be closer to $7/10$. Once this critical threshold is crossed, for some $\left(p_A^t, p_B^t, \tilde{x}^{t-1}\right)$ switching occurs in both directions, whilst for others it only occurs in one direction. Consequently the two firms' demand elasticities are discontinuous in $\tilde{x}^{t-1}$, and this significantly complicates the analysis.

around 1/2. Consequently the prices set by the two firms also oscillate around J. This oscillatory behavior arises because in each period, the firm which previously sold to more than half of young consumers, exploits this fact by charging a higher price than its rival. As a result it then sells to fewer than half of the current young consumers.

4 The effect of switching costs on prices

4.1 Steady state

Remark 4. *The steady state price is decreasing in s at $s = 0$.*

Proof. According to Proposition 2, $K = 0$ when $s = 0$. Totally differentiate equation (11) with respect to s; after substituting in $s = K = 0$ this simplifies to $\partial K/\partial s|_{s=0} = 1/3$. Totally differentiate equation (10) with respect to s; after again substituting in $s = K = 0$ this simplifies to $\partial J/\partial s|_{s=0} = (\delta_f/2)(-1 + \partial K/\partial s|_{s=0}) = -\delta_f/3 < 0$. $\square$

Several recent papers have shown that starting from $s = 0$, steady state price is decreasing in the switching cost. The same is also true in our model. Other papers then use numerical simulations to show that the steady state price is lower for somewhat larger switching costs as well. However as Figure 1 makes clear, away from $s = 0$ the effect of switching costs on steady state price is ambiguous, and depends upon the specific parameter values that we choose. To interpret Figure 1 note that the discount factors δ_c and δ_f both affect the steady state price and can both take values anywhere on $(0, 1)$. Therefore for each switching cost, there is a whole set of possible steady state prices. Figure 1 plots the infimum and supremum of that set for every possible switching cost in $[0, 7/10]$. When $s = 0$ the steady state price is equal to 1. However even when the switching cost is close to zero, steady state price can be higher or lower than this. Moreover as the switching cost grows, so does the range of possible steady state prices. For example depending upon the specific values attached to δ_c and δ_f, when $s = 7/10$ steady state price can be as much as 8% higher or 17% lower than it is when $s = 0$. Therefore comparative statics around $s = 0$ are rather special.

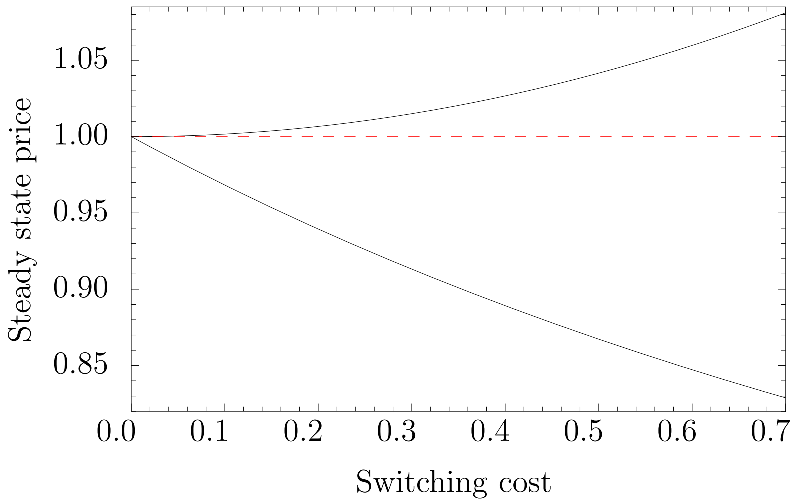


Figure 1: A plot showing the range of possible steady state prices.

It is convenient to split up the impact of switching costs on price, into the following four effects, all of which are mentioned in various parts of the literature. These are the *harvesting*, *poaching*, *investment*, and *consumer price* effects.[4] The first two reflect pricing incentives on old customers. According to the *harvesting effect*, firms should charge a high price and exploit their old customers' reluctance to switch away. However according to the *poaching effect*, firms should charge a low price and poach some of their rival's customers (using the low price to overcome their reluctance to switch). It turns out that since each firm has exactly half of the old customers, *in steady state* the harvesting and poaching effects cancel out.[5] Two assumptions are crucial in this respect. First we assumed that s is small enough to guarantee that switching actually occurs. By contrast in Beggs and Klemperer (1992) and To (1996), the switching cost is so large that nobody ever switches. Poaching is therefore impossible, and both firms

[4]Different papers use slightly different terminology. Within the context of two-period models, Fudenberg and Tirole (2000) discuss poaching, whilst Klemperer (1987) describes the other three effects.

[5]This intuition is similar to that given by Arie and Grieco (2012). They argue that a switching cost is like a subsidy to a firm's existing customers but a tax to everybody else. Since duopolists have exactly half the market in steady state, the tax and subsidy effects cancel.

just harvest. Second we assumed that preferences change independently over time. In Section 6 we show that if preferences are positively correlated, harvesting can dominate poaching.

Since the harvesting and poaching effects cancel, the steady state price is driven only by pricing incentives on young consumers. It is simple to show that $V_A^{t+1}(\tilde{x}^t)$ and $V_B^{t+1}(\tilde{x}^t)$ are respectively increasing and decreasing in $\tilde{x}^t$ i.e. market share in one period is valuable in the next. Therefore according to the *investment effect* firms should charge lower prices, as they try to win market share and thereby improve their future profitability. On the other hand if a firm cuts its price, consumers understand that it is only temporary and will be followed by a price increase in the next period (c.f. equations 3 and 4). Young consumers therefore have relatively inelastic demands, and according to the *consumer effect* firms should respond by charging higher prices.

Lemma 5. *The steady state price strictly decreases in δ_f and strictly increases in δ_c.*

Discount factors affect the steady state price as one would expect. A higher δ_f means that firms care more about future profits, and therefore both cut their prices in an attempt to increase their market shares. A higher δ_c means that consumers put less weight on temporary price cuts, such that firms face less elastic demand curves and therefore charge a higher price. The real question - which the next proposition addresses - is which of the investment and consumer effects is most likely to dominate.

Proposition 6. *For any δ_c and $s \in (0, 7/10]$, there exists a $\widetilde{\delta_f} \in (\delta_c s/2, 3\delta_c s/5)$ such that the steady state price is less than 1 if and only if $\delta_f > \widetilde{\delta_f}$.*

Proposition 6 confirms analytically that price can be either higher or lower depending upon parameters. However it also suggests that switching costs are usually pro-competitive. This is because even if consumers are twice as patient as firms - that is even if δ_c is as high as $2\delta_f$ - it will still be the case that $\delta_f \geq \widetilde{\delta_f}$. Indeed introspection suggests that in most cases consumers will not be more patient than firms. Therefore the investment effect is likely to outweigh the consumer effect, such that steady state price is lower with switching costs than without. The interpretation is that firms cut their price as a defensive measure, to prevent their rival from stealing valuable market share.

To understand why the investment effect usually dominates, recall from Section 3.1 that old consumers definitely buy product A if $x^{t+1} \leq (1-s)/2$ and definitely buy product B if $x^{t+1} \geq (1+s)/2$. We also know that if old consumers are in the "lock-in region" $x^{t+1} \in [(1-s)/2, (1+s)/2]$, they stay with their initial supplier. Consider the *investment effect*. If firm i captures a few extra young consumers, they are valuable in the next period if i). they buy product i when old and ii). if, but for buying i when young, they would buy $j \neq i$ when old. Equivalently these extra young consumers are valuable if and only if they lie in the lock-in region in the next period. The probability of actually being in the lock-in region is s. Moreover the value created for the firm by these additional young consumers is J since this is what they will contribute to future revenue. Therefore if a firm acquires a few extra young consumers, the direct effect on future profits is $J\delta_f s$.[6]

Now consider the *consumer effect*. According to equation (5) the marginal young consumer has location

$$\tilde{x}_t = \frac{1}{2} + \frac{p_B^t - p_A^t}{2 + 2K\delta_c s}$$

where the term $2K\delta_c s$ in the denominator measures the consumer effect. Intuitively if firm i slightly reduces its price it will increase its market share. Using equations (3) and (4) young consumers can infer that in the following period, i's price will be higher and $j\,(\neq i)$'s price will be lower. In particular $p_i^{t+1} - p_j^{t+1}$ increases in proportion to $2K$. However a young consumer who buys product i only incurs an expected future loss of $2K\delta_c s$. Intuitively only when a consumer finds herself in the lock-in region, does her initial decision to choose i over j actually *cause* her to pay the extra $2K$.[7] Since the

[6] In fact $\delta_f \times (dV_A/dx) = -K\delta_f + J\delta_f s$ because the rival firm will become more aggressive in the next period and reduce its price in proportion to K. However this additional (indirect, negative) effect on firm value does not qualitatively affect the intuition.

[7] For example suppose hypothetically that the young consumer knows that when she becomes old,

probability of ending up in the lock-in region is only s, the consumer effect is only on the order of $2K\delta_c s$.

Comparing the investment and consumer effects is just like comparing a level with a difference. Notice that everybody cares about what happens when a consumer becomes 'locked in'. However whilst the benefit of lock-in to a firm is a level (namely the price paid to it by the consumer), the cost to the consumer is only a difference (namely the *extra* amount she must pay). Intuitively if the switching cost is not too large, the level effect must swamp the difference effect. This is because for small s the link between current market share and future prices is small i.e. the additional amount that a locked-in consumer pays is also small. Put slightly differently a firm's incentive to lock in consumers, is very likely to outweigh a consumer's incentive to avoid being locked in. Consequently switching costs are pro-competitive in steady state unless consumers are for some reason much more patient than firms.

