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The Consumption-Wealth Ratio under Asymmetric Adjustment

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Abstract

This paper argues that nonlinear adjustment may provide a better explanation of fluctuations in the consumption-wealth ratio. The nonlinearity is captured by a Markov-switching vector error-correction model that allows the dynamics of the relationship to differ across regimes. Estimation of the system suggests that these states are related to the behaviour of financial markets. In fact, estimation of the system suggests that short-term deviations in the consumption-wealth ratio will forecast either asset returns or consumption growth; the first when changes in wealth are transitory, the second when changes in wealth are permanent. Our approach uncovers a richer and more complex dynamic in the consumption-wealth ratio than previous results in the literature, whilst being in accordance with theoretical predictions of a simple model of consumption under uncertainty.

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1. Introduction

There has been a renewed interest in the literature concerning the linkages between asset wealth and consumption. The preceding decade has witnessed remarkable changes in households' wealth, particularly due to stock market valuations, which may have had implications for the pattern of consumer spending (e.g. Poterba, 2000). On the other hand, movements in aggregate macroeconomic relationships, such as the consumption-wealth ratio, may provide some guidance on the future performance of asset markets. Lustig et al. (2008) show the importance of the consumption-wealth ratio in consumption-based asset pricing models, such as those in Campbell and Cochrane (1999) and Bansal and Yaron (2004). Furthermore, Campbell and Mankiw (1989) show that movements in the consumption-wealth ratio predict either asset returns or consumption growth. This result provides a link between the consumption-wealth ratio and the literature on the predictability of returns, fostered by the results presented in Campbell and Shiller (1988) — see Cochrane (2008) for a recent survey.

Lettau and Ludvigson (2001) (see also Lettau and Ludvigson, 2004) argue that, given the smoothness of consumption growth, the consumption-wealth ratio will essentially forecast returns. Lettau and Ludvigson (L&L henceforth) start from a fairly standard model of consumer behaviour involving consumption, asset wealth and labour income. In their empirical model, in principle, fluctuations in the consumption-wealth ratio could forecast changes in either of these variables. L&L estimate a vector error-correction model (VECM) and conclude that adjustment from shocks distorting the long-run equilibrium takes place mainly through asset returns, confirming their prior. This, in turn, means that deviations from the common trend embody agents' expectations of future returns on the market portfolio and, therefore, are a useful predictor of stock and excess returns.

However, given the nature of the variables, it is likely that these adjustments occur in different ways, depending on the state of economy and, in particular, on the phase of the stock market. In fact, asset wealth displays a more volatile behaviour than consumption or labour income, a feature that is clearly linked with the state of asset markets. Several papers document the existence of different regimes in financial markets; see Cecchetti et al. (1990), Bonomo and Garcia (1994) and Driffill and Sola (1998), for example. In addition, recent work on the predictability of returns has emphasized the role of non-linearities. Paye and Timmermann (2006) identify significant shifts in the coefficients relating stock returns to forecasting variables in several OECD countries. In a similar vein, Lettau and van Nieuwerburgh (2008) relate the performance of prediction equations to breaks in steady-state parameters. Their results also include evidence of Markov-switching in the mean dividend-price ratio. Therefore, in this paper, we argue that regime switching may provide a better explanation for fluctuations in the consumption-wealth ratio. We explicitly allow for different states, by postulating that the dynamics of the equilibrium errors follow a Markov-switching process. This, in turn, leads to a Markov-switching VECM (MS-VECM) representation of the trivariate relationship, which we use to investigate the possibility of nonlinear adjustment in the consumption-wealth ratio.

Estimation of this MS-VECM suggests that the mechanism through which deviations from the long-run relationship are eliminated depends on the state of the economy. Thus, we find a regime whereby wealth does most of the error-correction in the system, coinciding with periods of "turbulent" markets. However, we also identify a more "tranquil" state, where it is consumption growth that drives the system back to long-run equilibrium. Therefore, and unlike L&L, our findings suggest that short-term deviations in the trivariate relationship (consumption, labour income and non-human wealth) will forecast either asset returns or consumption growth, depending on the state of the economy.

These results seem to provide a more accurate description of the dynamics of the consumption-wealth ratio than the standard, linear specification, while being consistent with the theoretical framework employed by L&L. Our results also help to explain why other researchers — Davis and Palumbo (2001), or Mehra (2001), among others — found consumption to adjust sluggishly to shocks in income and wealth. In fact, single-equation error-correction models with consumption growth as the dependent variable will partly detect the adjustments in consumption that occur in the regime where markets are less volatile, although the main driving force of the system is the behaviour of asset wealth.

A Markov-switching type of asymmetric adjustment in cointegrated systems has been suggested by Psaradakis et al. (2004) and Camacho (2005). These papers form the basis of the methodology employed in this study. Paap and van Dijk (2003) employ a similar method, using a Bayesian approach to estimate possible Markov trends in the consumption-income relationship. However, they do not include asset wealth in their model and therefore they do not capture the dynamic features present in the cointegrated system studied by L&L.

Our paper is organised as follows. The next section briefly reviews the model employed by L&L, reassesses their results and argues that the characteristics of the data call for the estimation of a nonlinear specification. Section 3 presents a possible account of the switching nature of consumption-wealth adjustment. In section 4 we discuss econometric tests for nonlinear adjustment and apply them to the L&L data. System estimation is carried out in section 5. Section 6 summarises and concludes.

2. Background Discussion

In this section, we briefly review the model employed by L&L and point out why their results (and economic theory) suggest that a nonlinear framework may offer a better characterisation of the evolution of consumption and the components of wealth. We begin by considering a standard household budget constraint. Define W_t as the beginning of period aggregate wealth in period t, with an asset wealth component, A_t, and a human capital component, H_t. By letting C_t denote aggregate consumption in period t and $R_{w,t+1}$ denote the net return on W_t, a simple wealth accumulation equation is given by

$$W_{t+1} = (1 + R_{w,t+1})(W_t - C_t). \tag{2.1}$$

Based on this equation, Campbell and Mankiw (1989) derive an expression for the consumption-wealth ratio in logs. They take a first-order Taylor expansion of the equation, solve the difference equation forward and take expectations, resulting in

$$c_t - w_t = E_t \sum_{i=1}^{\infty} \rho_w^i (r_{w,t+i} - \Delta c_{t+i}), \tag{2.2}$$

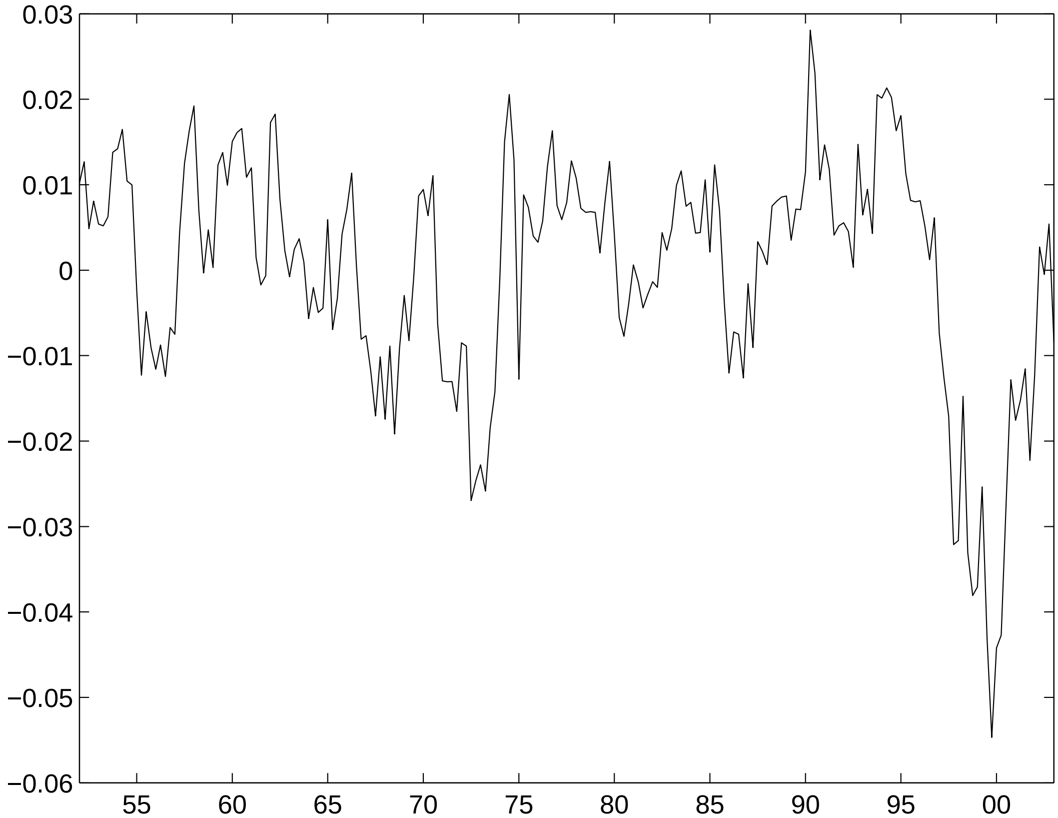
where $r = \log(1 + R)$, $\rho_w = (W - C)/W$ is the steady-state ratio of new investment to total wealth, and lower case letters denote variables in logs.

