DEMAND SHOCKS AND EQUILIBRIUM RELATIVE PRICES IN THE EURO AREA
PIETRO COVA*
BANCA D’ITALIA

This paper develops a two-sector, two-country model with monopolistic competition for a currency area. Within this framework we show that the same relative productivity and price trends suggested by the literature on the Balassa-Samuelson effect can be generated by shifts in expenditure shares that lead the existing capital stock in the currency area to move towards those sectors experiencing a rise in relative demand. Focusing on regional and sectoral inflation differentials across member countries of the European Monetary Union we find support for this demand-side explanation. JEL Classification [E31, F41]

Il presente lavoro sviluppa un modello a due settori, operanti in regime di concorrenza monopolistica, per due paesi appartenenti a un’area valutaria. Nell’ambito di questa impostazione teorica si dimostra che movimenti tendenziali della produttività relativa e dei prezzi relativi coerenti con l’effetto Balassa-Samuelson possono originare da variazioni nelle preferenze di spesa dei consumatori, in grado di generare uno spostamento dello stock di capitale esistente verso quei settori produttivi che sperimentano aumenti sostenuti della domanda relativa. Concentrando la nostra attenzione sui differenziali di inflazione regionali e settoriali tra i paesi membri dell’Unione Monetaria Europea siamo in grado di validare questa spiegazione dal lato della domanda.

* pietro.cova@bancaditalia.it; Servizio studi e relazioni internazionali.

The Author wishes to thank the editor, two anonymous referees and Behzad Diba, Matthew Canzoneri, and Robert Cumby for helpful comments and suggestions. Acknowledgments go also to seminar participants at the 2003 EEA conference in Stockholm, the European Central Bank, the Riksbank, and the Bank of England. All remaining errors are my own.
1. – Introduction

Fairly large and persistent cross-country inflation differentials have persisted within the European Union (EU) since the launch of the euro in 1999. Prices in member countries such as Ireland, Spain, Portugal, Greece, Italy (and in most Central and Eastern European Accession Countries) have been growing at much faster rates than in the rest of the union.1

CUMULATED PERCENT HICP INFLATION DIFFERENTIALS RELATIVE TO EU 15 AND RELATIVE TO GERMANY (G) (SOURCE: EUROSTAT) – TABLE 1

<table>
<thead>
<tr>
<th>Country</th>
<th>Relative to</th>
<th>1999-2005</th>
</tr>
</thead>
<tbody>
<tr>
<td>Germany</td>
<td>EU 15</td>
<td>-4.2</td>
</tr>
<tr>
<td></td>
<td>G</td>
<td>0.0</td>
</tr>
<tr>
<td>France</td>
<td>EU 15</td>
<td>-1.7</td>
</tr>
<tr>
<td></td>
<td>G</td>
<td>2.5</td>
</tr>
<tr>
<td>Italy</td>
<td>EU 15</td>
<td>2.2</td>
</tr>
<tr>
<td></td>
<td>G</td>
<td>6.4</td>
</tr>
<tr>
<td>Netherlands</td>
<td>EU 15</td>
<td>4.3</td>
</tr>
<tr>
<td></td>
<td>G</td>
<td>8.4</td>
</tr>
<tr>
<td>Portugal</td>
<td>EU 15</td>
<td>6.7</td>
</tr>
<tr>
<td></td>
<td>G</td>
<td>10.9</td>
</tr>
<tr>
<td>Spain</td>
<td>EU 15</td>
<td>7.5</td>
</tr>
<tr>
<td></td>
<td>G</td>
<td>11.6</td>
</tr>
<tr>
<td>Greece</td>
<td>EU 15</td>
<td>8.4</td>
</tr>
<tr>
<td></td>
<td>G</td>
<td>12.5</td>
</tr>
<tr>
<td>Ireland</td>
<td>EU 15</td>
<td>10.7</td>
</tr>
<tr>
<td></td>
<td>G</td>
<td>14.9</td>
</tr>
</tbody>
</table>

The vast academic and institutional literature on euro area inflation differentials that has followed focuses on the underlying causes, the dynamic and business-cycle properties, the sectoral distribution, the implications for the conduct of monetary policy, and, lastly, the implications for the joint conduct of the common monetary policy and the country-specific fiscal policies.2 These works can be further subdivided into purely empirical and more theoretical ones based on micro-founded dynamic stochastic general equilibrium models consistent with the by now standard New Open Economy Macro literature. The stylized

---

1 Throughout the paper our focus will be on the 1999q1-2005q4 period and only for the subset of euro area countries that constitute the "backbone" of the euro area during this period: Germany, France, Italy, Spain, the Netherlands, Portugal, Greece, and Ireland. The choice of the length of the period covered by the study has been dictated by the availability of reliable data over this time span. It may have been desirable to extend the starting period backwards in order to gain some more data points, but it was purposefully avoided so as not to have bilateral exchange rate movements affect the data. Note also that even though Greece only joined the euro area in 2001, the drachma has been only depreciating by 3.2 percent between 1999 and 2000 which should not affect the data for Greece presented and commented in the paper.

facts established by these studies can be briefly summarized as follows. First, there is no single dominant factor that can account for the observed differentials. Both supply side factors, mainly associated with differential productivity developments across countries and sectors, and demand side factors, mainly related to diverging output gaps ensuing the adoption of a common interest rate and to differential impacts of changes in the euro exchange rate due to differences in member countries' trade openness, have been highlighted as underlying causes. Second, there is by now a shared view that inflation differentials are procyclical and positively correlated with growth differentials across euro area countries. The latter fact has led some researchers to talk about a "cross-sectional Phillips curve for the euro area". Also, the persistence of European inflation differentials stands out as a peculiar feature, in comparison with the differentials observed in the U.S., with potentially non-trivial consequences for the size of the differentials themselves and for the correct conduct of monetary policy. Third, inflation differentials are spread across sectors, but they tend to be larger in the services sector. Fourth, even though a common monetary policy together with large and persistent inflation differentials map into correspondingly large and persistent real interest rate differentials in a highly integrated common trade and financial area, the welfare maximizing common central banker should aim at stabilizing the inflation rates of those countries exhibiting the highest degree of price and wage inflation dispersion due to the presence of nominal rigidities. In practice this entails assigning higher weights in the setting of the policy rate to the inflationary pressures, and hence differentials, stemming from larger and more rigid member countries. Fifth, fiscal policy should support the monetary lever so as to fend off the asymmetric shocks that hit the different countries, potentially exacerbating the inflation differentials. Yet to-date no consensus has emerged on the optimal fiscal-monetary policy mix in a common currency area. In fact, uncoordinated fiscal policies may even challenge the established view about the optimal monetary policy prescription mentioned in the previous point.

A common theme across these studies is the potential relevance of productivity differentials across countries and sectors, reflecting cross-country differences in economic structures, to account for the observed inflation area differentials in the euro area. At various points in time both the European Central Bank (ECB) and some academic researchers have stressed that the cross-country inflation differentials in Europe can be mainly attributed to sectoral productivity trends: the high-inflation member countries exhibit relative - tradables over nontradables - productivity trends that exceed the average trends across the Euro area. Assuming the law of one price holds for traded goods sold in the Euro area, any productivity gain in the traded goods sector induces the competitive firms in that sector to pass on their productivity improvements to the wages paid to their workers. Labor mobility across sectors and the fact that productivity growth in the

---

3 On the demand side, cross-country differences in the fiscal stance do not seem to be a major empirical contributing factor. Duarte M. and A. Wolman (2008) develop a positive analysis whereby national governments can use fiscal policy to protect a country's competitiveness when inflation differentials deviate excessively from the area-wide average.


5 Most studies use labor productivity as a proxy for total-factor-productivity.
nontraded goods sector is generally lower than in the traded goods sector forces then the competitive nontraded goods firms to raise their prices as well as their wages. This pushes the relative price of nontraded goods up and leads to the currently observed regional inflation differentials. The hypothesis that relative prices and inflation differentials adjust to trends in relative productivities is due to Balassa (1964) and Samuelson (1964). According to this long-run view relative price movements and regional inflation differentials can be attributed to supply-side factors.

As suggested by the above Figure 1 there is a strong cross-sectional evidence in favor of the standard Balassa-Samuelson explanation: over the 1999-2005 period the correlation between average annual changes in relative prices and in relative (labor) productivities was 0.77. A closer look at the European inflation differentials reveals, however, that these regional inflation differentials seem to be positively related to cross-country output growth differentials. The average correlation reported, for example, in Canzoneri, Cumby, Diba, and Mykhaylova (2005) and in Lane (2006) is 65 percent. This observation naturally

---

6 The original papers by BALASSA B. (1964) and SAMUELSON P. (1964) focused on equilibrium adjustments in relative prices and real exchange rates. In a currency union, such as the European Monetary Union, regional inflation differentials correspond to real exchange rate movements.

7 OECD data on gross value added by sector and number of employees underlies the construction of Figure 1. We are also aware that a new dataset for the 1980-2005 period, the EU KLEMS, specifically developed for the analysis of productivity developments across euro area countries and sectors has become available. See euklems.net and ECB (2008). Using this alternative data set does not affect the conclusions that can be inferred using the OECD data and that are subsumed in Figure 1.

8 There is also convincing prima facie evidence form HONOHAN P., AND P. LANE (2003) that relative productivity developments do not play a statistically significant role, at least over the short time span (1999-2001), considered by these authors. ANGELONI I., AND M. EHRMANN (2004) find however that the significance of these catching-up effects strengthens considerably as longer time intervals are considered.
invites a demand-side explanation, alongside the standard supply-side Balassa-Samuelson view.\footnote{Evidence on a cross-sectional Phillips Curve for the Euro area is reported, among others, also in Angeloni I., and M. Ehrmann (2004) and in ECB (2003).}

This paper rationalizes the demand-side explanation for relative price changes and regional inflation differentials by focusing on the interplay between demand-side factors - e.g. persistent shifts in expenditure shares - and equilibrium relative productivity movements in a currency area, such as EMU, with well-integrated capital markets. In our model an increase in Europe-wide demand for, say, Irish tradables leads to a capital inflow into Ireland. This raises labor productivity in Ireland. If the traded goods sector is more capital intensive than the nontraded goods sector then Irish labor productivity rises faster in the traded goods sector, and prices rise faster in the nontraded goods sector leading to a positive inflation differential between Ireland and the rest of the currency union. These are the same relative productivity and price trends suggested by the literature on the Balassa-Samuelson effect, but the trends are demand driven in our model.

