Monetary policy and welfare in a monetary union with labour market heterogeneity*

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Abstract

How should monetary policy be conducted in a monetary union when labour market structures differ across member countries as is arguably the case in the euro area? This paper develops a DSGE model of a two-country monetary union with labour market heterogeneity to answer this question. Asymmetries in labour market structures are proxied for by different degrees of nominal flexibility. A welfare loss function derived as a second-order approximation to household utility is evaluated. As is well-known from the previous literature, price inflation targeting may lead to welfare losses compared to monetary policy alternatives when the important nominal rigidity is in the labour market. Welfare may be improved in such a case by targeting wage inflation. This paper shows the mistake is larger when shocks are common to the monetary union than when they are country-specific due to an international externality. Also, the mistake is larger if labour market structures are very asymmetric, especially when shocks are highly correlated. Welfare improvements can be obtained by putting more weight on fighting wage inflation in the more rigid labour market if shocks are less than perfectly correlated.

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1 Introduction

This paper addresses the question of how monetary policy should be conducted in a monetary union in which labour market structures are different across member countries. It it well known that such labour market asymmetries are characteristic for the European Monetary Union (EMU), not least in terms of labour market flexibility, cf. for instance Arpaia and Pichelmann (2007) or Holden and Wulfsberg (2008) for recent documentation. While these labour market asymmetries are frequently mentioned in discussions of labour market policies, surprisingly little research effort has been devoted to understanding the implications of labour market asymmetries for monetary policy. In particular, little is known about how monetary policy should be designed in a monetary union with labour market asymmetries.[1]

This paper takes a first step in providing answers to this question. It develops a dynamic stochastic general equilibrium (DSGE) model of a two-country monetary union with monopolistic competition in labour markets and nominal wage rigidities. Labour market asymmetries are modelled by allowing the degrees of monopolistic competition and of nominal rigidity to differ across the monetary union's member countries. Disturbances to the union may be common across the member countries or idiosyncratic. The paper proceeds by evaluating the welfare of alternative monetary policy rules that the central bank may contemplate, emphasising the potential need to deal with labour market asymmetries across the union. Following the approach of Rotemberg and Woodford (1999), welfare is evaluated using a quadratic loss function derived from second-order Taylor approximations to the levels of expected household utility. This exercise is a first important step in understanding how monetary policy is best designed when monetary policy is common across structurally diverse, interdependent economic regions.

The current practice in most central banks pursuing an independent monetary policy is to emphasise the stabilisation of goods price inflation based on

[1] Abbritti and Mueller (2007) study optimal monetary policy in an asymmetric monetary union with unemployment, search frictions and real wage rigidities. Beetma and Jensen (2004) allow for labour market asymmetries in a model of a monetary union but focus on monetary and fiscal policy interaction. Dellas and Tavlas (2005) present a three-country model allowing for asymmetries in nominal wage flexibility, and find that countries with a high degree of nominal wage rigidity are better off in a monetary union. Fahr and Smets (2008) allow for asymmetries in downward wage rigidities in a study of the transmision of shocks in a monetary union with optimal monetary policy.

the development of a consumer price index. This includes the European Central Bank (ECB) responsible for monetary policy in the EMU. This practice finds theoretical justification in the New Keynesian literature in which nominal goods price rigidity is a distinguishing feature, see for instance Woodford (2003). In the basic New Keynesian model, nominal price rigidities lead to an inefficient allocation of resources in the economy unless the constraints represented by nominal rigidities do not bind. This happens in the model when inflation is stabilised.

As argued by Woodford (2003), allowing for nominal wage rigidities is probably not crucial if the objective of the analysis is to construct a positive model of the co-movements of output and inflation. But it is by no means obvious that nominal price rigidities are more important than nominal wage rigidities from an empirical perspective. Christiano et al. (2005), for instance, provide evidence of the contrary using US data. Nor does it appear to be empirically realistic to abstract from nominal wage rigidities. Though Smets and Wouters (2003) estimate a higher degree of nominal price rigidity than nominal wage rigidity for the euro area, their estimates suggest that nominal wage rigidities are substantial; the expected duration of wage contracts is estimated to be about one year.

Furthermore, nominal wage rigidities matter in important ways for a welfare-theoretic assessment of the proper goals of monetary policy as shown by Erceg et al. (2000). In a closed-economy with both nominal price and wage rigidities, they show that the optimal monetary policy places greater weight on stabilising inflation in the more rigid variable, prices or wages. That is, from a welfare-theoretic perspective, central banks should stabilise the nominal variables that fail to adjust so as to make the adjustment constraint non-binding, preventing misallocation in the markets characterised by rigidities. Indeed, they find that inflation targeting of the sort that is often considered to be a good approximation to actual central bank behaviour may induce substantial welfare losses when the important nominal rigidity is in the labour market as opposed to the goods market. A central bank operating with a seemingly empirically successful sticky price model may therefore seriously misjudge the welfare implications of its actions if sizeable nominal wage rigidities are present.

In a monetary union, the assessment of the welfare implications of alternative monetary policy prescriptions is further complicated by the fact that member countries often have very different characteristics along a number of dimensions of importance for the economic decision making of agents in the economy. Benigno (2004) addresses this question for the case of structural differences in goods markets. Specifically, he shows that optimal policy in a two-country monetary union in which nominal price rigidities differ across the two member countries is such that a higher weight is given to fighting goods price inflation in the country with the highest degree of nominal price rigidity. Moreover, Lombardo (2006) assesses the welfare losses of simple monetary policy rules, including goods price inflation targeting allowing for asymmetries also in the degree of monopolistic competition in goods markets. He shows that the union's central bank should put a higher weight on the more competitive country. In fact, if the two countries differ sufficiently in terms of market power in goods markets, Benigno's (2004) result may be overturned as the central bank might optimally assign a higher weight to the country with more flexible prices if competition is sufficiently fierce in this country. This indicates that asymmetries in structural characteristics have important implications for the design of monetary policy.

As shown by Andersen and Seneca (2008), differences in structural features in labour markets across a monetary union have non-trivial implications for the propagation of shocks across the union and for the incentives for structural labour market reforms. This indicates that labour market asymmetries may be important for monetary policy, and providing a better understanding of these implications therefore seems an urgent task that should be of particular interest to policy makers in Europe, where such labour market asymmetries are present. To do so, this paper abstracts from other asymmetries that may characterise the monetary union so as to isolate the effects stemming from labour market asymmetries. The interesting question of how these asymmetries interact with other potential asymmetries is left for future research. Hence, structural characteristics in goods markets, preferences etc. will be symmetric across the union. In addition, goods prices will be assumed to be perfectly flexible. While this is a strong assumption, it serves the purpose of keeping a strict labour market perspective. Thus, what is allowed to differ are characteristics of the labour markets in which households offer their labour services. That is, in an important market for household welfare, households face different immediate economic environments as both the degree of market power in their wage setting and the expected duration of their wage contracts - or more generally employment contracts - may differ across the monetary union, though the emphasis will be on asymmetries in wage rigidities.

In this paper, the relative welfare under alternative simple monetary policy rules is investigated under the assumption that taxes are designed to off-set the distortions from monopolistic competition in the economy's steady state. This serves to focus attention on distortions induced by differing degrees of nominal wage rigidity. The monetary policies considered are, first, flexible monetary policy rules according to which the central bank may respond to price inflation, union-wide or national wage inflation, and the welfare-relevant output gap. Second, strict targeting rules are considered that lead to successful stabilisation of either aggregate price inflation, aggregate or country-specific wage inflation, or the output gap.

An important question for this paper to answer is whether inflation targeting by a monetary union's central bank similar to the one used by the ECB leads to substantial welfare losses when the important nominal rigidities are in the labour market, and, most importantly, when labour markets are characterised by differing degrees of nominal rigidity.

As is well-known from the previous literature, the results show that when prices are flexible and wages rigid, flexible as well as strict price inflation targeting regimes lead to non-negligible welfare losses. Welfare may be noticeably improved by targeting wage inflation. This paper shows that the mistake made by by targeting price inflation rather than wage inflation is larger as shocks become more highly correlated across the monetary union's member countries, and as the degree of heterogeneity in the labour market structure increases. Moreover, further welfare improvements can be obtained by putting more weight on fighting wage inflation in the country with a more rigid labour market, especially if shocks are idiosyncratic and labour market structures highly asymmetric. Alternatively, price inflation targeting may not induce large welfare costs in an monetary union characterised by nominal rigidities in labour rather than in the goods market if labour markets are not too heterogenous and if shocks are mostly country-specific.

The paper is organised as follows. Section 2 presents the model and derives its log-linear representation. Section 3 presents the welfare function used to evaluate alternative monetary policy rules. Section 4 presents the welfare analysis for a calibrated version of the model. Section 5 concludes.

2 The model

The model economy consists of two countries in a monetary union. Each country has a large number of households and a large number of firms. There is one central bank responsible for monetary policy throughout the union. In particular, the central bank sets the risk-free interest rate R_t earned on one-period risk-free bonds in the union's single financial market. There is no active fiscal policy for stabilisation purposes, and the union is closed to the outside world.

Labour is immobile across borders, and national labour markets are characterised by monopolistic competition and nominal rigidities in the form of Calvo (1983) wage contracts of random duration. With monopolistic competition, each household supplies a differentiated labour service and has a certain degree of market power in setting the wage it demands for this service. Given the wage chosen, the household stands ready to supply the work hours demanded by firms. These assumptions lead to a downward-sloping demand curve for each household's labour service. Although both the degree of market power in wage setting and the degree of the nominal rigidities are allowed to differ between the two countries, emphasis is put on implications of differences in the expected duration of wage contracts. Such differences are taken to represent heterogeneity in labour market structure across the monetary union.