Despite the fact that H_t is not observable, L&L show that an empirically valid approximation may be obtained by using labour income, Y_t, as a proxy for human capital, H_t, resulting in the following log consumption-wealth ratio

$$c_t - \alpha_a a_t - \alpha_y y_t \approx E_t \sum_{i=1}^{\infty} \rho_w^i ((1 - v) r_{at+i} - \Delta c_{t+i} + v \Delta y_{t+1+i}), \tag{2.3}$$

where $(1-v)$ and v represent the steady-state shares of the wealth components a_t and y_t, respectively, and r_{at+i} denotes the net returns on asset wealth. The L&L papers provide a detailed discussion of the assumptions employed in the approximation. L&L then show that c_t, a_t and y_t share a common trend, with cointegration vector $(1, -\alpha_a, -\alpha_y)$ and cointegration residual $c_t - \alpha_a a_t - \alpha_y y_t$

Figure 1: Estimated equilibrium deviations for *cay* (log consumption-wealth ratio in the USA)



(cay_t in brief). Importantly for our argument, equation (2.3) implies that fluctuations in the consumption-wealth ratio will reflect future changes in asset wealth, consumption or labour income.

L&L proceed with their analysis by testing for the number of cointegration vectors, which they conclude to be only one. The cointegrating vector is estimated by the Dynamic OLS method of Stock and Watson (1993) as $(1, -0.3, -0.6)$, but the results appear to be robust with respect to the estimation method; therefore, our analysis will also employ this estimate. Secondly, L&L estimate a vector error-correction model (VECM) of the trivariate system, with the estimated cointegration vector imposed as the long-run attractor. The authors conclude that when a shock occurs, it is asset wealth that does most of the subsequent adjustment in order to restore the common trend.

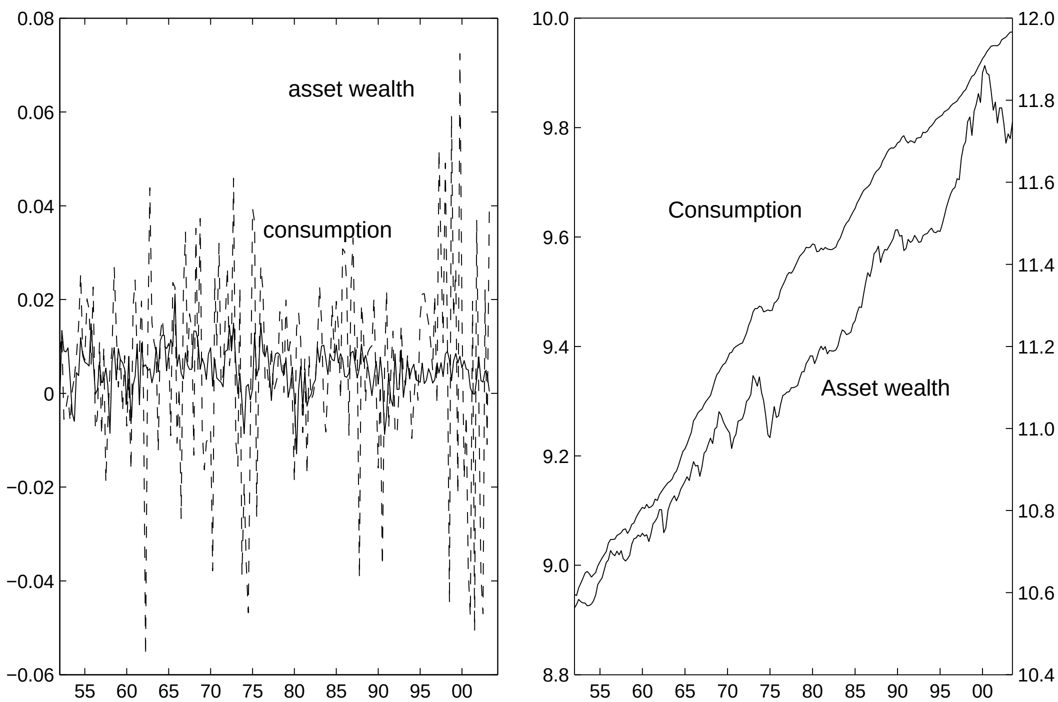
Table 1: Linear VECM

Equation	Δc_t	Δa_t	Δy_t
$\widehat{cay}_{t-1}$	-0.0211 (-0.955)	0.3337 (2.228)	0.0117 (0.326)
Δc_{t-1}	0.1996 (2.953)	0.0458 (0.141)	0.4957 (3.82)
Δa_{t-1}	0.0456 (3.219)	0.0924 (1.085)	0.0918 (2.44)
Δy_{t-1}	0.0763 (1.726)	-0.0656 (-0.369)	-0.1222 (-0.97)
(t-ratios based on HAC standard errors)			
Equations specification tests (p-values in square brackets)			
AR 1-5	3.039 [0.012]	0.718 [0.611]	0.923 [0.467]
Normality	5.822 [0.054]	25.532 [0.000]	48.653 [0.000]
ARCH	0.323 [0.863]	6.352 [0.000]	1.725 [0.146]
Heteroskedasticity	0.948 [0.478]	5.439 [0.000]	1.531 [0.149]
System specification tests (p-values in square brackets)			
Autocorrelation	Normality	Heterosced.	Heterosced.
1.374 [0.058]	70.828 [0.000]	1.744 [0.002]	1.653 [0.000]
Log likelihood	AIC	BIC	HQ
-2630.909	-5219.819	-5149.934	-5191.555
Johansen cointegration tests (p-values in square brackets)			
$H_0 : r =$	Trace	Max	
0	52.861 [0.000]	35.526 [0.478]	
1	17.335 [0.121]	13.726 [0.106]	
2	3.609 [0.473]	3.609 [0.473]	

However, a closer look at the results of L&L seems to suggest that the dynamic structure of the system may be further explored.[1] Take, for instance, the estimated equilibrium error $cay_t = c_t - 0.3a_t - 0.6y_t$ depicted in Figure

[1]In what follows, we resort to an updated version of the dataset used in Lettau and Ludvigson (2004). A detailed description of the data can be found in their Appendix A. The data itself is available from Ludvigsons's webpage (http://www.econ.nyu.edu/user/ludvigsons/). The results do not change if the actual data in Lettau and Ludvigson (2004) is used instead. The dataset comprises quarterly data on aggregate consumption, asset wealth and labour income, spanning from the fourth quarter of 1951 to the third quarter of 2003.

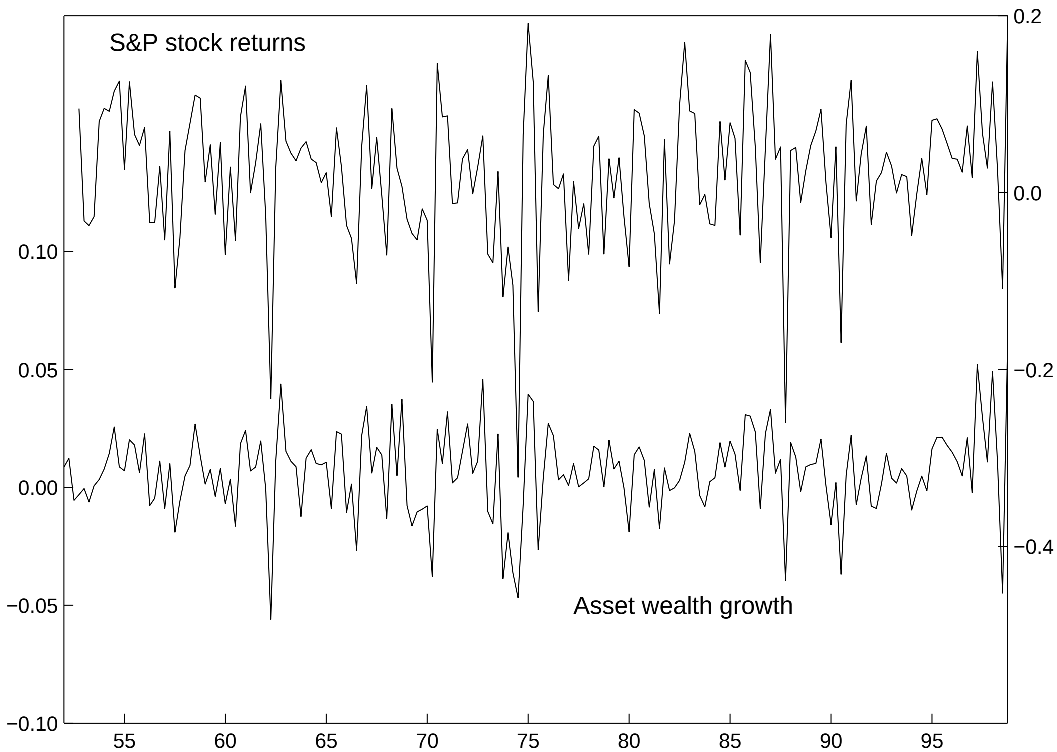
Figure 2: Consumption and asset wealth in the USA: growth rates (left panel) and logs (right panel)