Our results depend critically on the endogenous response of sectoral (labor) productivities to the capital flows - across sectors and member countries of the currency area - that arise from a demand shock. We think that this is a very plausible explanation for the regional inflation differentials that persist in EMU, given the empirical evidence on the importance of the Single Market in encouraging more foreign direct investment into and across the European Union, particularly into the 'smaller' and more peripheral countries, such as Ireland, Spain, Portugal and Greece.\footnote{See for example Neven D., and G. Siotis (1996), Pain N. (1997), Barry F. (1999), Neary P. (2002) and Blanchard O., and F. Giavazzi (2002).}

The model in this paper focuses on endogenous changes in sectoral labor productivities. The transmission mechanism, from demand-side factors to relative productivities, can also involve changes in measured total-factor-productivities (TFP). To model changes in measured TFP, we would need to add an unobserved factor, such as variable capital utilization rates or labor effort, to the model.\footnote{Main contributions to the empirical literature on productivity measurement that discuss extensively the role of unobserved factors such as variable capital utilization rates and labor hoarding include Basu et al. (2001, 2004) and Baxter M., and D. Farr (2005).} This would then allow us to show that even when labor has both an effort-dimension and an hours-dimension, demand-side disturbances can lead to inflation differentials consistent with the Balassa-Samuelson hypothesis through adjustments in (relative) measured TFP.

The rest of this paper is structured as follows: In section 2 we offer some casual empirical evidence that motivates our work by looking at data on sectoral relative price and output trends for a sample of EMU member countries (Germany, France, Italy, Spain, Ireland, Portugal, Greece and the Netherlands) which exhibit persistent and widely differing regional inflation differentials (see Table 1). Section 3 introduces our model, which deviates from the "standard" general equilibrium Balassa-Samuelson model in three ways: we assume that tradables are not perfect substitutes; we allow for sectoral differences in capital intensity, and we focus both on supply and on demand-side shocks.\footnote{See Asea P., and E. Mendoza (1994), Stockman A., and L. Tesar (1995) and Rebelo S. (1993) for models which can account for the standard (i.e. supply side) Balassa-Samuelson effect using a standard Real Business Cycle setup.}

\footnote{We do not want to impose the law of one price for tradables, given that we are focusing on a medium-run period (1999q1-2005q4) during which there is ample evidence that traded goods prices do not equalize. See footnote 16 for further references on this point.}
Section 4 presents closed-form solutions for the symmetric equilibrium of the model and explains how, in this setup, the interaction between demand-side factors and relative prices can lead to equilibrium inflation differentials. Section 5 concludes and discusses necessary extensions and some work in progress.

2. – Motivation

If the original Balassa-Samuelson view applied to Europe, then only supply-side factors should drive the currently observed relative price trends and inflation differentials (cfr. Figure 1 and Table 1). Thus, for example, if Ireland is experiencing an exogenous productivity growth in its traded goods sector, then the terms of trade for Ireland, that is, the price of Irish tradables, relative to say, German tradables, should remain unchanged.14 There is, however, some empirical evidence that terms of trade do not remain constant over fairly long horizons.15 This has led to some recent work on the Balassa-Samuelson effect by Benigno and Thoenissen (2003), MacDonald and Ricci (2002) and Natalucci and Ravenna (2008), who model traded goods as imperfect substitutes. An acceleration in Irish productivity leads then to a depreciation of the terms of trade due to the decrease in the marginal cost incurred by the Irish traded goods producers.16 On the other hand, if Ireland is experiencing strong relative demand for its tradables, then the Irish terms of trade should appreciate.

In Table 2 we choose Germany as the reference country, and compute inflation differentials - by comparing cumulated gross domestic product deflator growth rates - and terms of trade appreciations - by comparing cumulated export deflator growth rates - for a selected group of European countries. France is the only country in our sample whose relative price pattern - positive inflation differential and terms of trade depreciation - fully conforms to modern renditions of the (supply driven) Balassa-Samuelson effect, which allow for deviations from the law of one price in the traded goods sector. All the other countries have experienced positive inflation differentials and terms of trade appreciations relative to Germany, suggesting that supply-side explanations may not be enough to understand these persistent regional inflation differentials across the Euro area. A demand-side view, consistent with both the terms of trade appreciations and the positive inflation differentials reported in Table 2, might offer an important complementary explanation.17

14 Recall that, as we state in the introduction, underlying the original Balassa-Samuelson view is the assumption that traded-goods are perfect substitutes.
16 Recent empirical tests find a statistically strong support for the Balassa-Samuelson proposition when accounting for deviations from the law of one price in the traded goods sector. See for example Alberola E., and T. Tyrväinen (1998); Alberola E., and Marques J. (1999); Estrada A., and D. Lopez-Salido (2004); Sondergaard J. (2002); MacDonald R., and L. Ricci (2002).
17 We wish to stress that we consider this only as a complementary explanation for the observed intra euro area inflation differentials. In looking at Table 2 one might be tempted to argue that labor productivity developments in Germany in comparison with other euro area countries might be important alternative explanatory facts. There is, for example, enough evidence on the relative cost improvements attained by the German traded sector over the recent past.
Focusing on a period of relative inter-country currency stability - the 1999q1-2005q4 period - we want to verify whether these two implications of the demand-side view - a terms of trade appreciation and a positive inflation differential relative to Germany - are empirically supported by the data.\textsuperscript{18}

Figure 2 complements the information conveyed by Table 1, suggesting that on average the prices of the traded goods produced by some EMU member countries (Spain, Ireland, Portugal, Italy, Greece and the Netherlands) have risen faster than the corresponding prices for the German tradables. This could point to a shift in demand across the European currency union towards the traded goods produced by some of the higher inflation countries.

In order to gain some \textit{prima facie} evidence on this claim we compute for each country in our sample the ratio of nominal gross value added in the traded sectors over the corresponding variable for the entire EMU and use these ratios as proxies for 'expenditure shares'.\textsuperscript{19} A few caveats on our way of measuring the expenditure shares should be noted at this point. First, we choose to express expenditure shares in nominal terms as their variation

\textsuperscript{18} Since January 1999 the Euro has replaced the national currencies of all the European countries in our sample eliminating any residual exchange rate fluctuations between these countries.

\textsuperscript{19} We use OECD data on nominal gross value added for the "Industry; including energy" sector.
over time should really proxy shifts in relative demand across countries and sectors due to quality improvements, changes in number of varieties and tastes or cost differentials. In other words, we do not assume that the underlying basket of traded goods is invariant across countries and time. Similar relative demand shocks are also considered in Corsetti, Martin, and Pesenti (2007) and in Coeurdacier, Kollmann, and Martin (2007). Second, we intentionally do not use directly data on nominal exports to proxy expenditure shares as export values are not corrected for re-exports (the foreign value added contained in domestic exports), which may lead to serious misrepresentations of export shares as explained, among others, by Chen, Kondratowicz and Yi (2005). Third, changes in value added-based expenditure shares obviously also capture own-demand dynamics for domestic traded goods. Nonetheless, to the extent that tradability of traded output (i.e. the share of tradable goods output that can be assumed to be effectively exported) is constant over time for each country, these measures of expenditure shares are also a good proxy for country-specific international market shares.

That said, an increase over time in the expenditure share of one member country corresponds then also to a rise in the demand (or 'market share') across EMU for its tradables. In particular, Figure 3 shows that for Ireland, Greece, Spain and the Netherlands expenditure shares have risen strongly relative to Germany between 1999 and 2005.

---


21 Using export values confirms that over the same period Ireland, Greece and Spain have experienced an increase in their export shares relative to Germany.

22 Computing a simple tradability index using the World Bank's World Trade and Production Database, as in GUVEN C., AND VOLLRATH D. (2007), we are able to confirm that this is indeed the case for the countries in our sample.
differentials in the growth rates of relative expenditure shares are even stronger if we look over the 1996-2005 period (not reported). This latter observation suggests, as noted for example by Lane (2006) and Micco, Stein, and Ordonez (2003), that the entry into EMU has exerted a pro-trade effect well before the effective adoption of the euro on January 1999.23 This fact may have impinged on the dynamics of observed inflation differentials, which started to diverge again in 1999, as noted for example by the European Central Bank (2003), Honohan and Lane (2003), and Angeloni and Ehrmann (2004).24 The possible link between changes in expenditure shares and inflation differentials suggested by Figure 3 does not apply to Portugal, and, especially, to Italy. These two countries have experienced persistent and sizeable inflation differentials and similar or lower growth rates in their expenditure shares relative to Germany. This might suggest that some additional factors (i.e. neither supply nor the type of demand socks considered by this study) might account for the terms of trade appreciations and positive inflation differentials exhibited by these countries relative to Germany during the 1999-2005 period (cfr. Table 2). France stands out as the only EMU country in our sample which might conform to a "modified" version of the Balassa-Samuelson hypothesis, that allows for imperfect substitutability across traded goods: its relative productivity has been growing more rapidly than the German one, causing a faster rise in the relative price of nontradables (Figure 1), a slight terms of trade deterioration and a positive inflation differential vis-à-vis Germany (Table 2).