All firms in a given country are assumed to produce the same internationally traded good in a competitive market, but this good is differentiated from the goods produced by firms abroad. This leads to a downward-sloping demand curve for each country's product. To focus on the labour market, prices of goods are assumed to be perfectly flexible. It is assumed that the weight of each product in the consumption bundle is the same in both countries. Hence, there is no home bias in consumption. The weight assigned to each product is interpreted as the relative size of the country producing it.

2.1 Firms

The representative firm in country $i \in \{0, 1\}$ produces output Y_{it} according to the production function

$$Y_{it} = A_{it} N_{it}^{\gamma} \tag{1}$$

where A_{it} is the stochastic period-t productivity of firms in country i, and $0 < \gamma < 1$ is the degree of returns to scale. W_{it} represents aggregate wages in country i paid for the aggregate labour input into production, N_{it}, as described below. Real capital is disregarded to simplify, but decreasing returns can be interpreted as arising from a second factor of production in fixed supply.

The representative firm in country i maximises profits, which it distributes to households. There are no nominal price rigidities, and the firm takes the price of its product, P_{it}, as given. The profit maximisation problem yields a demand for aggregated labour services defined by the relation

$$\frac{(1-t_i^e)\,W_{it}}{P_{it}} = \gamma A_{it} N_{it}^{\gamma-1} \tag{2}$$

when t_i^e is a fixed-rate employment subsidy paid to firms (and financed by lump-sum taxes, cf. below).[2] The labour demand relation equates the real wage (as perceived by firms) to the marginal product of labour.

2.2 Households

In each period t, each household h in country i supplies a differentiated labour service, $N_{it}(h)$. The labour used in production in country i, N_{it}, is assumed to be an aggregate of the continuum of labour services supplied by the households:

$$N_{it} = \left[\int_0^1 N_{it}(h)^{\frac{\xi_i-1}{\xi_i}}\,dh\right]^{\frac{\xi_i}{\xi_i-1}} \tag{3}$$

where $\xi_i > 1$ is the elasticity of substitution between labour services.

Each household sets the wage rate it demands for its labour service as described below and satisfies firms' labour demand at the chosen wage. That is, given existing wage contracts, household h's labour effort is determined by demand. This demand for household h's labour service is determined by the cost minimisation problems of the country's firms, which minimise costs taking households' wage rates, $W_{it}(h)$, as given. This leads to a demand for household h's labour service given by

$$N_{it}(h) = \left(\frac{W_{it}(h)}{W_{it}}\right)^{-\xi_i} N_{it} \tag{4}$$

when W_{it} is the wage index with the property that the minimum cost of N_{it} units of aggregate labour is given by $W_{it}N_{it}$. It follows that the demand

[2] The employment subsidy is used to neutralise the distorting effect from monopolistic competition in the steady state around which the model is log-linearised. This facilitates the welfare analysis of monetary policy alternatives emphasising implications of nominal wage rigidities.

for household h's labour service is a decreasing function of the household's relative wage with elasticity ξ_i. Hence, ξ_i is inversely related to the degree of market power in wage setting.

Household h in country $i \in \{1, 2\}$ has the utility function

$$E_t \sum_{\tau=0}^{\infty} \delta^{\tau} \left[\frac{\sigma}{\sigma - 1} C_{it+\tau} (h)^{\frac{\sigma-1}{\sigma}} - \frac{1}{1+\mu} N_{it+\tau} (h)^{1+\mu} \right] \tag{5}$$

where E_t is an operator representing expectations over all states of the economy conditional on period-t information, $\delta \in (0, 1)$ is the subjective discount factor, and $C_{it} (h)$ is a real consumption index. $\sigma > 0$ is the elasticity of intertemporal substitution of consumption, and $\mu > 0$ is the inverse of the Frisch labour elasticity.

The consumption index is defined over the differentiated commodities produced in the union's member countries. Specifically,

$$C_{it} = \left[v_1^{\frac{1}{\theta}} C_{i1t}^{\frac{\theta-1}{\theta}} + v_2^{\frac{1}{\theta}} C_{i2t}^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}} \tag{6}$$

where $\theta > 0$, v_j is the relative size of country $j \in \{1, 2\}$, and C_{ijt} represents consumption of country j's commodity by households in country i. In every period t, households choose C_{ijt} for a given level of real consumption to minimise consumption expenditures. This yields a demand for country j's product in country i given by

$$C_{ijt} = v_j \left(\frac{P_{jt}}{P_t} \right)^{-\theta} C_{it} \tag{7}$$

when P_t is the price index defined by

$$P_t = \left[v_1 P_{1t}^{1-\theta} + v_2 P_{2t}^{1-\theta} \right]^{\frac{1}{1-\theta}} \tag{8}$$

This price index has the property that the minimum cost of C_{it} units of real consumption is given by $P_{it} C_{it}$.

Asset markets are assumed to be complete, i.e., available financial assets completely span the possible states of the economy.[3] This assumption leads

[3]Note that this is likely to decrease the potential welfare improvements through stabilisation policy as it provides an insurance mechanism for households. In particular, households share consumption risk through these complete markets so that consumption levels are equalised across the monetary union.

to the following period-t flow budget constraint for a household in country i:

$$E_t\left[Q_{t,t+1}B_{it}(h)\right]+P_tC_{it}(h)+T_{it}=B_{it-1}(h)+W_{it}(h)N_{it}(h)+\Pi_{it} \quad (9)$$

The right-hand side gives available resources as the sum of initial financial wealth, $B_{it-1}(h)$, labour income, $W_{it}(h)N_{it}(h)$, and nominal profit income, Π_{it}. The left-hand side represents the allocation of resources to consumption, $P_tC_{it}(h)$, bond-holdings, $E_t\left[Q_{t,t+1}B_{it}(h)\right]$, where $Q_{t,t+1}$ is the asset pricing kernel, and to a lump-sum tax, T_{it}, used by the government to finance an employment subsidy paid to firms.

Households choose real consumption and wages to optimise expected utility (5) subject to the sequence of budget constraints (9) and labour demand (4).[4] Defining the net risk-free nominal interest rate R_t by the relation

$$(1+R_t)^{-1}=E\left[Q_{t,t+1}\right] \quad (10)$$

the first-order conditions determining the optimal choice of consumption and bond holdings can be combined to yield the Euler equation

$$C_t^{-\frac{1}{\sigma}}=\delta(1+R_t)E_t\left(C_{t+1}^{-\frac{1}{\sigma}}\frac{P_t}{P_{t+1}}\right) \quad (11)$$

where the international risk-sharing property that consumption is equalised across households and countries has been used.[5]

Wages are set by households in a staggered fashion. Following Calvo (1983), each household in country i is allowed to reset the wage rate it demands for its labour service with a fixed probability $(1-\alpha_i)$. Hence, the wage rate set by household h at time t, $W_{it}^*(h)$, is the prevailing wage rate for the household at time $t+\tau$, i.e., $W_{it+\tau}(h)=W_{it}^*(h)$, with probability α_i^τ, and the expected duration of a contract is given by $(1-\alpha_i)^{-1}$. The complete-markets assumption implies that the fraction $(1-\alpha_i)$ of households in country i changing their wage rates at time t choose the same rate W_{it}^*. The remaining fraction α_i of households continue with the wage rate

[4] Implicitly, optimisation is also subject to a solvency condition that may be used to transform the sequence of flow budget constraints into a single life-time budget constraint. This has no effect on the first-order conditions, and since a log-linearised version of the model around its steady state is analysed here, the bounded stochastic shock processes specified below will ensure solvency at all times and in all states.

[5] Throughout, aggregate variables are indicated by the omission of country subindices and are given as averages of national variables with relative country sizes as weights.

prevailing at time $t-1$ where the distribution of wage rates is unchanged. Hence, the law of motion of the aggregate wage index in country i is given by

$$W_{it} = \left[\int_0^1 W_{it}(h)^{1-\xi_i}\,dh\right]^{\frac{1}{1-\xi_i}} = \left[\alpha_i W_{it-1}^{1-\xi_i} + (1-\alpha_i)(W_{it}^*)^{1-\xi_i}\right]^{\frac{1}{1-\xi_i}} \qquad (12)$$

When a household resets its wage, it does so to maximise expected utility (5) subject to the demand for its labour (4), its budget constraint (9) and the price setting mechanism just described. For a household changing its wage rate at time t, this is equivalent to maximizing the following function with respect to $W_{it}^*(h)$ subject to (4) and (9):[6]

$$E_t\sum_{\tau=0}^{\infty}(\alpha_i\delta)^{\tau}\left[\frac{\sigma}{\sigma-1}C_{it+\tau}(h)^{\frac{\sigma-1}{\sigma}} - \frac{1}{1+\mu}N_{it+\tau}(h)^{1+\mu}\right] \qquad (13)$$

The first-order condition can be written as

$$E_t\sum_{\tau=0}^{\infty}(\alpha_i\delta)^{\tau}\left[\left(\frac{\xi_i}{1-\xi_i}N_{it+\tau}(h)^{\mu} + C_{t+\tau}^{-\frac{1}{\sigma}}\frac{W_{it}^*(h)}{P_{t+\tau}}\right)N_{it+\tau}(h)\right] = 0 \qquad (14)$$

It follows that the monopolistically competitive household sets its wage rate so that the marginal utility of income from an extra unit of labour effort is a constant mark-up over the marginal disutility in discounted expected value terms.

Note that in the special case with flexible wages in which all households are allowed to reset the wage each period, the first-order condition collapses to

$$\frac{W_{it}}{P_t} = \frac{\xi_i}{\xi_i - 1}N_{it}^{\mu}C_t^{\frac{1}{\sigma}} \qquad (15)$$

where $N_{it} = N_{it}(h)$ and $W_{it} = W_{it}^*(h)$ for all h. That is, wages are set so as to equalise the real wage (as perceived by the household) to a mark-up over the marginal rate of substitution.

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[6]This differs from (5) in that implicit terms representing states where the wage to be set is not the prevailing wage are excluded.