1. It suggests that the adjustment dynamics follows the cyclical patterns of asset markets, as recognised by Lettau and Ludvigson (2004, p. 291). This is natural, given the presence of a_t in the long-run relationship. The "bull markets" of the late 1960s and late 1990s, for example, are clearly identified as periods where wealth seems to be above its equilibrium path. Notice also that these cycles are irregular, thus implying that equilibrium is most likely being restored in an asymmetric fashion.

On the other hand, a more detailed inspection of the results of the linear VECM reveals some potential specification problems. Table 1 reports results of maximum likelihood estimation of a first-order VECM, as well as of standard single and multi-equation specification tests, using the package PC-GIVE. The order of the VECM was chosen to be 1 by all tests and information criteria employed. In addition, we report heteroskedastic and autocorrelation consistent (HAC) asymptotic standard errors, computed with the plug-in procedure and the Quadratic Spectral kernel, as suggested by Andrews and Monahan (1992). This table is comparable to Table 1 in Lettau and Ludvigson (2004).

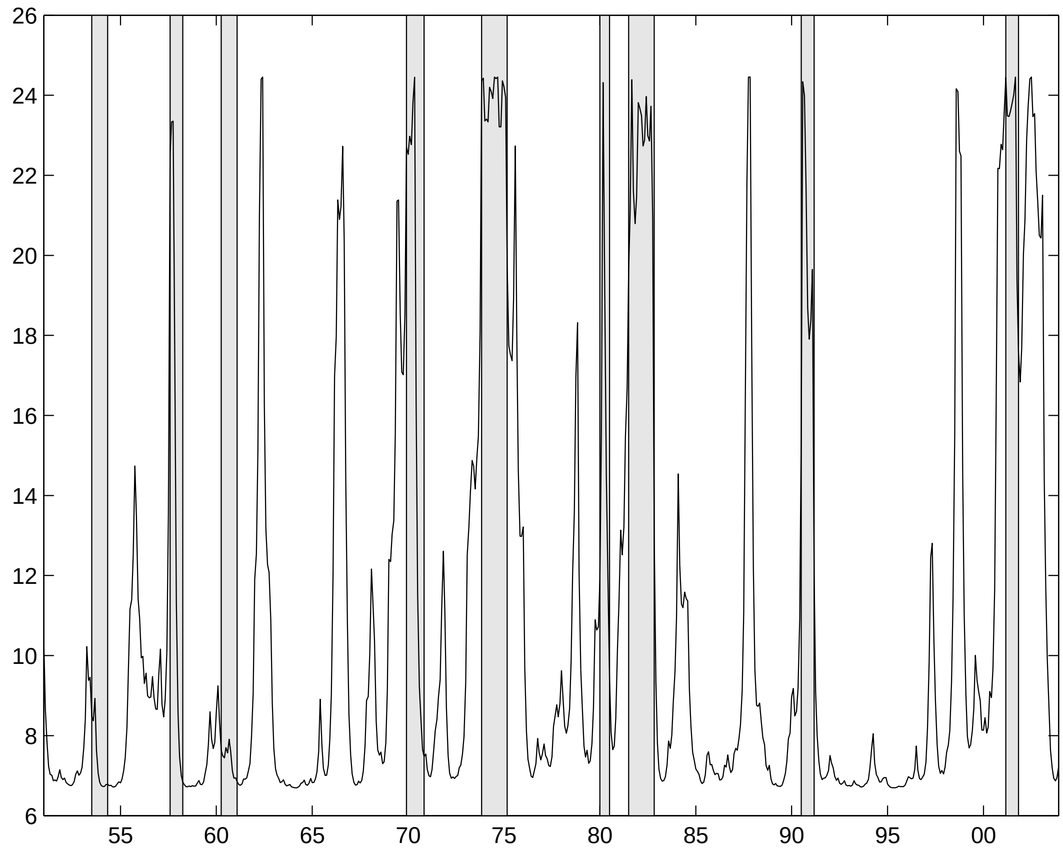
Figure 3: Standard &Poor stock returns and asset wealth growth



Analysing the results of the specification tests, it is clear that the estimated model appears to suffer from problems on all counts. Looking at individual equations, the LM test for autocorrelation up to 5 lags points to problems in the consumption equation, while heteroskedasticity (as revealed by a White test) and ARCH (LM statistic) mainly affects the wealth equation. Moreover, a Jarque-Bera test for normality indicates that the assumption of normal errors is violated. If the whole system is considered, the conclusions appear to be the same. Therefore, the use of HAC standard errors seems justified. Notice that, although the conclusions of L&L are not altered, the t-ratio (2.228) of the adjustment coefficient associated with wealth growth is significantly lower.

A possible explanation for these results lies in the stochastic properties of the variables in the system. Take, for example, consumption and wealth. It is clear from Figure 2, which represents the levels and growth rates of these variables, that a linear specification is hardly compatible with the exhibited dynamics. The reasonably stable path of consumption contrasts with that of

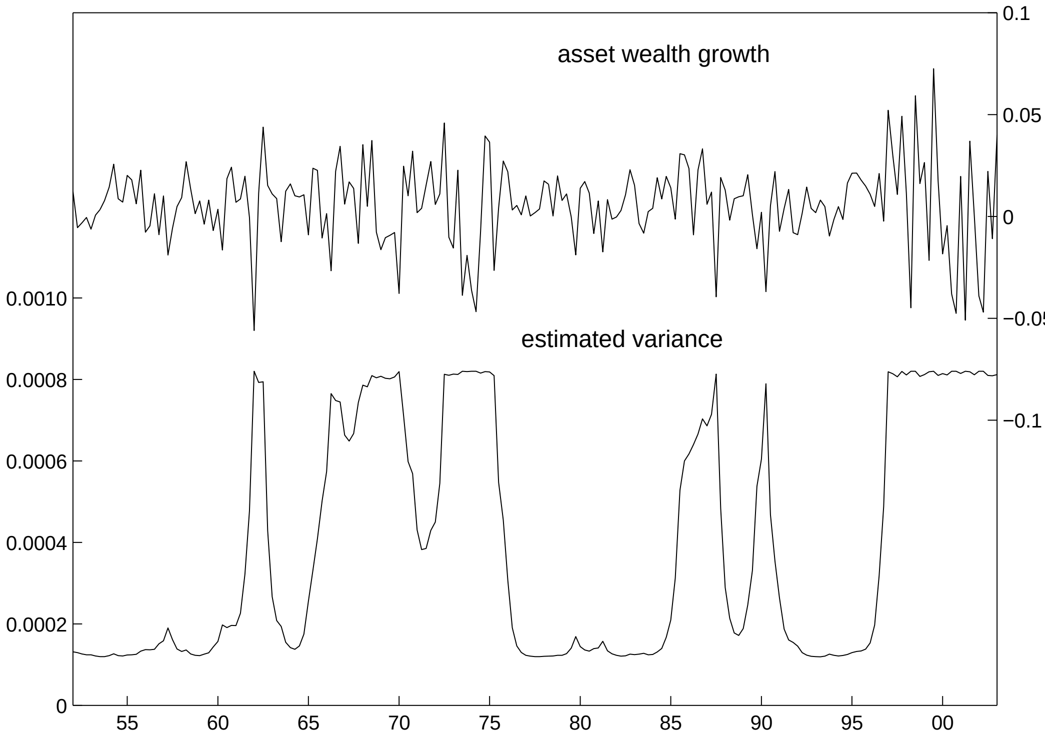
Figure 4: Variance of S&P excess returns estimated by a model with Markov switching mean and variance (NBER recession dates in shaded areas)



asset wealth. On the other hand, changes in asset wealth are closely linked to movements in stock market returns, as can be seen in Figure 3, depicting the log difference of quarterly asset wealth and stock returns from the Standard & Poor's Composite Index, making the similarities quite visible (the correlation is close to 0.9). Therefore, and given that the behaviour of financial markets, in particular returns volatility, is well characterised by regime switching (see the references above), we can expect asset wealth to follow a similar time-varying behaviour (this fact is acknowledged by Lettau and Ludvigson, 2004, p. 277, but not explicitly accounted for).