We conclude this section by summarizing the main stylized facts arising from our casual empirical analysis: Rising relative price trends and regional inflation differentials persist across EMU member countries since the adoption of the common currency in 1999. Moreover, most of the countries in our sample with inflation rates well above the EMU average - notably Ireland, Greece and Spain - are also experiencing terms of trade appreciations and rising expenditure shares. In the next two sections we present a simple model consistent with these empirical facts which is able to explain how demand-side factors can lead through changes in relative productivities to rising relative price trends and regional inflation differentials within a currency area, i.e. to inflation differentials which are consistent with the original Balassa-Samuelson effect.

3. – The Model

We develop a two-country model for a currency union in which both supply and demand shocks can play a role. The setup of our model is similar to standard New Open Economy Macro (NOEM) models (see the review article by Lane (2000)) except that we have removed the nominal rigidities, typically assumed in those models, to focus on medium run interactions.25 Thus, neither the exchange-rate regime nor, more in general, monetary

---

23 The period going from 1996 to the end of 1998 has been one of extreme exchange rate stability among the European countries considered in our study, such that the observed changes in expenditure shares cannot be ascribed to competitive devaluations. The exception is Greece, which has only stabilized its bilateral exchange rate vis-à-vis the EMU partners only starting from 1999 up to 2001 when it has formally joined the euro area.

24 Here we do not focus on the intertemporal dynamics of inflation differentials, i.e. on their persistence. This very important issue, which has non-trivial implications for the conduct of monetary policy in the euro area, is currently the focus of the Inflation Persistence Network at the ECB. See Altissimo et al. (2006) for a summary of their results.

25 In a different context MacDonald R., and L. Ricci (2002) also use the standard NOEM setup - a 'New Trade Theory' setup in their terminology - without assuming price stickiness.
factors matter for our discussion. Moreover, given our interest in the medium-run behavior of relative prices and regional inflation differentials, we assume that agents have perfect foresight.

The two countries, home and foreign, are assumed to be identical, populated by household-firm units distributed on the continuum [0,1]. Each economy, Home (H) and Foreign (F), has one traded, HT (FT), and one nontraded goods, HN (FN), sector. To simplify things we adopt a Yeoman-Farmer setup, i.e. each domestic (foreign) household h (f) owns a firm h (f) in the traded goods sector and a firm h (f) in the nontraded goods sector. Thus, each domestic (foreign) household-firm unit produces both one variety of traded goods, Y_{HT}(h) (Y_{FT}(f)), and one variety of nontraded goods, Y_{HN}(h) (Y_{FN}(f)).

The period specific utility function for the home and for the foreign representative agents is described by the following additively separable functional form:

\[ U(h) = \log C(h) - \frac{1}{1+\chi} \left[ L_{HT}(h) + L_{HN}(h) \right]^{1+\chi} \]

where aggregate consumption by each individual, \( C(h) \), is expressed as a Cobb-Douglas index:

\[ C(h) = C_{HN}(h)^{\gamma_{HN}} C_{HT}(h)^{\gamma_{HT}} C_{FT}(h)^{\gamma_{FT}} \]

Equation (2) implies that traded, as well as nontraded, goods are imperfect substitutes. The coefficients \( \gamma_{HT}, \gamma_{FT}, \gamma_{HN} \) denote the domestic expenditure shares of (home and foreign) tradables and nontradables, while the parameter \( \chi \) determines the curvature of the disutility of working hours. Note that the foreign household (f) has the same type of preferences and corresponding expenditure shares of tradables and nontradables \( \gamma_{HT}', \gamma_{FT}', \gamma_{HN}' \).

There are two production sectors, one for traded goods and one for nontraded goods. Each household-firm is initially endowed with half units of capital which fully depreciates in the process of production. Capital can be bought or sold across the currency area at price \( R \), i.e. capital is free to move both across sectors and across countries. Labor, the other input required for production, can only move across sectors but not across countries.

---

26 Note that it might seem odd to focus our attention on a 'currency' area, given that money does not appear in the setup of the model. We leave money out of the model for the sake of parsimony. Given our simplifying assumptions none of our results would be affected by the inclusion of money.

27 As explained in the previous section, 1999q1-2005q4 is the period we are focusing on. For details on data and data sources, see the figures at the end of the paper.


29 This logarithmic utility specification for consumption is not casual: It can be shown that within the class of utility functions with isoelastic preferences, employment is inversely related to productivity trends, if the intertemporal elasticity of substitution is greater than one. We want to avoid this controversial feature here, since productivity trends play an important role in our analysis.

30 Measurements of these expenditure shares are discussed in the previous section.

31 This is the simplest way to introduce capital by assuming it is a fixed factor, as in the two-sector closed economy model developed by Long J., and C. Plosser (1983). Allowing for full intra-period capital depreciation makes it possible to find simple closed form solutions for the relative prices in terms of the model's structural parameters. While this makes the intuition behind our results more transparent, working with a more elaborate dynamic model would not change our main qualitative conclusions.

32 Recent studies on the low degree of labor mobility across the European Union support this assumption. See for example Schmidt C., and M. Fertig (2002).
Production by each individual (household-)firm is then described by the following Cobb-Douglas functions:

\[
Y_{HT}(h) = A_{HT} \left[ K_{HT}(h) \right]^\alpha \left[ L_{HT}(h) \right]^{1-\alpha} \\
Y_{HN}(h) = A_{HN} \left[ K_{HN}(h) \right]^\beta \left[ L_{HN}(h) \right]^{1-\beta}
\]

where productivity shocks, \( A_{HT} \) and \( A_{HN} \), are sector- and country-specific, but do not depend on varieties \( h \).

The factor shares are constant, the same across countries and satisfy the standard assumptions, i.e.

\[
0 < \alpha < 1 \\
0 < \beta < 1
\]

We also assume that the traded goods sector is more capital-intensive relative to the nontraded goods one, such that the following condition always holds:³³

\[
\alpha > \beta
\]

Foreign (household-)firms, indexed by \( f \), have identical production functions, and the same restrictions discussed above apply to foreign factor shares. Thus, each household-firm in the currency area produces two particular varieties of goods, one for the traded and one for the nontraded goods sector. These varieties can be thought of as intermediate goods which are then sold to competitive firms. The competitive firms bundle these intermediate goods into final traded and final nontraded goods.³⁴ The domestic final goods can then be expressed in terms of the following CES indexes as:

\[
Y_{HT} = \left[ \int_0^1 Y_{HT}(h)^{\theta-1} dh \right]^{\frac{1}{\theta-1}} \\
Y_{HN} = \left[ \int_0^1 Y_{HN}(h)^{\theta-1} dh \right]^{\frac{1}{\theta-1}}
\]

where \( \theta \) measures the elasticity of substitution between goods which is assumed to be greater than one.³⁵ For simplicity, we also impose that \( \theta \) is the same for both domestic and foreign traded and nontraded goods varieties.

The corresponding home price indexes for the above final goods are:

\[
P_{HT} = \left[ \int_0^1 P_{HT}(h)^{1-\theta} dh \right]^{\frac{1}{1-\theta}} \\
P_{HN} = \left[ \int_0^1 P_{HN}(h)^{1-\theta} dh \right]^{\frac{1}{1-\theta}}
\]

³³ On this assumption see for example Obstfeld M., and K. Rogoff (1996) p.209. Some authors [Asea et al. (1994); Stockman A., and L. Tesar (1995)] document that the tradables sector is more labor-intensive. In a recent thorough study on the issue, Herrendorf B., and A. Valentin (2007) find for the U.S. that "tradeables are more capital intensive than nontradeables". We will discuss the implications of this finding for our results at the end of the next section.

³⁴ It might help to think about the competitive firms as supermarkets or department stores where a representative household usually buys his/her consumption "bundles". For a formal discussion of the "bundler" see Chari V., P. Kehoe, and E. McGrattan (2000) and Canzoneri M., R. Cumby, and B. Diba (2005).

³⁵ For a discussion on the role of the elasticity of substitution between varieties of differentiated goods see Obstfeld M., and K. Rogoff (1996), page 661.
Thus, each individual household-firm acts as a monopolistic producer and faces a downward sloping demand - derived from the competitive firm's profit maximization problem - for its variety of the following Dixit-Stiglitz (1977) form:

\[
Y_{HT}(h) = \left[ \frac{P_{HT}(h)}{P_{HT}} \right]^{-\theta} Y_{HT}
\]

(7)

\[
Y_{HN}(h) = \left[ \frac{P_{HN}(h)}{P_{HN}} \right]^{-\theta} Y_{HN}
\]

Demand for each variety of good is positively related to aggregate demand and inversely related to relative prices. The relative price elasticity of demand for a single variety of (traded or nontraded) good is increasing in the elasticity of substitution across varieties \( \theta \). Similar relationships to (5), (6) and (7) hold for the foreign economy.

We further assume the existence in each country of another competitive industry that combines \( Y_{HT}, Y_{HN}, Y_{FT} \) and \( Y_{FN} \) into the composite consumption goods:

\[
C_H = (C_{HN})^{\gamma_{HN}} (C_{HT})^{\gamma_{HT}} (C_{FT})^{\gamma_{FT}}
\]

(8)

\[
C_F = (C_{FN})^{\gamma_{FN}} (C_{HF})^{\gamma_{HF}} (C_{FT})^{\gamma_{FT}}
\]

and then sells them to the home and foreign households at the aggregate prices:

\[
P_H = \left( \frac{P_{HN}}{\gamma_{HN}} \right)^{\gamma_{HN}} \left( \frac{P_{HT}}{\gamma_{HT}} \right)^{\gamma_{HT}} \left( \frac{P_{PT}}{\gamma_{PT}} \right)^{\gamma_{PT}}
\]

(9)

\[
P_F = \left( \frac{P_{FN}}{\gamma_{FN}} \right)^{\gamma_{FN}} \left( \frac{P_{HF}}{\gamma_{HF}} \right)^{\gamma_{HF}} \left( \frac{P_{FT}}{\gamma_{FT}} \right)^{\gamma_{FT}}
\]

Finally the resource constraint of the domestic household-firm unit is given by:

\[
\frac{R}{2} + P_{HT}(h)Y_{HT}(h) + P_{HN}(h)Y_{HN}(h) \geq P_{HT}C_{HT}(h) + P_{HN}C_{HN}(h) + P_{FT}C_{FT}(h)
\]

(10)

\[+ R[K_{HT}(h) + K_{HN}(h)]\]

where the terms on the left side of the inequality represent the sources of income - the value of the capital endowment and total revenues from traded and nontraded goods sales - while the terms to the right of the inequality sign denote the expenditures of the representative household-firm on consumption bundles and on the required capital inputs for production. Foreign households face a similar budget constraint.