Two other points are worth briefly making. First recall that around $s = 0$ switching costs are pro-competitive *irrespective* of how large δ_f is relative to δ_c. The reason is that whilst the investment effect $J\delta_f s$ is first-order in s, the consumer effect $2K\delta_c s$ is only second-order because K is of the same order as s. Hence around $s = 0$ the investment effect must dominate. Secondly note that whilst the investment effect is roughly linear in s, the consumer effect is more-than-linear in s. This suggests that the investment effect will dominate initially, but then later the consumer effect will become more powerful i.e. steady state price should follow a U-shape. It also explains why in Proposition 6 the critical discount factor $\tilde{\delta}_f$ is generally higher for larger switching costs - although both investment and consumer effects tend to grow in s, the latter grows

her location will satisfy $x^{t+1} \notin [(1-s)/2, (1+s)/2]$. She therefore knows that her initial purchase decision will have no effect on her subsequent one, and moreover that she is equally likely to buy either of the two products. Hence her future payoff from locking in to i or j is the same. (Of course an infinitesimally small increase in the relative future price of good i is bad news if the consumer turns out to really like product i in the following period, and is good news if she ends up really liking product j, but this is immaterial ex ante.)

faster than the former.

Finally our intuition can also shed light on the broader question of why switching costs are often pro-competitive when they are relatively small, but (as shown by Beggs and Klemperer 1992 and To 1996) not when they are very large.[8] As discussed earlier, one reason is that when switching costs are very large, firms are unable to poach from their rival. Consequently they focus more on harvesting, which is a force for higher prices. A second related reason is that when switching costs are very large, the link between a firm's market share and its price will become stronger. Equivalently, the consumer effect will be stronger than in our model, and is likely to eventually dominate the investment effect. This again explains why very large switching costs lead to a higher steady state price. It is worth noting again, however, that all our results are valid (at a minimum) for any switching cost between about 0 and 60% of the steady state price. Therefore a very large switching cost is required to overturn our results.

4.2 Outside of steady state

In the short-run firms may start off with unequal market shares. Proposition 3 guarantees that the market will converge to a steady state where firms split the market equally. Nevertheless it is important to understand how switching costs affect competition before this steady state is reached. In general firms face different demand schedules and therefore do not charge the same price. The firm with the larger market share focuses more on harvesting and less on poaching, and consequently charges more than its

[8]Empirical evidence supports this distinction between small and (very) large switching costs. Dubé *et al* (2009) look at psychological costs of switching between brands of orange juice and margarine, and estimate that they reduce the market prices of these products by 3-6%. However Viard (2007) finds that number portability (i.e. a reduction in switching costs) led to a 14% reduction in prices charged to firms that had toll-free phone numbers. The difference may be that the market for toll-free calls has much larger switching costs and is therefore closer to the Beggs and Klemperer (1992) model; switching costs are likely to be substantial because a change in telephone number must be advertised to all potential customers. However in many other markets switching costs are significant yet much smaller (see the estimates provided in the introduction), so our results are more applicable.

smaller rival. In fact a simple calculation reveals that depending upon parameters, one firm might charge as much as 33% more than its rival.

One natural measure of market competition is the average (transaction) price. In period t the average price paid by consumers is

$$\frac{p_A^t D_A^t + p_B^t \left(2 - D_A^t\right)}{2} = J + \left(\tilde{x}^{t-1} - 1/2\right)^2 K \underbrace{\left(s - K \frac{2 + K\delta_c s}{1 + K\delta_c s}\right)}_{>0} \tag{13}$$

which (weakly) exceeds the average price J which consumers pay in the long-run steady state outcome. This is because even though $p_A^t + p_B^t = 2J$ in every time period, in the short-run one firm is able to both charge a higher price *and* sell to more than half of the market. However since $\left(\tilde{x}^{t-1} - 1/2\right)^2$ decreases monotonically over time (recall Proposition 3), it is clear from equation (13) that the average price decreases over time.

We showed earlier in Proposition 6 that the long-run price is below 1 if and only if δ_f exceeds a threshold $\tilde{\delta}_f$. The corresponding result for the short-run is:

Proposition 7. *For any δ_c, initial market shares, and $s \in (0, 7/10]$, there exists a $\hat{\delta}_f \in \left(\tilde{\delta}_f, 1\right)$ such that the average price is below 1 if and only if $\delta_f > \hat{\delta}_f$.*

Thus the short-run impact of switching costs is also ambiguous, and again depends upon a comparison of firm and consumer discount factors.[9] Compared to the case of no switching cost, average price is higher in all periods if $\delta_f < \tilde{\delta}_f$ but lower in all periods if $\delta_f > \hat{\delta}_f$. More interestingly when $\delta_f \in \left(\tilde{\delta}_f, \hat{\delta}_f\right)$ the average price starts out above 1, but then falls as the market matures and at some point drops below 1. Consequently under these circumstances switching costs are anti-competitive in the short-run and yet pro-competitive in the long-run.

[9] There is however a subtle difference between the short- and long-run. In the long-run switching costs are pro-competitive so long as δ_c/δ_f is large enough, even if in absolute terms δ_c and δ_f are both small. This is not necessarily true in the short-run - even if δ_c/δ_f is very large, switching costs can be anti-competitive if δ_f is sufficiently close to zero. The difference arises because in the long-run only the relative strengths of the investment and consumer effects matter. However in the short-run the larger firm's emphasis on harvesting provides an additional upward boost to prices, and hence the investment effect needs to be big in absolute terms (as well as big relative to the consumer effect).

In light of Proposition 7 it is natural to ask whether a switching cost could cause *both firms* to charge a lower price. This will clearly depend upon how mature the market is, so for simplicity we look at the most extreme case possible. The highest price one could ever observe in the market is $J + K/2$, which arises when all old consumers previously bought from the same firm. This could happen if one firm was a monopolist in the previous period, and V was sufficiently high to induce it to sell to all young consumers.[10]

Remark 8. *Start with $s = 0$ and introduce a small switching cost. The highest observable price $J + K/2$ decreases if and only if $\delta_f > 1/2$.*

Proof. This follows from Remark 4, where it was proved that $\partial K/\partial s|_{s=0} = 1/3$ and $\partial J/\partial s|_{s=0} = -\delta_f/3$. □

With small switching costs the price charged by an incumbent monopolist who faces a brand new entrant may be lower compared to the case of no switching cost. Out of everybody, a (recent) monopolist has the strongest incentive to harvest its customer base. Nevertheless if it cares enough about the future, it will follow the entrant and cut its price as a defensive measure to avoid losing too much market share. One would expect that as the switching cost grows, the incumbent's power over its old customers grows and therefore the harvesting effect should start to dominate. This is shown by Figure 2, which plots the range of prices which an incumbent monopolist might charge. As usual there is a whole set of possible prices, depending upon the values assigned to δ_c and δ_f. Figure 2 plots the infimum and supremum of that set. Clearly as s increases, the distribution of prices tends to shift up. However even for very high switching costs, there are combinations of δ_c and δ_f such that the incumbent's price is below the frictionless benchmark 1. Therefore even very large switching costs may cause both firms in the market to charge a lower price.

To summarize we have shown that the average price paid by consumers is higher in the short-run when the market is outside steady state. We then derived a condition

[10] Recall that $V - 1$ is the valuation of the consumer who is located farthest away from that firm.

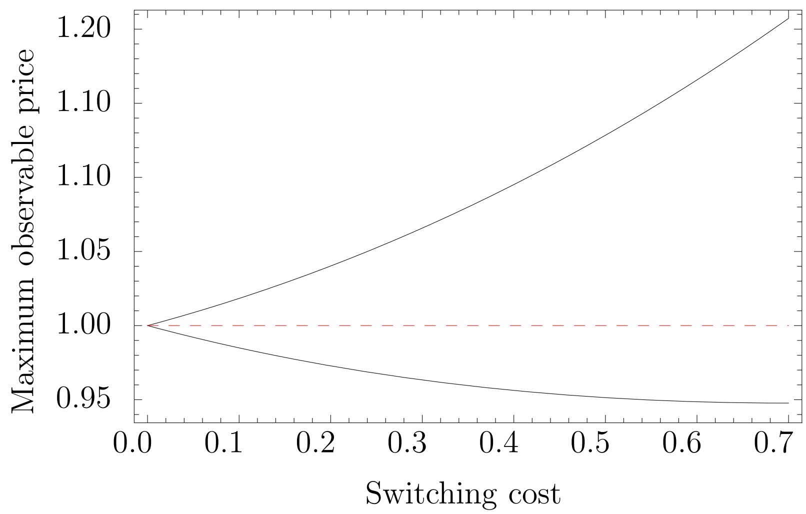


Figure 2: A plot showing the range of prices charged by an incumbent monopolist.

which determines whether this average price is higher or lower compared to a market where consumers do not incur switching costs. Finally we demonstrated that under certain conditions, even a firm with a very large customer base may respond to switching costs by lowering its price.

4.3 Profits

Since there is a close connection between prices and profits, the previous two sections suggest that firms are likely to be made worse off by switching costs. Firstly *in steady state* each firm charges a price J and sells to one unit of consumers in every period. Therefore switching costs reduce long-run profits if and only if they reduce long-run price. Proposition 6 then implies that switching costs are bad for firms except when consumers are especially patient. Secondly *outside of steady state*, total industry profit in any single period is equal to $p_A^t D_A^t + p_B^t D_B^t$. The latter is proportional to the average price charged in period t, which we defined earlier in equation (13). Therefore Proposition 7 says that unless firms are sufficiently impatient, industry profit will be lower in every period. Of course if firms start off with unequal market shares, the larger firm may still benefit from switching costs. However analogous to Remark 8, we can show that provided $\delta_f > 1/2$, a small switching cost reduces every firm's (discounted sum of) profits. In particular even a recent monopolist can be harmed by the introduction of a small switching cost.