Using a simple two-regime mean-variance switching representation to describe

Figure 5: Asset wealth in the USA: growth and volatility (variance of asset wealth growth estimated by a model with Markov switching mean and variance)



the changing behaviour of equity returns,[2] $r_t = \mu_{s_t} + \sigma_{s_t}\varepsilon_t$, we plot in Figure 4 the corresponding estimated variance ($p_{1,t}\sigma_1^2 + p_{2,t}\sigma_2^2$, where $p_{s,t}$ is the smoothed probability of state s in period t), together with NBER recession dates. As expected, we observe large swings in the variance of stock returns, with periods of higher volatility usually shorter than low volatility ones. Also, as noticed before by Hamilton and Lin (1996), many volatility spikes coincide with the largest contractions in the US economy. Applying a similar model to the first difference of log asset wealth, we see in Figure 5 the time-varying nature of asset wealth growth volatility. This simple description is able to pick up the major periods of high volatility captured by the model of equity returns

[2]Excess returns are computed as the log difference of the S&P 500 monthly stock index, plus the corresponding dividend yield minus the yield on 3-month Treasury Bills, quoted at monthly rates — see the accompanying files to the paper for more details and results.

(with the exception of the early 80s downturn). Naturally, the latter variable is subject to much larger and more frequent changes, but apart from some short-lived spikes in excess returns, the high volatility regimes for the two variables are quite comparable. Thus, it is likely that these features will have a non-negligible impact on the consumption-wealth ratio fluctuations. We discuss possible ways to account for these properties in the next section.

3. Regime Switching and Consumption

The literature on the interconnection between asset prices, wealth and consumption has been stressing two main issues. On the one hand, the short-run variance of asset wealth is essentially driven by asset-price volatility — see, for example, Poterba (2000). On the other hand, numerous papers have provided empirical evidence suggesting that stock market volatility may be far greater than can be justified by fundamentals — see, among others, Shiller (1989) and Campbell and Ammer (1993).

Rational bubbles, herd behaviour, fashions and fads have been suggested as explanations for episodes where misalignments of asset prices from their fundamental value seem to have appeared. Fads may be the result of waves of excessive pessimism or optimism. In fact, financial markets are known to experience "changes of mood", which some authors have modelled through regime switching — see, e.g., Driffill and Sola (1998). It is possible that this behaviour may be related to the finding (e.g. Mehra, 2001) that estimates of the wealth effect of asset prices on consumption also seem to vary significantly according to the time span and measures of asset wealth considered.

Guidolin and Timmermann (2005) argue that the optimal consumption behaviour of an investor depends on the nature of the regime switches of asset returns. In this paper, we illustrate this implication by means of a simple model of consumption behaviour under uncertainty concerning the nature of the driving force of asset wealth. In particular, in our model shocks to asset wealth are either transitory or permanent, but the agent may misinterpret them. Transitory shocks may be seen as representing misalignments from the equilibrium value as a result of fads or some kind of irrational exuberance. Permanent shocks represent changes in fundamentals such as a change in productivity growth. In this case, a permanent shock modifies the equilibrium value of asset prices and is accompanied by larger future dividends.

Assume that a consumer lives for two periods. In the first period there is a shock (ε_1) to the consumer's wealth as a result of an increase in asset prices. However, the consumer is unsure whether the shock is permanent or temporary, i.e., whether there will be an offsetting shock in the second period (ε_2). The problem of the consumer is to maximise expected life-time utility as of the first period:

$$\max E_1 \left[u \left(C_1 \right) + u \left(C_2 \right) \right], \tag{3.1}$$

subject to the budget constraints:

$$C_1 + A_1 = L_1 + A_0 + \varepsilon_1 \tag{3.2}$$

and

$$C_{2,s} = L_2 + A_1 + \varepsilon_{2,s}, \quad s = 1, 2, \tag{3.3}$$

where C_1 is consumption in the first period, $C_{2,s}$ is consumption in the second period when the second-period shock takes the value $\varepsilon_{2,s}$, A_0 is the initial asset wealth of the consumer, A_1 is asset wealth at the end of period 1/beginning of period 2 (excluding the second-period shock) and L_i is labour income in period i. The model incorporates several simplifications to allow the results to come through as clearly as possible; for instance, there is no time discounting and inflation is zero (all variables are in real terms). The consumer has to choose consumption and asset holdings in the first period, and consumption in the second period contingent on the second-period shock. The life-time budget constraint is:

$$C_1 + C_{2,s} = A_0 + L_1 + L_2 + \varepsilon_1 + \varepsilon_{2,s}, \quad s = 1, 2. \tag{3.4}$$

If the shock were temporary, call it state 1, then $\varepsilon_2 = \varepsilon_{2,1} = -\varepsilon_1$ and therefore lifetime wealth would be what it would have been in the absence of any shock: $A_0 + L_1 + L_2$. If the shock to wealth were permanent, call it state 2, then $\varepsilon_2 = \varepsilon_{2,2} = 0$. Given the second-period values, equation (3.3) implies $C_{2,2} = C_{2,1} + \varepsilon_1$.

Letting u_i denote the marginal utility of consumption in period i (as usual, assumed to be a decreasing function), the first-order conditions of the maximisation problem imply:

$$u_1 = E_1 \left(u_2 \right) \tag{3.5}$$

Note that we are implicitly keeping asset returns constant at unity, which may seem odd in the context of our study. This result is a consequence of assuming additive shocks and no discounting, which greatly simplifies the derivations and allows us to reach clear-cut conclusions. Moreover, it permits us to make use of a natural distinction between "temporary" and "permanent" shocks, as we did above.

Let P be the probability that the consumer assigns to the occurrence of state 2 and let $u_{2,i}$ denote the marginal utility of consumption in the second period in state i. The previous equation can be written as:

$$u_1 = (1 - P)\, u_{2,1} + P u_{2,2} \tag{3.6}$$

If the consumer correctly believes that the shock is permanent ($\varepsilon_2 = 0$, $P = 1$), then equation (3.6) becomes $u_1 = u_{2,2}$ and therefore $C_1 = C_{2,2}$, i.e., consumption in the first period will adjust fully to the new "long-run" value. In case the shock is wrongly believed to be permanent ($\varepsilon_2 = -\varepsilon_1$, $P = 1$), the consumer will first increase consumption and later, after the mistake is known, will decrease it. If the shock were correctly believed to be temporary ($\varepsilon_2 = -\varepsilon_1$, $P = 0$), then the consumer would not react to it. Instead, asset wealth would temporarily increase in the first period and then return to normal in the second period, i.e., wealth would be doing all of the adjustment. In the case where the consumer wrongly believes the shock to be temporary ($\varepsilon_2 = 0$, $P = 0$), the consumer will let wealth adjust in the first period. In the second period, after realising the true nature of the shock, the consumer will adjust consumption.

The message of this simple model is that the adjustment of consumption and wealth to shocks, and their relation with the consumption-wealth ratio, will depend on the nature of those shocks and on how they are perceived by the consumer. For instance, if an increase in wealth is temporary, and seen as such, the consumption-wealth ratio will initially decrease as a result of that increase in wealth. In this case, this change in the consumption-wealth ratio will signal a future decline in wealth, which will restore the long-run equilibrium, after the temporary nature of the shock reveals itself. On the contrary, if the shock is permanent, but viewed as temporary, then the consumption-wealth ratio will initially decrease (as a result of the increase in wealth), but subsequently it is consumption that will increase, i.e., in this case the movement in the consumption-wealth ratio would forecast the change in consumption.

If the nature of the shocks varies over time (probably accompanying changes in the state of financial markets), then the implications of the foregoing analysis are clear: the adjustment of consumption and wealth to shocks should be modelled with a nonlinear specification to accommodate changes in the dynamics, such as the ones described above.