Each domestic representative household chooses consumption, employment, (net) capital holdings and domestic prices (for both varieties of goods) to maximize (1) subject to the budget constraint (10), the production technologies (3) and the demand functions (7). The foreign household-firm faces a similar optimization problem.

Assuming for simplicity that the disutility of work is linear (\( \chi=0 \)) the following optimality conditions have to hold for each domestic representative agent:

\[36 \text{The assumption that production takes place under monopolistic competition is simply made to be in line with the current NOEM literature. It is by no means a key ingredient for our results.}\]

\[37 \text{When labor disutility is non-linear (} \chi \neq 0 \text{) only condition (iii) changes. It includes an additional non-linear term for aggregate employment. None of our results hinges on the value of } \chi.\]
(i) Nominal expenditures on each type of good are proportional to each other:

\[
\frac{1}{\gamma_{HN}} P_{HN} C_{HN}(h) = \frac{1}{\gamma_{HT}} P_{HT} C_{HT}(h)
\]

\[
\frac{1}{\gamma_{FF}} P_{FF} C_{FF}(h) = \frac{1}{\gamma_{HT}} P_{HT} C_{HT}(h)
\]

Notice that contrary to most of the current open economy macro literature we allow for shifts in the expenditure shares \((\gamma_{HT}, \gamma_{FF}, \gamma_{HN}, \gamma_{FN})\). Throughout this work - see Section 4.4 in particular - we assume that demand-side shocks are determined by exogenous shifts in the expenditure shares.38

(ii) The capital-labor ratios are always proportional across sectors:

\[
\frac{1 - \alpha}{\alpha} \frac{K_{HT}(h)}{L_{HT}(h)} = \frac{1 - \beta}{\beta} \frac{K_{HN}(h)}{L_{HN}(h)}
\]

(iii) Prices are set as a mark-up over nominal marginal costs:

\[
P_{HT} = \frac{\theta}{\theta - 1} A_{HT} \left[ \frac{K_{HT}(h)}{L_{HT}(h)} \right]^\alpha
\]

\[
P_{HN} = \frac{\theta}{\theta - 1} A_{HN} \left[ \frac{K_{HN}(h)}{L_{HN}(h)} \right]^\beta
\]

where \(\mu(h)\) is the domestic household's marginal utility of nominal wealth.39 Similar conditions hold for foreign agents.40

The model is closed by imposing that markets for goods and capital clear:

\[
\int_0^1 C_{HT}(h) dh + \int_0^1 C_{HT}(f) df = \int_0^1 Y_{HT}(h) dh
\]

\[
\int_0^1 C_{FF}(h) dh + \int_0^1 C_{FF}(f) df = \int_0^1 Y_{FF}(f) df
\]

\[
\int_0^1 C_{HN}(h) dh = \int_0^1 Y_{HN}(h) dh
\]

\[
\int_0^1 C_{FN}(f) df = \int_0^1 Y_{FN}(f) df
\]

\[
\int_0^1 K_{HT}(h) dh + \int_0^1 K_{HT}(h) dh + \int_0^1 K_{FF}(f) df + \int_0^1 K_{FN}(f) df = 1
\]

In what follows we will focus our attention on a symmetric equilibrium in which household variables are equal to aggregate variables.41

38 Stockman A., and L. Tesar (1995) model their taste shocks in a similar way. Contrary to them, we do not express expenditure shares in relative (traded versus nontraded) terms. Canzoneri et al. (2005) look at the implications of shocks to expenditure shares for the cyclical behavior of inflation differentials in a one-sector NOEM model for a currency area with capital accumulation and nominal rigidities.

39 Following Tille C. (2001), the above optimality condition states that in equilibrium output is inefficiently low, since the marginal utility of consumption exceeds the marginal cost of producing the output consumed by the mark-up \((\theta/(\theta - 1))\).

40 If we assumed a decentralized setup the labor supply decision of the household would link the marginal utility of nominal wealth to the wage rate and we would obtain a more familiar expression with wages instead of marginal utilities in the above pricing equation.
4. – Equilibrium relative price movements

4.1 Some definitions

Our primary interest in this work is to understand the equilibrium movements of relative prices and regional inflation differentials for a currency union, in response both to demand and supply-side shocks. In our framework, regional inflation differentials depend on how the ratio of aggregate prices, \( \frac{P_E}{P_H} \), responds to supply and to demand shocks. Given our assumptions, \( \frac{P_E}{P_H} \) can be expressed as:

\[
\frac{P_E}{P_H} = \Gamma \left( \frac{P_{FN}}{P_{HN}} \right)^{\gamma_{FN}} \left( \frac{P_{HT}}{P_{HT}} \right)^{\gamma_{HT}} \left( \frac{P_{FT}}{P_{FT}} \right)^{\gamma_{FT}}
\]

where \( \Gamma = \left( \gamma_{FN}^{*} \right)^{\gamma_{FN}} \left( \gamma_{HT}^{*} \right)^{\gamma_{HT}} \left( \gamma_{FT}^{*} \right)^{\gamma_{FT}} \). A fall in \( \frac{P_E}{P_H} \) corresponds to a positive domestic inflation differential. Using the fact that in each country the sum of the expenditure shares - \( \gamma_{HN}, \gamma_{HT}, \gamma_{FT} \) for the domestic and \( \gamma_{*FN}, \gamma_{*HT}, \gamma_{*FT} \) for the foreign country - adds up to unity, it is possible to rewrite the above price ratio as a function of a term involving the expenditure shares, the terms of trade - \( \frac{P_{FT}}{P_{HT}} \) - and the 'internal real exchange rate' - the ratio of country-specific relative prices (\( \frac{P_{CN}}{P_{CT}} \) where \( C=H,F \)):

\[
\frac{P_E}{P_H} = \Gamma \left( \frac{P_{FT}}{P_{HT}} \right)^{1-\gamma_{HT}-\gamma_{FT}} \left( \frac{P_{FN}}{P_{FT}} \right)^{\gamma_{FN}} \left( \frac{P_{HN}}{P_{HT}} \right)^{\gamma_{HN}}
\]

The above expression highlights that regional inflation differentials can be driven by changes in expenditure shares, in the terms of trade and in the internal real exchange rate. Sections 4.3 and 4.4 will explore the linkages between demand (supply) shocks and the components of \( \frac{P_E}{P_H} \). These linkages will in turn explain the regional inflation differentials that can arise in our currency area model.

4.2 Closed form solutions for inputs

The equilibrium values for relative prices and regional inflation differentials, as well as for all the other endogenous variables in the model, can be expressed in terms of the reduced-form solutions for both countries' inputs - capital and labor. To simplify the expressions of the closed-form solutions for capital and labor we further assume that both countries' expenditure shares of tradables are identical, i.e. that \( \gamma_{HT}=\gamma_{*HT} \) and \( \gamma_{FT}=\gamma_{*FT} \) (which implies that \( \gamma_{HN}=\gamma_{*FN}=\gamma_{N} \)). This strong assumption, which only allows for a distinction between the expenditure shares on domestic and foreign tradables, does not affect our qualitative results, as we show in the Appendix, and is widely used in the literature.\(^{42}\) Moreover, it is consistent with our way of measuring the expenditure shares (see the previous section),

\(^{41}\) This simplification results from (i) households being distributed on the unit interval, (ii) each household producing all of the goods, (iii) production functions being the same for all producers within each sector and (iv) productivity shocks being the same at the sector-level.

\(^{42}\) The seminal reference is CORSETTI G., AND P. PESENTI (2001).
which does not distinguish between the domestic and foreign demand for the traded goods produced by each country. What we really want to capture are shifts in the relative expenditure shares for the traded goods produced by the member countries of the currency area.\footnote{These shifts may initially only affect the home country’s export shares (i.e. the foreign expenditure shares on the home traded goods) as it joins the currency area due to the pro-trade effects associated to closer financial and real integration. Eventually, as positive income effects set in, the shifts due to these pro-trade effects would also result in increased home demand for the home traded goods.} The closed-form solutions for capital and labor are reported below in the next box:  

Closed form solutions for sectoral inputs – BOX 1

<table>
<thead>
<tr>
<th>Sector</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_{HT}$</td>
<td>$\gamma_{HT} \alpha \frac{\Psi}{\Omega}$</td>
</tr>
<tr>
<td>$K_{FT}$</td>
<td>$\gamma_{FT} \alpha \frac{\Psi}{\Omega}$</td>
</tr>
<tr>
<td>$K_{HN}$</td>
<td>$\gamma_{N} \beta \frac{\Sigma_{H}}{\Omega}$</td>
</tr>
<tr>
<td>$K_{FN}$</td>
<td>$\gamma_{N} \beta \frac{\Sigma_{F}}{\Omega}$</td>
</tr>
<tr>
<td>$L_{HT}$</td>
<td>$(1-\alpha) \gamma_{HT} \left( \frac{\theta-1}{\theta} \right) \frac{\Psi}{\Sigma_{H}}$</td>
</tr>
<tr>
<td>$L_{FT}$</td>
<td>$(1-\alpha) \gamma_{FT} \left( \frac{\theta-1}{\theta} \right) \frac{\Psi}{\Sigma_{F}}$</td>
</tr>
<tr>
<td>$L_{HN}$</td>
<td>$\left( \frac{1}{\theta} \right) \gamma_{N}$</td>
</tr>
<tr>
<td>$L_{FN}$</td>
<td>$\left( \frac{1}{\theta} \right) \gamma_{N}$</td>
</tr>
</tbody>
</table>

\(\Sigma_{H}, \Sigma_{F}, \Psi\) and \(\Omega\) are functions of the factor shares, of the elasticity of substitution across varieties and of the expenditure shares. Thus, capital in each sector depends both on supply- and on demand-side factors. The former are embodied in the mark-up \((\theta/(\theta-1))\) and factor-intensities \((\alpha, \beta)\) parameters, which reflect the market structure and the differences in production technologies across sectors. The latter are represented by the expenditure shares for tradables \((\gamma_{HT}, \gamma_{FT})\) and nontradables \((\gamma_{N})\). As will soon become clear, this dependence of equilibrium sectoral capital allocations on expenditure shares plays a key role in our setup, in order to understand the linkage between regional inflation differentials and demand-side shocks.\footnote{In solving the model we have taken \(R\), the rental price of capital, as the numeraire.} Similarly, employment across countries and sectors depends on demand- as well as on supply-side factors. Notice that neither the equilibrium values for capital nor the equilibrium values for labor depend on total-factor-productivities \(A_{cs}\), where \(cs=HT,FT,HN,FN\).