2.3 Log-linear representation

Welfare will be evaluated using solutions to the model in log-linear form in which variables are expressed in log-deviations from the steady state with stable wages.[7]

A log-linear version of the production function (1) is given by

$$y_{it} = a_{it} + \gamma n_{it} \tag{16}$$

Technology, a_{it}, is assumed follow a first-order autoregressive process

$$a_{it} = \rho_a a_{it-1} + \varepsilon_{it} \tag{17}$$

where the innovations, ε_{it}, are $N\left(0, \sigma_i^2\right)$ and may be correlated across countries as governed by the correlation parameter $\rho_\varepsilon \in [0,1]$ .These innovations are assumed to be the only shocks to the monetary union.

A log-linearisation of the labour demand relation gives

$$w_{it} - p_{it} = a_{it} - (1-\gamma)\, n_{it} \tag{18}$$

Note that the employment subsidy drops out of the log-linear labour demand relation. Hence, it has no effect on the dynamic responses to shocks in the log-linearised economy.

Log-linearisations of the first-order condition from the households' wage setting problem (14) and the law of aggregate wages (12) can be combined to yield a New Keynesian Phillips curve for wage inflation:[8]

$$\omega_{it} = \Lambda_i \left[\sigma^{-1} c_t + \mu n_{it} - (w_{it} - p_t)\right] + \delta E_t \omega_{it+1} \tag{19}$$

where $\omega_{it} = w_{it} - w_{it-1}$ is wage inflation in country i, and Λ_i is a decreasing function of the Calvo parameter α_i and of the elasticity of substitution between labour services ξ_i:

$$\Lambda_i = \frac{(1-\alpha_i)(1-\alpha_i \delta)}{\alpha_i (1+\mu \xi_i)} \tag{20}$$

[7]Steady state variables are indicated by the omission of time subscripts, and lowercase letters denote log-deviations from steady-state values of corresponding upper-case variables.

[8]See Andersen and Seneca (2008, app. A) for details on the derivation.

The supply side of the model is thus summarised by equations (16)-(19) for $i \in \{0,1\}$. Note that (19) collapses to the labour supply relation

$$w_{it} - p_t = \sigma^{-1} c_t + \mu n_{it} \tag{21}$$

in the virtual equilibrium with full wage flexibility, i.e. for $\alpha_i = 0$.

The demand side is represented by a log-linear version of the Euler equation (11) given as

$$c_t = E_t c_{t+1} - \sigma \left(r_t - E_t \pi_{t+1} \right) \tag{22}$$

where $\pi_t = p_t - p_{t-1}$ is price inflation, and a demand relation for the product produced in each country i

$$y_{it} = -\theta \left(p_{it} - p_t \right) + y_t \tag{23}$$

The latter is found by summing (7) over countries i, log-linearising, and imposing the equilibrium condition

$$y_{it}^d = y_{it} \tag{24}$$

where y_{it}^d is the aggregate demand in the monetary union for products produced in country i. Note that monetary policy affects the economy through the aggregate demand relation (22) only. In this sense, its effect on the two countries is symmetric.

2.4 Monetary policy

The model is closed by specifying the monetary policy reaction function. Two types of monetary policy rules are considered: Explicit flexible targeting rules and implicit strict targeting rules. Both types are within the class of simple monetary policy rules in the sense that they are operational rules that may be considered realistic in an actual monetary policy regime rather than the outcome of a Ramsey problem that the central bank may attempt to solve.[9]

With a flexible targeting rule, the central bank sets the risk-free interest rate in response to a vector of endogenous variables. Attention is restricted to rules that may be stated in log-linear form as

$$r_t = k_\pi \pi_t + k_1 \omega_{1t} + k_2 \omega_{2t} + k_y \hat{y}_t \tag{25}$$

[9]For a thorough discussion of optimal vs. simple policy rules, see Woodford (2003).

where $\hat{y}_t = y_t - y_t^n$ is the output gap given as output in excess of the level of output in the virtual flexible wage equilibrium. As shown in appendix A, this "natural" level of output can be found by combining equations (16), (18), (21) and (23). It reads

$$y_t^n = \frac{1+\mu}{(1-\gamma)+\mu+\gamma\sigma^{-1}} a_t$$

For $k_1 = k_2 = 0$, (25) reduces to the monetary policy rule specified by Taylor (1999), at least up to the definition of the output gap. Similarly, for $k_1 = k_2 = k_y = 0$, the rule reduces to a very simple rule according to which the central bank is only concerned about price inflation. More generally, however, (25) allows the central bank to respond not only to average wage inflation ($k_1 = k_2 > 0$), but also to place different weights on wage inflation in the two countries ($0 < k_1 \neq k_2 > 0$). In the welfare analysis below, it is of particular interest to investigate if welfare is improved by allowing the central bank to do so.

With a strict targeting rule, the central bank is assumed to successfully stabilise union-wide price inflation, national wage inflation in country 1, national wage inflation in country 2, aggregate wage inflation, or the output gap. Given these restrictions imposed on the equilibrium dynamics, the paths of the central bank's policy instrument will be complicated functions of the shocks to the economy. The evaluations of these rules are important benchmarks in an analysis of the relative importance of alternative target variables for monetary policy.

3 The welfare function

Following the approach of Rotemberg and Woodford (1999), the social welfare measure used to assess alternative monetary policy rules is derived from the utility functions of households. The welfare function gives the welfare losses experienced by households when the economy deviates from its efficient path expressed as a percentage of steady-state consumption. As shown in appendix B, under the assumption that the distortions associated with monopolistic competition have been eliminated in the steady-state by appropriate choices for the employment subsidies in the two countries (see further discussion below), the average period welfare loss can be expressed as

$$\mathcal{L}_t = \lambda_y VAR\left(\hat{y}_t\right) + \lambda_s VAR\left(\hat{s}_t^2\right) + \sum_{i=1}^{2} \lambda_{\omega i} VAR\left(\omega_{jt}^2\right) + t.i.p. \tag{26}$$

where $\hat{y}_t = y_t - y_t^n$ is the output gab, $\hat{s}_t = s_t - \tilde{s}_t^n$ is a terms-of-trade gap for the terms of trade defined as $s_t = p_{2t} - p_{1t}$, and $t.i.p.$ represents terms independent of policy that may safely be ignored when comparing monetary policy alternatives. The function parameters are given as

$$\begin{aligned}
\lambda_y &= \frac{1}{2}\left[\sigma^{-1} + \frac{(1-\gamma)+\mu}{\gamma}\right] \\
\lambda_s &= \frac{1}{2}\frac{(1+\mu)\,\theta^2}{\gamma}\upsilon_1\upsilon_2 \\
\lambda_{\omega i} &= \frac{1}{2}\frac{\upsilon_i\xi_i\gamma\alpha_i\,(1+\mu\xi_i)}{(1-\alpha_i)\,(1-\alpha_i\delta)} = \frac{1}{2}\frac{\upsilon_i\xi_i\gamma}{\Lambda_i}
\end{aligned}$$

Welfare will be evaluated by the solutions to the model in log-linear form as described above. Given the loss function derived here, this gives a valid welfare ranking of policy alternatives only when the steady state is efficient; i.e. when the distortion from monopolistic competition that leads to an inefficiently low output level (regardless of whether nominal rigidities might be present) has been eliminated, cf. Kim and Kim (2003) and Woodford (2003, ch. 6). Otherwise, fluctuations around the steady state would have an asymmetric impact on welfare as positive shocks would push the economy closer towards the efficient level and so actually improve welfare, and negative shocks would push the economy a long way away from the efficient level and so reduce welfare considerably. To take account of such effects, a second-order approximation to the model's structural relations is required.

The efficient allocation is characterised by the efficiency conditions that the marginal rate of substitution equals the marginal product of labour in each country.[10] Combining (2) and (15) gives

$$\gamma A_{it} N_{it}^{\gamma-1}\frac{P_{it}}{P_t} = \frac{(1-t_i^e)\,\xi_i}{\xi_i - 1} C_t^{\frac{1}{\sigma}} N_{it}^{\mu} \tag{27}$$

[10] Formally, this can be verified as the outcome of a sequence of static social planner's problems maximising a weighted average of the instantaneous utility functions of a generic household in each country subject to the economy's resource constraints.

which reduces to

$$\gamma A_i N_i^{\gamma-1} = \frac{(1-t_i^e)\,\xi_i}{\xi_i - 1} C^{\frac{1}{\sigma}} N_i^{\mu} \tag{28}$$

in the steady state. Hence, any remaining gap between the marginal rate of substitution and the marginal product of labour as a consequence of market power in wage setting can be eliminated in the steady state by setting $t_i^e = \xi_i^{-1}$. This assumption will be maintained in the welfare analysis.

From (26), welfare losses can be seen to be driven by variation in four variables, namely the union-wide output gap, the terms-of-trade gap, and the two national wage inflation levels. Variation in these variables are related to the remaining distortions in the economy. The sources of these remaining inefficiencies are the nominal rigidities represented by staggered wage contracts of random duration, and an international externality similar to the one identified by Corsetti and Pesenti (2001).

The wage inflation and aggregate output gap fluctuations appear in (26) for the same reasons that they appear in loss functions for closed economies with nominal wage rigidity, cf. for instance Erceg et al. (2000) and Galí (2008, ch. 6). With nominal wage rigidities, wages are not fully adjusted in response to shocks to the economy, and the average mark-up of the real wage (as perceived by the household) over the marginal rate of substitution will generally differ from the desired one prevailing in the flexible wage equilibrium. In addition, the staggered wage setting implied by the random duration of contracts under the Calvo wage-setting scheme leads to a relative wage distortion and so to a suboptimal allocation of hours across households. Only when the economy is stabilised at a point where households have no incentives to change their wages, given the prevailing wages and current shocks to the economy, will the constraint represented by the nominal rigidity become unbinding. In this case, wages are stabilised and the flexible wage allocation is attained. The only difference to the closed economy is that the loss function weighs the contribution from wage inflation in each of the two countries according to its size and structural characteristics.