The model we presented above is an extreme simplification of a more standard model of consumer behaviour — see the Appendix. The main difference is that in the model above either consumption or wealth adjusts in the second period. In a less simplified model (namely, one in which shocks are not additive), both variables would adjust, but the importance of the relative change would depend on the case considered: when the shock is temporary and viewed as such, both variables decrease in the second period and the change in wealth is larger than the change in consumption; when the shock is permanent and viewed as temporary, the opposite is true. Therefore, the consumption-wealth ratio may forecast changes in both variables, but the direction and magnitude of the changes to be forecast will differ according to the context. Thus, a linear empirical model is likely to give an incomplete, or even misleading, representation of the dynamics of the variables.

There already exists empirical work supporting the view that temporary and permanent asset wealth shocks lead to different consumption dynamics: Sousa (2007) estimates that variations in house prices may be associated with significant changes in consumption because a great component of the variation of housing wealth is permanent. The same author shows that large fluctuations in financial assets, which are mainly transitory, do not result necessarily in movements in consumption.

In the next section, we consider a formal approach to testing for nonlinear adjustment. We also introduce a multivariate Markov-switching representation of the trivariate relationship studied by L&L. This representation will be estimated and tested in section 5.

4. Testing for Asymmetric Adjustment

Following the discussion above, in this section we investigate the possibility of asymmetric adjustment in the consumption-wealth linkage. There is a difficulty in casting the testing problem in the usual framework (null of no cointegration vs. null of nonlinear cointegration), as some parameters will not be identified under the null. We follow the multi-step approach suggested in Psaradakis et al. (2004) to detect nonlinear error-correction.

As a first step, conventional procedures to establish the "global" properties of the series (such as unit root and cointegration tests) remain valid, as long as regularity conditions are obeyed (even though the deviations from the long-run equilibrium may be nonlinear). Once cointegration between the variables is discovered, a second step follows, focusing on the potential nonlinear "local" characteristics of the system, by looking at either the equilibrium error (in our case $cay_t = c_t - 0.3a_t - 0.6y_t$), or the associated error-correction model for signs of nonlinear adjustment. This task may be carried out by using a range of tests that include parameter instability tests (for example, those of Hansen, 1992b, or Andrews and Ploberger, 1994), general tests for neglected nonlinearity (e.g., RESET, White, Neural Networks) or nonlinearity tests designed to test linear adjustment against nonlinear error-correction alternatives, such as Markov switching (Hansen, 1992a) and threshold adjustment (Hansen, 1997, and Hansen, 1999). Moreover, and as suggested by Psaradakis et al. (2004), we may also resort to conventional model selection criteria such as the AIC (or BIC and Hannan-Quinn criteria), which was found to perform well in these circumstances.

If the analysis of the "local" features of the data points to nonlinearity, then a third step ensues, in which one should fit a MS model, either to cay_t or to the error-correction representation. However, in the case considered here, the results in L&L indicate that wealth does most of the adjustment towards equilibrium, meaning that a single-equation ECM with consumption as the dependent variable would be misspecified. Thus, one needs to analyse the whole system, which implies that a Markov-switching vector error-correction model should be employed instead.

Camacho (2005) shows that if the equilibrium errors z_t of a generic cointegrated system for the $m \times 1$ vector x_t follow a MS-(V)AR,

$$z_t = c_{s_t} + A_{s_t}(L)z_{t-1} + \theta_{st}\varepsilon_t \tag{4.1}$$

where c_{s_t} is the vector of Markov switching intercepts, $A_{s_t}(L) = (A^1_{s_t} + ... + A^p_{s_t}L^{p-1})$ and $\varepsilon_t|s_t \sim N(0, V_\varepsilon)$, then there is a corresponding MS-VECM representation

$$\Delta x_t = \mu_{s_t} + \Gamma_{s_t}z_{t-1} + \Pi_{s_t}(L)\Delta x_{t-1} + \sigma_{st}u_t \tag{4.2}$$

where Π_i's are $m \times m$ coefficient matrices, μ_{s_t} is a vector of intercepts, $u_t|s_t \sim N(0, V_u)$ and Γ_{s_t} is a regime-dependent long-run impact matrix. Indeed, the nonlinear dynamics of the equilibrium errors z_t may lead to a switching adjustment matrix Γ and to short-run dynamics of the endogenous variables (given by Π) that vary across regimes. Several possibilities may arise, including one where cointegration switches on and off, for example. The system may be estimated by a multi-equation version of the Hamilton filter and estimates of the (possibly different) adjustment coefficients obtained.

The second panel of Table 1 revisits the results in Lettau and Ludvigson (2004) regarding the long-run properties of the system, confirming that there is indeed cointegration among consumption, labour income and asset wealth, judging by the results of Johansen cointegration tests. Next, we focus on the local properties of the system. Using the estimated equilibrium error cay_{t-1}, we fit an over-parameterised linear $AR(p)$ for cay_{t-1} (initially with 4 lags, then tested down to 1), which was found to be an $AR(1)$ with autoregressive

Table 2: Stability and linearity tests of cay_t

Linear AR(1) model — specification tests				
Instability		Threshold		RESET
L_c (joint)	0.843	sup LM	9.943 [0.061]	1.755 [0.187]
L_c (var.)	0.541^{**}	avg LM	2.825 [0.064]	
avg F	3.305	exp LM	4.977 [0.043]	White
sup F	14.554^{**}	F_{12}	10.51 [0.061]	3.59 [0.029]
exp F	3.465^{**}	[bootstrapped p-values]		
Results from MS-AR(1) estimation				
$\mu_1 = 0.001$	$\phi_1 = \underset{(12.699)}{0.754}$	$\phi_2 = \underset{(8.374)}{0.826}$	$\sigma_1 = \underset{(7.315)}{0.059}$	$\sigma_2 = \underset{(3.302)}{0.101}$
$p_{11} = \underset{(60.91)}{0.981}$	$p_{22} = \underset{(13.652)}{0.931}$	LogL: 918.4	sup LR [p-value upper bound]	: 17.225 [0.022]
AIC MS model: −1820.7		AIC linear model: −1813.5		

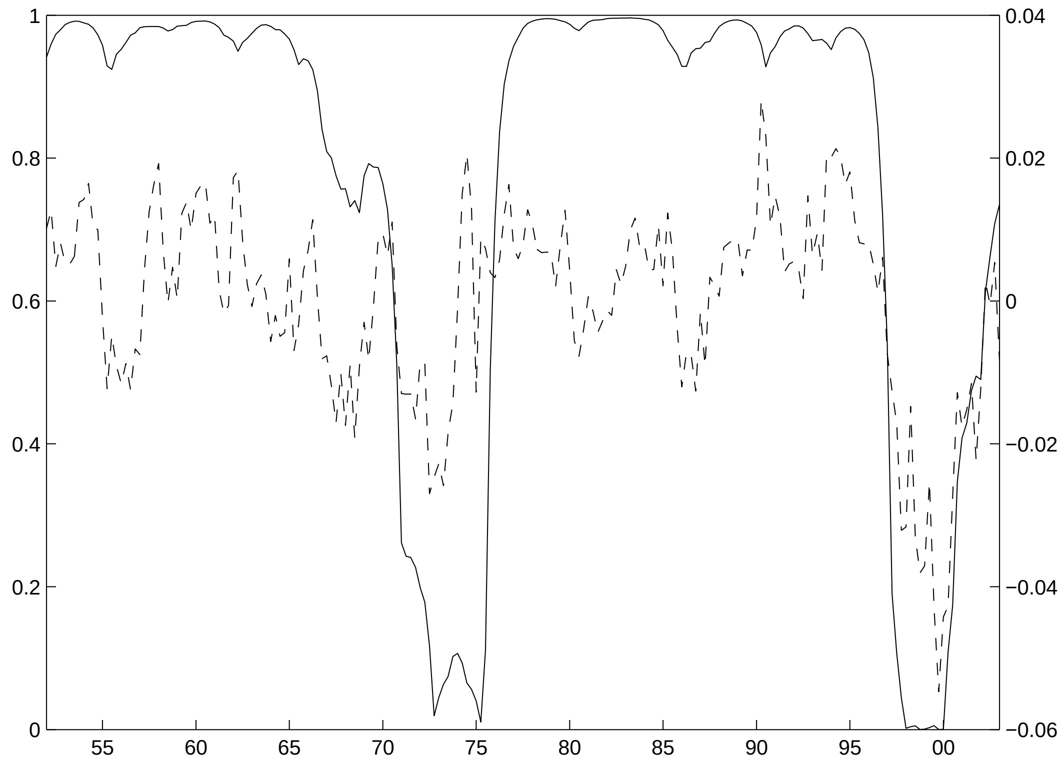
coefficient $\hat{\phi} = 0.851$. Then, we test for neglected instability and nonlinearity in this specification. The statistics include the L_c test of Hansen (1992b) against martingale parameter variation, Andrews and Ploberger (1994) sequential tests, the White test and the RESET test. Furthermore, Carrasco (2002) shows that tests for threshold effects will also detect MS behaviour, so we employ Hansen (1997) threshold tests. As recommended by Hansen (1999), we use bootstrapped p-values.