4.3 Equilibrium relative price movements and supply-side shocks

We start by focusing on how shocks to sectoral total-factor-productivities affect regional inflation differentials through changes in the two components highlighted by equation (16) - the terms of trade and the internal real exchange rate. For this purpose we assume that

\[\Sigma_{H}, \Sigma_{F}, \Psi \text{ and } \Omega \text{ are functions of the factor shares, of the elasticity of substitution across varieties and of the expenditure shares. Thus, capital in each sector depends both on supply- and on demand-side factors. The former are embodied in the mark-up } \left( \frac{\theta}{\theta-1} \right) \text{ and factor-intensities } (\alpha, \beta) \text{ parameters, which reflect the market structure and the differences in production technologies across sectors. The latter are represented by the expenditure shares for tradables } (\gamma_{HT}, \gamma_{FT}) \text{ and nontradables } (\gamma_{N}). \text{ As will soon become clear, this dependence of equilibrium sectoral capital allocations on expenditure shares plays a key role in our setup, in order to understand the linkage between regional inflation differentials and demand-side shocks. Similarly, employment across countries and sectors depends on demand- as well as on supply-side factors. Notice that neither the equilibrium values for capital nor the equilibrium values for labor depend on total-factor-productivities } (A_{cs}, \text{ where } cs=HT,FT,HN,FN).\]
total-factor-productivity in the home (foreign) traded goods sector, \( A_{HT} (A_{FT}) \), is growing relative to total-factor-productivity in the home (foreign) nontraded goods sector, \( A_{HN} (A_{FN}) \), and that \( A_{HT} \) is growing faster than \( A_{FT} \).\(^{46}\)

The first component on the right hand-side of the equality sign in equation (16) rises: the terms of trade, \( P_{FT}/P_{HT} \), deteriorate since domestic marginal costs in the traded goods sector are falling more rapidly than in the foreign traded goods sector. This can be easily seen by expressing the terms of trade in terms of the standard optimal pricing equations as:

\[
\frac{P_{FT}}{P_{HT}} = \frac{A_{HT}}{A_{FT}} \left[ \frac{K_{FT}/L_{FT}}{K_{HT}/L_{HT}} \right]^{1-\alpha}
\]

Recall from the solutions for factor inputs in Box (1), that neither capital nor employment are affected by shocks to total-factor-productivities. Thus, sectoral capital-labor ratios are not affected by supply-side shocks. In our model any rise in the ratio \( A_{HT}/A_{HT} \) translates one-to-one into a terms of trade deterioration.

Focusing next on the internal real exchange rate, the second component on the right hand-side of the equality sign in equation (16), it is straightforward to show that for the Cobb-Douglas specification of the production function chosen, the home (foreign) relative price of nontradables, \( P_{HN}/P_{HT} (P_{FN}/P_{FT}) \), is proportional to the ratio of average products of labor (\( APL \)) in each sector:

\[
\frac{P_{HN}}{P_{HT}} = \frac{1-\alpha}{1-\beta} \frac{APL_{HN}}{APL_{HT}} = \frac{A_{HN}}{A_{HT}} \left[ \frac{1-\alpha}{1-\beta} \right]^{1-\beta} \left[ \frac{\alpha}{\beta} \right]^{\frac{\gamma}{\alpha}} \left[ \frac{K_{HT}}{L_{HT}} \right]^{\gamma-\beta}
\]

\[
\frac{P_{FN}}{P_{FT}} = \frac{1-\alpha}{1-\beta} \frac{APL_{FN}}{APL_{FT}} = \frac{A_{FN}}{A_{FT}} \left[ \frac{1-\alpha}{1-\beta} \right]^{1-\beta} \left[ \frac{\alpha}{\beta} \right]^{\frac{\gamma}{\alpha}} \left[ \frac{K_{FT}}{L_{FT}} \right]^{\gamma-\beta}
\]

The second equality (in each equation) follows from the fact that, in equilibrium, capital-labor ratios are proportional across sectors (see equation (12)).\(^{47}\) The internal real exchange rate can thus be expressed as:

\[
\frac{P_{HN}/P_{HT}}{P_{FN}/P_{FT}} = \frac{P_{FT}}{P_{HT}} \frac{P_{HN}}{P_{HT}} = \frac{A_{HT}/A_{HN}}{A_{HT}/A_{HN}} \left[ \frac{K_{HT}/L_{HT}}{K_{HT}/L_{HT}} \right]^{1-\alpha}
\]

where we first simply rearrange the internal real exchange rate to express it as a function of the terms of trade and of the ratio of domestic over foreign nontraded goods prices. Equations (18) and (19) suggest that higher productivity growth rates in the traded goods sector, relative to the nontraded goods one, lead to a higher relative price for nontradables in each country. Moreover, given our assumption that the ratio of traded goods to nontraded goods productivity grows faster at home than abroad, the home country's relative price, \( P_{HN}/P_{HT} \), rises faster than the relative price in the foreign country, \( P_{FN}/P_{FT} \), so that in (20) the internal real exchange rate appreciates.

The benchmark preferences used in this paper - logarithmic utility function and Cobb-Douglas aggregate consumption aggregator - imply that a relative productivity shock leads to a terms of trade depreciation and to an internal real exchange rate appreciation.

---

46 This implies that \( A_{HT}/A_{HN} \) is growing faster than \( A_{FT}/A_{FN} \). These assumptions about sectoral productivity trends are often made in the literature on the Balassa-Samuelson hypothesis. See for example Canzoneri, M., R. Cumby, and B. Diba (1999); Obstfeld, M., and K. Rogoff (1996); Frenkel, J., and A. Razin (1996).

47 Canzoneri et al. (1999) explain that relative prices are proportional to average products of labor under assumptions about production functions that are less stringent than the Cobb-Douglas specification.
Moreover, under our assumptions of equal expenditure shares for all the countries belonging to the currency area equation (16) simplifies to:

\[
\frac{P_F}{P_H} = \left( \frac{P_{FT}}{P_{HT}} \right)^{\gamma_{HT} - \gamma_{FT}} \left( \frac{P_{FN}}{P_{FT}} \right)^{\gamma_{FT}} = \left( \frac{P_{FN}}{P_{HT}} \right)^{\gamma_{FT}}
\]

Equation (21) shows that - in the absence of major differences between the growth rates of domestic and foreign nontraded goods sectors’ productivities, \(A_{HN}\) and \(A_{FN}\) - the terms of trade depreciation exactly offsets the internal real exchange rate appreciation and no regional inflation differential arises in response to supply-side shocks.\(^{48}\)

Under more general assumptions about expenditure shares, which allow for differences between \(\gamma_{HT}, \gamma^*_{HT}, \gamma_{FT}\) and \(\gamma^*_{FT}\), changes in regional inflation differentials depend on which of the two effects - the terms of trade deterioration versus the internal real exchange rate appreciation - dominates. This can be seen by rearranging equation (16) as follows:

\[
\frac{P_F}{P_H} = \Gamma \left( \frac{P_{FT}}{P_{HT}} \right)^{\gamma^*_{FT} - \gamma_{FT}} \left( \frac{P_{FN}}{P_{FT}} \right)^{\gamma^*_{FT}} = \frac{\left( \gamma^*_{FN} \right)^{\gamma^*_{FT}}}{\left( \gamma_{HT} \right)^{\gamma_{HT}} \left( \gamma_{FT} \right)^{\gamma_{FT}}}
\]

where again \(\Gamma = \frac{\left( \gamma^*_{FN} \right)^{\gamma^*_{FT}}}{\left( \gamma_{HT} \right)^{\gamma_{HT}} \left( \gamma_{FT} \right)^{\gamma_{FT}}}\). Equation (22) suggests that with the benchmark preferences used in this paper a contemporaneous rise in \(A_{HT}/A_{HN}\) relative to \(A_{FT}/A_{FN}\) and in \(A_{HT}/A_{FT}\) can lead to a negative domestic inflation differential - a rise in \(P_F/P_H\) - when there is a home bias effect - \(\gamma_{HT} > \gamma^*_{HT}\) and \(\gamma_{FT} < \gamma^*_{FT}\) - and productivity growth rates in the domestic and foreign nontraded goods sectors are 'close'.\(^{49}\) In other words, in equation (22) above, the internal real exchange rate appreciation can be dominated by the terms of trade depreciation. This result differs from the original Balassa-Samuelson hypothesis according to which terms of trade changes are absent - traded goods prices obey the law of one price - and any internal real exchange rate appreciation always leads to a positive inflation differential (a fall in \(P_F/P_H\)). On the contrary, as we have just pointed out, when traded goods are imperfect substitutes relative supply-side shocks only lead to positive inflation differentials if the internal real exchange rate appreciation dominates the terms of trade depreciation. In light of the numerous papers documenting deviations from the law of one price for the traded goods produced across different national and regional borders, most of the new generation open economy models assume that traded goods are imperfect substitutes.\(^{50}\) As explained above this assumption of imperfect substitutability leads to Balassa-Samuelson type relative price trends and regional inflation differentials.