In the limiting case where $v_1 = 1$ and $v_2 = 0$, (26) reduces to the loss function for a closed economy with sticky wages and flexible prices (see for instance Galí, 2008, ch. 6):

$$\mathcal{L}_t = \lambda_y VAR\left(\hat{y}_t\right) + \lambda_{\omega 1} VAR\left(\omega_{1t}^2\right) \tag{29}$$

Similarly, when the two countries are identical and shocks are common to both countries, there are no terms-of-trade movements and the loss function becomes

$$\mathcal{L}_t = \lambda_y VAR\left(\hat{y}_t\right) + \lambda_\omega VAR\left(\omega_t^2\right) \tag{30}$$

where $\lambda_\omega = \lambda_{\omega 1} = \lambda_{\omega 2}$ and $\omega_t = \omega_{1t} = \omega_{2t}$. In this particular case, the monetary union is isomorphic to a standard New Keynesian economy with flexible prices and sticky wages, and the optimal policy takes a very simple form. The central bank should simply keep wages stable, and it follows from (19) that the flexible wage equilibrium will automatically be attained. Moreover, given that an employment subsidy ensures efficiency in the steady state, (27) implies that this flexible wage allocation will also be efficient.

This result can be generalised further to the case with structural heterogeneity. This is because wage changes are the only mechanism through which common shocks can be propagated differently in the two countries. The structural features that are allowed to differ across countries are all contained in the composite parameter Λ_i in (19). Hence, if wage inflation is somehow successfully stabilised in each of the two countries, common shocks will induce identical behaviour of other national variables. This means that there can be no terms-of-trade changes, and so the only remaining potential source of welfare loss would be the output gap. By a similar argument as above, however, zero wage inflation is associated with a zero output gap. Hence, in the case of common shocks and structural heterogeneity, optimal monetary policy would be one that successfully stabilises wage inflation across the monetary union.[11]

In the general case with less than perfectly correlated shocks, the central bank faces a trade-off between stabilisation of all the variables entering the social welfare function. This is because the central bank needs to consider an international externality in addition to the distortions created by the nominal rigidities themselves. This extra distortion is reflected in the terms-of-trade gap term in (26). Notice also from (27) that, as a consequence of movements in the terms of trade, the "natural" flexible wage equilibrium is generally inefficient even in the case of an employment subsidy ensuring the efficiency of the steady state around which the model's structural relations are approximated. Consequently, the terms-of-trade gap entering (26) is defined not simply as the deviation of the terms of trade from its "natural" level under flexible wages, but rather as the deviation from a multiple of this

[11] This leaves open the question of whether the central bank is able to do so given that it has one instrument to its disposal, the effect of which goes through aggregate demand, cf. (22). See the welfare analysis below for further discussion.

"natural" level. Only in the special case where the intertemporal elasticity of substitution is one will the flexible wage equilibrium be efficient in the general case (see the appendix for details).

The international externality arises because movements in the terms of trade works to shift demand between goods produced in each of the two countries. For instance, an increase in the terms of trade defined as $s_t = p_{2t} - p_{1t}$ works to shift demand from goods produced in country 2 to goods produced in country 1. This decreases the disutility from work in country 2 without reducing the utility from consumption, which stay constant as a consequence of international risk sharing. Hence, a positive technology shock in country 1, which increases output in country 1 and reduces the relative price of its product, effectively allows country 2 to act as a monopolist increasing its relative price and reducing its output. This means that more hours can be devoted to leisure without reducing the opportunity for consumption of other goods.[12] The flip-side of this, of course, is that households in country 1 work more without increasing their consumption. The presence of nominal rigidities works to amplify this effect on impact. This follows since wage adjustment costs will prevent workers in country 1 from increasing wages as much as they would have done without the restrictions of the Calvo wage-setting mechanism. As a consequence, firms demand more labour and the increase in output is larger than in the case of flexible wages. This, in turn, amplifies the terms-of-trade effect through the equilibrium effect on prices.

As noted above, the weights with which the output gap, the terms-of-trade gap and the wage inflation fluctuations enter the loss function depend on the model's structural parameters. Considering the weight on the output gap first, note that

$$\frac{\partial \lambda_y}{\sigma} < 0, \frac{\partial \lambda_y}{\gamma} < 0, \frac{\partial \lambda_y}{\mu} > 0 \tag{31}$$

That is, the weight on the output gap is decreasing in σ and γ, the elasticity of intertemporal substitution and the degree of returns to scale, respectively, and increasing in μ, the inverse of the Frisch labour elasticity. This is because

[12] In this economy with perfect risk sharing, consumption stays constant. In a more general setting, consumption of other goods may fall depending on the degree of substitutability, but as long as goods produced in the two countries are substitutes ($\theta > 1$), the reduction in utility from a fall in consumption will be smaller than the reduction in disutility from the change in workload. For $\theta = 1$, the two effects would cancel out. See De Paoli (2007) for a discussion in relation to a small open economy with sticky prices and flexible wages.

a reduction in σ or γ, or an increase in μ, increases the inefficiency gap between the marginal product of labour and the marginal rate of substitution for any given deviation from the flexible wage equilibrium.

Similarly,

$$\frac{\partial \lambda_s}{\mu} > 0, \frac{\partial \lambda_s}{\theta} > 0, \frac{\partial \lambda_s}{\gamma} < 0 \tag{32}$$

which means that the weight on the terms-of-trade gap is increasing in μ and θ, and decreasing in γ. Intuitively, an increase in θ increases the effect of terms-of-trade movements on demand, a reduction in γ increases the change in hours needed to adjust to the change in demand, and an increase in μ amplifies the utility effect of such changes. Moreover, this weight is decreasing in the degree of asymmetry in size as reflected in $\upsilon_1\upsilon_2$ (which takes its maximum for $\upsilon_1 = \upsilon_2 = 0.5$).

Finally, note that

$$\frac{\partial \lambda_{\omega i}}{\upsilon_i} > 0, \frac{\partial \lambda_{\omega i}}{\alpha_i} > 0, \frac{\partial \lambda_{\omega i}}{\xi_i} > 0, \frac{\partial \lambda_{\omega i}}{\mu} > 0, \frac{\partial \lambda_{\omega i}}{\gamma} > 0 \tag{33}$$

The first of these derivatives reflects the effect of relative size on the aggregate welfare function as described above, effectively because the central bank weighs welfare according to the mass of economic activity in the two countries. The second of the derivatives illustrates that an increase in the degree of nominal wage rigidity in a country increases the country's contribution to the monetary union's welfare loss. This indicates that the central bank should be more concerned about wage inflation in the country with the most rigid labour market as reflected in the expected duration of wage contracts. The third of the derivates suggests that the central bank should be more concerned about wage inflation in the country with the most competitive labour markets. In other words, the welfare implications of given nominal wage rigidities may be larger if the labour market is very competitive.[13] Finally, an increase in μ increases the effect of any given nominal rigidity by reducing the slope of the Phillips curve for wage inflation, while an increase in γ amplifies the effect of a given suboptimal allocation of hours across households through the aggregate labour input.

[13] This effect is similar to the finding of Lombardo (2006) that a central bank in a two-country monetary union with nominal price rigidities should be more concerned about price inflation arising in the member country with the most competitive goods market.

4 Welfare analysis

This section evaluates a number of monetary policy alternatives using the loss function derived in the previous section. The loss function is evaluated using a solution to the log-linearised model in section 2 when the model's parameters are set at values commonly used in the literature. The use of the log-linear structural relations means that the welfare analysis is validly ranking monetary policy alternatives according to their ability to counter distortionary effects from nominal rigidities. The analysis abstracts from welfare improvements that could be made by an appropriately designed monetary policy to undo the steady state distortions from monopolistic competition. Instead, it is assumed that fiscal policy neutralises this distortion in the steady state around which the model is log-linearised.

4.1 Calibration

The model's parameters are set to values that are within the ranges considered in the literature and not too far from those estimated for the euro area, for instance by Smets and Wouters (2003). It is important to note, however, that the objective here is to illustrate particular features of the economy, rather than to build a full empirical model.

Considering the parameters governing preferences first, the value for the subjective discount factor is set to $\delta = 0.99$, corresponding to a steady-state interest rate of four per cent with the interpretation that a period corresponds to a quarter. In the baseline calibration, utility is assumed to be logarithmic in consumption ($\sigma = 1$), and the Frisch elasticity of labour is set to unity ($\mu = 1$). The elasticity of substitution between goods produced in the two countries is set to $\theta = 4$ in the baseline calibration. This is probably in the high end of empirical estimates, but it serves to highlight the effects in play in the model. A lower value is considered in the sensitivity analysis, and though this variable has a substantial influence on the level of welfare losses, the qualitative implications for the choice between monetary policy alternatives are robust to the changes in θ considered. In the baseline calibration, the weights to the two countries' goods in the real consumption index are equalised to one half, i.e. $v_1 = v_2 = 0.5$.

The baseline calibration of the supply-side parameters is given by $\gamma = 0.7$, $\xi_1 = \xi_2 = 4, \rho_u = 0.9$ and $\sigma_1 = \sigma_2 = 0.015$. This corresponds to an economy with a labour share of 70 per cent and a wage mark-up of approximately 33 per cent. The technology shocks considered are temporary but very persistent. In the welfare analysis below, the size of these shocks as determined by the standard deviation are crucial in determining the levels of the welfare losses incurred under alternative monetary policy rules. For a quantitative assessment of the level of welfare losses it is therefore important to consider shocks of a size that is empirically plausible. However, this is not absolutely crucial for the purposes of this paper because the emphasis is on the contribution of labour market heterogeneity to the relative levels of welfare losses under alternative monetary policy rules. A more important qualification regards the absence of other shocks that may affect resource allocations under nominal rigidities. There is considerable controversy in the macroeconomic literature concerning the relative importance of different shocks in driving the business cycle (see for example Galí and Rabanal, 2005), and a quantitative assessment of the level of welfare losses would need to take the contribution of other shocks (e.g. demand shocks) to the fluctuations in endogenous variables into account. Concerning the correlation of shocks across countries, three cases are considered. In the first, $\rho_\varepsilon = 0$ and shocks are uncorrelated. This corresponds to purely country-specific shocks. In the second, $\rho_\varepsilon = 0.5$ and shocks are correlated put imperfectly so. And finally in the third, $\rho_\varepsilon = 1$ and shocks are perfectly correlated across countries. This corresponds to common shocks to the monetary union.