5 The effect of switching costs on consumer welfare

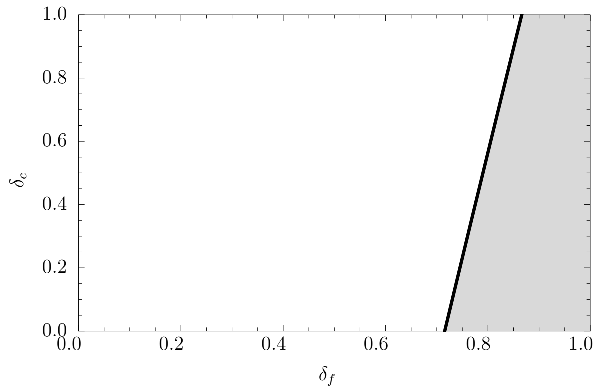
It is natural to ask whether switching costs could reduce prices so much, that they actually benefit consumers. To answer this question, we will focus on steady state consumer welfare.

In steady state both firms charge the same price, so equation (5) shows that *young consumers* buy from A if $x^t \leq 1/2$ and buy from B if $x^t > 1/2$. However these are exactly the same choices that they would make in a market without switching costs. Therefore when consumers are young, they benefit from switching costs if and only if the equilibrium price is lower. This is obviously not true for *old consumers*, because some of them incur the switching cost (a direct loss), whilst others of them keep buying an inferior product to avoid paying the switching cost (an indirect loss). In principle old consumers could still benefit from switching costs, if the steady state price falls enough to compensate them for these other losses. However the following lemma shows that this never happens:

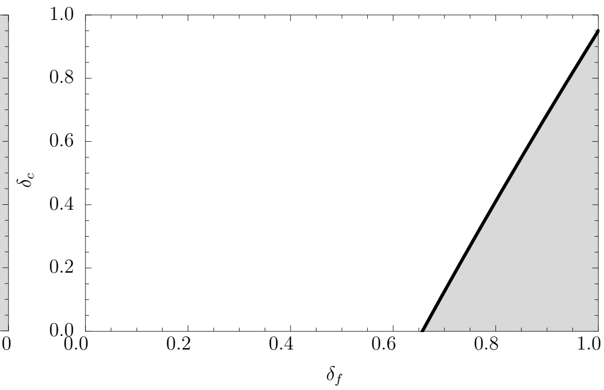
Lemma 9. *Switching costs always make old consumers worse off.*

Therefore in most interesting cases switching costs have three effects. 1). They benefit all consumers through a lower market price, 2). they harm old consumers who either have to incur these costs or avoid them by sticking with an inferior product, and 3). they transfer utility from the old to the young. The net effect will depend largely on how we weight the payoffs of young and old consumers. We now discuss two natural alternatives.

One natural way to measure consumer surplus, is to simply add the payoffs of young and old consumers i.e. look at consumer welfare at a specific point in time. In this case



(a) $s = 1/4$



(b) $s = 1/2$

Figure 3: Total (unweighted) consumer surplus is higher in the shaded region

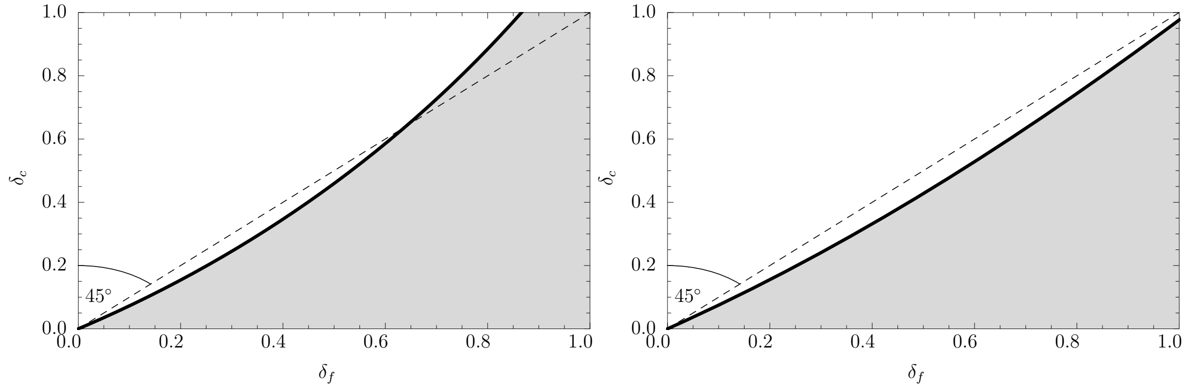
only the first two effects identified above are relevant. For example starting from $s = 0$, a small switching cost reduces the market price by $\delta_f/3$ but forces half of old consumers to incur the cost when switching suppliers. Consequently consumer surplus changes by $2(\delta_f/3) - 1/2$, which is positive provided $\delta_f > 3/4$. As another example when moving from $s = 0$ to $s = 1/4$, consumers are better off in aggregate provided that (δ_c, δ_f) lie in the shaded area in Figure 3(a). Figure 3(b) performs the same exercise when moving from $s = 0$ to $s = 1/2$. The boundary of the shaded area pivots as s increases, so it is a priori unclear whether consumers are better or worse off when they face a larger switching cost.[11]

Another natural measure of consumer surplus is the ex ante lifetime expected utility of a young consumer who is about to enter the market (i.e. weights of 1 and δ_c on young and old consumption respectively). With this alternative measure, all three effects identified above are relevant.

Proposition 10. *For any δ_f and s, there exists a threshold $\widetilde{\delta_c} > 0$ such that switching costs raise discounted lifetime consumer surplus if and only if $\delta_c < \widetilde{\delta_c}$.*

Proposition 10 is intuitive. When δ_c is very low, a consumer's lifetime utility is

[11] There is no clear ranking because although the losses associated with switching costs are higher when s is larger, so are the gains from paying a lower price.



(a) $s = 1/4$ (b) $s = 1/2$

Figure 4: Lifetime discounted consumer surplus is higher in the shaded region

mainly affected by how well off she is when young. Moreover Proposition 6 says that for sufficiently small δ_c the steady state price must be lower with switching costs, and therefore that consumers are better off when young. Two things change as δ_c increases. Firstly the steady state price increases and so consumers become worse off in both periods of their life. Secondly consumers care more about utility in their second period. This implies that the intertemporal benefit of switching costs (namely transferring utility from the future to the present, when consumers value it most) becomes less important.

Example 11. *Starting from $s = 0$, a small switching cost increases discounted lifetime consumer surplus if and only if $\delta_c < \widetilde{\delta_c} = 2\delta_f / [3 - 2\delta_f]$.*

Note that the condition $\delta_c < 2\delta_f / [3 - 2\delta_f]$ is definitely satisfied if either $\delta_f > 3/4$, or if $\delta_f \in [1/2, 3/4]$ and firms are more patient than consumers. Therefore as expected, a small switching cost is more likely to increase consumer surplus under this alternative measure. To further illustrate this point, Figures 4(a) and 4(b) plot the critical threshold $\tilde{\delta}_c$ for the cases where $s = 1/4$ and $s = 1/2$. In both diagrams the respective switching cost benefits consumers whenever (δ_c, δ_f) lies in the shaded area. It turns out (although this is a little difficult to see from the diagrams) that when δ_f is low the critical discount factors satisfy $\left.\widetilde{\delta_c}\right|_{s=1/2} > \left.\widetilde{\delta_c}\right|_{s=1/4}$, whereas when δ_f is higher they satisfy $\left.\widetilde{\delta_c}\right|_{s=1/2} < \left.\widetilde{\delta_c}\right|_{s=1/4}$. In either case provided consumers discount the future a little more than firms, they are definitely better off.

In summary switching costs benefit the young but harm the old. There exists a wide range of parameters such that in any given period, the benefits to the young outweigh the costs to the old. This range of parameters is significantly enlarged if we instead measure consumer welfare by looking at lifetime discounted surplus instead. Overall the model suggests that switching costs will typically harm firms but benefit consumers.

6 Discussion: correlated preferences

We now relax the assumption that consumer preferences evolve independently over time. There is lots of evidence that a consumer's valuation for any given product is often serially correlated across time. Moreover Dubé *et al* (2010) have shown that after controlling for switching costs, this persistence in preferences offers an additional explanation for why consumers exhibit inertia in their brand choices.

In order to be as general as possible, we model correlation in the following way. We continue to assume that in the *population* both young and old consumers are uniformly distributed on the Hotelling line. However at the *individual level* when a young consumer with location x^t becomes old, she is assigned a new location x^{t+1} which is drawn using a conditional density $f\left(x^{t+1}\middle| x^t\right)$. This conditional density is continuous, atomless, strictly positive and also radially symmetric i.e. $f\left(y\middle| z\right) = f\left(1-y\middle| 1-z\right)$ for all $y, z \in [0,1]$. The latter is a natural way to model symmetry in a dynamic setting, and it ensures that two young consumers located at z and $1-z$ have future preferences which are mirror images of each other. The special case $f\left(x^{t+1}\middle| x^t\right) = 1$ gives our earlier set-up where x^t and x^{t+1} are independent.