\(^{48}\) Major differences between the home and foreign nontraded goods sectors productivity growth rates would lead, in equation (21), to regional inflation differentials through changes in the ratio of nontraded goods prices.

\(^{49}\) Under more general assumptions about preferences - when the intertemporal elasticity of substitution is higher than unity - it can be shown that if (i) the home bias effect is not too strong - i.e. \(\gamma_{HT} (\gamma^*_{FT})\) is not too high compared to \(\gamma^*_{HT} (\gamma_{FT})\) - and (ii) the weight of aggregate (home and foreign) tradables in the consumption index is not too high, then a contemporaneous rise in \(A_{HT}/A_{HN}\) relative to \(A_{FT}/A_{FN}\) and in \(A_{HT}/A_{FT}\) always leads to a positive domestic inflation differential, i.e. to a fall in \(P_F/P_H\). See BENIGNO G., AND C. THOENISSEN (2002) for a discussion of this point in a model with nominal rigidities. Warnock (1999) focuses on the role of the home bias effect in NOEM models.

\(^{50}\) ENGEL C., AND J. ROGERS (1996) and ENGEL C. (1999; 2000) are the standard references documenting deviations from the law of one price for traded goods.
only under fairly restrictive assumptions about sectoral productivity trends and expenditure shares.

The above analysis points out another important general observation. In our setup with log utility and imperfect substitutability across domestic and foreign tradables - the benchmark preferences used by the literature - (relative) supply-side disturbances affect relative prices through a transmission mechanism that differs completely from the original supply-side view by Balassa and Samuelson. According to the original Balassa-Samuelson hypothesis firms are perfectly competitive and all the traded goods obey to the law of one price. The terms of trade are always fixed and relative productivity improvements translate into positive regional inflation differentials - a fall in $P_T/P_H$ - via rising relative prices, solely driven by rising marginal costs in the nontraded goods sector: any productivity improvement in the traded goods sector relative to the nontraded goods sector leads to a one-to-one increase in the wages paid to workers in the traded goods sector. Labor mobility across sectors leads to wage equalization and rising marginal costs in the nontraded goods sector. On the contrary, with the benchmark preferences used in our model, labor mobility plays no role for the transmission of relative productivity shocks to relative prices and, hence, for the determination of regional inflation differentials. When traded goods are imperfect substitutes, any productivity improvement in the traded goods sector relative to the nontraded goods sector leads to a one-to-one decrease in traded goods prices. This price fall benefits each representative household-firm in exactly the same way, i.e. real wages increase for all households. There is then no need for labor to move across sectors and marginal cost conditions in the nontraded goods sector remain unaffected by productivity changes in the traded goods sector. Supply-side shocks and labor mobility are completely disconnected in our model. Thus even if our model is able to generate the original Balassa-Samuelson effects - rising relative prices and positive inflation differentials - the underlying transmission mechanism differs completely from the original one, which operates through the labor markets.

4.4 Equilibrium relative price movements and demand-side shocks

In this section we offer a complementary explanation as to how our simple setup can account for Balassa-Samuelson type equilibrium movements in relative prices and in regional inflation differentials when demand-side factors play an explicit role. Suppose that for some reason - e.g. due to an improvement in the quality or competitiveness of a country's exports as it joins the currency area which leads consumers in the entire area to 'demand more' of its tradables - the relative demand for the traded goods produced by the home country rises permanently, i.e. $\gamma_{HT}$ rises relative to $\gamma_{FT}$. In our model this will lead (i)

51 Mechanically this can be seen again by noting that in Box (1) none of the closed form solutions for employment depends on productivity.

52 When the intertemporal elasticity of substitution differs from unity this statement is no longer correct. It can be shown that when, e.g., the intertemporal elasticity of substitution exceeds one, supply side shocks to one sector affect (marginally), through labor mobility, employment and (nominal) wage levels in both sectors. Thus, when preferences are no longer logarithmic, shocks to the traded goods sector affect (nominal) marginal costs in the nontraded goods sector.

53 Labor markets (and labor mobility, in particular) are receiving an increasing attention in all EMU member countries. Particularly in countries such as Italy, Germany, France, Spain and Greece which exhibit persistent and large regional inflation differentials, often ascribed by policymakers to the low degree of labor market flexibility.
to a rise in the relative price of nontradables at home and (ii) to a positive inflation
differential relative to the foreign country, i.e. to a rise in the home aggregate price index
relative to the foreign aggregate price index. (i) and (ii) are the same relative price effects
implied by the original Balassa-Samuelson hypothesis, but in our setup they are entirely
demand driven.

As before we decompose the change in \( \frac{P_F}{P_H} \) into terms of trade and internal real
exchange rate changes. Using the solutions for capital and employment (the equations in
Box(1)) we can now express the terms of trade as:

\[
\frac{P_{FT}}{P_{HT}} = \frac{A_{HT}}{A_{FT}} \left[ \frac{K_{FT}}{K_{HT}} / \frac{L_{FT}}{L_{HT}} \right]^{1-\theta} = \frac{A_{HT}}{A_{FT}} \left[ \frac{2\theta(\gamma_{FT}) + (\theta - 1)(\beta\gamma_N + \alpha(\gamma_{HT} - \gamma_{FT}))}{2\theta(\gamma_{HT}) + (\theta - 1)(\beta\gamma_N - \alpha(\gamma_{HT} - \gamma_{FT}))} \right]^{1-\theta} \\
= \frac{A_{HT}}{A_{FT}} \left[ \frac{\sum_F}{\sum_H} \right]^{1-\theta}
\]

and noting the fact that

\[
\frac{d\Sigma_H}{d\gamma_{HT} \mid \gamma_{HT} \rightarrow -d\gamma_{FT}} > 0 \\
\frac{d\Sigma_F}{d\gamma_{HT} \mid \gamma_{HT} \rightarrow -d\gamma_{FT}} < 0
\]

it follows from equation (23) that the terms of trade appreciate - \( P_{FT}/P_{HT} \) falls - in response
to a relative demand shock to the home traded goods sector.\(^54\) Focusing next on the internal
real exchange rate, we can combine (18), (19) and the solutions for capital and employment
in the home and foreign traded goods sectors to express domestic and foreign relative
prices respectively as:

\[
\frac{P_{HN}}{P_{HT}} = \frac{A_{HT}}{A_{HN}} \left[ \frac{1-\alpha}{1-\beta} \right]^{1-\beta} \left[ \frac{\alpha}{\beta} \right]^{\beta} \left[ \frac{K_{HT}}{L_{HT}} \right]^{a-\beta} = \Phi \frac{A_{HT}}{A_{HN}} \left[ \frac{\sum_H}{\Omega} \right]^{a-\beta}
\]

\[
\frac{P_{FN}}{P_{FT}} = \frac{A_{FT}}{A_{FN}} \left[ \frac{1-\alpha}{1-\beta} \right]^{1-\beta} \left[ \frac{\alpha}{\beta} \right]^{\beta} \left[ \frac{K_{FT}}{L_{FT}} \right]^{a-\beta} = \Phi \frac{A_{FT}}{A_{FN}} \left[ \frac{\sum_F}{\Omega} \right]^{a-\beta}
\]

where \( \Phi \) is a constant that only depends on factor shares.\(^55\) It is then easy to show that for a
permanent rise in the relative demand for the traded goods produced by the home country:

\[
\frac{d\Omega}{d\gamma_{HT} \mid \gamma_{HT} \rightarrow -d\gamma_{FT}} = 0
\]

which together with (24) implies that domestic relative average labor productivities
\( (APL_{HN}/APL_{HT}) \), and domestic relative prices, \( P_{HN}/P_{HT} \), rise, while foreign relative average
labor productivities \( (APL_{FN}/APL_{FT}) \), and foreign relative prices, \( P_{FN}/P_{FT} \), fall, i.e. the
internal real exchange rate, \( (P_{FN}/P_{FT})(P_{HN}/P_{HT}) \), appreciates. Finally, combining
expressions (23), (25) and (26) we can look at regional inflation differentials by rewriting
\( P_{F}/P_{H} \) (equation (21)) as:

\[^54\] We denote with \( (dx)/(d\alpha_1) \mid d\alpha_1 = -d\alpha_2 \) the variation in \( x = f(\alpha_1, \alpha_2) \) due to contemporaneous changes in \( \alpha_1 \) and
\( \alpha_2 \), \( d\alpha_1 \) and \( d\alpha_2 \) respectively, such that \( d\alpha_1 = -d\alpha_2 \).

\[^55\] Complete derivations of all the results that follow can be found in the Appendix.
Demand shocks lead, thus, in our model to regional inflation differentials through adjustments in the terms of trade and in the internal real exchange rate which occur in the same directions. An increase in the domestic traded goods expenditure share, $\gamma_{HT}$, relative to the foreign traded goods expenditure share, $\gamma_{FT}$, leads both to a terms of trade and to an internal real exchange rate appreciation. The next proposition re-states our main result more formally.

**Proposition:** If preferences are isoelastic with the intertemporal elasticity of substitution equal to unity (log utility), the elasticity of substitution across consumption goods is unity and home tradables are more capital intensive than home nontradables, then a permanent rise in the expenditure share of home tradables, leads to a rise (fall) in the domestic (foreign) relative price of nontradables and to a rise in the domestic aggregate price level relative to the foreign one, i.e. to a positive regional inflation differential.