Concerning the degree of nominal rigidity, nine cases are considered that differ with respect to the average degree of nominal rigidity in the monetary union and the degree of heterogeneity across countries. The average degree of nominal rigidity is measured by the average expected duration of wage contracts under the Calvo wage setting scheme:

$$AED = \frac{v_1}{1-\alpha_1} + \frac{v_2}{1-\alpha_2} \tag{34}$$

Similarly, the degree of heterogeneity in nominal rigidities is measured by the relative expected duration defined as

$$RED = \frac{1-\alpha_1}{1-\alpha_2} \tag{35}$$

Welfare is evaluated for combinations of $AED \in \{3, 4, 5\}$ and $RED \in \{1, 2, 3\}$.

Finally, six combinations of values for the parameters in the flexible targeting rules are considered. First, a price inflation rule with $(k_\pi, k_1, k_2, k_y) =$

$(1.5, 0, 0, 0)$, a symmetric wage inflation rule with $(k_\pi, k_1, k_2, k_y) = (0, 1.5, 1.5, 0)$, and an asymmetric wage inflation targeting rule with $(k_\pi, k_1, k_2, k_y) = (0, 1.1, 2, 0)$ so that the central bank puts more weight on the member country with the more rigid labour market (which is always country 2). Second, each of these rules are combined with a positive parameter on the output gap; i.e. three additional rules are considered with $(k_\pi, k_1, k_2, k_y) = (1, 5, 0, 0, 0.5/4)$, $(k_\pi, k_1, k_2, k_y) = (0, 1.5, 1.5, 0.5/4)$, and $(k_\pi, k_1, k_2, k_y) = (0, 1.1, 2, 0.5/4)$. The values of the parameters are of the size suggested by Taylor (1993).

4.2 Main results

Results from the baseline calibration are presented in tables 1-3 for RED given by 1, 2 and 3 quarters, respectively. That is, table 1 presents results for a symmetric monetary union, table 2 presents results for an asymmetric union where wage contracts are expected to be twice as long in country 2 as in country 1, and table 3 presents results for a highly heterogeneous monetary union with wage contracts thrice as long in country 2 as in country 1. In each table, the left panel gives results for $AED = 3$ corresponding to an average expected duration of wage contracts of three quarters, the middle panel gives results for $AED = 4$ equivalent to an average expected duration of one year, and the right panel for $AED = 5$. In each panel, the first column shows results for country-specific shocks with $\rho_\varepsilon = 0$, the second column for imperfectly correlated shocks with $\rho_\varepsilon = 0.5$, and the third column for common shocks with $\rho_\varepsilon = 1$.

Consider the symmetric case in table 1 first. When shocks are common, welfare losses can be eliminated if the central bank avoids targeting price inflation – at least up to the fourth decimal place of a percentage of steady state consumption. In this flexible-price economy, when the central bank does not interfere with the price adjustment process, movements in wages that would lead to an inefficient allocation can be avoided. All the central bank needs to do is to stand ready to respond to deviations from zero wage inflation so as to ensure determinacy; i.e. to rule out the possibility of ending up in a situation with inherent instability. The welfare losses resulting from both a flexible and a strict inflation rule are high compared to the other cases considered, and they are increasing in the average degree of nominal rigidity. This indicates that if shocks are common to the monetary union and the important nominal rigidity is in the labour market, a central bank responsible for monetary policy in this union may inflict non-negligible welfare losses on the economy if its key target variable is price inflation.

When shocks are idiosyncratic, the welfare losses from inflation targeting policies are smaller than in the case of common shocks. Again, welfare losses are increasing the average expected duration of contracts. The efficient allocation can no longer be achieved, but welfare may be improved by letting the central bank respond to wage inflation instead of price inflation. In this case, the lowest welfare loss is obtained by targeting aggregate wage inflation, and a flexible wage inflation rule results in the same welfare loss as a strict aggregate wage inflation rule. In addition, the same welfare loss is achieved by allowing the central bank to respond to the output gap in addition to aggregate wage inflation in the flexible rule and to strictly target the output gap. This suggests that these alternatives may be close to achieving the best monetary policy can do within the class of monetary policy rules considered. Not surprisingly, given the symmetry of labour market structures in this case, welfare losses increase if the central bank is more concerned about wage inflation in one of the two member countries. Note also that strictly targeting wage inflation in one country increases welfare losses in comparison with targeting aggregate wage inflation. This is because stabilisation in one country comes at the expense of greater fluctuations in the other country.

Results for imperfectly correlated shocks are in between the two cases with idiosyncratic and common shocks, respectively. Welfare losses are higher under price inflation targeting than when shocks are uncorrelated, but they are noticeably smaller under wage inflation targeting regimes. Again, the results suggest that the central bank should target wages either by following a flexible wage inflation rule responding to aggregate wage inflation, or by strictly targeting aggregate wage inflation or the output gap. Consequently, as the gap between welfare losses under price and wage inflation targeting increases in the correlation of shocks, it appears that the mistake made by targeting price inflation rather than wage inflation when the important nominal rigidity relates to wages and not goods prices is more serious when shocks are highly correlated. The reason for this is the international externality working through the terms of trade, which is absent when shocks are common to the two countries. By dampening price inflation, the central bank also dampens the terms-of-trade movements. This in itself has a beneficial effect on welfare off-setting some of the welfare losses resulting from the interference with the price adjustment process that would otherwise facilitate wage stability.

Table 2 presents results for the case with labour market heterogeneity in the sense that wage contracts can be expected to last twice as long in country 2 as in country 1. As discussed above, the optimal monetary policy in case of common shocks and labour market heterogeneity is one that successfully stabilises inflation in each of the two member countries. From table 2, it is seen that the optimal policy can be implemented as long as the central bank does not target price inflation but rather stands ready to react to deviations from the zero wage inflation target. Note also that the mistake made by targeting price inflation is larger in the case with labour market heterogeneity than in a symmetric monetary union. As before, welfare losses under price inflation targeting are increasing in the average degree of nominal rigidity.

When shocks are uncorrelated, welfare losses are all positive. Price inflation targeting either through a flexible rule or through strict targeting leads to considerably higher welfare losses than the other policy alternatives considered. Thus, the symmetric flexible wage inflation rule performs better than the flexible price inflation rule. Moreover, welfare may be noticeably improved by putting more weight on wage inflation in the more rigid country and less on the more flexible one. In addition, responding to the output gap as well further improves welfare. Notice also that strictly targeting wage inflation in the more rigid country comes close to achieving the same welfare loss as the asymmetric flexible rules, while the best policy response is one that stabilises the output gap completely. This indicates that further improvements in welfare may be achieved by carefully selecting the values of parameters in the flexible targeting rule. It this connection, it is important to note that it may not be possible for an actual central bank to respond to the output gap as it is defined here. The output gap entering the loss function is the welfare relevant output gap defined as output in deviations from the virtual flexible wage output. This generally makes the output gap unobservable, and targeting an output gap derived, say, by filtering an output series will be a very poor approximation to targeting $\hat{y}_t$, cf. for instance Sbordone (2002).

Results for shocks with $\rho_\varepsilon = 0.5$ fall in between results for idiosyncratic and common shocks with heterogeneous labour markets as well. Again, the best performing monetary policy under the baseline calibration is strict output gap targeting, and wage inflation targeting performs noticeably better than price inflation targeting. But although welfare is improved by taking the differences in wage rigidity into account, the benefit from doing so is smaller in this case than in the case with idiosyncratic shocks.

Table 3 presents results for a highly asymmetric monetary union with a relative expected duration of wage contracts set to three. The main results discussed above go through to this case. For common shocks, welfare losses can be eliminated by abstaining from price inflation targeting. For idiosyncratic shocks, wage inflation targeting is better than price inflation targeting, and welfare can be further improved by assigning a higher weight to the country in which labour markets are more rigid. In addition, also targeting the output gap improves welfare, and the lowest welfare loss is obtained by strictly targeting the output gap. Again, welfare losses are increasing in the degree of nominal rigidity. The main difference is that the welfare improvement obtained by following an asymmetric wage inflation rule is higher in this case, where labour market structures are very asymmetric. Similarly, strictly targeting wage inflation in the more rigid country becomes more appealing as labour markets are more heterogeneous, and this is second only to the strict output targeting rule in this case.

4.3 Sensitivity analysis

Table 4 presents a sensitivity analysis for the case with $AED = 4$ and $RED = 2$ (cf. the middle panel in table 2). Alternative values are considered for σ, μ and θ, in particular $\sigma = 2/3$, $\mu = 0.2$ and $\theta = 1.5$.[14]

The left panel of table 4 presents results for $\sigma = 2/3$, keeping the remaining parameters at the baseline values. This change in the value of the intertemporal elasticity of substitution has no effect on the main results presented above. The main difference is that welfare losses are larger under the price inflation targeting regimes. A contributing factor to this result is the curvature effect from changing this parameter discussed in relation to the loss function. A lower σ increases the inefficiency gap and hence welfare losses for a given deviation from the flexible wage equilibrium. Hence, a monetary policy that fails to close this inefficiency gap will result in higher welfare losses when σ is low. Notice, however, that for $\sigma = 2/3$, the flexible wage targeting rules are unable to stabilise wages enough to completely eliminate welfare losses up to the fourth decimal place when shocks are common.