Results are presented in Table 2. Some procedures fail to reveal mis-specifications, namely the RESET test, the L_c test for joint stability and the avg F test. However, all other tests reject their respective nulls at the 5% or 10% significance levels, so, overall, the evidence for nonlinear behaviour is sufficiently compelling.

Due to computational difficulties, we do not use the Hansen (1992a) test.[3] Nevertheless, the standard likelihood ratio (LR) of linear specification against the estimated MS-AR(1) model favours the latter (although the usual asymptotic distribution for the LR statistic is not strictly valid). Thus, we compute the upper bound on the significance level of the test using the approach in Davies (1987), which confirms the initial result. Alternatively, using Garcia

[3]The test involves defining a five dimensional grid for $\{\mu, \phi, \sigma, p, q\}$, with a total g^5 grid points considering g points for each parameter. The size of the grid, combined with a ill-behaved likelihood function (a considerable proportion of grid points did not achieve convergence) meant that the computational time would be prohibitive: results could only be obtained after several days and with little guarantee of being reliable.

Figure 6: cay_t (right scale, dashed line) and smoothed probabilities estimated by a Markov switching AR(1) model with switching mean and variance (left scale, state 1 — cf. Table 2)



(1998)'s critical values (Table 3, for the case $\phi = 0.8$) as an approximation for the distribution LR test, the same conclusion emerges. The bottom panel of Table 2 reports results on the estimation of a MS-AR(1) with changes in mean and variance for cay_{t-1}, while Figure 6 depicts the corresponding regime probabilities against cay_{t-1}. It is apparent that the MS model is picking up distinguished periods of large and volatile deviations from equilibrium. Thus, and following Camacho (2005), one should investigate the error-correction representation of the system, which is likely to offer a more complete description of the dynamics of the relationship.

5. A MS-VECM for the Consumption-Wealth Ratio

In order to estimate a Markov-switching vector error-correction model for the consumption-wealth ratio, one must consider carefully the dimension of the model. Indeed, even in a simple trivariate system, if all parameters are allowed to switch, identification problems may occur and estimation will be intractable. Hence, we opt to restrict matrix Π in (4.2) to be constant across regimes. Additionally, we follow L&L in estimating a first-order VAR system. More importantly, we specify Γ_{s_t} in (4.2) as a regime-dependent long-run impact matrix defined as

$$\Gamma_{s_t} = \alpha_{s_t}\beta \tag{5.1}$$

with cointegration vector β and adjustment matrix α_{s_t}. Note that we assume an invariant long-run relationship, while allowing the adjustment towards equilibrium to be state-dependent. This has a plausible interpretation, since it is consistent with both the theoretical framework discussed in section 2 and the empirical evidence concerning the "global" statistical properties of the consumption-wealth ratio discussed in section 3. Furthermore, modelling the weighting matrix as state-dependent implies that shocks to any of the three variables can have different effects across regimes through α_{s_t}, even in the long run. For example, shocks to asset wealth can have different effects on consumption depending on whether markets are in a boom or in a recession, or, alternatively, whether these shocks are permanent or temporary. In addition, the coefficients in α_{s_t} can also capture the speed at which agents learn the nature of the shocks.

Thus, we initially allow μ and Γ in (4.2) to be state-dependent (as well as the variance of the error term), and then exploit potential parameter restrictions in order to achieve a more parsimonious MS-VECM specification. The model to be estimated is therefore

$$\Delta x_t = \mu_{s_t} + \gamma_{s_t} cay_{t-1} + \pi(L)\Delta x_{t-1} + \sigma_{st} u_t, \tag{5.2}$$

where $x_t = \{c_t, a_t, y_t\}$, with 35 parameters. Estimation is carried out in GAUSS, using the multi-equation version of the Hamilton filter, as explained in Camacho (2005).

Table 3 displays results of the estimation of (5.2), using heteroskedasticity-robust standard errors based on the Outer-Product-Gradient matrix. We begin by noting that the results indicate the existence of two regimes with different characteristics. The most striking difference is that in the first regime it is asset wealth that reacts to *cay*, as in the linear model, albeit at a faster rate

Table 3: MS-VECM(1) estimates

	State 1		
Equation	Δc_t	Δa_t	Δy_t
μ (intercept)	0.0037 (6.569)	0.0089 (2.205)	0.0048 (2.871)
$\widehat{cay}_{t-1}$	0.0129 (0.618)	0.4784 (2.263)	0.0722 (1.059)
	State 2		
Equation	Δc_t	Δa_t	Δy_t
μ (intercept)	0.0041 (5.428)	0.0049 (1.978)	0.0023 (1.738)
$\widehat{cay}_{t-1}$	-0.1361 (-2.011)	0.3021 (1.293)	0.0328 (0.266)
	Short run dynamics		
Equation	Δc_t	Δa_t	Δy_t
Δc_{t-1}	0.2206 (3.048)	-0.186 (-0.196)	0.50 (3.346)
Δa_{t-1}	0.0424 (3.256)	0.1287 (1.893)	0.093 (2.672)
Δy_{t-1}	0.0485 (1.295)	0.0172 (0.427)	-0.1056 (-1.039)
(t-ratios based on heteroskedasticity-robust standard errors)			
Log-Lik.	AIC	BIC	HQ
-2669.521	-5269.043	-5151.567	-5221.936

(0.478 against 0.33), while in the second regime it is the adjustment coefficient on consumption growth that is significant (negative coefficient of -0.136). This, of course, contrasts with the results for the linear model, which does not allow for switching adjustment. On the other hand, note that the estimated Π matrix presents values similar to those found for the linear model, which suggests that the restrictions imposed may be valid.

As in the previous section, it is not straightforward to test the appropriateness of the MS-VECM over the linear model. A likelihood ratio test of a linear vs Markov specification is clearly favourable to the MS model, producing 77.224 with an upper-bound p-value of 0.000. This test is not usually valid, since the regularity conditions that justify the usual χ^2 approximation do not hold. However, the very large value of the statistic seems to offer support to the MS model. In addition, all of the model selection criteria favour the MS-VECM specification, when compared to those in Table 1. Although the transition probabilities ($p_{11} = 0.927$, $p_{22} = 0.952$) are estimated imprecisely (standard

Table 4: Restricted MS-VECM(1) estimates

	State 1			State 2		
Equation	Δc_t	Δa_t	Δy_t	Δc_t	Δa_t	Δy_t
$\widehat{cay}_{t-1}$	–	0.3662 (2.166)	–	−0.1328 (−3.523)	–	–

Intercepts and short run dynamics

Equation	Δc_t	Δa_t	Δy_t
μ	0.0038 (8.43)	0.0071 (5.667)	0.003 (3.467)
Δc_{t-1}	0.2137 (3.007)	−0.100 (−0.238)	0.506 (3.439)
Δa_{t-1}	0.041 (3.100)	0.0981 (1.572)	0.0823 (2.577)
Δy_{t-1}	0.0459 (1.115)	0.0172 (0.425)	−0.108 (−1.032)
$p_{11} =$	0.9174 (2.415)	$p_{22} =$	0.9475 (1.99)

(t-ratios based on heteroskedasticity-robust standard errors)

Log-Lik.	AIC	BIC	HQ
−2664.319	−5272.637	−5179.457	−5223.952

errors of 0.631 and 0.60), a multi-equation version of a Hamilton-White test of Markov specification (see Hamilton, 1996) with a p-value of 0.70 reveals that the Markov assumption should not be rejected. Nevertheless, there seems to be scope for simplification through the imposition of restrictions on redundant parameters.

Thus, in order to arrive at a more parsimonious specification, we employ a sequence of LR tests on model (5.2), based on the statistic

$$2\left(\log L_U - \log L_R\right) \sim \chi^2(n) \tag{5.3}$$

where L_U is the likelihood of the unrestricted model, L_R is the likelihood of the restricted model and n is the number of restrictions imposed.

We start by estimating a restricted version in which the non-significant adjustment coefficients in state 1 and state 2 are zero, achieving a log-likelihood of −2666.91 and producing a LR statistic with a p-value of 0.27 from a $\chi^2(4)$ distribution. In addition, imposing equal intercepts across regimes delivers a log-likelihood of −2664.319, with a LR test supporting these restrictions (p-value of 0.17 from a $\chi^2(7)$). Hence, our final model has 28 parameters in total, with estimates presented in Table 4. Notice that both the regime probabilities and the adjustment coefficients are now estimated much more precisely. The short-run matrix displays practically the same values, as well as the consumption adjustment coefficient, while the wealth adjustment parameter is now closer to the value in the linear model. Again, the Hamilton-White Markov specification test produces a p-value of 0.68, confirming the superiority relatively to the linear model. All model selection criteria continue to favour the restricted model.