A more complete derivation of this result can be found in the Appendix. Here we will instead focus on the intuition behind our main result and discuss its robustness.

Full integration of capital markets in the currency union assures that, in equilibrium, the rental price of capital always equalizes across member countries, i.e. at any point in time the following equilibrium condition must hold in all the sectors of the currency union:

$$
R = P_c MPK_{cs}
$$

where $MPK$ is the marginal product of capital and $cs$ equals $HT$, $HN$, $FT$, $FN$. In words, condition (29) states that in equilibrium the rental price of capital $R$ has to be equalized across countries and sectors.

As the demand for home tradables increases relative to the demand for foreign tradables, prices for home tradables rise (a terms of trade appreciation). According to equation (29) the rental price of capital in the home traded goods sector rises. Given our assumptions about preferences and the aggregate consumption index, as the prices of home tradables rise relative to the prices of nontradables, domestic consumers respond by demanding also more nontraded goods. *Ceteris paribus*, the price of nontraded goods rises in response to this additional demand by domestic residents. Thus, according to equation (29) the rental price of capital rises temporarily above its initial equilibrium value in both domestic sectors. Capital immediately shifts to the home country, in order to take advantage of the higher home rental price of capital. Since tradables production is more capital intensive than the

---

56 This result suggests further, that in equations (23), (25), (26) and (28), the sectoral capital-labor ratios should be positively related with the terms of trade, with the relative prices of nontraded goods and, hence, with the relative (domestic over foreign) aggregate price differentials. On the implications of supply side factors, in the standard Balassa-Samuelson model, for sectoral capital-labor ratios see KAKKAR V. (2002).

57 It can also be shown that this result does not depend on the value of the Frisch labor supply elasticity, $\chi$.

58 One could also assume that the increase in the expenditure shares of home tradables is, entirely or partly, offset by a fall in the expenditure shares of nontradables, but as we discuss later this seems at odds with the empirical evidence.

59 Given that this is a simple one-period model all of the adjustments described in this paragraph will clearly take place instantaneously, but it seems nonetheless useful to highlight them separately so as to gain some more intuition on our main result. We are really describing the transition from one equilibrium to another as the economy is hit by a permanent demand shock.
production of nontradables, as foreign capital flows into the home country, labor productivity rises more in the traded than in the nontraded goods sector. This leads to an equilibrium increase in the relative price of domestic nontradables and, together with the initial terms of trade appreciation, to a positive inflation differential across the two countries. The flow of capital continues until rental prices equalize again across the currency area, i.e. until the capital markets equilibrium condition (29) is restored.

The capital market plays thus an explicit role in our model; it offers, besides the labor market, an additional transmission channel between the consumption- and the production-side. Moreover, adjustments on the capital market also lead to labor movements across sectors - due to the fact that the capital flows have asymmetric effects on sectoral labor productivities - which in turn affect relative prices. Thus both capital and labor movements play a key role for the transmission of relative demand shocks on relative prices and regional inflation differentials. Marginal costs in the nontraded goods sector are no longer unaffected by shocks to the traded goods sector as when disturbances originate only on the supply-side.

The model accounts qualitatively, from a demand-side perspective, for the stylized facts emphasized by the empirical literature on the original Balassa-Samuelson hypothesis: rising labor productivities, rising relative labor productivity, rising relative prices and positive regional inflation differentials. It also yields equilibrium adjustments in home production which are consistent with another important stylized fact: the rise, over time, in nontradables consumption alongside relative prices. Following the demand shock, the home economy attains a new equilibrium with higher output and consumption of both tradables and nontradables.

Two important caveats are in order at this point. First, if the nontraded goods sector is, contrary to our assumptions, more capital intensive than the traded goods sector, then the relative price adjustments initiated by the demand-shock are reversed. Thus, unless factor intensities differ across sectors, demand-side factors play no role for the determination of equilibrium relative prices and inflation differentials. Moreover, even if demand-side factors can offer a complementary explanation for relative price movements, for these to be consistent with the Balassa-Samuelson effect it must be the case that labor-shares are lower in the traded than in the nontraded goods sector. Measurement of sectoral labor-shares is a very controversial issue. Several studies find evidence that labor-shares are higher for tradables than for nontradables. Sectoral labor-shares are usually estimated using real unit labor costs, which for a generic sector $S$ can be expressed as:

\[ \text{real unit labor costs for sector } S \]
\[ \alpha_s = \frac{W_SL_s}{P_SY_s} \]

where \( \alpha_s \) measures the labor share in the sector. Such estimates could be severely biased by the sectoral price indexes, \( P_s \), used to deflate nominal unit labor costs, if mark-ups differ 'significantly' across sectors. As pointed out in a recent paper by Basu, Fernald and Shapiro (2004), mark-up estimates for nontraded manufactures are relatively higher than those for the traded manufactures. We suspect that this might be true more in general, not merely for manufactured (traded vs. nontraded) goods. Thus, by accounting for different sectoral mark-ups, it could well be the case that traded goods are more capital-intensive than nontraded ones as we assume throughout this work.64

Finally, if prices in both sectors are sticky or sluggish then the demand-side effects on relative prices and inflation differentials will reflect both changes in tastes, as before, and sector-specific nominal price rigidities with possibly interesting normative implications.65

5. – Conclusion

The present work presents a simple open economy model that shows how demand shocks can affect relative prices and lead to inflation differentials across the member countries of a currency union. We address this question by focusing on the regional and sectoral inflation differentials that persist across some member countries of the European Monetary Union. Our work offers a complementary explanation for these regional inflation differentials: we argue that it is not enough to rely on changes in relative labor productivities to conclude that persistent regional inflation differentials are simply due to supply-side shocks in accordance with the standard Balassa-Samuelson hypothesis. In our model the same relative productivity and price trends suggested by the literature on the Balassa-Samuelson hypothesis can be generated by shifts in expenditure shares that lead the existing capital stock in the currency union to move towards those sectors experiencing a rise in relative demand. A first-pass look at the data available so far supports our point of view that the demand-side explanation is plausible and that it needs to receive more consideration in future empirical and theoretical work.66

64 For estimates on mark-ups for manufactures see Table 1 at the end of BASU et AL. (2004). CANZONERI et AL. (2002) refer to the "protected service sector hypothesis", i.e. to the possibility that mark-ups in the nontraded sectors are higher than in the traded sectors, because of a lower exposure of nontraded sectors (private and government services in particular) to external competitive pressures. The measurement of sectoral labor-shares is clearly a too important point for our analysis and we plan to investigate this issue more carefully in the course of future work.

65 We plan to investigate this claim shortly by extending our model to a dynamic setting with staggered prices. Altissimo et al. (2004) develop a dynamic model with nominal price rigidities for a currency area, which has no capital accumulation but assumes diminishing returns to labor (which can be interpreted as fixed firm-specific capital). Demand shocks are accounted for by shocks to government expenditures and not by shifts in consumers' tastes as is the case in our model.

66 On the empirical front it would be interesting to assess whether the type of demand shocks which are the focus of the paper can be properly identified. Following, for example, the work by GALI J. (1999) one could try to augment the identifying framework developed in that paper by allowing for demand shocks which have long-lasting effects on prices and permanent effects on output. A clear concern with doing this type of exercise is the lack of an adequately long time series. On the theoretical front, it would be interesting to explore the policy implications of such shocks in a calibrated DSGE model, along the lines of BENIGNO P. (2004). That model has complete markets. An extension to the incomplete markets case, as suggested by GALI J., AND T. MONACELLI (2008), could yield further interesting results.
Another possibly interesting way to show how demand-side shocks can explain regional inflation differentials by leading to the same Balassa-Samuelson effects (rising relative productivities and relative prices) relies on assuming that sectoral measured total-factor-productivities (TFP) can be affected by the response of some unobserved factor - such as variable capital utilization rates and labor effort - to a persistent demand-side disturbance. We explore this alternative formulation of the problem in a separate paper (available upon request). Empirically it is much harder to verify this type of linkage between demand-side disturbances and inflation differentials.

We also focus on the nexus between supply-side shocks and relative price trends in our optimizing model with the benchmark preferences used in many of the existing contributions to the NOEM literature (unitary inter- and intratemporal elasticity of substitution). Compared to the original Balassa-Samuelson hypothesis, two main differences stand out: First, when traded goods are imperfect substitutes, as is the case in our setup, relative productivity shocks lead always to a terms of trade depreciation and to a rise (fall) in domestic (foreign) relative prices - i.e. to an 'internal real exchange rate' appreciation. We show that these two effects tend to offset each other and that they lead to regional inflation differentials only under fairly restrictive assumptions about expenditure shares and the relative magnitudes of sectoral TFP growth rates. By contrast, when traded goods are perfect substitutes, as in the original Balassa-Samuelson hypothesis, any relative productivity shock leads to an inflation differential through an 'internal real exchange rate' appreciation. Second, when traded goods are imperfect substitutes the transmission mechanism between productivity shocks and regional inflation differentials does not depend critically upon the reallocation of employment across sectors. This is another major departure from the original Balassa-Samuelson model in which the assumption of perfect labor mobility across sectors within each country plays a key role for the transmission of productivity shocks to inflation differentials.

Our model addresses the above two observations. First, we show that demand-side shocks lead to relative price trends and inflation differentials both through a terms of trade and through an 'internal real exchange rate' appreciation. There is ample evidence that most of the euro area countries exhibiting high inflation differentials are also experiencing substantial terms of trade appreciations. Second, our demand-side approach to explaining regional inflation differentials highlights the importance of factor (capital and labor) reallocations across sectors and countries. These factor reallocations are receiving increasing attention by researchers and policymakers as the European economic integration process is progressively gaining pace.