The middle panel of table 4 gives results for $\mu = 0.2$, keeping the remaining parameters at the baseline values (so that now again $\sigma = 1$). In this case, welfare losses are generally lower. A higher labour elasticity decreases

[14] Other values of these parameters have been considered, but the signs of derivatives of changes are the same as those discussed here.

the curvature effect contributing to this result, now along with the labour response effects working through the slope of the Phillips curve and with the direct utility effects of inefficiency as discussed in section 3.1. The main story that wage inflation targeting is better than price inflation targeting, and that welfare may be improved by following an asymmetric policy rule, is confirmed in this case. Moreover, the best monetary policy alternative considered is now the one that leads the central bank to stabilise wage inflation in the more rigid country.

The right panel of table 4 gives results for $\theta = 1.5$ keeping the remaining parameters at the baseline values. The main effect of this change in the international elasticity of substitution is to lower the welfare losses by reducing the international externality. As the benefit from targeting price inflation goes through the terms of trade, it follows that the largest relative reduction in welfare losses occurs under the wage targeting policy regimes. Again, an asymmetric wage targeting rule is preferable when shocks are idiosyncratic, and welfare losses can be eliminated by targeting wage inflation when shocks are common. It is clear, however, that once wages are targeted rather than prices, differences across the monetary policy alternatives are small.

5 Conclusion

This paper has addressed the question of how monetary policy should be conducted in a monetary union in which labour market structures are different across member countries. The analysis has maintained the assumption that the important nominal rigidities are in the labour market rather than in the goods market, and alternative simple monetary policy rules have been evaluated under this assumption that prices are flexible and wages sticky.

As is known from the previous literature, price inflation targeting may result in non-negligible welfare losses if nominal wage rigidities are important and nominal price rigidities are not. In this paper, it is found that the mistake made by targeting price inflation under these assumptions is larger when shocks are common to the monetary union than when they are specific to the member countries due to an international externality. Also, the mistake is larger if the monetary union's member countries are characterised by different degrees of nominal wage rigidity; i.e. when labour market structures are asymmetric.

Finally, the results suggest that the central bank can improve the welfare of the monetary union's citizens by putting more weight on wage inflation in the country with the more rigid labour market. The welfare improvements from doing so are larger when labour markets are very heterogeneous, and when shocks are idiosyncratic rather than correlated.

An interesting topic for further research is to further characterise the optimal monetary policy responses in a monetary union of the kind described in this paper as well as the weights in simple policy rules.

A Natural equilibrium

In the virtual or "natural" equilibrium with flexible wages, the supply side is summarised by the production function

$$\begin{aligned} & y_{it} = a_{it} + \gamma n_{it} \\ \Leftrightarrow \quad & n_{it} = \gamma^{-1} \left(y_{it} - a_{it} \right) \end{aligned} \tag{36}$$

and the labour demand relation

$$w_{it} - p_{it} = a_{it} - \left(1 - \gamma \right) n_{it} \tag{37}$$

while the labour supply relation is

$$w_{it} - p_t = \sigma^{-1} y_t + \mu n_{it} \tag{38}$$

and the relative demand

$$\begin{aligned} & y_{it} = -\theta \left(p_{it} - p_t \right) + y_t \\ \Leftrightarrow \quad & p_{it} - p_t = -\theta^{-1} \left(y_{it} - y_t \right) \end{aligned} \tag{39}$$

all for $i \in \{1, 2\}$.

Combining (36) and (37) gives

$$\begin{aligned} w_{it} - p_{it} &= a_{it} - \left(1 - \gamma \right) n_{it} \\ &= a_{it} - \left(1 - \gamma \right) \gamma^{-1} \left(y_{it} - a_{it} \right) \\ &= \frac{1}{\gamma} a_{it} - \frac{1 - \gamma}{\gamma} y_{it} \end{aligned}$$

and after rearranging

$$y_{it} = \frac{1}{1 - \gamma} a_{it} - \frac{\gamma}{1 - \gamma} \left(w_{it} - p_{it} \right)$$

Substituting (38) and (39) into this relation gives

$$\begin{aligned} y_{it} &= \frac{1}{1 - \gamma} a_{it} - \frac{\gamma}{1 - \gamma} \left(\left(\sigma^{-1} y_t + \mu \gamma^{-1} \left(y_{it} - a_{it} \right) \right) + \theta^{-1} \left(y_{it} - y_t \right) \right) \\ &= \frac{1}{1 - \gamma} \left[\left(1 + \mu \right) a_{it} + \left(\theta^{-1} - \sigma^{-1} \right) \gamma y_t - \left(\mu + \gamma \theta^{-1} \right) y_{it} \right] \end{aligned}$$

Rearranging this expression gives the equilibrium value of country-specific output:

$$
\begin{aligned}
& y_{it}\left(1+\frac{\mu+\gamma\theta^{-1}}{1-\gamma}\right)=\frac{1}{1-\gamma}\left((1+\mu)\,a_{it}+\left(\theta^{-1}-\sigma^{-1}\right)\gamma y_t\right) \\
\Leftrightarrow\quad & y_{it}\frac{(1-\gamma)+\mu+\gamma\theta^{-1}}{1-\gamma}=\frac{1}{1-\gamma}\left((1+\mu)\,a_{it}+\left(\theta^{-1}-\sigma^{-1}\right)\gamma y_t\right) \\
\Leftrightarrow\quad & y_{it}^{n}=\frac{1+\mu}{(1-\gamma)+\mu+\gamma\theta^{-1}}a_{it}+\frac{\left(\theta^{-1}-\sigma^{-1}\right)\gamma}{(1-\gamma)+\mu+\gamma\theta^{-1}}y_t^{n} \qquad (40)
\end{aligned}
$$

where a superscript n has been added to indicate equilibrium values in this "natural" equilibrium with flexible wages.

Aggregate output in the flexible wage equilibrium is found by aggregating (40) over countries and rearranging:

$$
\begin{aligned}
& y_t^{n}=\frac{1+\mu}{(1-\gamma)+\mu+\gamma\theta^{-1}}a_t+\frac{\left(\theta^{-1}-\sigma^{-1}\right)\gamma}{(1-\gamma)+\mu+\gamma\theta^{-1}}y_t^{n} \\
\Leftrightarrow\quad & \frac{(1-\gamma)+\mu+\gamma\theta^{-1}-\left(\theta^{-1}-\sigma^{-1}\right)\gamma}{(1-\gamma)+\mu+\gamma\theta^{-1}}y_t^{n}=\frac{1+\mu}{(1-\gamma)+\mu+\gamma\theta^{-1}}a_t \\
\Leftrightarrow\quad & y_t^{n}=\frac{1+\mu}{(1-\gamma)+\mu+\gamma\sigma^{-1}}a_t \qquad (41)
\end{aligned}
$$

The equilibrium terms of trade under flexible wages can be found by combining (39) with (40):

$$
\begin{aligned}
& s_t^{n}=p_{2t}-p_{1t}=-\theta^{-1}\left(y_{2t}-y_{1t}\right) \\
\Leftrightarrow\quad & s_t^{n}=-\frac{1+\mu}{(1-\gamma)\,\theta+\mu\theta+\gamma}\left(a_{2t}-a_{1t}\right) \qquad (42)
\end{aligned}
$$

B Welfare function

This section derives a second order approximation to the average welfare losses incurred by a generic household. The derivation follows the approach of Galí (2008).

In each country $i\in\{1,2\}$, there is a continuum of households indexed by $h\in[0,1]$ with an instantaneous utility function

$$
U_{it}\left(h\right)=U\left(C_{it}\left(h\right),N_{it}\left(h\right)\right) \qquad (43)
$$

where $C_{it}(h) = C_t$ owing to perfect risk-sharing.

A second-order Taylor expansion of $U_t(h)$ around the steady state $U \equiv U(C,N)$ is given as

$$\begin{aligned} U_{it}(h) &= U + U_c(C_t - C) + U_n(N_{it}(h) - N) \\ &+\frac{1}{2}U_{cc}(C_t - C)^2 + \frac{1}{2}U_{nn}(N_{it}(h) - N)^2 \\ &+U_{cn}(C_t - C)(N_{it}(h) - N) + \mathcal{R}[(C_t, N_{it}(h)), (C,N)] \end{aligned}$$

where $\mathcal{R}(.,.)$ is a remainder term satisfying

$$\frac{\mathcal{R}[(C_t, N_{it}(h)), (C,N)]}{\|(C_t - C, N_{it}(h) - N)\|^3} \to 0$$

as $(C_t - C, N_{it}(h) - N) \to (0,0)$. Noting that $U_{cn} = 0$ by assumption, it follows that a second-order Taylor approximation expressed in terms of log-deviations from the steady state is given as

$$\begin{aligned} U_{it}(h) - U &\approx U_c C\left(\frac{C_t - C}{C}\right) + U_n N\left(\frac{N_{it}(h) - N}{N}\right) \\ &+\frac{1}{2}U_{cc}C^2\left(\frac{C_t - C}{C}\right)^2 + \frac{1}{2}U_{nn}N^2\left(\frac{N_{it}(h) - N}{N}\right)^2 \\ &\approx U_c C\left(c_t + \frac{1}{2}c_t^2\right) + U_n N\left(n_{it}(h) + \frac{1}{2}n_{it}(h)^2\right) \\ &+\frac{1}{2}U_{cc}C^2 c_t^2 + \frac{1}{2}U_{nn}N^2 n_{it}(h)^2 \\ &= U_c C\left(y_t + \frac{1-\sigma^{-1}}{2}y_t^2\right) + U_n N\left(n_{it}(h) + \frac{1+\mu}{2}n_{it}(h)^2\right) \end{aligned}$$

where the second (approximate) equality makes use of the second-order approximations $C_t \approx C\left(1 + c_t + \frac{1}{2}c_t^2\right)$ and $N_{jt}(h) \approx N\left(1 + n_{jt}(h) + \frac{1}{2}n_{jt}(h)^2\right)$, and the third equality of the definitions $\sigma^{-1} = -U_{cc}C/U_c$ and $\mu = U_{nn}N/U_n$ as well as of the equilibrium condition $c_t = y_t$. Integrating over households gives

$$\begin{aligned} \int_0^1 (U_{it}(h) - U)\,dh &\approx U_c C\left(y_t + \frac{1-\sigma^{-1}}{2}y_t^2\right) \\ &+U_n N\left(E_h n_{it}(h) + \frac{1+\mu}{2}E_h n_{it}(h)^2\right) \end{aligned}$$