Note that a firm's demand is no longer a linear function of its past market share, and consequently there does not exist an equilibrium in linear strategies. The difficulty of formally proving existence of a Markovian equilibrium in this more general setting is well-known (see Dutta and Sundaram (1998) for a comprehensive discussion). For this reason we take the following approach. When $s = 0$ there are no payoff-relevant state variables, so a MPE does trivially exist, and it involves the two firms playing the (static) Hotelling equilibrium in each period. Assuming that a (continuous) MPE also exists in the neighborhood of $s = 0$, we can derive first order conditions and use them to study comparative statics in this neighborhood.

Proposition 12. *Starting from $s = 0$, a small switching cost reduces steady state price if and only if*

$$\frac{1}{2}\frac{\partial \Pr\left(X \geq 1/2|Y = 1/2\right)}{\partial Y} + \frac{\delta_c}{2}\frac{\partial \Pr\left(X \geq 1/2|Y = 1/2\right)}{\partial Y} - \delta_f \frac{f\left(1/2|1/2\right)}{3} < 0 \qquad (14)$$

To interpret Proposition 12, when preferences are independent $\Pr\left(X \geq 1/2|Y\right) = 1/2$, and so the inequality (14) definitely holds. When instead preferences are positively correlated, we expect that a consumer who is more attached to product B in one period, is also more likely to prefer B over A in another period. Equivalently we expect that $\partial \Pr\left(X \geq 1/2|Y\right)/\partial Y \geq 0$, in which case (14) might not be satisfied. Before commenting further on this, we provide some brief intuition behind expression (14).

The first term in (14) is a combination of the harvesting and poaching effects, whilst the second term is the consumer effect, and the third term is the investment effect. The *harvesting* and *poaching effects* are therefore now (weakly) positive, and the intuition is as follows. As explained earlier, firms can exploit their own old consumers with a high price, or poach some of their rival's customers with a low price. With independent preferences half of old consumers are locked in to the 'wrong' firm, and the incentives to exploit and poach cancel. When instead preferences are positively correlated, fewer marginal consumers are locked in to the 'wrong' firm. This makes it more profitable to harvest and less profitable to poach, so the former effect now dominates.

The second term of (14) is the *consumer effect* and it too is now positive. As reported earlier, the standard explanation is that young consumers are less responsive to price cuts, because they expect a price rise to follow in the next period. Starting from $s = 0$, we showed that this effect is only second-order when consumer tastes evolve independently over time; for similar reasons it is also second-order even when tastes are correlated across time. Instead the positive consumer effect in (14) is caused by a quite different mechanism which, to our knowledge, has not previously been mentioned in the literature. It arises due to expected changes in future preferences. For example suppose that firm A reduces p_A^t and tries to attract some young consumers located slightly to the right of $x^t = 1/2$. Since preferences are positively correlated, these young consumers expect to prefer product B in the next period. This makes them more reluctant to buy A now, which causes demand to become less elastic.

The final term of (14) is the *investment effect*. As in the base model, firms compete for the marginal young consumer who is located at $x^t = 1/2$. As argued previously, this marginal consumer is valuable in the future if she turns out to be located in the lock-in region. Starting from $s = 0$, a small increase in the switching cost changes her probability of being in the lock-in region by $f(1/2|1/2)$. Since preferences are correlated, it is also likely that $f(1/2|1/2) > 1$ i.e. the investment effect is stronger than in our earlier model.

To summarize when consumer preferences are correlated over time, the first two terms of inequality (14) are positive, and therefore even very small switching costs are not necessarily pro-competitive. Note however that only the behavior of $f(x^{t+1}|\, x^t)$ around the point $x^t = 1/2$, is relevant for whether (14) holds. Correlation on the other hand is a global concept, which summarizes the behavior of $f(x^{t+1}|\, x^t)$ for all x^t. This immediately implies that the amount of correlation has *no* direct bearing on whether switching costs are pro- or anti-competitive. What matters instead is whether $\partial \Pr(X \geq 1/2 | Y = 1/2) / \partial Y$ is large or small. As an example, suppose that if a young consumer is almost indifferent about which product to buy, she is also equally likely to prefer A or B when she becomes old. Then $\partial \Pr(X \geq 1/2 | Y = 1/2) / \partial Y$ is zero and switching costs are definitely pro-competitive, even if in the wider population there is a strong positive correlation between x^t and x^{t+1}. Therefore although our earlier assumption of independence is not innocuous, it can be substantially relaxed without changing the conclusion that small switching costs are likely to be pro-competitive in the long-run. Furthermore, whenever consumer tastes are correlated, fewer old consumers need to actually incur the switching cost because they are already locked into the 'correct' firm. This means that conditional on switching costs being pro-competitive, there is again a good chance that they improve consumer welfare.

7 Conclusion

We have presented a tractable model of dynamic competition, and solved it for a very wide and empirically relevant set of switching costs. In general the long-run impact of switching costs is ambiguous and depends upon how patient are firms and their consumers. We provided a condition which determines whether in the long-run switching costs are pro- or anti-competitive. Using this condition, we found a presumption that in steady state switching costs lead to lower prices. This is because a firm's incentive to lock-in consumers, strongly outweighs any single consumer's incentive to avoid being locked in. We then used the model to address some other issues which have been largely neglected by the previous literature. We showed that short-run prices can be extremely heterogeneous, and that focusing on steady state may lead to biased conclusions about the pro-competitiveness of switching costs. We also examined the wider effects of switching costs, on for example consumer welfare. Switching costs often act as a way of transferring surplus from old to young consumers. When consumers are relatively impatient, this trade-off is favorable and consumer welfare is increased. Finally we investigated how our conclusions might change when consumer tastes are correlated over time.

Throughout the paper we have assumed that the switching cost is exogenously given. However in practice manufacturers can often choose to make their products more or less compatible with those of their rivals. Retailers can also make it harder for their customers to cancel subscriptions or move to another provider. Therefore an interesting way to extend the current model, would be to allow each firm to influence how easily its customers can switch to its rival. Our existing results already show that profits are usually maximized when consumers are able to switch costlessly. However we conjecture that firms may end up playing a Prisoner's dilemma. In particular it seems plausible that each firm might benefit from making it slightly more difficult for its existing customers to switch. However once both firms do this, price competition is intensified and they both earn less profit. A natural implication is that firms might try to 'collude' and establish industry standards that make it easier for consumers to change providers. We hope to think more about this in future work.

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A Main Proofs

Proof of Lemma 1. The expected lifetime payoff from buying product A in period t is:

$$V - x^t - p_A^t + \delta_c \left[\int_0^{\dot{x}^{t+1}} \left(V - y - Ep_A^{t+1} \right) dy + \int_{\dot{x}^{t+1}}^1 \left(V - (1-y) - Ep_B^{t+1} - s \right) dy \right] \tag{A.1}$$

where $\dot{x}^{t+1} = \left(1 + Ep_B^{t+1} - Ep_A^{t+1} + s \right)/2$. (A.1) can be rewritten as

$$V - x^t - p_A^t + \delta_c \left[V - \frac{1}{2} - Ep_A^{t+1} + \int_{\dot{x}^{t+1}}^1 \left[-(1-2y) + Ep_A^{t+1} - Ep_B^{t+1} - s \right] dy \right] \tag{A.2}$$

Similarly the expected lifetime payoff from buying product B in period t is

$$V - \left(1 - x^t \right) - p_B^t + \delta_c \left[V - \frac{1}{2} - Ep_B^{t+1} + \int_0^{\ddot{x}^{t+1}} \left[(1-2y) + Ep_B^{t+1} - Ep_A^{t+1} - s \right] dy \right] \tag{A.3}$$

where $\ddot{x}^{t+1} = \left(1 + Ep_B^{t+1} - Ep_A^{t+1} - s \right)/2$. The difference between (A.2) and (A.3) is clearly decreasing in x^t. Therefore provided $|p_B^t - p_A^t|$ is not too large, there exists an $\tilde{x}^t \in (0,1)$ such that (A.2) and (A.3) are equal when evaluated at $x^t = \tilde{x}^t$. (This also means that (A.2) exceeds (A.3) when $x^t < \tilde{x}^t$, whilst the opposite is true when $x^t > \tilde{x}^t$.) To get equation (1), equate (A.2) and (A.3), then substitute in $x^t = \tilde{x}^t$, then simplify. □

Proof of Proposition 2. Lemma A.1 below derives expressions for J, M, N, R as a function of K, which must hold in any equilibrium. Lemma A.2 then shows there is a unique K consistent with our problem, and shows that it lies on $[s/3, 3s/8)$. □

Lemma A.1. *J and K satisfy equations (10) and (11), and in addition*

$$M = \frac{J}{1-\delta_f} \tag{A.4}$$

$$N = \frac{2s\left(1 + K\delta_c s\right) - K\left(2 + K\delta_c s\right)}{2 + K\delta_c s + \delta_f s} \tag{A.5}$$

$$R = \frac{K^2}{2} \left(\frac{2 + K\delta_c s}{1 + K\delta_c s} \right) \tag{A.6}$$