Overall, it seems that the MS-VECM captures the main dynamic features in the trivariate system, and does that better than a linear VECM. Our findings also suggest that short-term deviations in the relationship will forecast either asset returns or consumption growth, depending on the state of the economy. These results differ from the conclusions of L&L, but note that the theoretical relationship in (2.3) does not preclude our findings. Indeed, fluctuations in *cay* may be related to future values of either r_t, Δc_t or Δy_t. We believe our results allow us to make an empirical point: if we allow for nonlinear adjustment, the data reveals two possible channels to restore equilibrium, that will be "switched on/off" according to the phase of the business cycle.

The smoothed probabilities of the second regime,[4] depicted in Figure 7, pick up very well the phases that one usually associates with "turbulent" markets. Indeed, these probabilities display a pattern that matches almost exactly the behaviour of asset wealth growth volatility in Figure 5. This fact appears to indicate that the regime switching in the system is being driven by asset wealth (and, therefore, by financial markets).

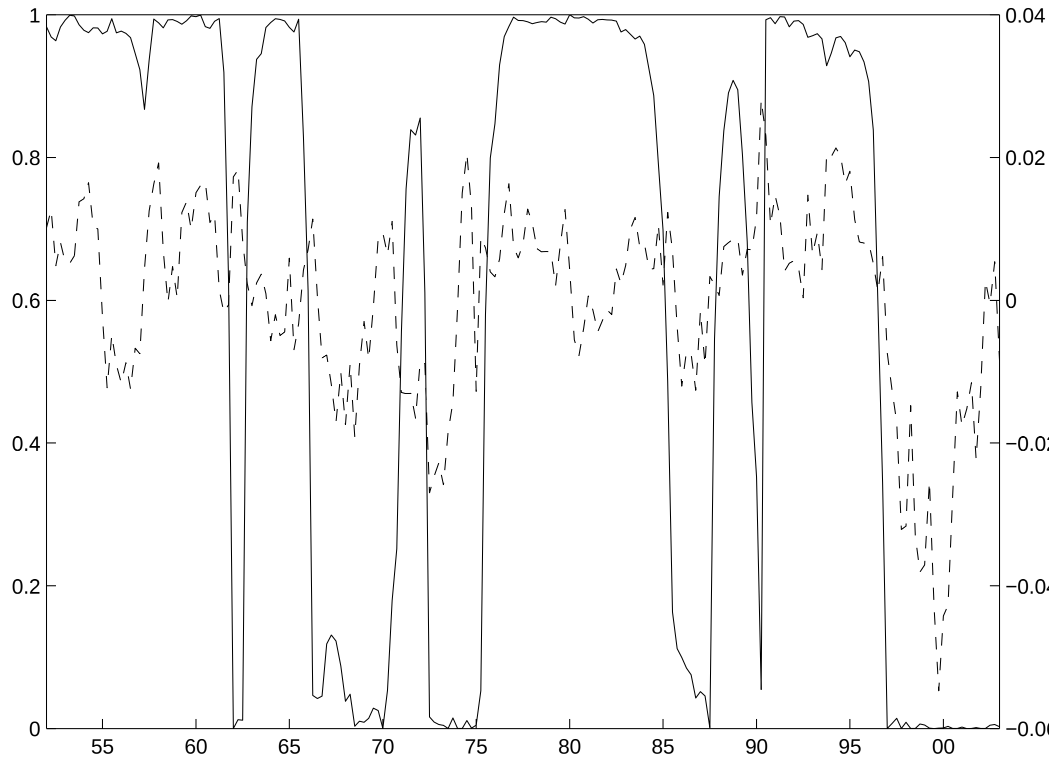
In view of the previous discussion, a possible interpretation of regime 1 is that in this state consumers are able to recognise periods of transitory growth in wealth. In accordance with the theoretical models discussed in sections 2 and 3, consumers let wealth vary until it eventually returns to its equilibrium path and the long-run equilibrium is restored. In state 2, consumption does adjust: when the consumers adjust their views on the nature of variations in wealth, they adjust their consumption paths accordingly. Thus, the results derived from the MS-VECM seem to be interpretable in the light of simple models of consumption, such as the one in section 3, which suggest varying adjustment dynamics.

6. Concluding Remarks

The behaviour of consumption is one of the most studied issues in economics. It is a matter of importance to policy-making, especially in an era in which

[4]These are very similar those obtained with the unrestricted MS-VECM, not reported here.

Figure 7: cay_t (right scale, dashed line) and smoothed probabilities estimated for regime 2 of the MS-VECM (left scale, cf. Table 4)



a consensus appears to have emerged concerning the desirability of keeping inflation low. The extraordinary movement in asset prices in the late 1990s raised the problem of knowing whether it heralded a new period of high inflation, due to demand pressures fuelled by the "wealth effect" of asset prices on consumption. In face of this, the traditional linear model of consumption and wealth, as the one discussed at length by Lettau and Ludvigson, reveals an intriguing picture: a picture in which consumption appears not to adjust to deviations of the consumption-wealth ratio from its long-run trend; instead, wealth does all the adjustment.

Theoretical models of consumption suggest that consumption should react to movements in wealth. We have shown that the reaction depends on whether the shocks are viewed as more likely temporary or more likely permanent, which in turn should depend on the state of financial markets. Based on this insight, we estimated a Markov-switching vector error-correction model of consumption, labour income and asset wealth.

Our theoretical and empirical models deliver results consistent with those of the reference papers, such as Lettau and Ludvigson (2001, 2004), provided one takes into account the fact that the financial markets seem to go through different regimes. L&L conclude that most of the variation in wealth is transitory and unrelated to variations in consumption. The theoretical model discussed in this paper leads to the same conclusion: when the shock to wealth is transitory, the consumption-wealth ratio should forecast the subsequent change in wealth. However, when the change in wealth is permanent but initially viewed as temporary, the theoretical model predicts that consumption could be forecast by the consumption-wealth ratio. Our empirical model allows for these different adjustment dynamics and therefore nests that of L&L. Unsurprisingly, our model provides a better description of the data than the traditional linear model. Namely, as mentioned above, it helps to explain recent controversial results, concerning the adjustment of the variables to deviations from the long-run equilibrium and the forecasting ability of the system.

Our interpretation of the results rests on the possibility of the agents incorrectly viewing permanent shocks as temporary. Many of the references cited in sections 1-3 argue that agents in financial markets do make mistakes. Naturally, a two-regime view of the world, such as the one resulting from our estimations, is perhaps too simplistic. Future research should evaluate whether the estimates and the interpretation we offer in this paper are robust to alternative models, samples, estimation methods, datasets, etc., and consistent with other pieces of evidence.

7. Appendix

7.1 Base scenario

In this Appendix we present a model of consumer behaviour in an attempt to show how the mechanism described in section 3 of the paper might work in a more realistic setting.

Assume the household maximizes the standard lifetime utility function:

$$\sum_{t=0}^{\infty} \beta^t u(c_t), \quad u(c_t) = \frac{c_t^{1-\sigma} - 1}{1-\sigma}, \quad \sigma > 1, \quad 0 < \beta < 1 \tag{7.1}$$

subject to the budget constraint

$$c_t + A_t Q_t = A_{t-1}(Q_t + D_t) \tag{7.2}$$

In the expressions above, c_t is consumption and A_t is the quantity of assets bought in period t, which cost Q_t and will pay dividends D_{t+1} in $t+1$. To simplify the derivations, we ignore "labour income". Introducing labour income, besides making the derivations much more complex, would make the results presented below depend on the actual values of the parameters. It would also raise the issue of whether labour supply should be modelled as exogenous or endogenous, which might influence the results. Note also that we do not use expectations notation. To study the extreme (and clearer) cases discussed in the paper, we assume the agent believes she has perfect foresight.