Noting that

$$n_{it} + \frac{1}{2}n_{it}^2 \approx E_h n_{it}(h) + \frac{1}{2}E_h n_{it}(h)^2$$

and

$$E_h n_{it}(h)^2 \approx n_{it}^2 + \xi_i^2 VAR_h(w_{it}(h))$$

up to a second-order approximation, we can write this as

$$\begin{aligned}\int_0^1 (U_{it}(h) - U)\,dh \;&\approx\; U_c C\left(y_t + \frac{1-\sigma^{-1}}{2}y_t^2\right) \\ &\quad + U_n N\left(n_{it} + \frac{1+\mu}{2}n_{it}^2 + \frac{\xi_i^2\mu}{2}VAR_h(w_{it}(h))\right)\end{aligned}$$

From the definition of aggregate employment, it follows that $\gamma n_{it} = y_{it} - a_{it} + d_{it}$ where

$$d_{it} \approx \frac{\xi_i}{2}VAR_h(w_{it}(h))$$

Using this gives

$$\begin{aligned}\int_0^1 (U_{it}(h) - U)\,dh \;&\approx\; U_c C\left(y_t + \frac{1-\sigma^{-1}}{2}y_t^2\right) \\ &\quad + \frac{U_n N}{\gamma}\left(y_{it} + \frac{1+\mu}{2\gamma}(y_{it} - a_{it})^2 + \frac{(1+\xi_i\mu)\gamma\xi_i}{2}VAR_h(w_{it}(h))\right) \\ &\quad + t.i.p.\end{aligned}$$

where $t.i.p.$ represents terms that are independent of policy (here terms in a_{it}) and so irrelevant for the evaluation of monetary policy alternatives.

Under the assumption of an efficient steady state (so that $-U_n/U_c = \gamma Y/N$), we can express the deviations of utility in percentage terms of steady-state consumption as follows:

$$\begin{aligned}\frac{U_{it} - U}{U_c C} \;&\approx\; y_t + \frac{1-\sigma^{-1}}{2}y_t^2 - y_{it} - \frac{1+\mu}{2\gamma}(y_{it} - a_{it})^2 \\ &\quad - \frac{(1+\xi_i\mu)\gamma\xi_i}{2}VAR_h(w_{it}(h))\end{aligned}$$

where terms independent of policy have been dropped for convenience. Aggregating over countries gives

$$\frac{U_t - U}{U_c C} = \frac{1-\sigma^{-1}}{2} y_t^2 - \frac{1+\mu}{2\gamma} \sum_{i=1}^{2} v_i \left(y_{it} - a_{it}\right)^2 - \sum_{i=1}^{2} v_i \frac{\left(1+\xi_i \mu\right) \gamma \xi_i}{2} VAR_h \left(w_{it}\left(h\right)\right) \tag{44}$$

By inserting the expression (40) for country-specific output, the first two terms of this relation become[15]

$$\begin{aligned}
& \frac{1-\sigma^{-1}}{2} y_t^2 - \frac{1+\mu}{2\gamma} \sum_{i=1}^{2} v_i \left(-\theta\left(p_{it} - p_t\right) + y_t - a_{it}\right)^2 \\
= & \frac{1-\sigma^{-1}}{2} y_t^2 - \frac{1+\mu}{2\gamma} \sum_{i=1}^{2} v_i \left(\left(\theta\left(p_{it} - p_t\right) + a_{it}\right)^2 + y_t^2 - 2\left(\theta\left(p_{it} - p_t\right) + a_{it}\right) y_t\right) \\
= & -\frac{1}{2}\left(\sigma^{-1} + \frac{(1-\gamma)+\mu}{\gamma}\right) y_t^2 + \frac{1+\mu}{\gamma} y_t a_t \\
& -\frac{1+\mu}{2\gamma} \sum_{i=1}^{2} v_i \left(\left(\theta\left(p_{it} - p_t\right) + a_{it}\right)^2\right) + t.i.p.
\end{aligned}$$

After using (41) to replace a_t (and dropping $t.i.p.$), this expression can be written as

$$\begin{aligned}
& -\frac{1}{2}\left(\sigma^{-1} + \frac{(1-\gamma)+\mu}{\gamma}\right) y_t^2 + \frac{(1-\gamma)+\mu+\gamma\sigma^{-1}}{\gamma} y_t y_t^n \\
& -\frac{1+\mu}{2\gamma} \sum_{j=1}^{2} v_j \left(\theta\left(p_{it} - p_t\right) + a_{it}\right)^2 \\
= & -\frac{1}{2}\left[\left(\sigma^{-1} + \frac{(1-\gamma)+\mu}{\gamma}\right)\left(y_t - y_t^n\right)^2 + \frac{1+\mu}{\gamma} \sum_{j=1}^{2} v_j \left(\theta\left(p_{it} - p_t\right) + a_{it}\right)^2\right]
\end{aligned}$$

[15] Second-order terms from a second-order equivalent to (40) would only add third-order term to (44) due to the quadratic terms in this relation.

Since

$$
\begin{aligned}
& \sum_{i=1}^{2} v_i \left(\theta \left(p_{it} - p_t\right) + a_{it}\right)^2 \\
= & \left(\sum_{i=1}^{2} v_i \left(\theta \left(p_{it} - p_t\right) + a_{it}\right)\right)^2 \\
& + v_1 v_2 \left[\left(\theta \left(p_{2t} - p_t\right) + a_{2t}\right) - \left(\theta \left(p_{1t} - p_t\right) + a_{1t}\right)\right]^2 \\
= & \; v_1 v_2 \left[\theta \left(p_{2t} - p_{1t}\right) + \left(a_{2t} - a_{1t}\right)\right]^2 + t.i.p. \\
= & \; v_1 v_2 \theta^2 \left(s_t - \frac{(1-\gamma) + \mu + \gamma\theta^{-1}}{1+\mu} s_t^n\right)^2 + t.i.p.
\end{aligned}
$$

where $s_t \equiv p_{2t} - p_{1t}$ represents the terms-of-trade, we can write (44) as

$$
\begin{aligned}
\frac{U_t - U}{U_c C} = & -\frac{1}{2}\left[\left(\sigma^{-1} + \frac{(1-\gamma)+\mu}{\gamma}\right)\left(y_t - y_t^n\right)^2 + \frac{(1+\mu)\,\theta^2}{\gamma} v_1 v_2 \left(s_t - \tilde{s}_t^n\right)^2\right] \\
& - \sum_{i=1}^{2} v_i \frac{(1+\xi_i\mu)\,\gamma\xi_i}{2} VAR_h\left(w_{it}(h)\right) \qquad (45)
\end{aligned}
$$

where

$$
\tilde{s}_t^n = \frac{(1-\gamma) + \mu + \gamma\theta^{-1}}{1+\mu} s_t^n
$$

Taking the expected value then gives the following second-order approximation to the consumer's lifetime utility:

$$
\begin{aligned}
\mathcal{W}_t = & \; E_t \sum_{\tau=t}^{\infty} \delta^{\tau} \frac{U_t - U}{U_c C} \\
= & -\frac{1}{2} E_t \sum_{\tau=t}^{\infty} \delta^{\tau} \left\{\left(\sigma^{-1} + \frac{(1-\gamma)+\mu}{\gamma}\right)\left(y_t - y_t^n\right)^2 + \frac{(1+\mu)\,\theta^2}{\gamma} v_1 v_2 \left(s_t - \tilde{s}_t^n\right)^2 \right. \\
& \left. + \sum_{i=1}^{2} v_i \left(1+\xi_i\mu\right)\gamma\xi_i VAR_h\left(w_{it}(h)\right)\right\} \qquad (46)
\end{aligned}
$$

By applying proposition 6.3 in Woodford (2003), we may write

$$
VAR_h\left(w_{it}(h)\right) \approx \alpha_i VAR_h\left(w_{it-1}(h)\right) + \frac{\alpha_i}{1-\alpha_i}\omega_t^2
$$

which is accurate up to a remainder term of third order. Integrating this expression forward and taking the discounted value yields the second-order accurate relation

$$\sum_{\tau=t}^{\infty} \delta^{\tau} VAR_h \left(w_{it}\left(h\right)\right) \approx \frac{\alpha_i}{\left(1-\alpha_i\right)\left(1-\alpha_i \delta\right)} \sum_{\tau=t}^{\infty} \delta^{\tau} \omega_t^2 + t.i.p.$$

Consequently, $\mathcal{W}_t$ may be written as

$$\mathcal{W}_t = -E_t \sum_{\tau=t}^{\infty} \delta^{\tau} \left[\lambda_y \left(y_t - y_t^n\right)^2 + \lambda_s \left(s_t - \tilde{s}_t^n\right)^2 + \sum_{i=1}^{2} \lambda_{\omega i} \omega_{jt}^2 \right] \tag{47}$$

where

$$\begin{aligned} \lambda_y &= \frac{1}{2}\left[\sigma^{-1} + \frac{(1-\gamma)+\mu}{\gamma}\right] \\ \lambda_s &= \frac{1}{2}\frac{(1+\mu)\,\theta^2}{\gamma} v_1 v_2 \\ \lambda_{\omega i} &= \frac{1}{2}\frac{v_i \xi_i \gamma \alpha_i \left(1+\mu \xi_i\right)}{\left(1-\alpha_i\right)\left(1-\alpha_i \delta\right)} = \frac{1}{2}\frac{v_i \xi_i \gamma}{\Lambda_i} \end{aligned}$$

Finally it follows that the *average period* welfare *loss* is

$$\mathcal{L}_t = \lambda_y VAR\left(\hat{y}_t\right) + \lambda_s VAR\left(\hat{s}_t^2\right) + \sum_{i=1}^{2} \lambda_{\omega i} VAR\left(\omega_{jt}^2\right) \tag{48}$$

where $\hat{y}_t = y_t - y_t^n$ is the welfare relevant output gap and $\hat{s}_t = s_t - \tilde{s}_t^n$ is a terms-of-trade gap.