Proof of Lemma A.1. Let $\pi_A^t\left(p_A^t, p_B^t, \tilde{x}^{t-1}\right) = p_A^t D_A^t\left(p_A^t, p_B^t, \tilde{x}^{t-1}\right)$ be flow profit in period t. Firm A chooses p_A^t to maximize $\pi_A^t\left(p_A^t, p_B^t, \tilde{x}^{t-1}\right) + \delta_f V_A^{t+1}\left(\tilde{x}^t\right)$. Take $\pi_A^t\left(p_A^t, p_B^t, \tilde{x}^{t-1}\right) + \delta_f V_A^{t+1}\left(\tilde{x}^t\right)$ and use equations (5) and (6) to substitute out for $V_A^{t+1}\left(\tilde{x}^t\right)$. Then maximize with respect to p_A^t to get a first order condition[12]

$$D_A^t\left(p_A^t, p_B^t, \tilde{x}^{t-1}\right) - \frac{p_A^t}{2}\frac{2+K\delta_c s}{1+K\delta_c s} - \frac{\delta_f N}{2\left(1+K\delta_c s\right)} - \frac{\delta_f R\left(p_B^t - p_A^t\right)}{2\left(1+K\delta_c s\right)^2} = 0 \tag{A.7}$$

Substitute out p_A^t and p_B^t using equations (3) and (4), collect terms and rewrite (A.7) in the form $\alpha_1 + \alpha_2\left(\tilde{x}^{t-1} - \frac{1}{2}\right) = 0$. Setting $\alpha_1 = \alpha_2 = 0$ gives the following conditions

$$1 - \frac{J}{2}\frac{2+K\delta_c s}{1+K\delta_c s} - \frac{\delta_f N}{2\left(1+K\delta_c s\right)} = 0 \tag{A.8}$$

$$s - \frac{3K}{2}\frac{2+K\delta_c s}{1+K\delta_c s} + \frac{\delta_f R K}{\left(1+K\delta_c s\right)^2} = 0 \tag{A.9}$$

To find an expression for A's period-t valuation, take $\pi_A^t\left(p_A^t, p_B^t, \tilde{x}^{t-1}\right) + \delta_f V_A^{t+1}\left(\tilde{x}^t\right)$ and again use equations (5) and (6) to substitute out for $V_A^{t+1}\left(\tilde{x}^t\right)$. Then use equations (3) and (4) to eliminate p_A^t and p_B^t. After collecting terms, A's period-t valuation can be expressed in the form $\alpha_3 + \alpha_4\left(\tilde{x}^{t-1} - \frac{1}{2}\right) + \alpha_5\left(\tilde{x}^{t-1} - \frac{1}{2}\right)^2$. Since we assumed in equation (6) that this value equals $M + N\left(\tilde{x}^{t-1} - \frac{1}{2}\right) + R\left(\tilde{x}^{t-1} - \frac{1}{2}\right)^2$, we can equate coefficients and get three equations

$$\alpha_3 = J + \delta_f M = M \tag{A.10}$$

$$\alpha_4 = Js - JK\frac{2+K\delta_c s}{1+K\delta_c s} + K - \frac{\delta_f K N}{1+K\delta_c s} = N \tag{A.11}$$

$$\alpha_5 = Ks - K^2\frac{2+K\delta_c s}{1+K\delta_c s} + \frac{\delta_f R K^2}{\left(1+K\delta_c s\right)^2} = R \tag{A.12}$$

Since $s > 0$ equation (A.9) implies that $K \neq 0$. Therefore rewrite equation (A.9) as

$$R = \frac{3\left(2+K\delta_c s\right)\left(1+K\delta_c s\right)}{2\delta_f} - \frac{s\left(1+K\delta_c s\right)^2}{\delta_f K} \tag{A.13}$$

[12]We can also prove that $\pi_A^t\left(p_A^t, p_B^t, \tilde{x}^{t-1}\right) + \delta_f V_A^{t+1}\left(\tilde{x}^t\right)$ is globally quasiconcave in p_A^t. However since the proof is lengthy, we omit it. The proof is complicated by the fact that each of the three different groups of consumers (young, old locked to A, old locked to B) has a different demand elasticity. Following a non-infinitesimal price deviation, a firm may for example stop selling to one or more of these groups, and thus its demand elasticity will jump. Full details are available on request.

and then substitute this into equation (A.12) and rearrange to find $\phi(K) = 0$ where

$$\phi(K) = \delta_f K^3 (2 + K\delta_c s) - 3K (2 + K\delta_c s)(1 + K\delta_c s)^2 + 2s (1 + K\delta_c s)^3 \tag{A.14}$$

Setting $\phi(K) = 0$ gives equation (11) in the text. Substituting equation (A.13) into the lefthand side of (A.12) gives the expression for R in equation (A.6). To get the expressions for J and N in (10) and (A.5), jointly solve equations (A.8) and (A.11). $\square$

Lemma A.2. *Equation (11) has a unique solution consistent with our problem, and it lies in* $[s/3, 3s/8)$.

Proof of Lemma A.2. The demand expression (8) is valid if and only if $|K| \leq 1 - s$. To see this, note firstly that (8) is only well-defined if $1 + K\delta_c s \neq 0$, which is satisfied provided $|K| \leq 1 - s$. Secondly (8) assumes that each firm sells to a positive mass of young consumers. This requires that $\tilde{x}^t \in (0, 1)$ which, using equation (5), is equivalent to $|K| < 1 + K\delta_c s$. This is again satisfied provided $|K| \leq 1 - s$. Thirdly (8) assumes that generically (i.e. whenever $\tilde{x}^{t-1} \notin \{0, 1\}$) each firm has old consumers both switching to and away from it. Using Section 3.1 this requires $\ddot{x} > 0$ and $\dot{x} < 1$. $\ddot{x} > 0$ is equivalent to $1 - s > 2K(\tilde{x}^{t-1} - 1/2)$: a necessary and sufficient condition for this to hold for any $\tilde{x}^{t-1} \in (0, 1)$, is that $|K| \leq 1 - s$. $\dot{x} < 1$ is also shown to hold under the same condition.
Aim: show that equation (11) has exactly one solution on the interval $[-(1-s), 1-s]$, and that it lies in $[s/3, 3s/8)$. Note then that $K \leq 1 - s$ hold, because by assumption $s \leq 7/10$.
Step 1: Show that equation (11) has exactly one solution on the interval $[0, 1-s]$.
Step 1a. Show that $\frac{\partial \phi(K)}{\partial K} < 0$ for all $K \in [0, 1-s]$. Using equation (A.14):

$$\frac{1}{2}\frac{\partial \phi(K)}{\partial K} = K^2 \delta_f (3 + 2K\delta_c s) + 3(1 + K\delta_c s)\left[-1 - 4K\delta_c s - 2(K\delta_c s)^2 + \delta_c s^2 + K\delta_c^2 s^3\right]$$

Since we are considering $K \geq 0$, $3 + 2K\delta_c s \leq 3(1 + K\delta_c s)$ and therefore

$$\frac{1}{2}\frac{\partial \phi(K)}{\partial K} \leq 3(1 + K\delta_c s)\left[K^2\delta_f - 1 - 4K\delta_c s - 2(K\delta_c s)^2 + \delta_c s^2 + K\delta_c^2 s^3\right]$$

Notice that $-4K\delta_c s + K\delta_c^2 s^3 < 0$ since $\delta_c s^2 < 4$, and that $K^2\delta_f - 1 + \delta_c s^2 \leq (1-s)^2 - 1 + s^2 = 2s(s-1) < 1$. Therefore $\frac{\partial \phi(K)}{\partial K} < 0$ for all $K \in [0, 1-s]$.

Step 1b. Note that $\phi(0) = 2s > 0$. Now prove that $\phi(1-s) < 0$. Substituting $K = 1-s$ into $\phi(K)$ and then simplifying, we find that:

$$\phi(1-s) = \delta_f (1-s)^3 [2 + \delta_c s(1-s)] - [1 + \delta_c s(1-s)]^2 \left(6 - 8s + 3\delta_c s - 8\delta_c s^2 + 5\delta_c s^3\right)$$

Clearly $[2 + \delta_c s(1-s)] / [1 + \delta_c s(1-s)]^2 \leq 2$ because $[2+X]/[1+X]^2$ decreases in X for any $X \geq 0$. Therefore to show that $\phi(1-s) < 0$, it is sufficient to prove that

$$2(1-s)^3 < 6 - 8s + 3\delta_c s - 8\delta_c s^2 + 5\delta_c s^3 \tag{A.15}$$

First if $s \in [0, 3/5]$ then the righthand side of (A.15) increases in δ_c. Therefore if (A.15) holds for $\delta_c = 0$ it will also hold for any $\delta_c \in [0,1]$. When $\delta_c = 0$ (A.15) becomes $2(1-s)^3 < 6 - 8s$, which is easily shown to hold for any $s \in [0, 3/5]$. Second if instead $s \in [3/5, 7/10]$ then the righthand side of (A.15) decreases in δ_c. Therefore if (A.15) holds for $\delta_c = 1$ it will also hold for any $\delta_c \in [0,1]$. When $\delta_c = 1$ (A.15) becomes $0 < 4 + s - 14s^2 + 7s^3$ which is easily seen to hold for any $s \in [3/5, 7/10]$. Therefore $\phi(1-s) < 0$ for any $s \in [0, 7/10]$.

Step 1c. Combining steps 1a and 1b, there is a unique $K \in [0, 1-s]$ that solves $\phi(K) = 0$.

Step 2: Show that the solution on $[0, 1-s]$ to $\phi(K) = 0$, actually lies on $[s/3, 3s/8)$.

Step 2a. Show that $\phi(s/3) > 0$. The terms $-3K(2 + K\delta_c s)(1 + K\delta_c s)^2 + 2s(1 + K\delta_c s)^3$ become $(1 + K\delta_c s)^2 K\delta_c s^2\big|_{K=s/3} > 0$. Also $\delta_f K^3 (2 + K\delta_c s) > 0$ since $K > 0$.