Future work should extend the present model to a dynamic setting, while relaxing some of the simplifying assumptions used - the absence of nominal rigidities, the full depreciation of capital within each period, the perfect mobility of labor across sectors within each country. This will allow us to address some more policy-oriented questions related to the demand management problem faced by the European Central Bank and the policymakers of the prospective new entrants into the euro area.67

---

67 See for example NATALUCCI F., AND F. RAVENNA (2008) who show that even the standard supply-side Balassa-Samuelson type real appreciation might lead to serious monetary policy formulation trade-offs during the incumbent accession process. MASTEN I. (2008) qualifies their results by showing that the monetary policy trade-offs critically depend on the nature of the productivity disturbances (permanent versus temporary). CA’ZORZI M., R. DE SANTIS, AND F. ZAMPOLLI (2005) analyze the welfare implications of joining a currency area in a simplified framework which does not include capital. BRUHA J., AND J. POPDIERA (2007)
REFERENCES


develop a two-country NOEM model which also includes investment into quality improvements to account for the observed real exchange rate appreciations in the European transition countries.


ROGERS J., "Monetary Union, price level convergence, and inflation: how close is Europe to the United States?", *Journal of Monetary Economics*, April, no. 54 (3), 2007, pages 785-796.


APPENDIX 1

This appendix shows that when the consumption expenditure share of home tradables $\gamma_{HT}$ rises relative to the consumption expenditure share of foreign tradables $\gamma_{FT}$, the domestic capital-labor ratio in the traded goods sector unambiguously increases (and correspondingly the capital-labor ratio in the foreign traded goods sector unambiguously falls). This result, in turn, implies that in our setup demand shocks can lead to regional inflation differentials through rising relative productivities and rising relative prices as in the original (i.e. supply-driven) Balassa-Samuelson hypothesis, provided the production of traded goods is more capital-intensive than the production of nontraded goods, i.e. if $\alpha > \beta$.

First, note from the solutions in Box (1) that the capital-labor ratio in the domestic traded goods sector can be expressed as:

\[
\left( A1 \right) \quad \frac{K_{HT}}{L_{HT}} = \frac{1}{\beta \gamma_N} \left( \frac{\alpha}{1 - \alpha} \frac{\theta}{\theta - 1} \right) K_{HN}
\]

or:

\[
\left( A2 \right) \quad \frac{K_{HT}}{L_{HT}} = \frac{1}{2} \left( \frac{\alpha}{1 - \alpha} \frac{\theta}{\theta - 1} \right) \frac{2 \theta (\gamma_{HT}) + (\theta - 1) \left[ \beta \gamma_N - \alpha (\gamma_{HT} - \gamma_{FT}) \right]}{\theta (\gamma_{HT} + \gamma_{FT}) + (\theta - 1) \beta \gamma_N \left[ \alpha (\gamma_{HT} + \gamma_{FT}) + \beta \gamma_N \right]}
\]

and taking logs:

\[
\log \left( \frac{K_{HT}}{L_{HT}} \right) = \log \left( \frac{1}{2} \left( \frac{\alpha}{1 - \alpha} \frac{\theta}{\theta - 1} \right) \right) + \log \left( \frac{2 \theta (\gamma_{HT}) + (\theta - 1) \left[ \beta \gamma_N - \alpha (\gamma_{HT} - \gamma_{FT}) \right]}{\theta (\gamma_{HT} + \gamma_{FT}) + (\theta - 1) \beta \gamma_N \left[ \alpha (\gamma_{HT} + \gamma_{FT}) + \beta \gamma_N \right]} \right)
\]

Since we are interested in changes of expenditure shares such that:

\[d_\gamma_{HT} = -d_\gamma_{FT}\]

we can disregard (the constant and) the last term in expression (A2), i.e. we can simply look at the derivative with respect to $\gamma_{HT}, \gamma_{FT}$ of

\[\frac{\partial \log \left( \frac{K_{HT}}{L_{HT}} \right)}{\partial \gamma} \bigg|_{\gamma_{HT} = d_\gamma_{HT}} = \frac{\partial \log \left( \frac{2 \theta (\gamma_{HT}) + (\theta - 1) \left[ \beta \gamma_N - \alpha (\gamma_{HT} - \gamma_{FT}) \right]}{\theta (\gamma_{HT} + \gamma_{FT}) + (\theta - 1) \beta \gamma_N \left[ \alpha (\gamma_{HT} + \gamma_{FT}) + \beta \gamma_N \right]} \right)}{\partial \gamma} \bigg|_{\gamma_{HT} = d_\gamma_{HT}}\]

which equals to

\[\left( A3 \right) \quad \frac{\partial \log \left( \frac{K_{HT}}{L_{HT}} \right)}{\partial \gamma} \bigg|_{\gamma_{HT} = d_\gamma_{HT}} = \frac{2 \theta (1 - \alpha) + \alpha}{2 \theta (\gamma_{HT}) + (\theta - 1) \beta \gamma_N \left[ \alpha (\gamma_{HT} + \gamma_{FT}) + \beta \gamma_N \right]} > 0\]

Using the above equation (A3) we can then easily show that, assuming $\alpha > \beta$, the domestic (foreign) relative price of nontradedables, i.e. equation (18) (equation (19)) in the main text

\[
\frac{P_{HN}}{P_{HT}} = \frac{1 - \alpha}{1 - \beta} \frac{APL_{HN}}{APL_{HT}} = \frac{A_{HN}}{A_{HT}} \frac{1 - \alpha}{1 - \beta} \left[ \frac{\alpha}{\beta} \right]^\gamma \left[ \frac{K_{HT}}{L_{HT}} \right]^{1 - \beta}
\]

rises (falls) as the consumption expenditure share of domestic tradables rises, due to an increase in relative productivity.

Finally, equation (A3) also implies that a permanent positive shock to the consumption expenditure share of domestic tradables also leads to a terms of trade, $\frac{P_{FT}}{P_{HT}}$, appreciation (see equation (23) in the main text). Equation (28) in the main text shows then that both terms of trade and 'internal real exchange rate' (the ratio of domestic over foreign relative prices of nontraded goods) appreciations contribute to regional inflation differentials.
APPENDIX 2

This appendix shows that the main result of the paper shown in Appendix 1, i.e.
that a permanent positive shock to the consumption expenditure share of domestic tradables leads both to a rise in the domestic relative price of nontradables and to a terms of trade appreciation, does not depend on the simplifying assumption followed in the main text that the expenditure shares on domestic (foreign) traded goods are identical in the home and foreign consumption bundle, i.e. that $\gamma_{HT} \equiv \gamma_{HT}^*$ ($\gamma_{FT} \equiv \gamma_{FT}^*$). Assuming now that $\gamma_{HT} \neq \gamma_{HT}^*$ and $\gamma_{FT} \neq \gamma_{FT}^*$ (and hence $\gamma_{HN} \neq \gamma_{FN}$), the solution for the capital-labor ratio in the home traded sector, can still be expressed as in equation (A1) above:

$$\frac{K_{HT}}{L_{HT}} = \frac{1}{\beta \gamma_N} \frac{\alpha - \theta}{1 - \alpha \theta - 1} K_{HN}$$

The new solution for $K_{HN}$ is now:

$$(A4) \quad K_{HN} = \frac{\gamma_{HN}}{\gamma_{FN}} \beta \gamma_{FN} \left[ \frac{(\alpha - \beta)(\gamma_{HT} + \gamma_{FT}^*) + \beta K_{FN}}{[(\alpha - \beta)(\gamma_{HT} + \gamma_{FT}^*) + \beta]}ight]$$

$K_{FN}$ now equals:

$$(A5) \quad K_{FN} = K_{FN}^{\gamma_{HT} \neq \gamma_{FT}^*} \cdot \Psi$$

where we have denoted the previous solution for $K_{FN}$ under the assumption of equal expenditure shares ($\gamma_{HT} \equiv \gamma_{HT}^*$ and $\gamma_{FT} \equiv \gamma_{FT}^*$) as $K_{FN}^{\gamma_{HT} = \gamma_{FT}^*}$.

Also, in (A5) we have defined:

$$\Psi = \frac{\left[\theta \left(\gamma_{HT}^* + \gamma_{FT}^*\right)^2 + \beta (\gamma_{HT}^* \gamma_{HN} + \gamma_{FT}^* \gamma_{FN}) + (\theta - 1) \beta \gamma_{HN} \gamma_{FN}\right] - \alpha \left(\gamma_{HT} \gamma_{FN} + \gamma_{FT} \gamma_{HN}\right)}{\theta \left(\gamma_{HT}^* + \gamma_{FT}^*\right)^2 + \beta (\gamma_{HT}^* \gamma_{HN} + \gamma_{FT}^* \gamma_{FN}) + (\theta - 1) \beta \gamma_{HN} \gamma_{FN}}$$

Note that (A5) collapses to $K_{FN}^{\gamma_{HT} = \gamma_{FT}^*}$ when $\gamma_{HT} \equiv \gamma_{HT}^*$ and $\gamma_{FT} \equiv \gamma_{FT}^*$ (since then $\Psi = 1$). It is straightforward to show that for changes in expenditure shares such that there is a permanent relative increase in the demand for the home traded good in the foreign country, i.e.

$$d \gamma_{HT}^* = -d \gamma_{FT}^*$$

then,

$$\frac{\partial K_{FN}}{\partial \gamma} \bigg|_{\gamma_{HT} = \gamma_{HT}^*} < 0$$

since,

$$\frac{\partial \Psi}{\partial \gamma} \bigg|_{\gamma_{HT} = \gamma_{HT}^*} < 0$$

Finally, note that $K_{FN}$ is also decreasing for changes in expenditure shares such that $d \gamma_{HT} = -d \gamma_{FT}$.

Applying the latter results to equation (A4) above, it is then easy to show that also under this more realistic assumption about the expenditure shares both the terms of trade and the 'internal real exchange rate' appreciate in response to a permanent increase in the relative demand of home traded goods.