The first-order condition is thus:

$$u'(c_t) = \beta u'(c_{t+1}) \frac{Q_{t+1} + D_{t+1}}{Q_t} \tag{7.3}$$

This implies that, in steady state,

$$\frac{Q}{Q+D} = \beta \Leftrightarrow Q = \frac{\beta D}{1-\beta} \tag{7.4}$$

If the agent expects D_t (Q_t) to be constant and equal to D_0 $(Q_0 = \beta(1-\beta)^{-1}D_0)$, the budget constraint gives:

$$c_0 = A_{-1}D_0 \tag{7.5}$$

The intertemporal budget constraint is:

$$\sum_{t=0}^{\infty} c_t \phi_t = A_{-1}(Q_0 + D_0) \tag{7.6}$$

where

$$\phi_0 = 1, \phi_1 = \frac{Q_0}{Q_0 + D_0} = \beta, \phi_t = \left(\frac{Q_0}{Q_0 + D_0}\right)^t = \beta^t \tag{7.7}$$

which leads to another version of equation (7.5):

$$\frac{c_0}{1-\beta} = A_{-1}(Q_0 + D_0) \tag{7.8}$$

The right-hand side of this equation is initial wealth (W_0). We thus have a permanent-income consumption function, in which consumption is a constant fraction of wealth. The consumption-wealth ratio is $1-\beta$.

For future reference, note that in the present scenario, the value function is:

$$V_0(A_{-1}) = \sum_{t=0}^{\infty} \beta^t u(c_0) = \frac{u(c_0)}{1-\beta} = \frac{u(A_{-1}D_0)}{1-\beta} \tag{7.9}$$

Note also that assets at the beginning of time $t = 1$ is $A_0 = A_{-1}$.

7.2 Temporary shock

The temporary shock in our model changes, in period $t = 1$, Q_0 and D_0 to $Q^* = Q_0\frac{\delta}{\beta}$ and $D^* = D_0\frac{\delta}{\beta}$, with $\delta > \beta$ (a boom). The following identities hold:

$$\frac{Q^*}{Q_0 + D_0} = \delta \tag{7.10}$$

$$\frac{Q^*}{Q_0} = \frac{\delta}{\beta} \tag{7.11}$$

$$\frac{Q^*}{D_0} = \frac{\delta}{1-\beta} \tag{7.12}$$

The value function is now given by:

$$V_1(A_{-1}) = \max\left\{u(c_1) + \beta V_0(A_1)\right\} = \max\left\{u(c_1) + \beta\frac{u\left(A_1 D_0\right)}{1-\beta}\right\} \tag{7.13}$$

with

$$A_1 = \frac{A_{-1}(Q^* + D^*) - c_1}{Q^*} \tag{7.14}$$

The first-order condition is:

$$u'(c_1) = \beta\frac{u'\left(A_1 D_0\right)}{1-\beta}\frac{D_0}{Q^*} \Leftrightarrow c_1^{-\sigma} = \frac{\beta}{\delta}\left(A_1 D_0\right)^{-\sigma} \tag{7.15}$$

which yields:

$$c_1 = \frac{1-\beta}{1 + \beta\left[\left(\frac{\beta}{\delta}\right)^{\frac{1}{\sigma}-1} - 1\right]}W_1 \tag{7.16}$$

Wealth in period $t = 1$ is:

$$W_1 = A_{-1}(Q^* + D^*) \tag{7.17}$$

Note that

$$W_1 = \frac{\delta}{\beta}W_0 > W_0 \tag{7.18}$$

i.e, wealth has increased following the temporary shock.

The consumption-wealth ratio when there is a temporary shock is:

$$\frac{1-\beta}{1 + \beta\left[\left(\frac{\beta}{\delta}\right)^{\frac{1}{\sigma}-1} - 1\right]} \tag{7.19}$$

which is less than $1-\beta$, since

$$\sigma > 1 \Rightarrow \left(\frac{\beta}{\delta}\right)^{\frac{1}{\sigma}-1} - 1 > 0 \tag{7.20}$$

This means that the relative change in consumption is less than the relative change in wealth, i.e., the impact of the temporary shock is larger on wealth than on consumption. In the main paper, with further simplifying assumptions, only wealth changes.

In period $t = 2$, asset prices and dividends revert to pre-shock values: $Q_2 = Q_0$ and $D_2 = D_0$. Note that period $t = 2$ is identical to period $t = 0$ except for the value of initial assets, which is A_1 instead of A_{-1}.

Wealth becomes:

$$\begin{aligned} W_2 &= A_1(Q_0 + D_0) & (7.21)\\ &= \frac{\left(\frac{\beta}{\delta}\right)^{\frac{1}{\sigma}}}{1+\beta\left[\left(\frac{\beta}{\delta}\right)^{\frac{1}{\sigma}-1} - 1\right]} W_1 & (7.22)\end{aligned}$$

Wealth in period $t = 2$ is less than W_1, since

$$\sigma > 0 \Rightarrow \left(\frac{\beta}{\delta}\right)^{\frac{1}{\sigma}} < 1 \tag{7.23}$$

Consumption will also decrease:

$$\begin{aligned} c_2 &= (1-\beta)W_2 & (7.24)\\ &= \left(\frac{\beta}{\delta}\right)^{\frac{1}{\sigma}} c_1 < c_1 & (7.25)\end{aligned}$$

Consumption in period $t = 2$ will nevertheless be larger than consumption in period $t = 0$:

$$c_2 = \frac{\left(\frac{\beta}{\delta}\right)^{\frac{1}{\sigma}-1}}{1+\beta\left[\left(\frac{\beta}{\delta}\right)^{\frac{1}{\sigma}-1} - 1\right]} c_0 > c_0 \tag{7.26}$$

Again, in absolute value, the relative change in consumption is less than the relative change in wealth. In the main paper, only wealth adjusted, implying that the movement in the consumption-wealth ratio in period $t = 1$ would be signalling the change in wealth. In the present model, the change in wealth will be larger than the change in consumption and thus the consumption-wealth ratio will *essentially* forecast the former rather than the latter.

7.3 Permanent shock

We now turn to the case where the shock in period $t = 1$ makes prices and dividends permanently change to $Q^* = Q_0 \frac{\delta}{\beta}$ and $D^* = D_0 \frac{\delta}{\beta}$. If the household realises the shock is permanent, both consumption and wealth adjust, with no change in the consumption-wealth ratio. The case we wish to analyse is the one where the household mistakes the permanent shock for a temporary shock.

To recapitulate, in the initial period, we have

$$c_0 = (1-\beta)A_{-1}(Q_0 + D_0) \tag{7.27}$$
$$W_0 = A_{-1}(Q_0 + D_0) \tag{7.28}$$
$$\frac{c_0}{W_0} = 1-\beta \tag{7.29}$$

In period $t = 1$, there is a shock to wealth that the household perceives as temporary. This shock increases wealth and decreases the consumption-wealth ratio:

$$W_1 = A_{-1}(Q^* + D^*) \tag{7.30}$$
$$\frac{c_1}{W_1} = \frac{1-\beta}{1+\beta\left[\left(\frac{\beta}{\delta}\right)^{\frac{1}{\sigma}-1} - 1\right]} < 1-\beta \tag{7.31}$$

At the beginning of period $t = 1$, A_1 is therefore:

$$A_1 = \frac{A_{-1}(Q^* + D^*) + y - c_1}{Q^*} \tag{7.32}$$
$$= \frac{\left(\frac{\beta}{\delta}\right)^{\frac{1}{\sigma}-1}}{1+\beta\left[\left(\frac{\beta}{\delta}\right)^{\frac{1}{\sigma}-1} - 1\right]} A_{-1} > A_{-1} \tag{7.33}$$

In period $t = 2$, the household realises the shock is permanent. The problem she solves is the same as in the base scenario, with Q^*, D^* and A_1 instead of Q_0, D_0 and A_{-1}:

$$c_2 = (1-\beta)A_1(Q^* + D^*) \tag{7.34}$$
$$W_2 = A_1(Q^* + D^*) \tag{7.35}$$
$$\frac{c_2}{W_2} = 1-\beta \tag{7.36}$$

Both wealth and consumption increase:

$$c_2 = \left(\frac{\beta}{\delta}\right)^{\frac{1}{\sigma}-1} c_1 > c_1 \tag{7.37}$$

$$W_2 = \frac{\left(\frac{\beta}{\delta}\right)^{\frac{1}{\sigma}-1}}{1+\beta\left[\left(\frac{\beta}{\delta}\right)^{\frac{1}{\sigma}-1}-1\right]} W_1 > W_1 \tag{7.38}$$

However, the relative change in consumption is now larger than the relative change in wealth. In this case, the decrease in the consumption-wealth ratio in period $t = 1$ would *essentially* forecast the change in consumption rather than the change in wealth. In this scenario, in the main paper, only consumption adjusted in the second period; wealth had already adjusted fully at the time of the shock.

We do not analyse the case in which the temporary shock is perceived as permanent. The reason is that in this case, as in the case where the shock is truly permanent, the consumption-wealth ratio would not change.

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