Step 2b. Show that $\phi(3s/8) < 0$. Letting $Y = 3\delta_c s^2/8$,

$$\begin{aligned}
\phi\left(\frac{3s}{8}\right) &= \left(\frac{27}{512}\right)\delta_f s^3 (2+Y) - \frac{9s}{8}(2+Y)(1+Y)^2 + 2s(1+Y)^3 \\
&= \left(\frac{27}{512}\right)\delta_f s^3 (2+Y) - \frac{13s}{500}(2+Y)(1+Y)^2 - \frac{1099s}{1000}(2+Y)(1+Y)^2 + 2s(1+Y)^3
\end{aligned}$$

The first two terms are negative provided that $27s^2/512 < 13/500$, and this holds because $s < 7/10$. The final two terms are negative if and only if $Y < 198/901 \approx 0.21$, and this holds because $Y \leq (3/8)(7/10)^2 = 0.18375$. Therefore $\phi(3s/8) < 0$.

Step 2c. Since $\phi(K)$ strictly decreases on $[0, 1-s]$, and $\phi(s/3) > 0$ but $\phi(3s/8) < 0$, the solution to $\phi(K) = 0$ must lie on $[s/3, 3s/8)$.

Step 3. Now show there is no solution to $\phi(K) = 0$ for $K \in [-(1-s), 0]$.

Step 3a. Using Step 1a

$$\frac{1}{6}\frac{\partial^2 \phi(K)}{\partial K^2} = 2K\delta_f (1 + K\delta_c s) + \delta_c s \left(-5 - 12K\delta_c s - 6(K\delta_c s)^2 + 2\delta_c s^2 + 2K\delta_c^2 s^3\right)$$

Since $K \in [-(1-s), 0]$, the first term $2K\delta_f (1 + K\delta_c s)$ is negative. To show the remainder is also negative, it is sufficient to show that $-5 - 12K\delta_c s + 2\delta_c s^2 < 0$. The latter is toughest to satisfy when K is very negative and δ_c is large, so substitute in $K = -(1-s)$ and $\delta_c = 1$. It it is then sufficient to prove that $-5 + 12s(1-s) + 2s^2 < 0$: this is easily shown to hold for any $s \in [0, 7/10]$. Therefore $\partial^2\phi(K)/\partial K^2 < 0$ ($\phi(K)$ is concave) for all $K \in [-(1-s), 0]$.

Step 3b. Show that $\phi(-(1-s)) > 0$. Rewrite $\phi(K)$ as

$$\begin{aligned} & -K(2 + K\delta_c s)\left[3(1 + K\delta_c s)^2 - \delta_f K^2\right] + 2s(1 + K\delta_c s)^3 \\ = \;& (1-s)(2 - \delta_c s(1-s))\left[3(1 - \delta_c s(1-s))^2 - \delta_f^2 (1-s)^2\right] + 2s(1 - \delta_c s(1-s)) \end{aligned}$$

which by inspection in positive. We also showed in Step 1b that $\phi(0) > 0$. Therefore since $\phi(K)$ is concave on $[-(1-s), 0]$ and positive at the boundaries of that set, it must be true that $\phi(K) > 0 \forall K \in [-(1-s), 0]$, hence there is no root to $\phi(K)$ on that interval. $\square$

Proof of Lemma 5. Step 1a. According to equation (10) $dJ/d\delta_f < 0 \iff \partial K/\partial\delta_f < \alpha_1/\alpha_2$ where

$$\alpha_1 = 2s(1 + K\delta_c s) - K(2 + K\delta_c s), \quad \alpha_2 = 2\left[(2\delta_c s + \delta_f)\left(1 + \frac{s}{2}\delta_f\right) - \delta_c s\right]$$

Step 1b. Firstly the derivative of α_1 with respect to K is $2\delta_c s^2 - 2 - K\delta_c s$, which is negative because $\delta_c s^2 < 1$. Secondly by inspection α_2 is increasing in δ_c, δ_f and s. Therefore

$$\frac{\alpha_1}{\alpha_2} > \frac{\alpha_1|_{K=3s/8}}{\alpha_2|_{s=7/10, \delta_c=\delta_f=1}} = \frac{s\left(\frac{5}{4} + \frac{39}{64}\delta_c s^2\right)}{5.08} = \frac{s}{5.08}\left(\frac{5}{4} + \frac{39}{64}\delta_c s^2\right)$$

Step 1c. According to equation (A.14) $\partial K/\partial\delta_f = \alpha_3/\alpha_4$ where

$$\begin{aligned}
\alpha_3 &= K^3\left(2+K\delta_c s\right)\\
\alpha_4 &= 3\left(1+K\delta_c s\right)^2\left[2\left(1-\delta_c s^2\right)+K\delta_c s\right]+3K\delta_c s\left(1+K\delta_c s\right)\left(5+3K\delta_c s\right)-\alpha_5\\
\alpha_5 &= \delta_f K^2\left(6+4K\delta_c s\right)
\end{aligned}$$

Step 1d. Firstly α_3 increases in K therefore $\alpha_3 < \alpha_3|_{K=3s/8} = \frac{27s^3}{512}\left(2+\frac{3\delta_c s^2}{8}\right)$. Secondly $\alpha_4+\alpha_5$ increases in K therefore

$$\alpha_4+\alpha_5 \geq \left(\alpha_4+\alpha_5\right)|_{K=s/3} = \left(1+\frac{\delta_c s^2}{3}\right)\left[6+2\delta_c s^2-\frac{2}{3}\left(\delta_c s^2\right)^2\right]$$

which by inspection increases in s, so $\alpha_4+\alpha_5 \geq \left(\alpha_4+\alpha_5\right)|_{K=s/3,s=0} = 6$. Thirdly α_5 increases in K, δ_c, δ_f, s and so $\alpha_5 \leq \alpha_5|_{K=3s/8,s=7/10,\delta_c=\delta_f=1} < 1/2$. Consequently $\alpha_4 \geq 6-1/2 = 11/2$ and

$$\frac{\partial K}{\partial\delta_f} = \frac{\alpha_3}{\alpha_4} \leq \frac{2}{11}\frac{27s^3}{512}\left(2+\frac{3\delta_c s^2}{8}\right) = \frac{27s^3}{2816}\left(2+\frac{3\delta_c s^2}{8}\right)$$

Since $s \leq 7/10$ a simple calculation shows that this bound on $\partial K/\partial\delta_f$ is less than the bound on α_1/α_2 given in step 1b. Hence J decreases in δ_f as claimed.

Step 2. According to equation (10) $dJ/d\delta_c > 0 \iff \beta_1+\beta_2\left(\partial K/\partial\delta_c\right) > 0$ where

$$\beta_1 = Ks\left(2+2\delta_f s-K\delta_f\right), \quad \beta_2 = 2\delta_f+\left(\delta_f\right)^2 s+2\delta_c s+2\delta_c\delta_f s^2$$

Clearly $\beta_1, \beta_2 > 0$ so it is sufficient to show that $\partial K/\partial\delta_c > 0$. According to equation (A.14) $\partial K/\partial\delta_c = \beta_3/\alpha_4$ where

$$\beta_3 = \delta_f K^4 s+3Ks\left(1+K\delta_c s\right)^2\left(2s-K\right)-6K^2 s\left(1+K\delta_c s\right)\left(2+K\delta_c s\right)$$

We already know that $\alpha_4 > 0$ so $\partial K/\partial\delta_c > 0$ if and only if $\beta_3 > 0$, which holds provided

$$3Ks\left(1+K\delta_c s\right)^2\left(2s-K\right) > 6K^2 s\left(1+K\delta_c s\right)\left(2+K\delta_c s\right)$$

After cancelling terms and noting that $K \in [s/3, 3s/8)$, this is seen to hold. $\square$

Proof of Proposition 6. From equation (10) steady state price is 1 when $s = 0$. First, using equation (10) $J < 1$ if and only if

$$K\delta_c s - \delta_f (s - K) < 0 \tag{A.16}$$

which is harder to satisfy as K increases. We also know from Proposition 2 that $K < 3s/8$. Therefore if inequality (A.16) holds when evaluated at $K = 3s/8$, it always holds. Substituting $K = 3s/8$ into (A.16), we get a condition $\delta_f > (3\delta_c s)/5$. Second, $J > 1$ if and only if

$$K\delta_c s - \delta_f (s - K) > 0 \tag{A.17}$$

which is easier to satisfy as K increases. We again know from Proposition 2 that $K \geq s/3$. Therefore if inequality (A.17) holds when evaluated at $K = s/3$, it always holds. Substituting $K = s/3$ into (A.17), we get a condition $\delta_f < (\delta_c s)/2$. Therefore steady state price is definitely lower if $\delta_f > (3\delta_c s)/5$ and definitely higher if $\delta_f < (\delta_c s)/2$. Lemma 5 says that the steady state price strictly decreases in δ_f. Therefore there exists a unique threshold between $(\delta_c s)/2$ and $(3\delta_c s)/5$ such that $J = 1$ when $\delta_f = \widetilde{\delta}_f$. $\square$

Proof of Proposition 7. Step 1. Show that the average price strictly decreases in δ_f. Totally differentiate (13) with respect to δ_f:

$$\frac{dJ}{d\delta_f} + \left(\tilde{x}^{t-1} - 1/2\right)^2 \left[s - 2K - K\frac{2 + K\delta_c s}{(1 + K\delta_c s)^2}\right] \frac{\partial K}{\partial \delta_f}$$

We know that $dJ/d\delta_f < 0$ from Lemma 5. Adapting the proof of the same lemma, we can also show that $\partial K/\partial\delta_f > 0$.[13] Therefore it is sufficient to show that the square-bracketed term is negative, which is easily done.