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Table 1: Welfare losses for RED=1

$\mathcal{L}$	$AED = 3$			$AED = 4$			$AED = 5$		
Flexible rules	$\rho_\varepsilon = 0$	$\rho_\varepsilon = 0.5$	$\rho_\varepsilon = 0$	$\rho_\varepsilon = 0$	$\rho_\varepsilon = 0.5$	$\rho_\varepsilon = 1$	$\rho_\varepsilon = 0$	$\rho_\varepsilon = 0.5$	$\rho_\varepsilon = 1$
$(1.5; 0; 0; 0)$	0.1659	0.2008	0.2358	0.2565	0.3266	0.3967	0.3346	0.4362	0.5378
$(0; 1.5; 1.5; 0)$	0.0480	0.0240	0.0000	0.0581	0.0291	0.0000	0.0656	0.0328	0.0000
$(0; 1.1; 2; 0)$	0.0488	0.0244	0.0000	0.0590	0.0295	0.0000	0.0665	0.0332	0.0000
$(1.5; 0; 0; 0.125)$	0.1318	0.1497	0.1676	0.1678	0.1936	0.2194	0.1835	0.2096	0.2357
$(0; 1.5; 1.5; 0.125)$	0.0480	0.0240	0.0000	0.0581	0.0291	0.0000	0.0656	0.0328	0.0000
$(0; 1.1; 2; 0.125)$	0.0486	0.0243	0.0000	0.0588	0.0294	0.0000	0.0662	0.0331	0.0000
$\mathcal{L}$									
Strict rules									
$\pi_t = 0$	0.1816	0.2243	0.2671	0.2375	0.2981	0.3587	0.2815	0.3567	0.4318
$\omega_t = 0$	0.0480	0.0240	0.0000	0.0581	0.0291	0.0000	0.0656	0.0328	0.0000
$\omega_{1t} = 0$	0.0637	0.0319	0.0000	0.0770	0.0385	0.0000	0.0866	0.0433	0.0000
$\omega_{2t} = 0$	0.0637	0.0319	0.0000	0.0770	0.0385	0.0000	0.0866	0.0433	0.0000
$\hat{y}_t = 0$	0.0480	0.0240	0.0000	0.0581	0.0291	0.0000	0.0656	0.0328	0.0000

Note: Benchmark calibration and $RED = 1$. Flexible rules refer to monetary policy rules of the form (25) and are indicated by (k_π, k_1, k_2, k_y).

Table 2: Welfare losses for RED=2

$\mathcal{L}$	$AED=3$			$AED=4$			$AED=5$		
Flexible rules	$\rho_\varepsilon=0$	$\rho_\varepsilon=0.5$	$\rho_\varepsilon=0$	$\rho_\varepsilon=0$	$\rho_\varepsilon=0.5$	$\rho_\varepsilon=1$	$\rho_\varepsilon=0$	$\rho_\varepsilon=0.5$	$\rho_\varepsilon=1$
$(1.5;0;0;0)$	0.1745	0.2184	0.2623	0.2764	0.3612	0.4460	0.3667	0.4892	0.6116
$(0;1.5;1.5;0)$	0.0459	0.0229	0.0000	0.0556	0.0278	0.0000	0.0629	0.0315	0.0000
$(0;1.1;2;0)$	0.0423	0.0211	0.0000	0.0524	0.0262	0.0000	0.0601	0.0301	0.0000
$(1.5;0;0;0.125)$	0.1329	0.1570	0.1810	0.1697	0.2021	0.2345	0.1855	0.2181	0.2508
$(0;1.5;1.5;0.125)$	0.0449	0.0224	0.0000	0.0545	0.0273	0.0000	0.0618	0.0309	0.0000
$(0;1.1;2;0.125)$	0.0419	0.0210	0.0000	0.0521	0.0260	0.0000	0.0598	0.0299	0.0000
$\mathcal{L}$									
Strict rules									
$\pi_t=0$	0.1673	0.2035	0.2397	0.2209	0.2741	0.3273	0.2634	0.3306	0.3978
$\omega_t=0$	0.0533	0.0266	0.0000	0.0640	0.0320	0.0000	0.0720	0.0360	0.0000
$\omega_{1t}=0$	0.0770	0.0385	0.0000	0.0892	0.0446	0.0000	0.0976	0.0488	0.0000
$\omega_{2t}=0$	0.0443	0.0221	0.0000	0.0581	0.0290	0.0000	0.0687	0.0343	0.0000
$\hat{y}_t=0$	0.0403	0.0202	0.0000	0.0511	0.0256	0.0000	0.0592	0.0296	0.0000

Note: Benchmark calibration and $RED=2$. Flexible rules refer to monetary policy rules of the form (25) and are indicated by (k_π, k_1, k_2, k_y).

Table 3: Welfare losses for RED=3

$\mathcal{L}$	$AED = 3$			$AED = 4$			$AED = 5$		
Flexible rules	$\rho_\varepsilon = 0$	$\rho_\varepsilon = 0.5$	$\rho_\varepsilon = 0$	$\rho_\varepsilon = 0$	$\rho_\varepsilon = 0.5$	$\rho_\varepsilon = 1$	$\rho_\varepsilon = 0$	$\rho_\varepsilon = 0.5$	$\rho_\varepsilon = 1$
$(1.5; 0; 0; 0)$	0.1829	0.2370	0.2912	0.3000	0.4029	0.5057	0.4092	0.5590	0.7088
$(0; 1.5; 1.5; 0)$	0.0437	0.0218	0.0000	0.0528	0.0264	0.0000	0.0598	0.0299	0.0000
$(0; 1.1; 2; 0)$	0.0369	0.0185	0.0000	0.0464	0.0232	0.0000	0.0539	0.0269	0.0000
$(1.5; 0; 0; 0.125)$	0.1323	0.1633	0.1943	0.1698	0.2100	0.2502	0.1859	0.2266	0.2673
$(0; 1.5; 1.5; 0.125)$	0.0414	0.0207	0.0000	0.0501	0.0251	0.0000	0.0569	0.0285	0.0000
$(0; 1.1; 2; 0.125)$	0.0357	0.0179	0.0000	0.0452	0.0226	0.0000	0.0527	0.0264	0.0000
$\mathcal{L}$									
Strict rules									
$\pi_t = 0$	0.1487	0.1763	0.2039	0.1989	0.2423	0.2857	0.2390	0.2956	0.3521
$\omega_t = 0$	0.0580	0.0290	0.0000	0.0693	0.0347	0.0000	0.0776	0.0388	0.0000
$\omega_{1t} = 0$	0.0822	0.0411	0.0000	0.0938	0.0469	0.0000	0.1015	0.0507	0.0000
$\omega_{2t} = 0$	0.0306	0.0153	0.0000	0.0443	0.0221	0.0000	0.0550	0.0275	0.0000
$\hat{y}_t = 0$	0.0301	0.0150	0.0000	0.0415	0.0208	0.0000	0.0502	0.0251	0.0000

Note: Benchmark calibration and $RED = 3$. Flexible rules refer to monetary policy rules of the form (25) and are indicated by (k_π, k_1, k_2, k_y).

Table 4: Sensitivity analysis

$\mathcal{L}$	$\sigma = 2/3$			$\mu = 0.2$			$\theta = 1.5$		
Flexible rules	$\rho_\varepsilon = 0$	$\rho_\varepsilon = 0.5$	$\rho_\varepsilon = 0$	$\rho_\varepsilon = 0$	$\rho_\varepsilon = 0.5$	$\rho_\varepsilon = 1$	$\rho_\varepsilon = 0$	$\rho_\varepsilon = 0.5$	$\rho_\varepsilon = 1$
$(1.5; 0; 0; 0)$	0.3382	0.4545	,05708	0.1335	0.1568	0.1802	0.2254	0.3329	0.4404
$(0; 1.5; 1.5; 0)$	0.0554	0.0281	0.0009	0.0440	0.0220	0.0000	0.0054	0.0027	0.0000
$(0; 1.1; 2; 0)$	0.0526	0.0267	0.0008	0.0430	0.0215	0.0000	0.0051	0.0025	0.0000
$(1.5; 0; 0; 0.125)$	0.2173	0.2738	0.3302	0.1020	0.1101	0.1181	0.1216	0.1773	0.2330
$(0; 1.5; 1.5; 0.125)$	0.0545	0.0276	0.0007	0.0437	0.0218	0.0000	0.0053	0.0026	0.0000
$(0; 1.1; 2; 0.125)$	0.0523	0.0265	0.0007	0.0429	0.0214	0.0000	0.0051	0.0025	0.0000
$\mathcal{L}$									
Strict rules									
$\pi_t = 0$	0.2597	0.3320	0.4044	0.1235	0.1408	0.1582	0.1637	0.2400	0.3163
$\omega_t = 0$	0.0634	0.0317	0.0000	0.0454	0.0227	0.0000	0.0060	0.0030	0.0000
$\omega_{1t} = 0$	0.0876	0.0438	0.0000	0.0494	0.0247	0.0000	0.0081	0.0041	0.0000
$\omega_{2t} = 0$	0.0574	0.0287	0.0000	0.0410	0.0205	0.0000	0.0059	0.0030	0.0000
$\hat{y}_t = 0$	0.0511	0.0256	0.0000	0.0423	0.0211	0.0000	0.0050	0.0025	0.0000

Note: Parameters other than those mentioned are at their baseline values. Flexible rules refer to monetary policy rules of the form (25) and are indicated by (k_π, k_1, k_2, k_y).