Step 2. It is simple (though tedious) to show that for any δ_c, $s \in (0, 7/10]$, and $\tilde{x}^{t-1}$: (a). average price exceeds 1 as $\delta_f \to \tilde{\delta}_f$ and (b). average price is below 1 as $\delta_f \to 1$. Since average price strictly decreases in δ_f, the threshold $\hat{\delta}_f$ exists and is unique. $\square$

[13] In particular using Step 1c of the proof, we know that $\partial K/\partial\delta_f = \alpha_3/\alpha_4$. By inspection $\alpha_3 > 0$, and $\alpha_4 \geq 6\left(1 - \delta_c s^2\right) - \alpha_5$ which is easily shown to be strictly positive.

Proof of Lemma 9. Old consumers who previously bought product A will i). stay with A and earn $V - J - x^t$ if their location satisfies $x^t \leq (1+s)/2$ or ii). switch to B and earn $V - J - (1 - x^t) - s$ otherwise. Integrating over all possible values of x^t, their expected surplus is $V - J - 1/4 + s^2/4 - s/2$. By similar logic old consumers who previously bought B also have expected surplus $V - J - 1/4 + s^2/4 - s/2$. Therefore when $s = 0$ consumer surplus is $V - 5/4$ (because $J = 1$ when $s = 0$). Consequently switching costs make old consumer worse off if and only if $s^2/4 - s/2 < J - 1$. Using equation (10) we know that

$$J - 1 = \frac{K\delta_c s + \delta_f (K - s)}{2 + K\delta_c s + \delta_f s} > \frac{\delta_f (K - s)}{2 + K\delta_c s + \delta_f s} > \frac{-\frac{2}{3}\delta_f s}{2 + K\delta_c s + \delta_f s} > \frac{-\frac{2}{3}\delta_f s}{2 + \delta_f s} > \frac{-2s}{3(2+s)}$$

therefore in order to prove $s^2/4 - s/2 < J - 1$, it is sufficient to prove that $s^2/4 - s/2 < -2s/[3(2+s)]$ which is easily shown to hold for any $s \in (0, 7/10]$. $\square$

Proof of Proposition 10. The proof follows from arguments in the text, and Lemma 5 which says that J increases in δ_c. $\square$

B Other proofs

Proof of Proposition 12 To simplify the notation, we use Δ^t as a shorthand for $p_B^t - p_A^t$, and also Δ_e^{t+1} as a shorthand for $Ep_B^{t+1} - Ep_A^{t+1}$. We start with some preliminary lemmas.

Lemma B.1. *Suppose that in period $t-1$ all young consumers with $x^{t-1} \leq \tilde{x}^{t-1}$ bought from A and all others bought from B. Then demand for product A in period t is:*

$$D_A^t = \int_0^{\tilde{x}^{t-1}} F\left(\left.\frac{1+\Delta^t+s}{2}\right| z\right) dz + \int_{\tilde{x}^{t-1}}^1 F\left(\left.\frac{1+\Delta^t-s}{2}\right| z\right) dz + \tilde{x}^t \tag{B.1}$$

where $\Delta^t = p_B^t - p_A^t$, and where $\tilde{x}^t$ is implicitly defined by the equation

$$1 - 2\tilde{x}^t + \Delta^t + \delta_c \left[s + \int_{\dot{x}^{t+1}}^1 f\left(z|\tilde{x}^t\right)\left(2z - 1 - \Delta_e^{t+1} - s\right) dz\right] \\ - \delta_c \left[\int_{\ddot{x}^{t+1}}^1 f\left(z|\tilde{x}^t\right)\left(2z - 1 - \Delta_e^{t+1} + s\right) dz\right] = 0 \tag{B.2}$$

and where $\dot{x}^{t+1}$ and $\ddot{x}^{t+1}$ are as defined earlier, namely $\dot{x}^{t+1} = \left(1 + \Delta_e^{t+1} + s\right)/2$ and $\ddot{x}^{t+1} = \left(1 + \Delta_e^{t+1} - s\right)/2$.

Proof. The first two terms of (B.1) are demand from old consumers. In the previous period A sold to all young consumers with $x^{t-1} \leq \tilde{x}^{t-1}$; as shown earlier they will buy A in period t if and only if $x^t \leq \left(1 + p_B^t - p_A^t + s\right)/2$. In the previous period B sold to all young consumers with $x^{t-1} \geq \tilde{x}^{t-1}$; as shown earlier they will switch to A in period t if and only if $x^t \leq \left(1 + p_B^t - p_A^t - s\right)/2$. The last term of (B.1) is demand from young consumers. Let us define $W_A^{t+1} = V - E\left(x^{t+1} \middle| x^t\right) - Ep_A^{t+1}$. Using the same arguments as when proving Lemma 1, a young consumer in period t with location x^t has expected payoffs from buying A and B given by:

$$V - x^t - p_A^t + \delta_c \left[W_A^{t+1} + \int_{\dot{x}^{t+1}}^1 f\left(z|x^t\right)\left(2z - 1 - \Delta_e^{t+1} - s\right) dz\right] \tag{B.3}$$

$$V - \left(1 - x^t\right) - p_B^t + \delta_c \left[W_A^{t+1} - s + \int_{\ddot{x}^{t+1}}^1 f\left(z|x^t\right)\left(2z - 1 - \Delta_e^{t+1} + s\right) dz\right] \tag{B.4}$$

Now (B.3) minus (B.4) is strictly decreasing in x^t when $s = 0$ (and therefore by continuity, when s is sufficiently close to zero). Hence there exists an $\tilde{x}^t$ such that all consumers in period t with location $x^t \leq \tilde{x}^t$ buy A, and all others buy B. Substituting $x^t = \tilde{x}^t$ into (B.3) and (B.4) and then equating them, gives equation (B.2) in the lemma. $\square$

Lemma B.2. *Recall the definition of the marginal young consumer $\tilde{x}^t$ in Lemma B.1. In steady state i). when $s = 0$, $d\tilde{x}^t/dp_A^t = -1/2$, and ii). the following holds:*

$$\left.\frac{\partial \left(d\tilde{x}^t/dp_A^t\right)}{\partial s}\right|_{s=0} = \frac{\delta_c}{2}\frac{\partial \Pr\left(z \geq 1/2|x^t = 1/2\right)}{\partial x^t} \tag{B.5}$$

Proof. Totally differentiate (B.2) with respect to p_A^t at the steady state, to get:

$$\frac{d\tilde{x}^t}{dp_A^t} = -\frac{1}{\bar{\gamma}}$$

$$\begin{aligned}\bar{\gamma} = 2 - \delta_c \int_{\frac{1+s}{2}}^{1} \frac{\partial f\left(z|x^t = 1/2\right)}{\partial x^t}\left(2z - 1 - s\right) dz + \delta_c \int_{\frac{1-s}{2}}^{1} \frac{\partial f\left(z|x^t = 1/2\right)}{\partial x^t}\left(2z - 1 + s\right) dz \\ - \delta_c \frac{d\Delta_e^{t+1}\left(x^t = 1/2\right)}{dx^t} \int_{\frac{1-s}{2}}^{\frac{1+s}{2}} f\left(z|x^t = 1/2\right) dz \end{aligned} \tag{B.6}$$

For part i). note that $\bar{\gamma} = 2$ when $s = 0$. For part ii). note that when $s = 0$ there is a MPE in which $\Delta_e^{t+1} = 0$ for all x^t. Therefore $d\Delta_e^{t+1}\left(x^t = 1/2\right)/dx^t$ is zero when $s = 0$. Also note that the integral in the final term of (B.6) is zero when $s = 0$. Therefore the derivative of the final term in (B.6) with respect to s around $s = 0$, is zero. Consequently:

$$\begin{aligned}\frac{1}{\delta_c}\left.\frac{\partial \bar{\gamma}}{\partial s}\right|_{s=0} &= 2\int_{1/2}^{1} \frac{\partial f\left(z|x^t = 1/2\right)}{\partial x^t} dz = 2\frac{\partial \Pr\left(z \geq 1/2|x^t = 1/2\right)}{\partial x^t} \\ \implies \left.\frac{\partial \left(d\tilde{x}^t/dp_A^t\right)}{\partial s}\right|_{s=0} &= \frac{1}{\left(\bar{\gamma}|_{s=0}\right)^2}\left.\frac{\partial \bar{\gamma}}{\partial s}\right|_{s=0} = \frac{\delta_c}{2}\frac{\partial \Pr\left(z \geq 1/2|x^t = 1/2\right)}{\partial x^t}\end{aligned}$$

$\square$

Lemma B.3. *In steady state i). when $s = 0$, $dD_A^t/dp_A^t = -1$ and ii). the following holds*

$$\left.\frac{\partial \left(dD_A^t/dp_A^t\right)}{\partial s}\right|_{s=0} = \frac{1+\delta_c}{2}\frac{\partial \Pr\left(z \geq 1/2|x^t = 1/2\right)}{\partial x^t}$$

Proof. Differentiate the demand expression in equation (B.1) with respect to p_A^t at the steady state. Using $\bar{\gamma}$ defined in Lemma B.2 this gives:

$$\frac{dD_A^t}{dp_A^t} = -\frac{1}{\bar{\gamma}} - \frac{1}{2}\int_0^{1/2} f\left(\left.\frac{1+s}{2}\right| z\right) dz - \frac{1}{2}\int_{1/2}^{1} f\left(\left.\frac{1-s}{2}\right| z\right) dz \tag{B.7}$$

To prove part i). note that $f\left(x^{t+1}\middle| x^t\right) f_{x^t}\left(x^t\right) = f\left(x^t\middle| x^{t+1}\right) f_{x^{t+1}}\left(x^{t+1}\right)$, where $f_{x^i}\left(x^i\right)$ is the marginal density for x^i. Since x^t and x^{t+1} are uniformly distributed, $f\left(x^{t+1}\middle| x^t\right) = f\left(x^t\middle| x^{t+1}\right)$ and therefore $\int_0^1 f\left(1/2\middle| z\right) dz = \int_0^1 f\left(z\middle| 1/2\right) dz = 1$. To prove part ii). differentiate equation (B.7) with respect to s around $s = 0$:

$$\left.\frac{\partial\left(dD_A^t/dp_A^t\right)}{\partial s}\right|_{s=0} = \frac{\delta_c}{2}\frac{\partial \Pr\left(x^{t+1} \geq 1/2 | x^t = 1/2\right)}{\partial x^t} + \frac{\int_{1/2}^1 f'\left(\frac{1}{2}\middle| z\right) dz - \int_0^{1/2} f'\left(\frac{1}{2}\middle| z\right) dz}{4}$$

Radial symmetry implies $\int_0^{1/2} f'\left(\frac{1}{2}\middle| z\right) dz = -\int_{1/2}^1 f'\left(\frac{1}{2}\middle| z\right) dz$. Also as argued above, $f\left(1/2|z\right) = f\left(z|1/2\right)$. Therefore:

$$\int_{1/2}^1 f'\left(\frac{1}{2}\middle| z\right) dz \equiv \int_{1/2}^1 \frac{\partial f\left(y = \frac{1}{2}\middle| z\right)}{\partial y} dz = \int_{1/2}^1 \frac{\partial f\left(z\middle| y = \frac{1}{2}\right)}{\partial y} dz = \frac{\partial \Pr\left(z \geq \frac{1}{2} | y = \frac{1}{2}\right)}{\partial y}$$

□

Lemma B.4. *In steady state:*

$$\left.\frac{\partial D_A^t/\partial \tilde{x}^{t-1}}{\partial s}\right|_{s=0} = f\left(\frac{1}{2}\middle|\frac{1}{2}\right) \tag{B.8}$$

Proof. Differentiate the demand expression (B.1) with respect to $\tilde{x}^{t-1}$:

$$\frac{\partial D_A^t}{\partial \tilde{x}^{t-1}} = F\left(\left.\frac{1+\Delta^t+s}{2}\right| \tilde{x}^{t-1}\right) - F\left(\left.\frac{1+\Delta^t-s}{2}\right| \tilde{x}^{t-1}\right) \tag{B.9}$$

Substitute in $\tilde{x}^{t-1} = 1/2$ and $\Delta^t = 0$, then differentiate the resulting expression with respect to s. □

Lemma B.5. *In steady state the derivative of* $\frac{\partial\left(dD_A^t/dp_A^t\right)}{\partial \tilde{x}^{t-1}}$ *with respect to s is zero at $s = 0$.*

Proof. Differentiate equation (B.9) with respect to p_A^t to get

$$\frac{\partial\left(dD_A^t/dp_A^t\right)}{\partial \tilde{x}^{t-1}} = \frac{1}{2}\left[-f\left(\left.\frac{1+\Delta^t+s}{2}\right| \tilde{x}^{t-1}\right) + f\left(\left.\frac{1+\Delta^t-s}{2}\right| \tilde{x}^{t-1}\right)\right]$$

Then substitute in $\tilde{x}^{t-1} = 1/2$ and $\Delta^t = 0$. Now differentiate with respect to s, and note that radial symmetry implies $f'\left(1/2|1/2\right) = 0$. □

Now for the main proof.

Proof of Proposition 12. In this problem the payoff-relevant state variable in period t is $\tilde{x}^{t-1}$. We again look for a symmetric MPE which is continuous around $s=0$, and where the steady state has $\tilde{x}^{t-1}=1/2$ and both firms charging the same price.

Firm A chooses p_A^t to maximize $p_A^t D_A^t + \delta_f V_A^{t+1}(\tilde{x}^t)$, which gives a *F.O.C.*

$$D_A^t + p_A^t \frac{dD_A^t}{dp_A^t} + \delta_f \frac{dV_A^{t+1}(\tilde{x}^t)}{d\tilde{x}^t}\frac{d\tilde{x}^t}{dp_A^t} = 0 \tag{B.10}$$

Firstly Impose steady state on the *F.O.C.*, and then differentiate it with respect to s around $s=0$. In doing this, note i). that in steady state $D_A^t = 1$, ii). the properties of dD_A^t/dp_A^t given in Lemma B.3, iii). that when $s=0$ firm A charges a price of 1 and has value $V_A^{t+1} = 1/(1-\delta_f)$ which is not a function of $\tilde{x}^t$, and iv). that $d\tilde{x}^t/dp_A^t = -1/2$ when $s=0$ (Lemma B.2). Letting $\bar{p}$ be the steady state price, we find that:

$$\left.\frac{\partial \bar{p}}{\partial s}\right|_{s=0} = \frac{1+\delta_c}{2}\frac{\partial \Pr\left(x^{t+1}\geq 1/2 | x^t = 1/2\right)}{\partial x^t} - \frac{\delta_f}{2}\left[\left.\frac{\partial\left(dV_A^{t+1}(1/2)/d\tilde{x}^t\right)}{\partial s}\right|_{s=0}\right] \tag{B.11}$$

Secondly differentiate the *F.O.C.* in equation (B.10) with respect to $\tilde{x}^{t-1}$ to get:

$$\frac{d(F.O.C.)}{dp_A^t}\frac{dp_A^t}{d\tilde{x}^{t-1}} + \frac{d(F.O.C.)}{dp_B^t}\frac{dp_B^t}{d\tilde{x}^{t-1}} + \left[\frac{\partial D_A^t}{\partial \tilde{x}^{t-1}} + p_A^t\frac{\partial\left(dD_A^t/dp_A^t\right)}{\partial\tilde{x}^{t-1}}\right] = 0$$

The aim now is to differentiate this equation with respect to s around $s=0$. To do this note that when $s=0$, i). $F.O.C. = 1 + p_B^t - 2p_A^t$, ii). p_A^t and p_B^t both equal 1 and so are not a function of $\tilde{x}^{t-1}$, and iii). that according to Lemma B.5, both $\partial\left(dD_A^t/dp_A^t\right)/\partial\tilde{x}^{t-1}$and its derivative with respect to s, are zero. Also use Lemma B.4 which gives an expression for the derivative of $\partial D_A^t/\partial\tilde{x}^{t-1}$ with respect to s. Then

$$-2\left.\frac{\partial\left(dp_A^t/d\tilde{x}^{t-1}\right)}{\partial s}\right|_{s=0} + \left.\frac{\partial\left(dp_B^t/d\tilde{x}^{t-1}\right)}{\partial s}\right|_{s=0} + f\left(\frac{1}{2}\middle|\frac{1}{2}\right) = 0$$

We look for a symmetric equilibrium in which $p_A^t(\tilde{x}^{t-1}) = p_B^t(1-\tilde{x}^{t-1})$ therefore $dp_A^t/d\tilde{x}^{t-1}\left(\frac{1}{2}\right) = -dp_B^t/d\tilde{x}^{t-1}\left(\frac{1}{2}\right)$. So differentiating the above equation with respect to s at $s=0$, gives:

$$\left.\frac{\partial\left(dp_A^t\left(\frac{1}{2}\right)/d\tilde{x}^{t-1}\right)}{\partial s}\right|_{s=0} = \frac{f\left(\frac{1}{2}\middle|\frac{1}{2}\right)}{3} \tag{B.12}$$

Thirdly by the principle of optimality

$$V_A^t\left(\tilde{x}^{t-1}\right)=\max_{p_A^t} p_A^t D_A^t\left(\cdot\right)+\delta_f V_A^{t+1}\left(\tilde{x}^t\right)$$

Totally differentiating with respect to $\tilde{x}^{t-1}$and using the envelope theorem, gives:

$$\frac{dV_A^t}{d\tilde{x}^{t-1}}=\left[p_A^t\frac{dD_A^t}{dp_B^t}+\delta_f\frac{dV_A^{t+1}}{d\tilde{x}^t}\frac{d\tilde{x}^t}{dp_B^t}\right]\frac{dp_B^t}{d\tilde{x}^{t-1}}+p_A^t\frac{\partial D_A^t}{\partial\tilde{x}^{t-1}}$$

Since $D_A^t\left(\cdot\right)$ and $\tilde{x}^t$ only depend upon current prices through Δ^t, $dD_A^t/dp_B^t=-dD_A^t/dp_A^t$ and $d\tilde{x}^t/dp_B^t=-d\tilde{x}^t/dp_A^t$. Therefore using A's *F.O.C.* (B.10), the above becomes:

$$\frac{dV_A^t}{d\tilde{x}^{t-1}}=D_A^t\frac{dp_B^t}{d\tilde{x}^{t-1}}+p_A^t\frac{\partial D_A^t}{\partial\tilde{x}^{t-1}} \tag{B.13}$$

Next impose a steady state on (B.13) and substitute in $dp_B^t/d\tilde{x}^{t-1}\left(\frac{1}{2}\right)=-dp_A^t/d\tilde{x}^{t-1}\left(\frac{1}{2}\right)$. Then differentiate the resulting equation with respect to s at $s=0$. Finally combine this last equation with equations (B.12) and (B.11), to get the required expression for $\partial\bar{p}/\partial s|_{s=0}$. □